DynamicEarthNet: towards FedSPD

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Overview

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DynamicEarthNet dataset

Multi-spectral images

- 75 Areas of Interest (AOI)
- Satellite images collection: daily 2018/01/01 $\rightarrow$ 2019/12/31
- 730 images/AOI $\rightarrow$ $54,750$ satellite images in total
- 4 channels: RGB+NIR
- Spatial resolution: 3 meters
- Image size: 1024×1024

One time-series: $x \in \mathbb{R}^{T \times H \times W \times 4}$ with T:number of days in the month, $H = W = 1024$.

Land Use/Land Cover (LULC)

- Labels: $24 \times 75 = 1,800$
- 7 different (imbalanced) classes:

class name	%	#AOIs	color
impervious surface	7.1	70	
agriculture	10.3	37	
forest & other vegetation	44.9	71	
wetlands	0.7	24	
soil	28.0	75	
water	8.0	58	
snow & ice	1.0	2	

No single AOI contains all 7 classes.
For a given time series, label:

$$y \in \{0, \ldots, 6\}^{T \times H \times W}$$

Challenges

Issue 1 Voluminous data (hundreds of GB)

Issue 2 Slow to move all images to a single processing server

Why Federated Learning is relevant?

Solution Issue 1 client $\rightarrow$ smaller dataset & own stat. heterogeneity with clear interpretation

Solution Issue 2 No exchange of raw datasets, just each client's updates

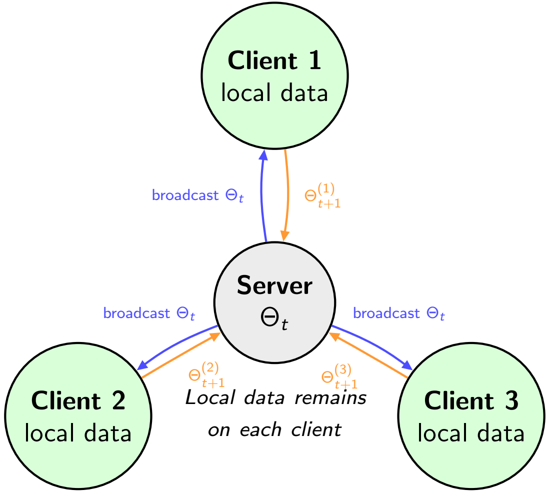
- Each AOI $\rightarrow$ local phenomena
- FL: adapt to local distrib and aggregate knowledge for better generalization

  ⋆ Idea 1 one AOI = one client $\rightarrow$ too many clients (75);

  ⋆ Idea 2 create clients based on dominant LULC categories across AOI

    1. Urban areas
    2. Agricultural areas
    3. Forest-dominated areas
    4. Water-dominated areas
    5. Mixed land-use areas

Federated Learning Overview



Framework

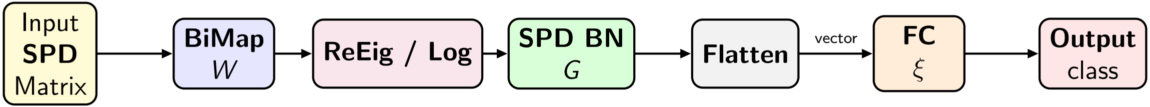
K clients. Each client k has dataset D_k of n_k pixel annotation pairs $\{x_i^{(k)}, y_i^{(k)}\}_{i=1,\ldots,n_k}$, where

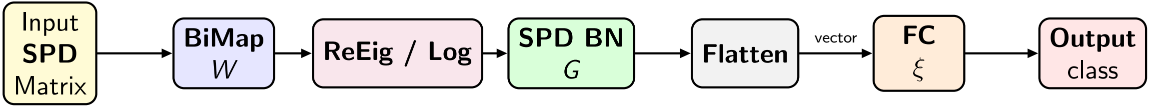
- $x_i^{(k)}$ is a covariance matrix in $\mathbb{R}^{4T\times 4T}$ (with $T = 31$),
- $y_i^{(k)} \in \{0, \ldots, 6\}$ is a semantic mask

Total number of samples $n := \sum_{k=1}^{K} n_k$

Loss function ℓ (CE)

Forward pass SPD-Net $f_\Theta(\cdot)$





Each client's SPD-net with L layers has learnable parameters

$$\Theta = (\underbrace{W_1, \ldots, W_L}_{\textit{BiMap}}, \underbrace{G_1, \ldots, G_L}_{\textit{BN}}, \underbrace{\xi_1, \ldots, \xi_d}_{\textit{FC}}) \in \mathcal{M}$$

with

$$\mathcal{M} := \left(\prod_{l=1}^{L} \mathrm{St}(d_l, d_{l+1}) \right) \times \left(\prod_{l=1}^{L} \mathrm{SPD}_{d_l} \right) \times \mathbb{R}^d.$$

Local objective functions For each client $k = 1, \ldots, K$,

$$F_k(\Theta) := \frac{1}{n_k} \sum_{i=1}^{n_k} \ell\big(f_\Theta(x_i^{(k)}),\, y_i^{(k)}\big).$$

Global objective function

$$\arg\min_{\Theta \in \mathcal{M}} F(\Theta) := \sum_{k=1}^{K} \frac{n_k}{n} F_k(\Theta).$$

Federated procedure

Algorithm

$\star$ **Initialization** Θ_0 to all clients (random)

$\star$ **Outer loop** For $t = 0, \ldots, T-1$ (rounds):

1. A subset $\mathcal{S}_t$ of $r \leq K$ clients is sampled (random)
2. Server broadcasts Θ_t to clients.
3. Each client $k \in \mathcal{S}_t$ runs E_k local updates on Θ with initialization Θ_t.
4. Aggregation of each client's local updated parameters via barycenters $\rightarrow \Theta_{t+1}$

$\star$ **Global output** Final aggregated param. Θ_T or $\mathrm{Unif}(\{\Theta_0, \ldots, \Theta_T\})$

If each local function F_k is L_k-smooth and geodesically convex, then the algorithm converges in $O(1/T)$ (provided some curvature/manifold additional assumptions).

Riemannian local updates at client $k \in \mathcal{S}_r$

Let θ denote a Riemannian parameter (either one of the W_l or G_l).

- Initialize $\theta_0^{(k)} = \theta_t$
- For $s = 1, \ldots, E_k$:

$$\theta_{s+1}^{(k)} \;=\; R_{\theta_s^{(k)}}\Big(-\eta^{(k)}\, g^{(k)}\Big),$$

  where the local Riemannian gradient is

$$g^{(k)} \;=\; \operatorname{grad} F_k(\theta_s^{(k)}) \;-\; \mathcal{T}_{\theta_t \to \theta_s^{(k)}}\Big(\operatorname{grad} F_k(\theta_t) \;-\; \operatorname{grad} F(\theta_t)\Big),$$

  - The blue term is the optional **SVR correction** (variance reduction).

Differential Privacy as a model variant

Besides the variance-reduction approach, we can also inject **DP noise** on each local gradient to protect sensitive data.

Key idea: Add tangent-space Gaussian noise before retraction/exponential update.

- **DP** can be combined with or without the SVR term.
- Each client k ensures local DP by perturbing its gradient:

$$g_{\mathrm{DP}}^{(k)} = g^{(k)} + \varepsilon_t, \quad \text{where} \quad \varepsilon_t \sim \mathcal{N}_{\theta_s^{(k)}}(0, \sigma^2).$$

- This noise is drawn from a *tangent-space Gaussian* distribution at $\theta_s^{(k)}$.

Privacy Guarantee: With an appropriate choice of σ and composition rules, the entire FL pipeline can provide an (ε, δ)-DP guarantee.

Riemannian local updates with optional DP

- Initialize $\theta_0^{(k)} = \theta_t$
- For $s = 1, \ldots, E_k$:

$$g^{(k)} \;=\; \operatorname{grad} F_k(\theta_s^{(k)}) \;-\; \mathcal{T}_{\theta_t \to \theta_s^{(k)}}\Big(\operatorname{grad} F_k(\theta_t) \;-\; \operatorname{grad} F(\theta_t)\Big),$$

$$g_{\mathrm{DP}}^{(k)} \;=\; g^{(k)} \;+\; \varepsilon_s, \qquad \varepsilon_s \;\sim\; \mathcal{N}_{\theta_s^{(k)}}(0, \sigma^2).$$

Then update via retraction (or exponential map):

$$\theta_{s+1}^{(k)} \;=\; R_{\theta_s^{(k)}}\Big(-\eta^{(k)}\, g_{\mathrm{DP}}^{(k)}\Big).$$

- **Red** term is the **DP noise** addition (optional)
- **Blue** term is still the **SVR correction** (optional).

Local updates for Euclidean FC weights (SVR + DP)

Let $\xi \in \mathbb{R}^{d_{\mathrm{FC}}}$ be the parameter vector for the final fully-connected layer. During each local epoch on client k:

$$\xi_0^{(k)} = \xi_t, \quad \text{and for } s = 0, \ldots, E_k - 1 :$$

$$\xi_{s+1}^{(k)} = \xi_s^{(k)} - \eta^{(k)} \Big(\nabla F_k\big(\xi_s^{(k)}\big) - \big[\nabla F_k(\xi_t) - \nabla F(\xi_t)\big] + \varepsilon_s \Big),$$

where:

- $\nabla F_k(\xi_s^{(k)})$: local Euclidean gradient on client k.
- $\big[\nabla F_k(\xi_t) - \nabla F(\xi_t)\big]$ is the **SVR correction** term.
- $\varepsilon_s \sim \mathcal{N}(0, \sigma^2 I)$ is the **DP noise** term.
- After E_k steps, client k sends $\xi_{E_k}^{(k)}$ to the server.

Server aggregation (Karcher flow) for $\theta = G$ SPD

After each client $k \in \mathcal{S}_r$ sends back $\theta_{E_k}^{(k)}$, we compute the Riemannian (Fréchet) mean on the manifold:

$$\Theta_{t+1} \;=\; \arg\min_{\Theta \in \mathcal{M}} \sum_{k \in \mathcal{S}_r} p_k \, d\big(\Theta,\ \theta_{E_k}^{(k)}\big)^2.$$

- In practice, we use a gradient-descent-based procedure (Karcher flow):

$$\Theta_{t+1} \;\leftarrow\; \mathrm{Exp}_{\Theta_t}\Big(-\gamma \sum_{k \in \mathcal{S}_r} p_k \, \mathrm{Log}_{\Theta_t}(\theta_{E_k}^{(k)})\Big),$$

  where γ is a step size for the Karcher flow.

Server aggregation (projection-based) for $\mathbf{W} \in \mathrm{St}(d_l, d_{l+1})$

For the BiMap parameters W_l living on the Stiefel manifold, we can use:

- **Karcher flow** or Riemannian gradient-based aggregator:

$$W_{t+1} \;=\; \arg\min_{W \in \mathrm{St}(d_l, d_{l+1})} \sum_{k \in S_t} p_k \, d^2\big(W,\; W_\tau^{(k)}\big),$$

  which can be iteratively approximated (exp/log maps).

- **Projection-based aggregator**:

$$W_{t+1} \;=\; \mathrm{uf}\Big(\underbrace{\sum_{k \in S_t} p_k \, W_\tau^{(k)}}_{\text{arithmetic mean}}\Big) \quad \text{or} \quad W_{t+1} \;=\; \mathrm{qf}\Big(\sum_{k \in S_t} p_k \, W_\tau^{(k)}\Big).$$

  No iterative scheme needed, computationally cheaper (simple retractions/liftings).

Server aggregation of FC weights

At round t, we receive each client's updated $\xi_{E_k}^{(k)}$ for $k \in \mathcal{S}_t$. To get ξ_{t+1}, standard weighted average (FedAvg):

$$\xi_{t+1} \;=\; \sum_{k \in \mathcal{S}_t} \frac{n_k}{\sum_{j \in \mathcal{S}_t} n_j} \, \xi_{E_k}^{(k)}$$

No need for any Riemannian retraction or manifold-based aggregator.

References