TASK2_structuring_products

January 27, 2025

[22]:
```
# Importing libraries
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import yfinance as yf
from scipy.stats import norm
import seaborn as sns
```

[23]:
```
# Importing data
coffee = yf.Ticker('KC=F')
coffee_data = yf.download('KC=F', start='2010-01-01', end='2025-01-26')

# Calculate daily price change as a percentage
coffee_data['Price Change'] = coffee_data['Close'].pct_change()
# Fill missing values (e.g., for the first row) with 0 or NaN as appropriate
coffee_data['Price Change'].fillna(0, inplace=True)

coffee_data.head()
```

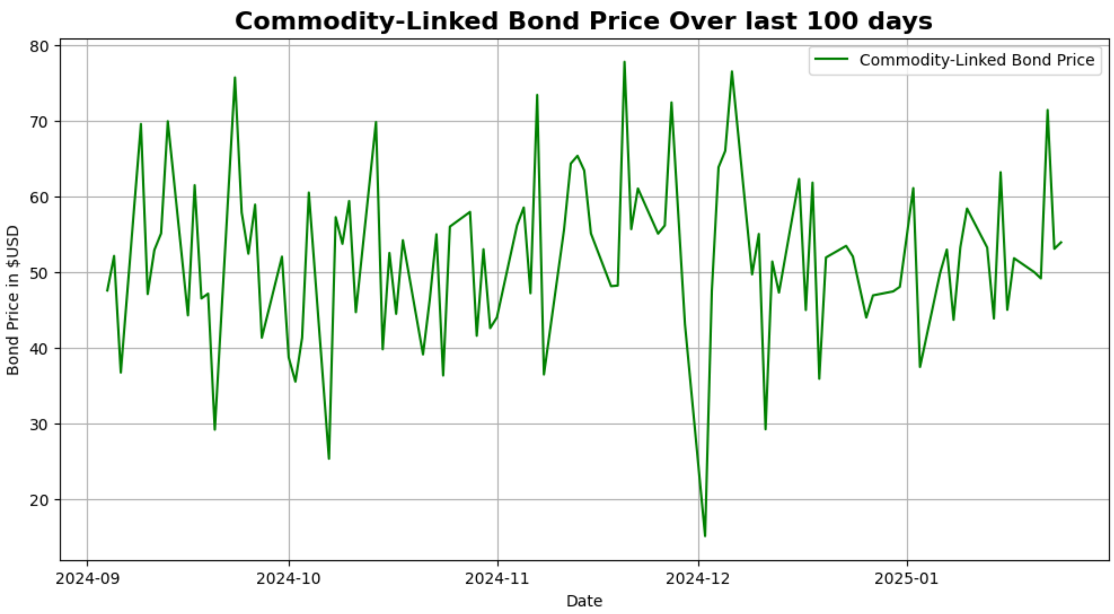
```
[*********************100%***********************]  1 of 1 completed
/var/folders/0v/nd2019hd3lxb6r07g9ywbv500000gn/T/ipykernel_1937/464887490.py:8:
SettingWithCopyWarning:
A value is trying to be set on a copy of a slice from a DataFrame

See the caveats in the documentation: https://pandas.pydata.org/pandas-
docs/stable/user_guide/indexing.html#returning-a-view-versus-a-copy
  coffee_data['Price Change'].fillna(0, inplace=True)
```

[23]:
```
Price                       Adj Close       Close        High         Low  \
Ticker                           KC=F        KC=F        KC=F        KC=F
Date
2010-01-04 00:00:00+00:00  141.850006  141.850006  142.449997  136.000000
2010-01-05 00:00:00+00:00  141.000000  141.000000  142.699997  140.399994
2010-01-06 00:00:00+00:00  141.600006  141.600006  142.649994  140.050003
2010-01-07 00:00:00+00:00  141.899994  141.899994  142.399994  139.850006
2010-01-08 00:00:00+00:00  145.350006  145.350006  146.000000  141.949997

Price                            Open Volume Price Change
```

```
Ticker                          KC=F   KC=F
Date
2010-01-04 00:00:00+00:00  136.000000  14005          NaN
2010-01-05 00:00:00+00:00  141.850006   9109    -0.005992
2010-01-06 00:00:00+00:00  141.600006   9547     0.004255
2010-01-07 00:00:00+00:00  141.550003   7353     0.002119
2010-01-08 00:00:00+00:00  142.449997  16035     0.024313
```

[24]:

```
# Scenario 1: Conservative investor

# Selected product: Commodity-Linked Bond
fixed_coupon = 0.05 # Fixed component
sensitivity = 0.5 # Sensitivity to commodity price changes
face_value = 1000     # Bond face value (example)

# Function to calculate bond price
def calculate_bond_price(fixed_coupon, sensitivity, price_change, face_value):
    P_f = fixed_coupon * face_value
    delta_C = price_change * face_value
    return P_f + sensitivity * delta_C

# Adding bond price to the dataset
coffee_data['Bond Price'] = coffee_data['Price Change'].apply(
    lambda change: calculate_bond_price(fixed_coupon, sensitivity, change,␣
↪face_value)
)

# Plotting the bond price over time
plt.figure(figsize=(12, 6))
plt.plot(coffee_data.index[-100:], coffee_data['Bond Price'].iloc[-100:],␣
↪label='Commodity-Linked Bond Price', color='green')
plt.title('Commodity-Linked Bond Price Over last 100 days', fontsize=16,␣
↪fontweight='bold')
plt.xlabel('Date')
plt.ylabel('Bond Price in $USD')
plt.legend()
plt.grid()
plt.show()

# Displaying the data
print(coffee_data[['Close', 'Price Change', 'Bond Price']].tail())

print(f'\nThe bond price for a ${face_value:.2f} bond with {fixed_coupon*100:.
↪2f}% annual return (today: {coffee_data.index[-1].date()}) is␣
↪${coffee_data["Bond Price"].iloc[-1]:.2f}$.')
```




```
Price                               Close Price Change Bond Price
Ticker                               KC=F
Date
2025-01-20 00:00:00+00:00  328.350006      0.000000  50.000000
2025-01-21 00:00:00+00:00  327.799988     -0.001675  49.162451
2025-01-22 00:00:00+00:00  341.850006      0.042862  71.430779
2025-01-23 00:00:00+00:00  343.950012      0.006143  53.071531
2025-01-24 00:00:00+00:00  346.649994      0.007850  53.924962
```

```
The bond price for a $1000.00 bond with 5.00% annual return (today: 2025-01-24)
is $53.92$.
```

[25]:

```
# Scenario 2: Moderate Risk Taker

# Selected product: Structured Notes

# Relevant parameters:
C_0 = (coffee_data['Close'].iloc[-1]).iloc[0] # Initial price, here spot price
F = 0.05 * C_0 # Fixed component (5% return)
r = 0.02 # Risk-free rate (2%)
alpha = 1 # Participation rate
daily_returns = coffee_data['Close'].pct_change() # Daily returns of coffee
sigma = (daily_returns.std() * np.sqrt(252)).iloc[0] # Annualized volatility of␣
↪coffee
mu = (daily_returns.mean() * 252).iloc[0] # Annualized drift of coffee
T = 1 # Simulated period (in years)
num_sims = 10000 # Number of simulations
```

```python
num_steps = 252 # Number of steps in each simulation (daily)

# Simulating future prices
def MonteCarlo(C_0, mu, sigma, T, num_sims, num_steps):
    np.random.seed(42)  # Set seed for reproducibility
    dt = T / num_steps  # Time increment
    price_paths = np.zeros((num_steps, num_sims))  # Initialize price paths array
    price_paths[0] = C_0  # Set the initial price for all simulations

    for t in range(1, num_steps):
        z = np.random.standard_normal(num_sims)  # Generate random normal variables
        price_paths[t] = price_paths[t-1] * np.exp((mu - 0.5 * sigma**2) * dt + sigma * np.sqrt(dt) * z)

    return price_paths  # Return the prices at maturity (final time step)

C_T = MonteCarlo(C_0, mu, sigma, T, num_sims, num_steps)

# Plotting the Monte Carlo simulation (1 every 10 simulations)
plt.figure(figsize=(12, 3))
for i in range(0, num_sims, 10):  # Plot 1 every 10 simulations
    plt.plot(C_T[:, i], linewidth=1)
plt.title(f"Monte Carlo Simulation of Coffee Future Prices\n({num_sims} Simulations, {T} Years)", fontsize=16, fontweight='bold')
plt.xlabel("Days")
plt.ylabel("Price in $USD")
plt.grid(alpha=0.3)
plt.show()

# Calculate payoff for each simulation
P_n = F + alpha * (C_T[-1] - C_0)

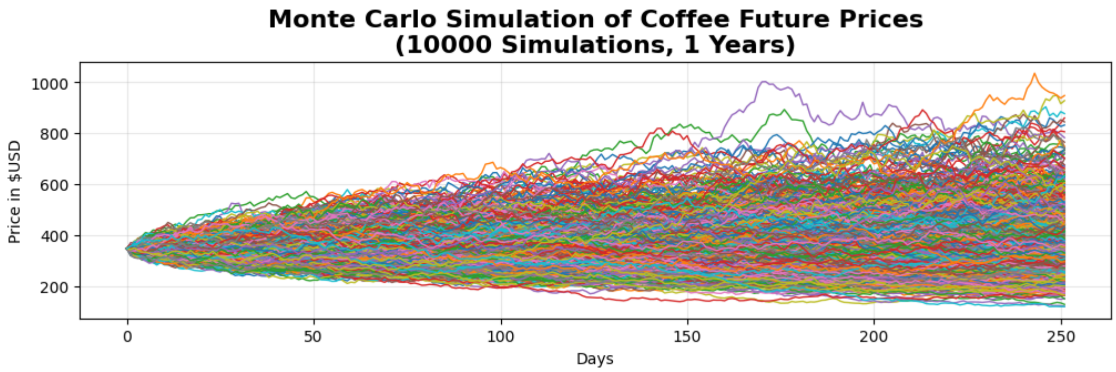
# Expected payoff
expected_payoff = np.mean(P_n)

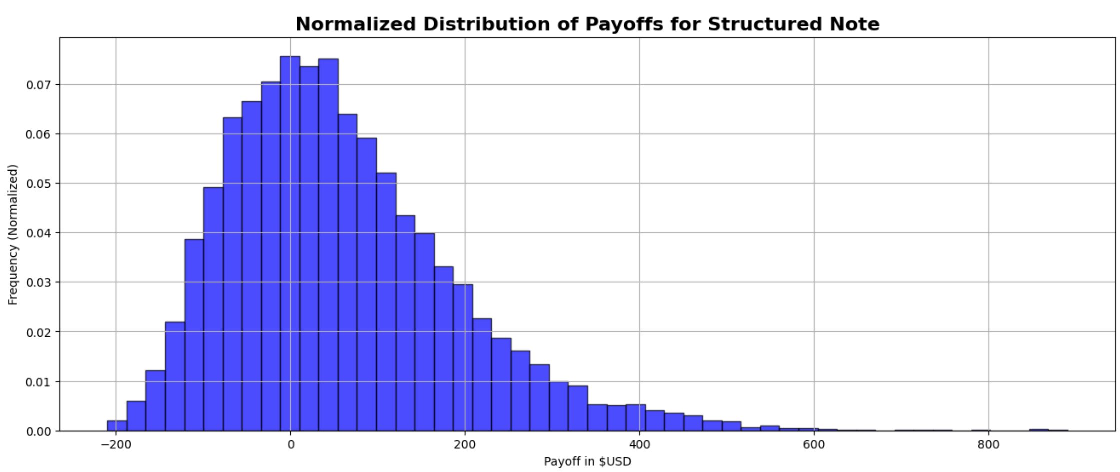
# Histogram Calculation
counts, bins = np.histogram(P_n, bins=50)  # Raw counts and bin edges
normalized_counts = counts / num_sims  # Normalize frequencies to sum to 1
bin_centers = (bins[:-1] + bins[1:]) / 2  # Calculate bin centers for plotting

# Plot the normalized distribution of payoffs
plt.figure(figsize=(16, 6))
plt.bar(bin_centers, normalized_counts, width=np.diff(bins), color='blue', alpha=0.7, edgecolor='black')
```

```
plt.title('Normalized Distribution of Payoffs for Structured Note',␣
    ↪fontsize=16, fontweight='bold')
plt.xlabel('Payoff in $USD')
plt.ylabel('Frequency (Normalized)')
plt.grid()
plt.show()

# Display results
print(f"Initial Coffee Price on {coffee_data.index[-1].date()}: C_0 = ${C_0:.
    ↪2f} ")
print(f"Expected Payoff for Structured Note in {T} years: ${expected_payoff:.
    ↪2f}")
```






```
Initial Coffee Price on 2025-01-24: C_0 = $346.65
Expected Payoff for Structured Note in 1 years: $58.12
```

[78]:
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# Scenario 3: High Risk Investor

# Selected Product: Digital Options

# Parameters:
S_0 = (coffee_data['Close'].iloc[-1]).iloc[0] # Initial price, here spot price
S_x = S_0 * (1 + .05) # Strike price of coffee
r = 0.02 # Risk-free rate (2%)
T = .5 # Time to maturity (6 months)
daily_returns = coffee_data['Close'].pct_change() # Daily returns of coffee
sigma = (daily_returns.std() * np.sqrt(252)).iloc[0] # Annualized volatility of␣
↪coffee

# Define the reward (price paid if the option is exercised)
reward = 1 # Reward for the option

# Calculate d1 and d2
def options_parameters(S_0, S_x, r, T, sigma):
    d1 = (np.log(S_0 / S_x) + (r + 0.5 * sigma**2) * T) / (sigma * np.sqrt(T))
    d2 = d1 - sigma * np.sqrt(T)
    return d1, d2

# Defining the strategy
call_strat = True # True for call option, False for put option

# Option pricing using Black-Scholes formula
def digital_option_pricer(S_0, S_x, r, T, sigma, d1, d2, call_strat=True,␣
↪reward=1):
    if call_strat:
        return reward * np.exp(-r * T) * norm.cdf(d2)  # Multiply call price by␣
↪reward
    else:
        return reward * np.exp(-r * T) * norm.cdf(-d2)  # Multiply put price by␣
↪reward

d1, d2 = options_parameters(S_0, S_x, r, T, sigma)
V_call = digital_option_pricer(S_0, S_x, r, T, sigma, d1, d2, call_strat,␣
↪reward)
V_put = digital_option_pricer(S_0, S_x, r, T, sigma, d1, d2, not call_strat,␣
↪reward)

# Display results
print(f"Digital Call Option Value: ${V_call:.2f}")
print(f"Digital Put Option Value: ${V_put:.2f}")
print(f'Parameters:')
print(f'  Initial Price: ${S_0:.2f}')
print(f'  Strike Price: ${S_x:.2f}')
```

```
print(f'  Reward: ${reward:.2f}')
print(f'  Risk-free Rate: {r:.2%}')
print(f'  Time to Maturity: {T:.1f} years')
print(f'  Volatility: {sigma:.2%}')

fig, axes = plt.subplots(1, 2, figsize=(16, 6), constrained_layout=True)

# Payoff Diagram
S_T = np.linspace(S_x * 0.8, S_x * 1.2, 500)  # Asset price range
digital_call_payoff = np.where(S_T > S_x, reward, 0) - V_call
digital_put_payoff = np.where(S_T < S_x, reward, 0) - V_put

axes[0].plot(S_T, digital_call_payoff, label='Digital Call Payoff',␣
 ↪color='blue')
axes[0].plot(S_T, digital_put_payoff, label='Digital Put Payoff', color='red')
axes[0].axvline(S_x, color='black', linestyle='--', label='Strike Price')
axes[0].set_title('Digital Option Payoff Diagram (Reward=1)', fontsize=14,␣
 ↪fontweight='bold')
axes[0].set_xlabel('Asset Price at Maturity ($)')
axes[0].set_ylabel('Payoff ($)')
axes[0].legend()
axes[0].grid()

# Option Value vs. Strike Price
strike_prices = np.linspace(S_0 * 0.8, S_0 * 1.2, 500)
call_values = []
put_values = []

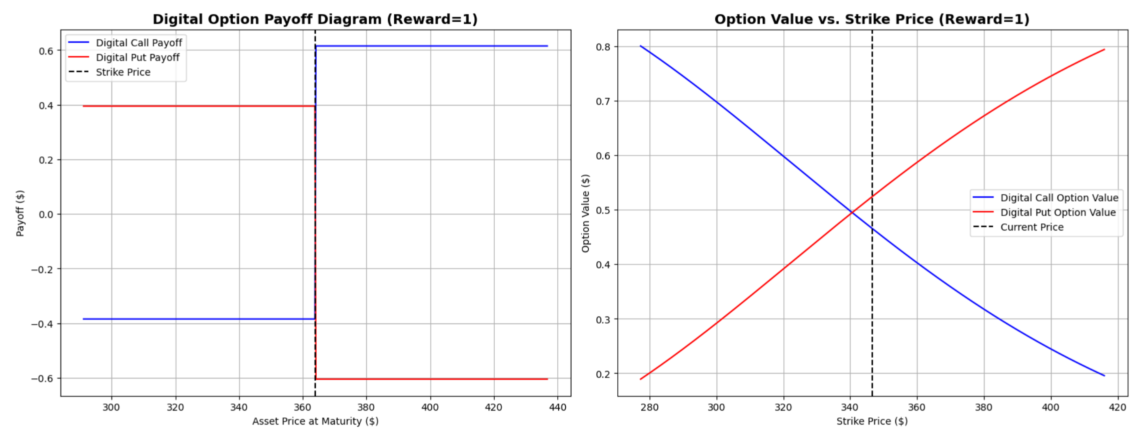
for S_x in strike_prices:
    d1, d2 = options_parameters(S_0, S_x, r, T, sigma)
    call_values.append(digital_option_pricer(S_0, S_x, r, T, sigma, d1, d2,␣
 ↪call_strat=True, reward=reward))
    put_values.append(digital_option_pricer(S_0, S_x, r, T, sigma, d1, d2,␣
 ↪call_strat=False, reward=reward))

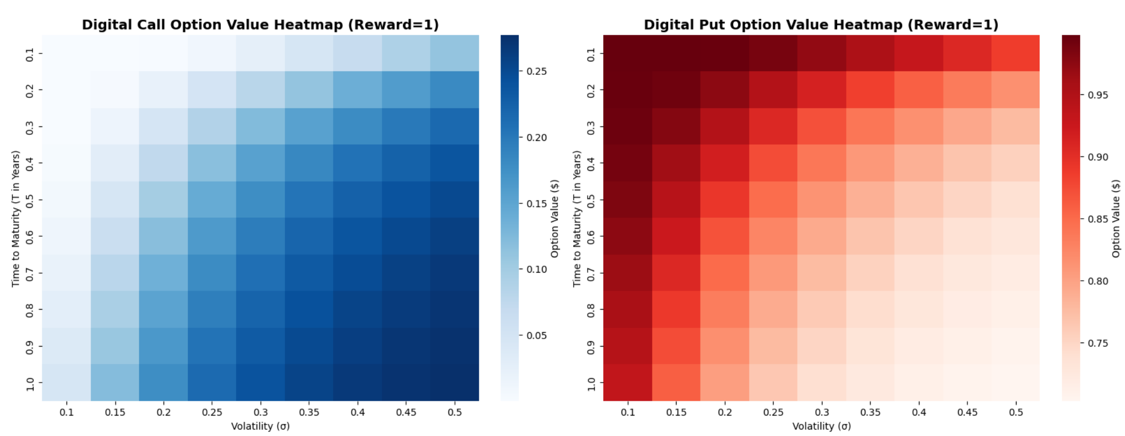
axes[1].plot(strike_prices, call_values, label='Digital Call Option Value',␣
 ↪color='blue')
axes[1].plot(strike_prices, put_values, label='Digital Put Option Value',␣
 ↪color='red')
axes[1].axvline(S_0, color='black', linestyle='--', label='Current Price')
axes[1].set_title('Option Value vs. Strike Price (Reward=1)', fontsize=14,␣
 ↪fontweight='bold')
axes[1].set_xlabel('Strike Price ($)')
axes[1].set_ylabel('Option Value ($)')
axes[1].legend()
axes[1].grid()
```

```python
plt.show()

# Define ranges for T and sigma
T_range = np.linspace(0.1, 1, 10)  # Time to maturity (0.01 to 1 year)
sigma_range = np.linspace(0.1, 0.5, 9)  # Volatility (10% to 50%)

# Create a grid of option values
call_values = np.zeros((len(T_range), len(sigma_range)))
put_values = np.zeros((len(T_range), len(sigma_range)))

for i, T in enumerate(T_range):
    for j, sigma in enumerate(sigma_range):
        d1, d2 = options_parameters(S_0, S_x, r, T, sigma)
        call_values[i, j] = digital_option_pricer(S_0, S_x, r, T, sigma, d1,␣
↪d2, call_strat=True, reward=reward)
        put_values[i, j] = digital_option_pricer(S_0, S_x, r, T, sigma, d1, d2,␣
↪call_strat=False, reward=reward)

# Plot heatmaps side by side
fig, axes = plt.subplots(1, 2, figsize=(16, 6), constrained_layout=True)

# Heatmap for Digital Call Option
sns.heatmap(call_values, ax=axes[0],
            xticklabels=np.round(sigma_range, 2),
            yticklabels=np.round(T_range, 2),
            cmap='Blues', cbar_kws={'label': 'Option Value ($)'})
axes[0].set_title('Digital Call Option Value Heatmap (Reward=1)', fontsize=14,␣
↪fontweight='bold')
axes[0].set_xlabel('Volatility ( )')
axes[0].set_ylabel('Time to Maturity (T in Years)')

# Heatmap for Digital Put Option
sns.heatmap(put_values, ax=axes[1],
            xticklabels=np.round(sigma_range, 2),
            yticklabels=np.round(T_range, 2),
            cmap='Reds', cbar_kws={'label': 'Option Value ($)'})
axes[1].set_title('Digital Put Option Value Heatmap (Reward=1)', fontsize=14,␣
↪fontweight='bold')
axes[1].set_xlabel('Volatility ( )')
axes[1].set_ylabel('Time to Maturity (T in Years)')

plt.show()
```

```
Digital Call Option Value: $0.38
Digital Put Option Value: $0.61
Parameters:
```

```
Initial Price: $346.65
Strike Price: $363.98
Reward: $1.00
Risk-free Rate: 2.00%
Time to Maturity: 0.5 years
Volatility: 33.25%
```





[82]:
```python
# Scenario 4: Diversification Seeker

# Product selected: Exchange Traded Fund (ETFs)

from sklearn.linear_model import LinearRegression

# Parameters
np.random.seed(42)
num_assets = 10  # Number of assets in the ETF
volatility = 0.3  # Volatility of the ETF
```

```
def generate_etf_portfolio(underlying, num_assets, volatility):
    # Ensure coffee_data['Close'] is one-dimensional
    underlying_close = underlying['Close'].values.flatten()
    portfolio_weights = np.random.random(num_assets)
    portfolio_weights /= portfolio_weights.sum()  # Normalize weights to␣
 ↪1

    # Simulating ETF asset prices
    etf_portfolio = pd.DataFrame({
        f'Asset_{i+1}': underlying_close * (1 + np.random.normal(0, 0.05,␣
 ↪len(underlying_close)))
        for i in range(num_assets)
    }, index=underlying.index)

    # Drop any NaN values
    etf_portfolio = etf_portfolio.dropna()

    # ETF Portfolio Price: Weighted sum of assets
    etf_portfolio['ETF_Price'] = etf_portfolio.dot(portfolio_weights)

    return etf_portfolio

etf_portfolio = generate_etf_portfolio(coffee_data, num_assets, volatility)
etf_portfolio.head()
```

[82]:

```
                              Asset_1     Asset_2     Asset_3     Asset_4  \
Date
2010-01-04 00:00:00+00:00  138.520259  140.684459  146.549043  143.537136
2010-01-05 00:00:00+00:00  144.825048  142.495258  135.198866  139.205014
2010-01-06 00:00:00+00:00  138.319009  134.567026  134.242327  133.743971
2010-01-07 00:00:00+00:00  138.595641  134.692334  147.703573  129.970625
2010-01-08 00:00:00+00:00  147.108467  135.258773  148.230603  145.289244

                              Asset_5     Asset_6     Asset_7     Asset_8  \
Date
2010-01-04 00:00:00+00:00  135.893743  137.045814  137.324544  142.839676
2010-01-05 00:00:00+00:00  137.396708  142.475948  150.131133  140.555621
2010-01-06 00:00:00+00:00  115.862787  137.710372  141.689305  132.098002
2010-01-07 00:00:00+00:00  156.743433  136.653167  132.894868  142.508928
2010-01-08 00:00:00+00:00  149.351179  140.408348  142.559575  146.097706

                              Asset_9    Asset_10   ETF_Price
Date
2010-01-04 00:00:00+00:00  138.146658  142.505116  141.705398
2010-01-05 00:00:00+00:00  148.711957  149.667142  142.561027
2010-01-06 00:00:00+00:00  143.621305  142.407366  136.012082
```

```
2010-01-07 00:00:00+00:00  134.343861  152.003906  140.579283
2010-01-08 00:00:00+00:00  138.340456  138.227648  142.316033
```

[83]:
```python
# Tracking Error Calculation
coffee_prices = coffee_data['Close']
def compute_tracking_error(etf_portfolio, underlying_prices, optimized):
    aligned_prices = underlying_prices.reindex(etf_portfolio.index)['KC=F']
    if optimized:
        squared_diff = (etf_portfolio['Optimized_ETF_Price'] - aligned_prices)␣
↪** 2
    else:
        squared_diff = (etf_portfolio['ETF_Price'] - aligned_prices) ** 2
    return np.sqrt(squared_diff.mean())  # Mean of squared differences, then␣
↪sqrt

tracking_error = compute_tracking_error(etf_portfolio, coffee_prices,␣
↪optimized=False)

# Optimize Portfolio Weights (Linear Regression for Best Fit)
X = etf_portfolio.drop(columns=['ETF_Price']).values  # Asset prices
y = coffee_prices.values.flatten()  # Benchmark prices

reg = LinearRegression(fit_intercept=False)  # No intercept, weights must sum␣
↪to 1
reg.fit(X, y)
optimized_weights = reg.coef_
optimized_weights /= optimized_weights.sum()  # Normalize weights to sum to 1

# Optimized ETF Portfolio Price
etf_portfolio['Optimized_ETF_Price'] = etf_portfolio.
↪drop(columns=['ETF_Price']).dot(optimized_weights)

# Recalculate Tracking Error for Optimized Portfolio
optimized_tracking_error = compute_tracking_error(etf_portfolio, coffee_prices,␣
↪optimized=True)
etf_portfolio
```

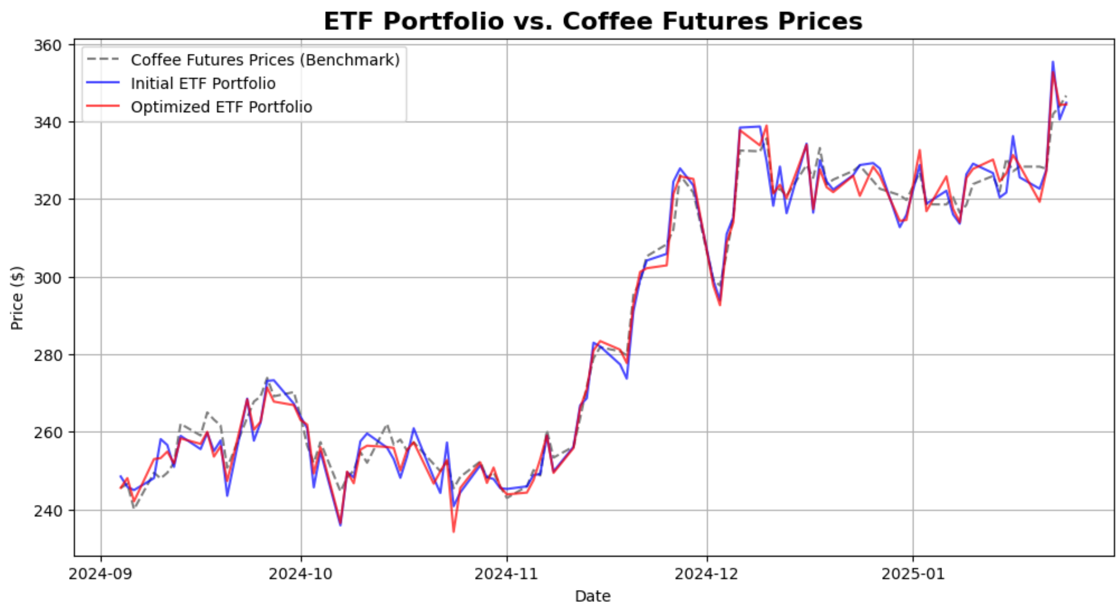
[83]:
```
                              Asset_1     Asset_2     Asset_3     Asset_4  \
Date
2010-01-04 00:00:00+00:00  138.520259  140.684459  146.549043  143.537136
2010-01-05 00:00:00+00:00  144.825048  142.495258  135.198866  139.205014
2010-01-06 00:00:00+00:00  138.319009  134.567026  134.242327  133.743971
2010-01-07 00:00:00+00:00  138.595641  134.692334  147.703573  129.970625
2010-01-08 00:00:00+00:00  147.108467  135.258773  148.230603  145.289244
…                                   …           …           …           …
2025-01-20 00:00:00+00:00  321.579202  304.890964  356.562489  328.705061
2025-01-21 00:00:00+00:00  332.015479  347.291409  306.662197  322.202468
```

```
2025-01-22 00:00:00+00:00  337.736437  353.254894  363.279468  357.114932
2025-01-23 00:00:00+00:00  344.085532  343.464735  330.796517  325.752460
2025-01-24 00:00:00+00:00  341.006340  370.297178  315.521124  353.424746

                              Asset_5     Asset_6     Asset_7     Asset_8  \
Date
2010-01-04 00:00:00+00:00  135.893743  137.045814  137.324544  142.839676
2010-01-05 00:00:00+00:00  137.396708  142.475948  150.131133  140.555621
2010-01-06 00:00:00+00:00  115.862787  137.710372  141.689305  132.098002
2010-01-07 00:00:00+00:00  156.743433  136.653167  132.894868  142.508928
2010-01-08 00:00:00+00:00  149.351179  140.408348  142.559575  146.097706
…                                 …          …          …          …
2025-01-20 00:00:00+00:00  313.939097  308.741943  300.489183  327.138840
2025-01-21 00:00:00+00:00  327.874223  340.495080  322.289903  324.189910
2025-01-22 00:00:00+00:00  353.770102  343.866069  345.116049  358.069147
2025-01-23 00:00:00+00:00  331.470343  340.692070  384.832328  329.270257
2025-01-24 00:00:00+00:00  346.004915  341.967444  352.171238  340.740420

                              Asset_9    Asset_10   ETF_Price  \
Date
2010-01-04 00:00:00+00:00  138.146658  142.505116  141.705398
2010-01-05 00:00:00+00:00  148.711957  149.667142  142.561027
2010-01-06 00:00:00+00:00  143.621305  142.407366  136.012082
2010-01-07 00:00:00+00:00  134.343861  152.003906  140.579283
2010-01-08 00:00:00+00:00  138.340456  138.227648  142.316033
…                                 …          …          …
2025-01-20 00:00:00+00:00  287.478030  338.073192  322.652781
2025-01-21 00:00:00+00:00  317.032919  334.736327  327.467388
2025-01-22 00:00:00+00:00  356.214010  357.684285  355.382147
2025-01-23 00:00:00+00:00  345.020599  365.337126  340.500579
2025-01-24 00:00:00+00:00  358.591555  328.607278  344.806464

                           Optimized_ETF_Price
Date
2010-01-04 00:00:00+00:00           140.384591
2010-01-05 00:00:00+00:00           143.121007
2010-01-06 00:00:00+00:00           135.547111
2010-01-07 00:00:00+00:00           140.740668
2010-01-08 00:00:00+00:00           143.167757
…                                        …
2025-01-20 00:00:00+00:00           319.285124
2025-01-21 00:00:00+00:00           327.144732
2025-01-22 00:00:00+00:00           352.695952
2025-01-23 00:00:00+00:00           344.228196
2025-01-24 00:00:00+00:00           344.390260

[3788 rows x 12 columns]
```

[84]:

```python
# Display Results
print(f"Initial Portfolio Tracking Error: ${tracking_error:.2f}")
print(f"Optimized Portfolio Tracking Error: ${optimized_tracking_error:.2f}")
print("Optimized Portfolio Weights:", optimized_weights)

# Plot Results
plt.figure(figsize=(12, 6))
plt.plot(coffee_prices[-100:], label='Coffee Futures Prices (Benchmark)',␣
 ↪color='grey', linestyle='--')
plt.plot(etf_portfolio['ETF_Price'][-100:], label='Initial ETF Portfolio',␣
 ↪color='blue', alpha=0.7)
plt.plot(etf_portfolio['Optimized_ETF_Price'][-100:], label='Optimized ETF␣
 ↪Portfolio', color='red', alpha=0.7)
plt.title('ETF Portfolio vs. Coffee Futures Prices', fontsize=16,␣
 ↪fontweight='bold')
plt.xlabel('Date')
plt.ylabel('Price ($)')
plt.legend()
plt.grid()
plt.show()
```

```
Initial Portfolio Tracking Error: $3.05
Optimized Portfolio Tracking Error: $2.64
Optimized Portfolio Weights: [0.10650711 0.09241898 0.105159   0.09942995
0.0965561  0.08965858
 0.1001723  0.1016026  0.10162214 0.10687323]
```



```
[ ]:
```