Aufgabe 5.1

$E(X) = 15$ - durchschnittliche Lebenserwartung $\implies \lambda = \frac{1}{15}$

a) $P(X \leq 10) = 1 - e^{-\frac{10}{15}} = 0.49 = 49\%$

b) $P(X > 30) = e^{-\frac{30}{15}} = 0.135 = 13.5\%$

c) $E(X > 30) = 15$

Aufgabe 5.2

a) $\lambda = \frac{1}{5} \quad \mu = \frac{1}{10} \quad \tau = \lambda + \mu = \frac{3}{10}$

$E(X) = \frac{1}{\tau} = 3.33$ Lebensdauer des Gesamtbauteils

durchschnittliche Lebensdauer $= E(X)$

b) Teilbauteile sind parallel eingeschaltet, also hängt die Lebensdauer des Gesamtessystems nicht von dem einzelnen Bauteil ab. Falls ...

Aufgabe 5.3

a) $E(X) = 3.15$ im Durchschnitt Tor/Spiel

$P(X > 2) = 1 - (P(0) + P(1) + P(2)) = 1 - (\frac{3.15^0}{0!} \cdot e^{-3.15} + \frac{3.15^1}{1!} \cdot e^{-3.15} + \frac{3.15^2}{2!} \cdot e^{-3.15}) = 1 - (e^{-3.15} +$

$+3.15 \cdot e^{-3.15} + \frac{9.9225}{2} \cdot e^{-3.15}) = 1 - (0.043 + 0.135 + 0.212) = 0.61 = 61\%$

b) $\lim_{n\to\infty} \binom{n}{k} \cdot (\frac{\lambda}{n})^k \cdot (1 - \frac{\lambda}{n})^{n-k} = \frac{\lambda^k}{k!} \cdot e^{-\lambda} \implies P(X > 2|n = 90) = 1 - (P(0) + P(1) + P(2)) = 61\%$

c) Wegen der Limes $P(X > 2|n = 5400) = P(X > 2|n = 90) = 61\%$

Aufgabe 5.4

$E(A) = 2 \quad E(B) = 1$

$P(X = 0) = \frac{2^0}{0!} \cdot e^{-2} = 0.135$

$P(X = 1) = \frac{1^1}{1!} \cdot e^{-1} = 0.37$

$P(1:0) = P(X = 0) \cdot P(X = 1) = 0.05 = 5\%$

Aufgabe 5.5

$E(X_i) = \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} jP(X = j) = \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n} j = \lim_{n\to\infty} \frac{1}{n} \sum_{j=1}^{n-k} j = \lim_{n\to\infty} \frac{n-k}{n} = ...$

Aufgabe 5.6

$X \sim N(-3, 5)$

a) $P(X \leq -8) = \Phi(\frac{-8+3}{5}) = \Phi(-1) = 0.1587 = 15.87\%$

b) $P(-3.5 \leq X \leq 3.5) = P(X \geq -3.5) - P(X \leq 3.5) = \Phi(\frac{3.5+3}{5}) - \Phi(\frac{-3.5+3}{5}) = \Phi(1.3) - \Phi(-0.1)$

$= 0.9032 - 0.4602 = 0.443 = 44.3\%$

c) $P(X \geq -3) = \Phi(\frac{-3+3}{5}) = \Phi(0) = 0.5 = 50\%$

d) $P(X \geq 22) = \Phi(\frac{22+3}{5}) = \Phi(5) = 0$

$x_{5\%} = -3 + 5 \cdot \Phi^{-1}(0.05) = -3 + 5 \cdot (-1.64) = -11.2$

$x_{99\%} = -3 + 5 \cdot \Phi^{-1}(0.99) = -3 + 5 \cdot 2.326 = 8.63$

Aufgabe 5.7

$X \sim N(100, 15)$

a) $P(X \leq 90) = \Phi(\frac{90-100}{15}) = \Phi(-0.67) = 0.2514 = 25.14\%$

b) $P(X > 110) = 1 - \Phi(\frac{110-100}{15}) = 1 - \Phi(0.67) = 1 - 0.7475 = 0.2525 = 25\%$

c) $P(X > 140) = 1 - \Phi(\frac{140-100}{15}) = 1 - \Phi(2.67) = 1 - 0.9961 = 0.0039 = 0.39\%$

d) $x_{1\%} = 100 + 15 \cdot \Phi^{-1}(0.01) = 100 + 15 \cdot (-2.326) = 65.11$

e) $x_{99.8\%} = 100 + 15 \cdot \Phi^{-1}(0.998) = 100 + 15 \cdot 2.878 = 143.17$

Aufgabe 5.8

$X \sim N(60, 5)$

a) $P(X \leq 55) = \Phi(\frac{55-60}{5}) = \Phi(-1) = 0.1587$

$P(55 \leq X \leq 65) = P(X \leq 65) - P(X \geq 55) = \Phi(1) - \Phi(-1) = 0.8413 - 0.1587 = 0.6826$

$P(65 \leq X \leq 70) = P(X \leq 70) - P(X \geq 65) = \Phi(2) - \Phi(1) = 0.9772 - 0.8413 = 0.1359$

$P(X \geq 70) = 1 - \Phi(\frac{70-60}{5}) = 1 - \Phi(2) = 0.0228$

b) $E(X) = 0.2 \cdot 0.1587 + 0.25 \cdot 0.6826 + 0.3 \cdot 0.1359 + 0.35 \cdot 0.0228 = 0.2511 = 25.11$ Euro

c) $x_{10\%} = 60 + 5 \cdot \Phi^{-1}(0.1) = 60 + 5 \cdot -1.28 = 53.6$

$x_{50\%} = 60 + 5 \cdot \Phi^{-1}(0.5) = 60 + 5 \cdot 0 = 60$

$x_{90\%} = 60 + 5 \cdot \Phi^{-1}(0.9) = 60 + 5 \cdot 1.29 = 66.45$

Gewichtsklasse in g	≤ 53	$[53, 60]$	$[60, 66]$	≥ 66
Preis in €	0.2	0.25	0.3	0.35

Aufgabe 5.9

$X \sim N(12.250, 1500)$

$P(X \leq 10.000) = \Phi(\frac{10.000-12.250}{1500}) = \Phi(-1.5) = 0.0668 = 6.68\%$

$P(X \geq 10.000) = 1 - \Phi(\frac{10.000-12.250}{1500}) = 1 - \Phi(-1.5) = 1 - 0.0668 = 0.9332$ ...