

Logistic Regression

TC

October 2021

1. Find the function:

$$f'(x) = f(x)(1 - f(x))$$

$$\iff \frac{d(f(x))}{dx} = f(x)(1 - f(x))$$

$$\longrightarrow \frac{d(f(x))}{f(x)(1 - f(x))} = dx$$

$$\longrightarrow \int \frac{d(f(x))}{f(x)(1 - f(x))} = \int dx$$

$$\longrightarrow \int \frac{1}{f(x)(1 - f(x))} d(f(x)) = \int dx$$

$$\longrightarrow \int \frac{1}{f(x)} + \frac{1}{1 - f(x)} d(f(x)) = \int dx$$

$$\iff \ln(f(x)) - \ln(1 - f(x)) = x$$

$$\iff \frac{f(x)}{1 - f(x)} = e^x$$

$$\longrightarrow f(x) = e^x(1 - f(x))$$

$$\iff f(x) = e^x - e^x f(x)$$

$$\iff 1 = \frac{e^x}{f(x)} - e^x$$

$$\iff f(x) = \frac{e^x}{1 + e^x} = \sigma(x)$$