­A hydrological calibration procedure for stochastic rainfall model

# Abstract

# Introduction

Long sequences of simulated streamflow are important in assessing future flood and drought risks. An approach to simulate streamflow is the use of hydrological model with simulated rainfall input. Stochastic rainfall modelling involves the simulation of sequences of rainfall at a scale of interest (e.g., sub-daily, daily, monthly, annual, multi-annual) that is statistically similar to observed rainfall timeseries, typically measured at rain gauges. Stochastic rainfall models (SRMs) are commonly fitted to observed rainfall data. However, when simulated rainfall data are used as input for hydrological models, it is possible that the simulated rainfall will not be translated to realisations of streamflow that are statistically similar to the observed streamflow ([Bennett et al., 2019](#_ENREF_4)). This can be due to several factors, including limited understanding of the rainfall-runoff process or the inability of SRMs to capture important rainfall attributes. This paper demonstrates the issues of potentially good-modelled rainfall translated to poor-modelled streamflow and introduces a new hydrological calibration procedure that allows SRMs to help capture streamflow statistics that cannot be achieved with a rainfall-based calibration approach.

In catchment hydrology, the term continuous simulation is the simulation of the wet and dry conditions of a catchment by estimating the loss in rainfall and generating streamflow at daily, hourly, or sub-hourly time scales. The process of continuous simulation requires a rainfall-runoff model with sequences of meteorological inputs such as rainfall, evapotranspiration or temperature depending on the specification of the model ([Beven, 2012](#_ENREF_5)). In contrast to the common event-based approach for hydrological risk assessment, continuous simulation does not require assumptions about the initial catchment soil moisture ([Boughton and Droop, 2003](#_ENREF_7)). Continuous simulation also allows the generation of long sequences of streamflow from which important flood or drought statistics can be extracted for the establishment of appropriate mitigation strategies, early warning, or long-term projection ([Linsley and Crawford, 1974](#_ENREF_21), [Boughton and Hill, 1997](#_ENREF_8), [Blazkova and Beven, 2002](#_ENREF_6), [Lamb, 2005](#_ENREF_19), [Viviroli et al., 2009](#_ENREF_36)).

However, one of many challenges to the continuous simulation approach is to assess the impact of uncertainties in rainfall input on rainfall-runoff models response. Poor understanding of the effect of rainfall variability or uncertainties in rainfall data on the output of rainfall-runoff models will hamper the ability of continuous simulation in providing reliable hydrological assessment ([Michaud and Sorooshian, 1994](#_ENREF_22), [Faurès et al., 1995](#_ENREF_14), [Andréassian et al., 2001](#_ENREF_1), [Cristiano et al., 2017](#_ENREF_10)). The sensitivity of rainfall-runoff models to the spatial variability of rainfall input has been investigated in several studies ([Wilson et al., 1979](#_ENREF_38), [Michaud and Sorooshian, 1994](#_ENREF_22)). However, [Nicótina et al. (2008)](#_ENREF_25) pointed out mixed conclusions in the effect of rainfall spatial variability on rainfall-runoff models response from multiple mesoscale catchments, where some catchments tend to dampen the effect of spatial distributions while others tend to amplify it and concluded that the sensitivity of runoff response to the spatial distribution of rainfall is also affected by catchment characteristics such as hillslope and routing time distribution. Sources of simulated streamflow deficiencies have also been reported to be caused by the deficiencies of simulated rainfall in different time periods. In particular, poor simulated streamflow within a month could be the result of rainfall deficiencies in the concurrent month, the preceding month, or a contiguous block of month ([Bennett et al., 2019](#_ENREF_4)). It was also found that in some cases, “good” simulated rainfall can create “poor” streamflow estimations while “poor” simulated rainfall can create “good” streamflow estimations ([Bennett et al., 2019](#_ENREF_4)). The case of “good” rainfall - “poor” streamflow has also been reported in ([Gao et al., 2020](#_ENREF_15)), in which a single site SRM was developed and was shown to capture well a range of rainfall statistics including the wet and dry spell distributions, the lower and the upper tails. However, when the model is used to generate rainfall input for a rainfall-runoff model, the streamflow estimation was shown to have relatively large underestimation in the high flow ranges (from the 90th to the 99th percentiles, with increasing magnitude of underestimation). It is evident that a SRM should not only be able to preserve observed rainfall attributes but also be able to capture observed streamflow characteristics for practical hydrological application.

There has been a substantial rainfall model development to capture both the spatial pattern ([Wilks, 1998](#_ENREF_37), [Qian et al., 2002](#_ENREF_27), [Leonard et al., 2008](#_ENREF_20), [Rasmussen, 2013](#_ENREF_28), [Baxevani and Lennartsson, 2015](#_ENREF_2), [Bennett et al., 2018](#_ENREF_3), [Evin et al., 2018](#_ENREF_13)), as well as the temporal pattern including daily ([Richardson and Wright, 1984](#_ENREF_29), [Semenov and Barrow, 1997](#_ENREF_31), [Sharma and Lall, 1999](#_ENREF_32), [Srikanthan and McMahon, 2001](#_ENREF_33)), sub-daily ([Gupta and Waymire, 1993](#_ENREF_17), [Khaliq and Cunnane, 1996](#_ENREF_18), [Cowpertwait, 2006](#_ENREF_9)), year to year variation ([Thyer and Kuzera, 1999](#_ENREF_35), [Srikanthan and Pegram, 2009](#_ENREF_34)) of the observed rainfall. The SRMs are evaluated with an extensive range of rainfall statistics. The statistics used for assessing SRM performance in several studies are tabulated in Table X.

Graphical user interface, table

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Figure X. Example figure of the table. The table will be updated in future version

However, it is shown that these studies on SRMs development generally considered solely observed rainfall statistics as benchmarks without any consideration on streamflow statistics. Hence, the aim of this paper is to develop a new approach to calibrate SRMs with both rainfall and streamflow statistics.

This paper has three key objectives:

1. Introduce the hydrological calibration procedure for SRMs.
2. Identify case study scenarios where simulated rainfall categorised as “good” translated to simulated runoff categorised as “bad”.
3. Demonstrate the improvements that the hydrological calibration procedure provided to the SRM through examining different rainfall and runoff statistics.

A range of catchments within Australia are examined with the use of a SRM and lumped conceptual rainfall runoff model.

This paper is structured as follows. Sect. 2 introduces the hydrological calibration procedure for SRMs. Sect. 3 presents the case studies that are used to demonstrate the hydrological calibration procedure. Results, discussion, and conclusion are in sect. 4, sect. 5, sect. 6 respectively.

# Hydrological calibration of SRMs

## Overview

A typical calibration procedure for SRMs involves computing some observed rainfall statistics which are then used as model parameter to generate simulated rainfall. Fig. 1 illustrates the general process of SRMs development.

Diagram

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Figure 2. Schematic of SRMs development with observed rainfall data.

For example, the popular WGEN models ([Richardson and Wright, 1984](#_ENREF_29)) has a daily rainfall generator component that requires 4 rainfall statistics: the probability of dry-wet event (PDW), the probability of wet-wet event (PWW), the shape (α) and rate (β) parameters of the gamma distribution. The first two probabilistic parameters PDW and PWW control the wet/dry pattern of the rainfall using a 1st order Markov chain while parameters α and β controls the amount of rainfall occurs on wet days. These parameters can be computed directly from the observed rainfall and used for simulating rainfall estimates.

Calibrating with observed rainfall data allows SRMs to preserve the observed rainfall statistics. The simulated rainfall time series can be used as input for hydrological models to produce streamflow time series for hydrological assessment. However, it is not necessarily given that simulated rainfall time series will translate to streamflow time series that preserve the properties of observed streamflow data. Therefore, another approach is to calibrate model parameters with streamflow statistics. Fig. 2 illustrates a schematic of the hydrological calibration procedure for SRMs. This approach utilises an optimization process based on targeted streamflow attributes to identify best suited set of SRM parameters. The use of optimization loop for SRM has also been used in previous studies to perturb rainfall data for scenarios-neutral climate impact assessment ([Guo et al., 2018](#_ENREF_16), [Culley et al., 2019](#_ENREF_11))

Diagram

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Figure 3. Schematic of the hydrological calibration for SRMs with observed runoff

In comparison to the typical procedure of SRMs development, the hydrological calibration of SRMs requires an additional rainfall-runoff model component to simulate runoff estimates and observed streamflow data. The proposed steps for the hydrological calibration procedure are as follows.

1. Generating a sequence of simulated rainfall from the SRM with a pre-defined set of parameters.
2. Generating a sequence of simulated runoff using the rainfall runoff model with the simulated rainfall as input.
3. Computing a pair of statistics from the simulated runoff and the observed runoff (e.g., the flow duration curve).
4. Computing an objective function with the simulated and observed runoff statistics (e.g., the sum of squared errors)
5. Finding the SRM parameters set that optimizes the objective function.

From Fig. 2, the simulated runoff statistic is compared against the observed runoff statistic. This comparison could affect the performance of the hydrological calibration procedure due to potential errors in the measurement of the observed runoff. To overcome this issue, the observed runoff time series is replaced by a virtual observed runoff time series. The virtual observed runoff is generated by inputting observed rainfall to the same rainfall-runoff model in previous step. Fig. 3 illustrates the hydrological calibration procedure with virtual observed runoff.

Diagram

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Figure 4. Schematic of the hydrological calibration for SRMs with virtual observed runoff.

To further illustrate the hydrological calibration procedure, an example with the WGEN model ([Richardson and Wright, 1984](#_ENREF_29)) and the GR4J rainfall-runoff model ([Perrin et al., 2003](#_ENREF_26)) that are used for this study is presented in the following sub-sections.

**Step 1 – rainfall simulation**

The WGEN mentioned above is used to generate simulated rainfall time series. To initiate the procedure, the 4 monthly model parameters (i.e., 4 parameters for each of the 12 month) are computed from the observed rainfall data. These values will be used as the starting point for the optimization process.

The occurrence parameters PDW and P­WW are computed using the following equations:

Where nDW is the number of wet days given a dry occurred previously, nDD is the number of dry days given a dry day occurred previously, nWW is the number of wet days given a wet day occurred previously and nWD is the number of dry days given a wet day previously.

The parameters α and β is computed by simply using the method of moment (MoM):

Where μ is the mean of wet days amount and ν is the variance of wet days amount.

In total, there will be 48 parameters computed as initial values for the optimization. The bounds of the parameters are tabulated in Table 1

Table 1. Bounds of the 4 WGEN model parameters

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| Parameter | PDW | P­WW | α | β |
| Lower bound | 0 | 0 |  |  |
| Upper bound | 1 | 1 | Inf | Inf |

**Step 2 – streamflow simulation**

The GR4J model is used to generate both simulated streamflow and virtual observed streamflow. The model is calibrated with observed runoff data which yields 4 parameters. These 4 parameters are fixed throughout the whole process.

In this study, the GR4J model is calibrated with the Nash-Sutcliffe Efficiency (NSE) ([Nash and Sutcliffe, 1970](#_ENREF_24)) as the objective function. The optimization of the NSE follow the procedure presented in ([Michel, 1991](#_ENREF_23)). In general, the algorithm combines a global screening of the fitness landscape and a steepest decent local search to identify the best-fit model parameters. The calibration is set up with 2-year warm up period.

**Step 3 – Runoff statistic**

The choice of runoff statistics within the hydrological calibration procedure is flexible and is dependent on modellers interest. For this study, the flow duration curve is selected as the target statistic to calibrate the SRM. The flow duration curve is a cumulative frequency curve that shows the exceedance probabilities for a range of runoff magnitudes ([Searcy, 1959](#_ENREF_30)). While the flow duration curve does not account for the chronological sequence of the observed runoff data, it can depict the long-term trend of the runoff and is useful for flood and drought studies.

The pair flow duration curves are calculated from both virtual observed runoff and simulated runoff. Fig.X shows the conceptual representation of the flow duration curves.

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Figure X. example only, future figure will contain two lines i.e. 1 realisation of sim FDC and 1 vir. Obs. FDC

**Step 4 – Objective function**

From Fig. 4, there are some deficiencies between the simulated and the virtual observed flow duration curve. In particular, both the lower tail and upper tail of the simulated runoff are not well-reproduced comparing to the virtual observed runoff. An approach to improve the reproduction of the simulated flow duration curve is to calibrate the SRM using goodness-of-fit objective functions. In this study the sum of squared errors (SSE) - a Least Squares - type objective function is used to find the best-fit SRM parameters. The SSE is denoted as:

Where *n* is the number of observations, is the virtual observed runoff, is the simulated runoff.

**Step 5 - Optimization**

The Shuffle Complex Evolution (SCE) optimization algorithm ([Duan et al., 1992](#_ENREF_12)) is used to find the best-fit SRM parameters. Given that the WGEN model has in total 48 parameters, the search space will become relatively large which could affect the efficiency of the optimization. Therefore, to reduce the size of the parameter space, the optimization process can be performed on a monthly basis. In this way, the parameters set for each optimization loop is reduced to 4 parameters only. Then the optimization can be run 12 times to obtain the parameters set for each month.

# Case study

In this study, a range of catchments spreading across Australia are randomly selected to first demonstrate the common issues of “good” modelled rainfall translated to “bad” modelled runoff. Then the hydrological calibration procedure is performed at each catchment to identified sets of SRM parameters that can improve the modelled runoff. The information of 9 catchments is tabulated in Table X.

Table 2

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| **Number** | **Catchment outlet** | **Latitude** | **Longitude** | **Jurisdiction** | **Catchment Area (km2)** | **Record Length** |
| 1 | Cotter River at Gingera | -35.59 | 148.82 | ACT | 130 | 1963-2019 |
| 2 | Corang River at Hockeys | -35.15 | 150.03 | NSW | 165.6 | 1950-2019 |
| 3 | Bremer River at Walloon | -27.6 | 152.69 | QLD | 628.1 | 1961-2019 |
| 4 | Barambah Creek at Litzows | -26.3 | 152.04 | QLD | 646.6 | 1964-2019 |
| 5 | Goulburn River at Dohertys | -37.33 | 146.13 | VIC | 700.2 | 1967-2019 |
| 6 | Happy Valley Creek at Rosewhite | -36.58 | 146.82 | VIC | 138 | 1961-2019 |
| 7 | Leven River at Bannons Bridge | -41.25 | 146.09 | TAS | 499.3 | 1963-2019 |
| 8 | Scott Creek at Scott Bottom | -35.1 | 138.67 | SA | 29 | 1969-2019 |
| 9 | Harvey River at Dingo Road | -33.09 | 116.04 | WA | 148 | 1970-2019 |

The observed daily runoff data at each catchment outlet are obtained from the hydrologic reference stations database (BOM). The observed daily rainfall data and the estimated observed daily potential evapotranspiration for each catchment are obtained from one nearest rainfall station to the catchment outlet.

The daily rainfall generator component of the WGEN model is fitted to the observed rainfall at each catchment following the procedure descripted in Step 1 (sect. 2) to generate simulated rainfall. The fitted parameters of the WGEN model for each catchment are tabulated in Table X. The GR4J model is calibrated to the observed daily runoff at each catchment following the procedure in Step 2 (sect. 2). The GR4J model parameters and the correspondent NSE are tabulated in Table XX. Together with the observed daily rainfall and estimated observed daily PET, the GR4J is used to generate the daily virtual observed runoff for each catchment. While the simulated runoff for each catchment is generated from the GR4J model with the simulated observed rainfall and the estimated observed PET.

Table X. Fitted WGEN parameters

Table XX. GR4J parameters and NSE

The performances of the WGEN model and the GR4J model in simulating rainfall and runoff are categorised using the CASE framework proposed by [Bennett et al. (2019)](#_ENREF_4). ***[this section will be completed in future version, including definition of the CASE framework]***

# Results

## Identifying cases of “good” modelled rainfall resulted in “poor” modelled runoff

*[this section presents two cases. 1 with deficiency in the upper tail of runoff, 1 with deficiency in the lower tail of runoff]*

|  |  |
| --- | --- |
|  |  |
|  |  |

Figure 5. deficiency in the upper tail of runoff.

|  |  |
| --- | --- |
|  |  |
|  |  |

Figure 6. deficiency in lower tail of runoff

## Performing hydrological calibration

*[This section will show the results after the hydrological calibration with the two sites mentioned in Sect. 4.1. Repeated 2 similar figures as in 4.1 with improved runoff stats*

* *Potential results: calibrating with a different objective function (Weighted SSE) and/or setting constraint to rainfall statistics.*
* *Note: Objective function with rainfall constraint has not been introduced in the methodology*

# Discussion

# Limitations and opportunities

# Conclusions

* Limitation and future opportunities:
* Rainfall runoff models choice
* Objective function choice
* Feasibility – runtime
* Other Rainfall model (extend WGEN to capture seasonal pattern - harmonic function)

*Data availability.* All the data used in this study can be requested by contacting the corresponding author Thien Nguyen at truonghuythien.nguyen@adelaide.edu.au.

*Author contributions.*

*Competing interest.* The authors declare that they have no conflict of interest.

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