A swap curve for Insurance Pricing

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Plan

1. Context

- Insurance business: **Selling options**
- What's **the 'premium'**? (Best Estimate reserve)
- Solvency II's Article 77.2: ... *shall correspond to the **probability**-weighted average of future cash-flows, taking account of time value of money (expected present value of future cash-flows), using the **relevant risk-free interest rate term structure***

2. The proposed methodology

- **No arbitrage short rate models**; include $t \mapsto b(t)$: for an exact **fitting of current term structure**
- **Example**: Hull & White (1990, 1994) extended Vasicek model

Discount factors at time t_0, for maturity t:

$$P(t_0, t) = exp\left(-X_0\phi(t - t_0) - a \int_{t_0}^{t} b(u)\phi(t - u)du - \psi(t - t_0)\right)$$

Where:

$$\phi(s) := \frac{1}{a}\left(1 - e^{-as}\right)$$

$$\psi(s) := -\int_0^s \left(\frac{\sigma^2}{2}\phi^2(s - \theta)\right) d\theta$$

2. The proposed methodology (cont'd)

$$P(t_0, t) = exp\left(-X_0\phi(t - t_0) - a\int_{t_0}^{t} b(u)\phi(t - u)du - \psi(t - t_0)\right)$$

- ▶ What we do: **piecewise-constant specification** of $t \mapsto b(t)$: at swaps' maturities $T_1, \ldots, T_n \longrightarrow b_1, \ldots, b_n$
- ▶ **Curve extrapolation**: additional parameter b_{n+1} based on an Ultimate Forward Rate
- ▶ $\int_{t_0}^{t} b(u)\phi(t - u)du$ becomes explicitly calculable
- ▶ $P(t_0, t)$ is expressed in a basis of functions

2. The proposed methodology (cont'd)

- **Parameters**: $X_0, a, \sigma, b_1, \ldots, b_n$
- **Parameters' calibration**: Iterative valuation of market swaps (OIS or EUR 6M IRS + Credit Risk Adjustment) using new $P(t_0, t)$ (depending on $a, \sigma, b_1, \ldots, b_n$) + Leave-One-Out cross validation
- **Forecasting and simulation**: Functional Principal Components Regression on parameters + Univariate time series forecasting
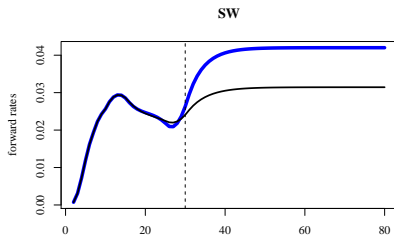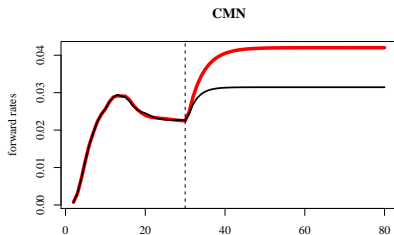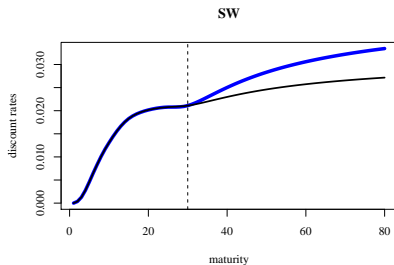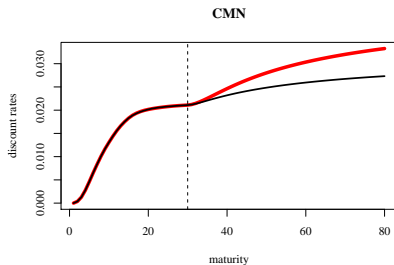
3. Some results: On curve construction

- Andersen & Piterbarg (2010) data
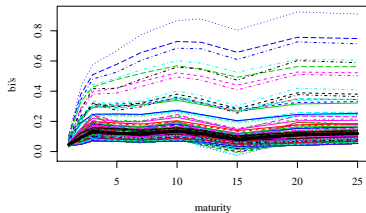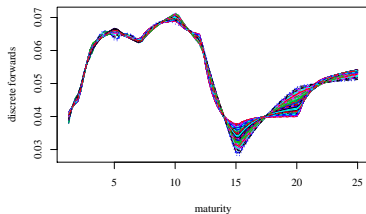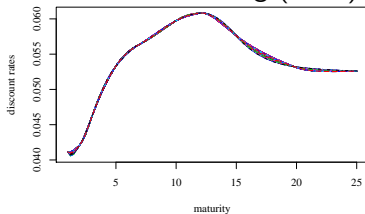
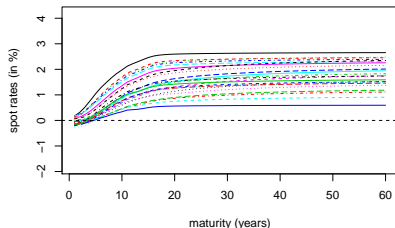# 3. Some results: On curve extrapolation, with market OIS and EUR 6M IRS + CRA
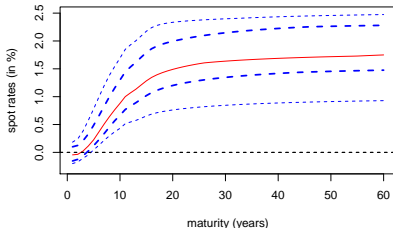
- Ametrano & Bianchetti (2013) data

3. Some results: On forecasting and simulation

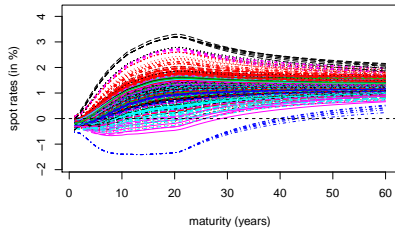▶ Market EUR 6M IRS data + Credit Risk Adjustment