'Sensibilité d'une rente viagère à l'extrapolation de la courbe des taux dans un contexte LTGA', JIRF 2015

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Solvency II economic balance sheet, technical provisions

- **Solvency II**? QIS5? LTGA?
- **Simplified balance sheet**

Assets	Liabilities
Assets	- Available capital
at	- **Solvency Capital Requirement** (SCR)
market	- Risk Margin
value	- **Best Estimate Liabilities**

- 'Fair' valuation $\sim$ **Assets** *marked to market* + **Liabilities** *marked to model*
- **Technical provisions** = **Best Estimate Liabilities** + Risk Margin

Best Estimate Liabilities (BEL) explained

$$BEL_t = \sum_T \mathbb{E}[D(t,T)CF_T|\mathcal{F}_t \otimes \mathcal{T}_t]$$

- $D(t,T)$: **stochastic discount factor**
- CF_T: **future cash-flows**
- $\mathcal{F}_t \otimes \mathcal{T}_t$: **financial** and **technical** information at t
- **Simple case**: CF_T deterministic/highly predictable
- **Difficult case**: CF_T depending on financial **and** technical information

In the article:

- CF_T: deterministic, mortality risk mutualized
- $d(t,T) := \mathbb{E}[D(t,T)|\mathcal{F}_t] = \mathbf{?} =$ **critical input**

Solvency II term structure of discount factors

How to derive $d(t, T)$ within Solvency II?

Liquid part

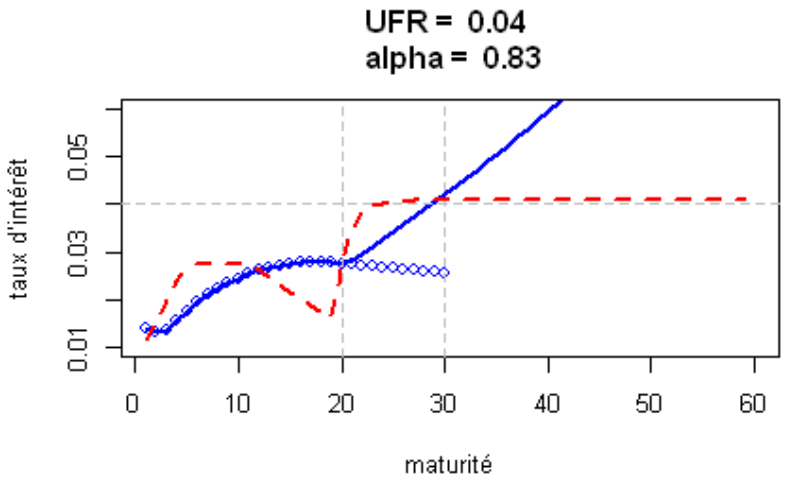
- Par **swap rates**
- Parallel shift for credit risk deduction (10 bps)
- $+$ Volatility adjustment (optional)
- $+$ Matching adjustment (optional)

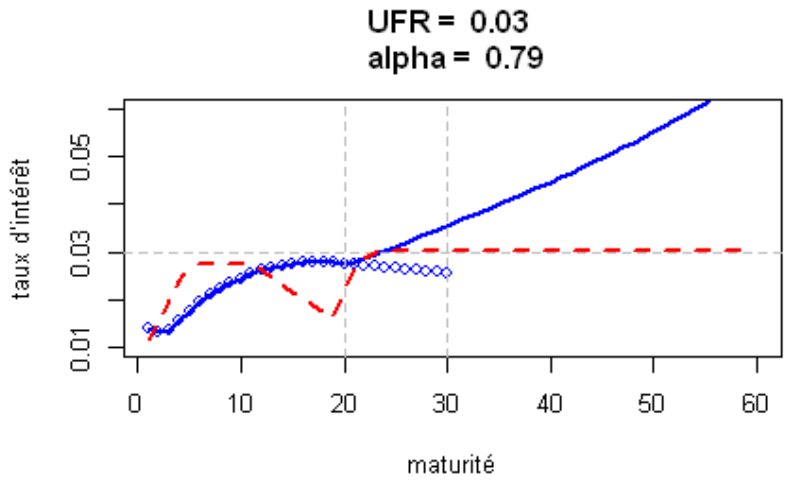
Extrapolated part

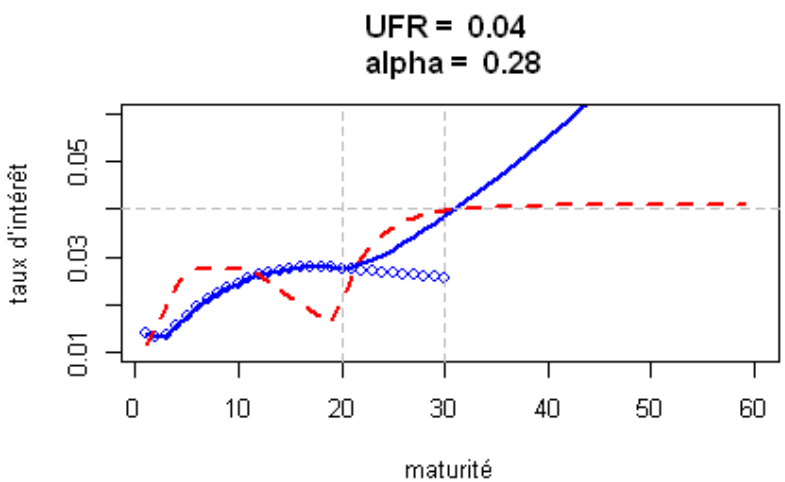
- Last Liquid Point (LLP)
- **Ultimate Forward Rate (UFR)**
- Convergence speed $(\alpha) \sim$ convergence period (years)
- Bootstrap, interpolation, **extrapolation**: **Smith-Wilson**

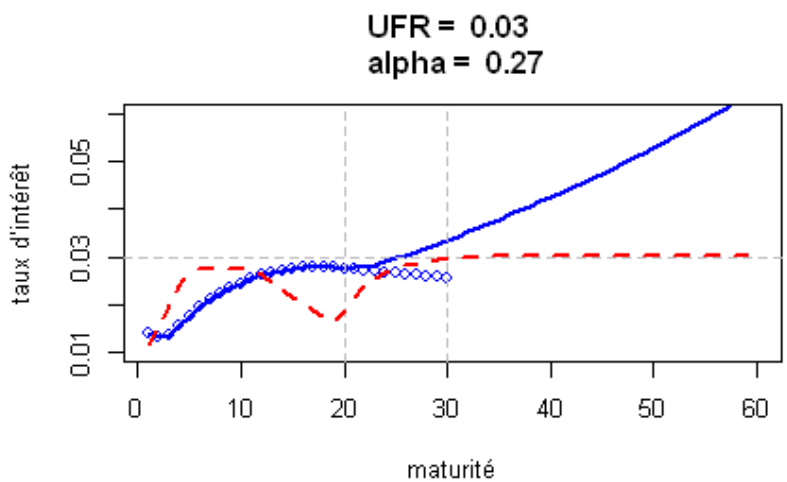
Solvency II term structure of discount factors (cont'd)

Solvency II interpolation/extrapolation explained









Solvency II term structure of discount factors (cont'd)

UFR?

- **Endogeneous** UFR $= \mathbf{f_\infty} = \mathbf{f_\infty(t)}$
  - Vasicek-Fong (1982): **French Institute of Actuaries**

$$d(t,s) = \sum_m \beta_{m,s} e^{-m\mathbf{f_\infty}(s-t)}$$

  - Nelson & Siegel (1987) or Svensson (1994): **Central Banks**

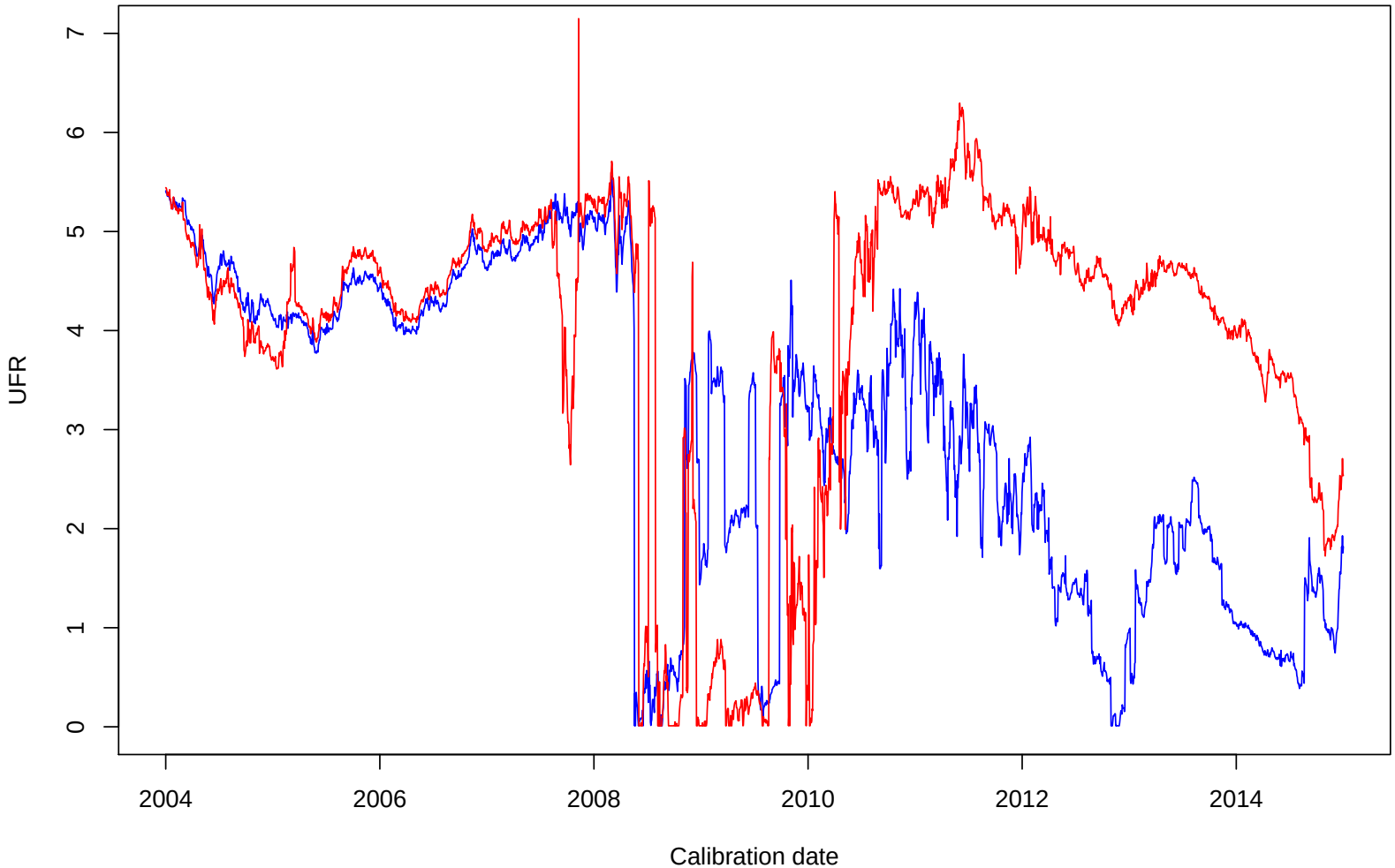
$$d(t,s) = exp\left(-(s-t)\left(\mathbf{f_\infty} + \beta_1 K_1(t,s) + \beta_2 K_2(t,s)\right)\right)$$

- **Exogeneous** UFR $= \mathbf{f_\infty} =$ constant
  - Smith-Wilson (2001): **Solvency II**

$$d(t,s) = e^{-\mathbf{f_\infty}(s-t)} + \sum_m \beta_m K_m(t,s)$$

Solvency II term structure of discount factors (cont'd)

- Problem with an **endogeneous** UFR: potential **volatility** induced in liabilities
- ECB data; calibration with Svensson method (*Blue* = AAA Bonds, *Red* = All Bonds)



Solvency II term structure of discount factors (cont'd)

The case for a **fixed exogeneous** UFR for insurance

- High demand on long-term swaps from pension funds, artificially driving long rates down?
- More stable liabilities valuation
- UFR = Expected **inflation** + Expected **real interest rates**
- UFR = fixed since 5 years = 4.2% = 2% + 2.2%

Results from the article

Based on a **fixed annuity** valuation

- **Fixed cash-flows** depending on a mortality table (mutualized mortality risk)
- **Additional benefits** cash-flows depending of implied forward
- **Aggregated cash-flows** discounted, using **Smith-Wilson** for, interpolation and extrapolation (swap curve at 12/31/2011)

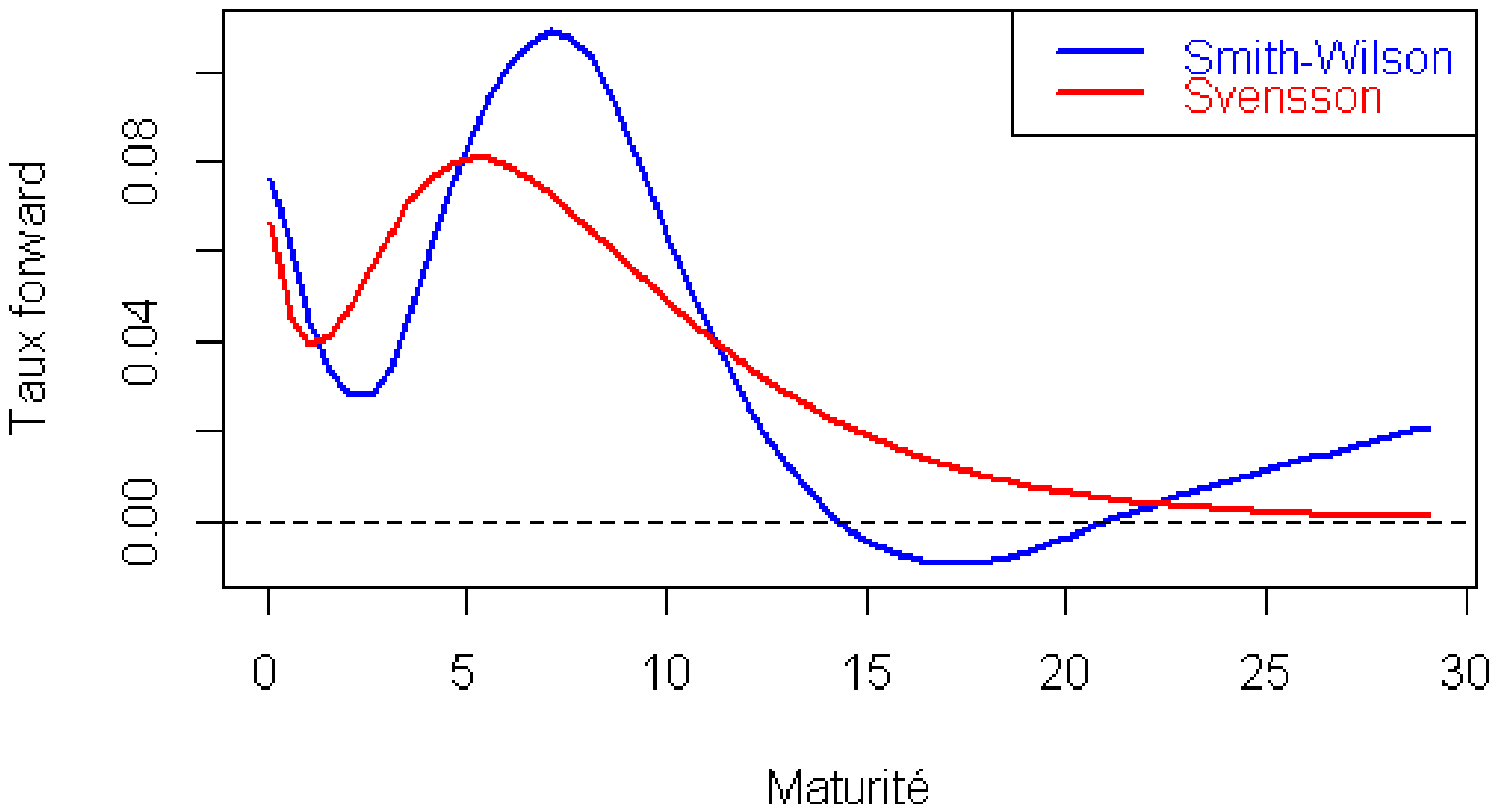
Results on the valuation

1. On **interpolation** with a 'complicated' (realistic, but not so common) benchmark curve
2. On the **dependence with UFR and convergence speed**
3. On the **dependence with LLP and convergence speed**

Results from the article (cont'd)

On interpolation

Implied forwards on "A Curve where all cubic splines produce negative rates" (Hagan & West (2006))



Results from the article (cont'd)

On interpolation (cont'd)

Maturité	BE Smith-Wilson	BE Svensson	Ecart relatif
0 à 5 ans	9 912,8	9 847,5	0,7%
5 à 10 ans	6 342,1	6 309,7	0,5%
10 à 15 ans	4 048,2	4 384,5	-7,7%
15 à 21 ans	5 668,9	5 580,9	1,6%
21 à 26 ans	3 949,1	3 679,4	7,3%
26 à 30 ans	3 329,4	3 218,0	3,5%
Total			
	33 250,6	**33 020,0**	**0,7%**

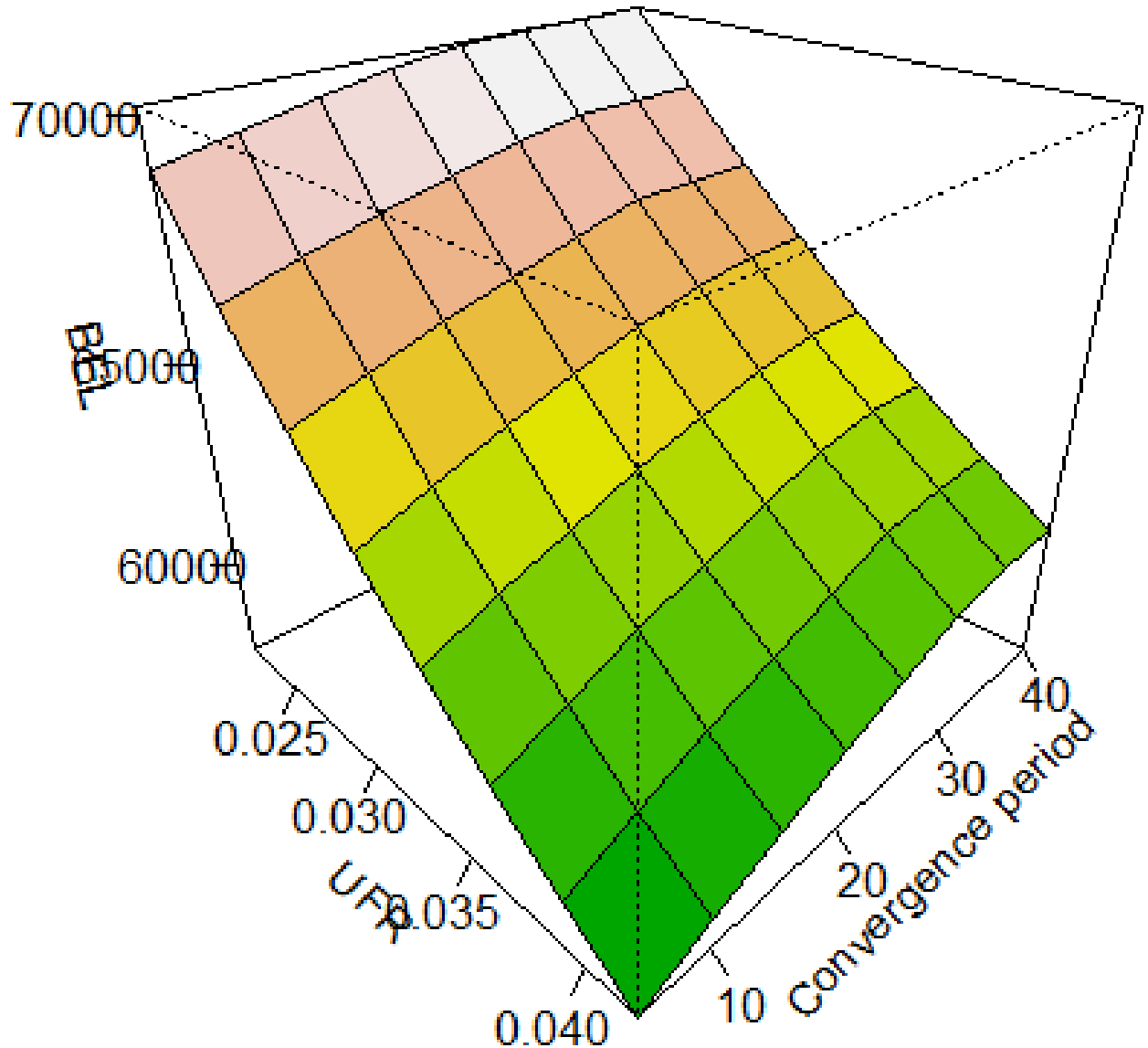
Table 3: Evaluations du Best Estimate par les méthodes de Smith-Wilson et de Svensson

Maturité	Reval. Smith-Wilson	Reval. Svensson	Ecart relatif
0 à 5 ans	218,8	256,9	-14,8%
5 à 10 ans	291,7	213,9	36,4%
10 à 15 ans	56,8	75,0	-24,3%
15 à 21 ans	-18,7	32,9	-156,8%
21 à 26 ans	11,7	5,9	98,3%
26 à 30 ans	27,8	1,8	1444,1%
Total			
	588,1	586,4	0,3%

Table 4: Evaluations des revalorisations de la rente par les méthodes de Smith-Wilson et de Svensson

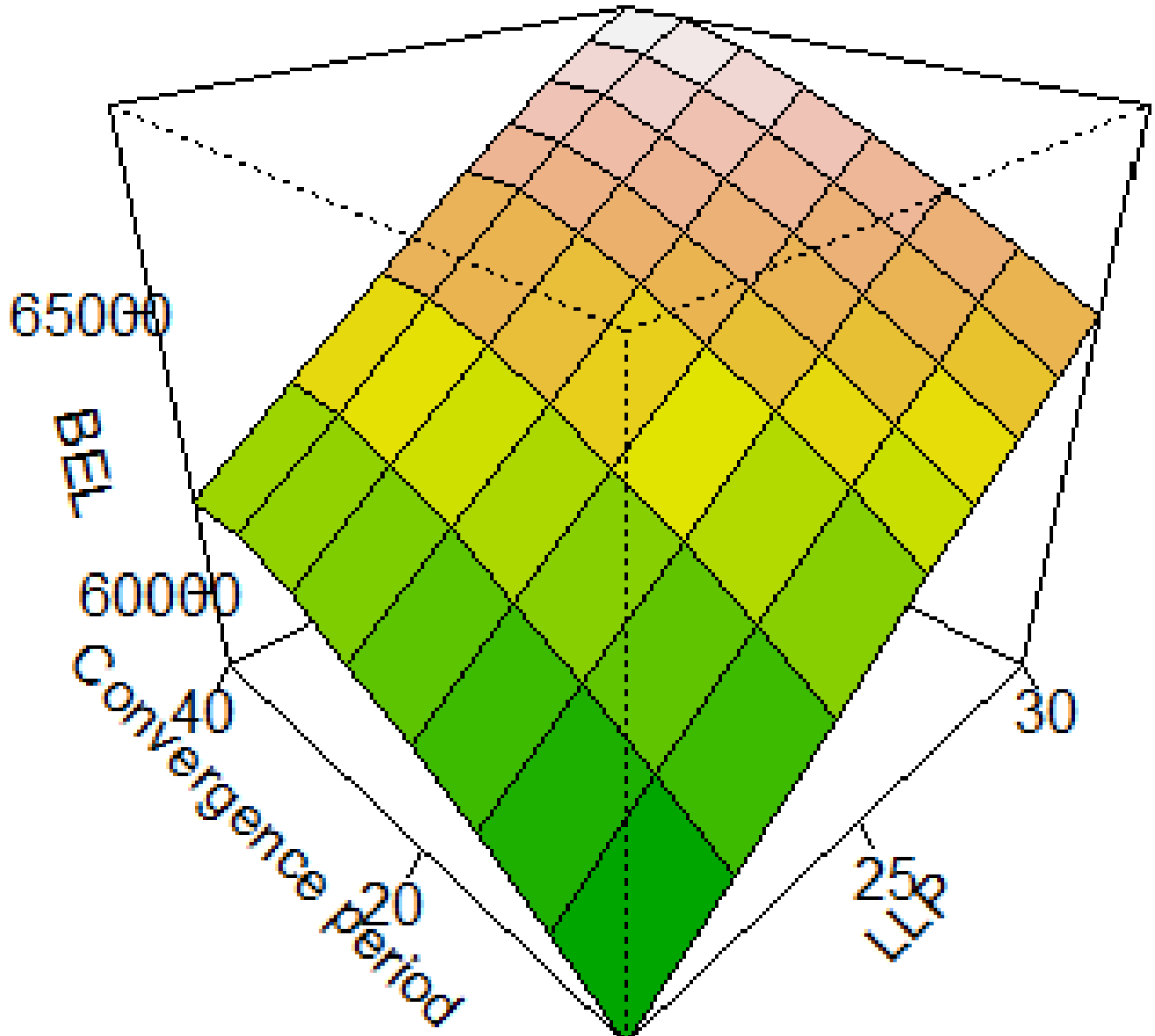
Results from the article (cont'd)

Extrapolation: UFR/convergence speed dependence (fixed LLP = 20)



Results from the article (cont'd)

Extrapolation: Cv. speed/LLP dependence (UFR = 4.2%)



Results from the article

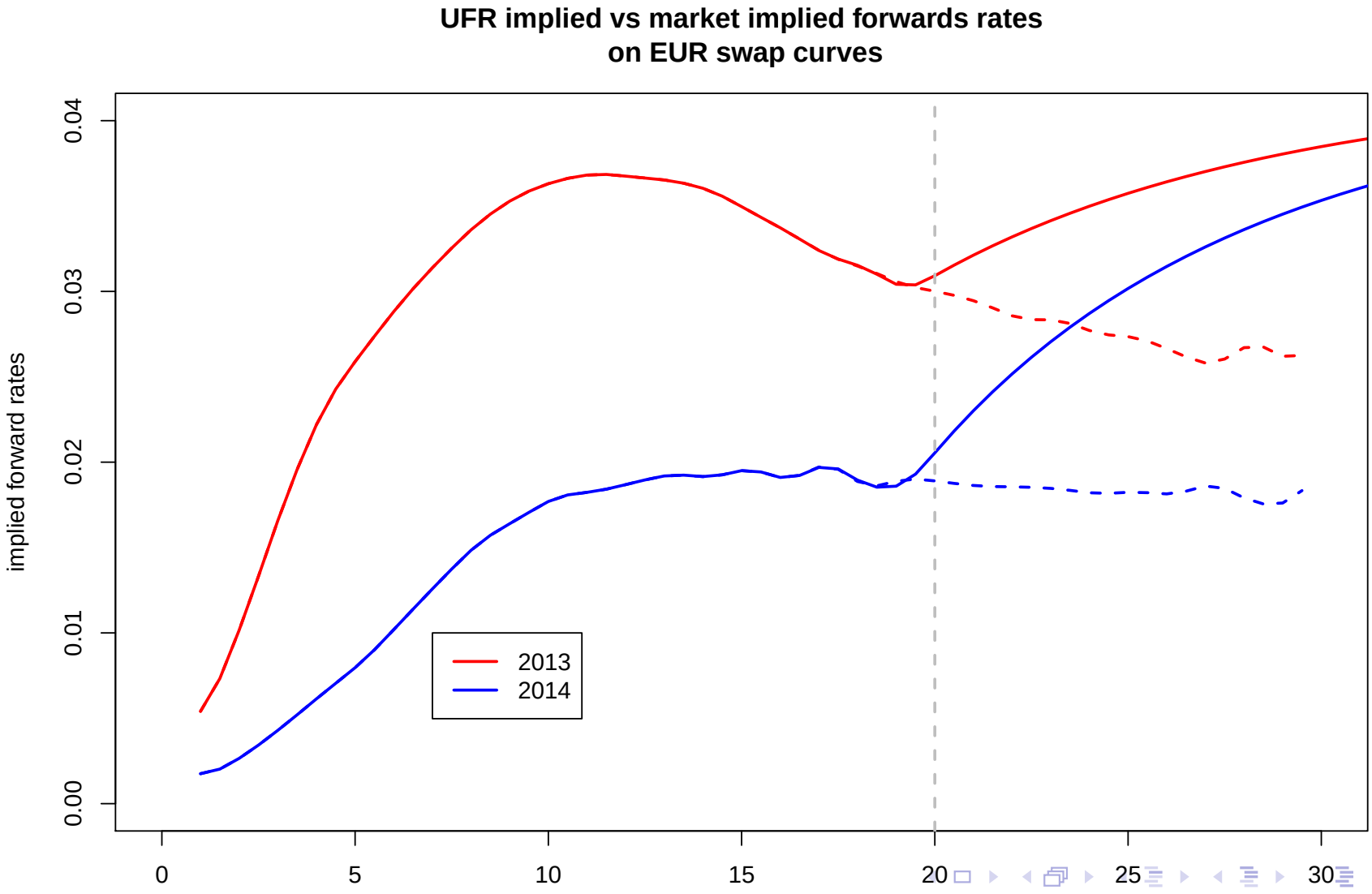
A 'Shiny App' showing the impact of LLP, UFR and α on a annuity

- Just run the following commands in R (internet connexion required, tested on Safari and Chrome)

```
# Loading Shiny package
library(shiny)
# Run the app in your browser; execs code from my Gist
runGist("ee4e7b9506a09e5d7cb8")
```

"Problem" with an **exogeneous** UFR

- Hedging current UFR (increasing basis with low rates)?
- Reasonable macroeconomic assumptions (2%+2.2%) ?



Future work

- About the inclusion of insurer's assets returns in a framework
- On interpolation/extrapolation for valuation
- On forecasting for the ORSA (Own Risk Solvency Assessment, Solvency II pillar 2)
- On low interest rates impacts