Presentation of the 5-bus test system

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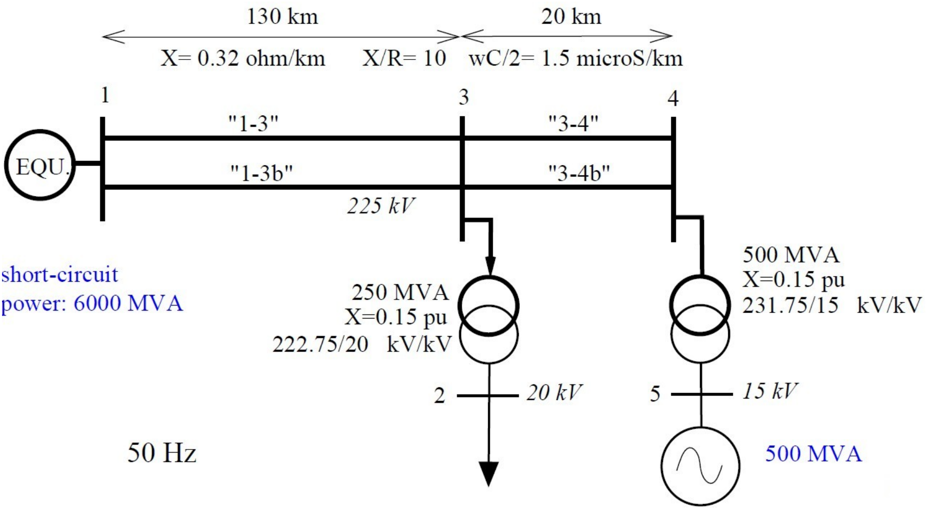
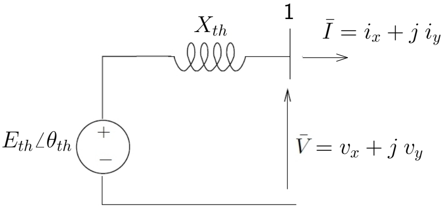
December 2024

System overview



Thévenin equivalent



$$X_{th} = \frac{1}{\frac{S_{sc}}{S_{base}}} = \frac{100}{6000} = 0.0167 \text{ pu}$$

S_{sc}: short-circuit power[1] S_{base}: base power of network = 100 MVA

$$\begin{aligned} E_{th}\angle\theta_{th} &= \bar{V} + j\, X_{th}\bar{I} \\ \iff \quad E_{th}\cos\theta_{th} + j\, E_{th}\sin\theta_{th} &= (v_x + j\, v_y) + j\, X_{th}(i_x + j\, i_y) \end{aligned}$$

Algebraic-only model:

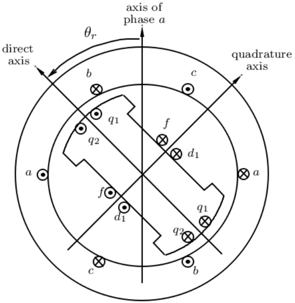
$$\begin{aligned} E_{th}\cos\theta_{th} - v_x + X_{th} i_y &= 0 \\ E_{th}\sin\theta_{th} - v_y - X_{th} i_x &= 0 \end{aligned}$$

[1] more precisely: contribution of Thévenin equivalent to short-circuit power at bus 1

Synchronous machine

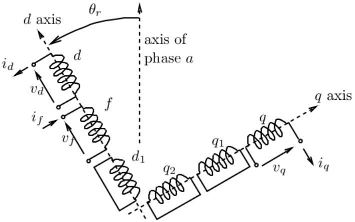
Round-rotor machine

typical of thermal power plants



f: field (or excitation) winding
d_1: direct-axis damper winding
q_1, q_2: quadrature-axis damper windings

After applying Park transformation:



3-phase stator replaced by d, q windings.

Inductance matrices:

$$\mathbf{L}_d = \begin{bmatrix} L_\ell + M_d & M_d & M_d \\ M_d & L_{\ell f} + M_d & M_d \\ M_d & M_d & L_{\ell d1} + M_d \end{bmatrix}$$

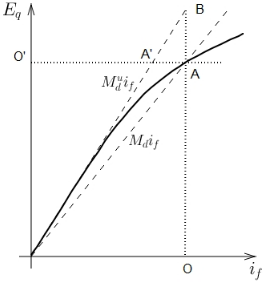
$$\mathbf{L}_q = \begin{bmatrix} L_\ell + M_q & M_q & M_q \\ M_q & L_{\ell q_1} + M_q & M_q \\ M_q & M_q & L_{\ell q2} + M_q \end{bmatrix}$$

- independent of rotor position
- d and q axes decoupled
- in per unit, using the proper base

Magnetic saturation of material

Open-circuit characteristic:

- machine operating at no load, rotating at nominal angular speed
- terminal voltage E_q measured for various values of the field current i_f



Saturation factor : $k = \dfrac{OA}{OB} = \dfrac{O'A'}{O'A} < 1$

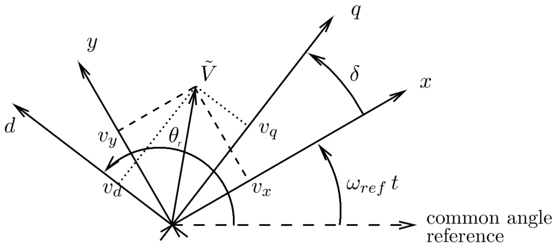
A standard model : $k = \dfrac{1}{1 + m(E_q)^n} \qquad m, n > 0$

Electrical part of model in (d,q) reference frame with phasor approximation

$$
\begin{aligned}
\psi_d &= L_\ell i_d + \psi_{ad} & \psi_q &= L_\ell i_q + \psi_{aq} \\
\psi_f &= L_{\ell f} i_f + \psi_{ad} & \psi_{q1} &= L_{\ell q1} i_{q1} + \psi_{aq} \\
\psi_{d1} &= L_{\ell d1} i_{d1} + \psi_{ad} & \psi_{q2} &= L_{\ell q2} i_{q2} + \psi_{aq} \\
\psi_{ad} &= M_d (i_d + i_f + i_{d1}) & \psi_{aq} &= M_q (i_q + i_{q1} + i_{q2}) \\
M_d &= \frac{M_d^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n} & M_q &= \frac{M_q^u}{1 + m\left(\sqrt{\psi_{ad}^2 + \psi_{aq}^2}\right)^n} \\
\frac{1}{\omega_N}\frac{d}{dt}\psi_f &= v_f - R_f i_f & \frac{1}{\omega_N}\frac{d}{dt}\psi_{q1} &= -R_{q1} i_{q1} \\
\frac{1}{\omega_N}\frac{d}{dt}\psi_{d1} &= -R_{d1} i_{d1} & \frac{1}{\omega_N}\frac{d}{dt}\psi_{q2} &= -R_{q2} i_{q2} \\
v_d &= -R_a i_d - \omega \psi_q - \cancel{\frac{d\psi_d}{dt}} & v_q &= -R_a i_q + \omega \psi_d - \cancel{\frac{d\psi_q}{dt}}
\end{aligned}
$$

- M_d^u (resp. M_q^u) non-saturated value of M_d (resp. M_q)
- ψ_{ad} (resp. ψ_{aq}): air-gap flux in d (resp. q) axis
- all variables in per unit, except time t
- $\omega_N = 2\pi f_N$ $\quad$ f_N: nominal frequency

Connecting the model to the (x, y) reference frame



δ: "rotor angle"

Motion equation

$$\frac{1}{\omega_N}\frac{d}{dt}\delta = \omega - \frac{\omega_{ref}}{\omega_N} \qquad \omega\text{: rotor speed in per unit}$$

$$2H\,\frac{d}{dt}\omega = T_m - T_e = T_m - (\psi_d i_q - \psi_q i_d)$$

T_m: mechanic torque given by turbine
T_e: electromagnetic torque opposed by machine

$$H = \frac{\frac{1}{2} I\,\left(\frac{\omega_N}{p}\right)^2}{S_N} \qquad \text{inertia constant (in s)}$$

I: moment of inertia of *all* rotating masses $\qquad$ p: number of pair of poles
S_N: nominal apparent power of machine (in MVA)

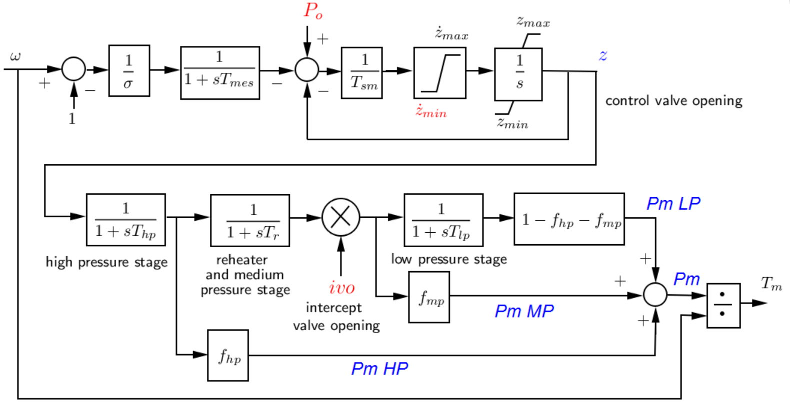
Data

$S_N = 500$ MVA $\qquad U_N = 15$ kV

stator (or armature):	resistance $R_a = 0.$	leakage react. $X_\ell = 0.15$ pu
synchronous reactance*:	d-axis $X_d = 2.20$ pu	q-axis $X_q = 2.00$ pu
transient reactance*:	d-axis $X'_d = 0.30$ pu	q-axis $X'_q = 0.40$ pu
subtransient reactance*:	d-axis $X''_d = 0.20$ pu	q-axis $X''_q = 0.20$ pu
open-circuit transient time constant*:	d-axis $T'_{do} = 7.00$ s	q-axis $T'_{qo} = 1.50$ s
open-circuit subtransient time constant*:	d-axis $T''_{do} = 0.05$ s	q-axis $T''_{qo} = 0.05$ s
saturation	$m = 0.10$	$n = 6.0257$
inertia:	$H = 4$ s	

*used at initialization to obtain $L_{\ell f}, L_{\ell d1}, M^u_d, R_f, R_{d1}, L_{\ell q1}, L_{\ell q2}, M^u_q, R_{q1}, R_{q2}$

Speed governor and steam turbine



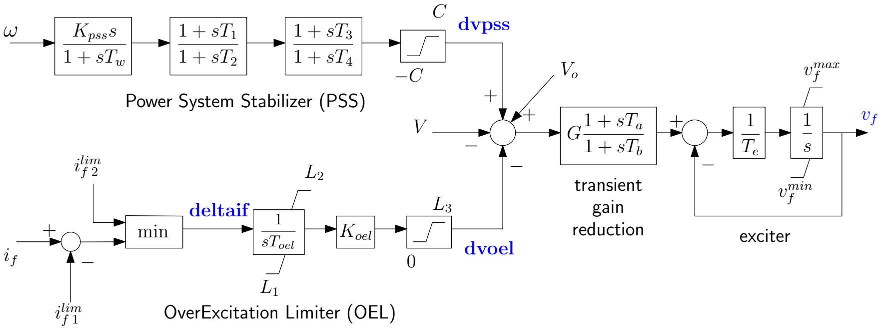
Turbine nominal power $P_{nom} = 460$ MW. All variables in pu on the P_{nom} base.

$$\sigma = 0.04 \qquad T_{mes} = 0.1\ s \qquad T_{sm} = 0.4\ s$$

$$\dot{z}_{min} = -0.05\ pu/s \qquad \dot{z}_{max} = 0.05\ pu/s \qquad z_{min} = 0. \qquad z_{max} = 1.\ pu$$

$$T_{hp} = 0.3\ s \qquad f_{hp} = 0.4 \qquad T_r = 5.0\ s \qquad f_{mp} = 0.3 \qquad T_{lp} = 0.3\ s \qquad ivo = 1$$

Automatic voltage regulator, excitation system and overexcitation limiter



All variables on the base (voltage and current) of the excitation system.

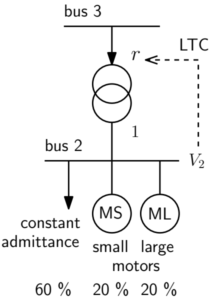
$$G = 70. \quad T_a = T_b = 1\ s \quad T_e = 0.4\ s \quad v_f^{min} = 0. \quad v_f^{max} = 5\ pu$$

$$K_{pss} = 50 \quad T_w = 5\ s \quad T_1 = T_3 = 0.323\ s \quad T_2 = T_4 = 0.0138\ s \quad C = 0.06\ pu$$

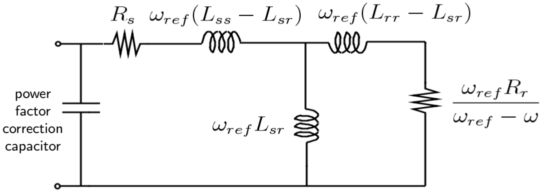
$$i_{f\,1}^{lim} = 2.90\ pu \quad i_{f\,2}^{lim} = 1.00\ pu \quad T_{oel} = 12\ s \quad K_{oel} = 2.0$$

$$L_1 = -1.1 \quad L_2 = 0.1 \quad L_3 = 0.2\ pu$$

Load



Induction motor model :



Dynamics of rotor speed : $2H\dfrac{d\omega}{dt} = T_e - T_{mo}(A\omega^2 + B)$

Constant admittance load: $P = GV^2 = P_o(V/V_o)^2 \qquad Q = -BV^2 = Q_o(V/V_o)^2$

MS: equivalent motor representing a population of "small motors"*:

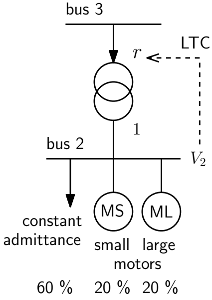
$$R_s = 0.031 \quad L_{ss} = 3.30 \quad L_{sr} = 3.20 \quad L_{rr} = 3.38 \quad R_r = 0.018 \; pu$$
$$H = 0.7 \; s \quad A = 0.5 \quad B = 0.5$$

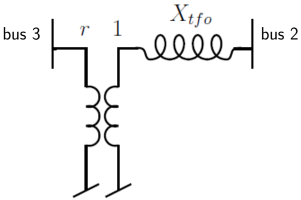
ML: equivalent motor representing a population of "large motors"*:

$$R_s = 0.013 \quad L_{ss} = 3.867 \quad L_{sr} = 3.80 \quad L_{rr} = 3.97 \quad R_r = 0.009 \; pu$$
$$H = 1.5 \; s \quad A = 0.5 \quad B = 0.5$$

*values in pu on the motor MVA base

Load Tap Changer (LTC)





Range of variation of ratio r:

- minimum: 0.81
- maximum: 1.10
- number of tap positions: 30

Voltage control logic:

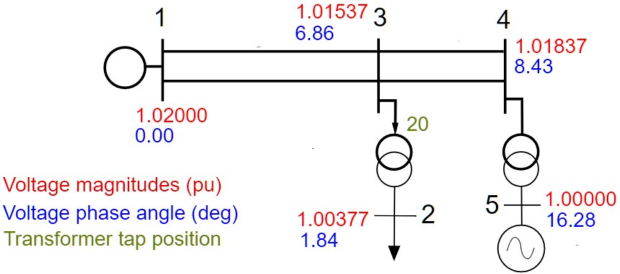
if $V_2 < V_2^o - \epsilon$ decrease r by one position
if $V_2 > V_2^o + \epsilon$ increase r by one position
else leave r unchanged

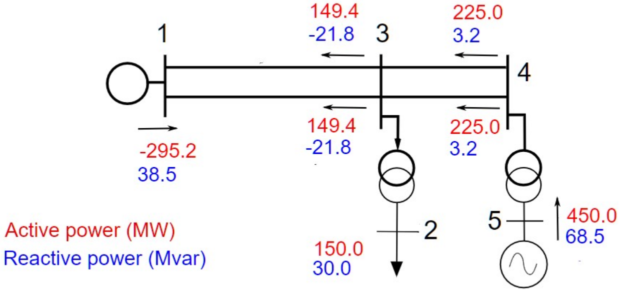
$$V_2^o = 1.00 \text{ pu} \qquad \epsilon = 0.01 \text{ pu}$$

Delays:

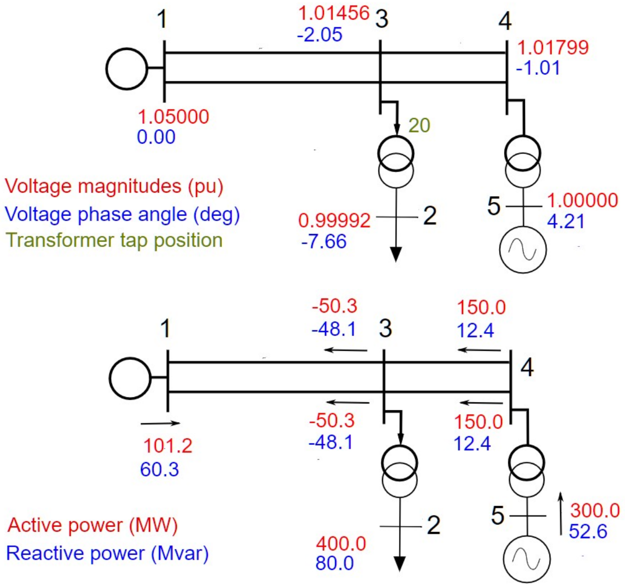
- before first tap change: 25 s
- between successive tap changes: 10 s

Operating point # 1





Operating point # 2



Operating point # 3