

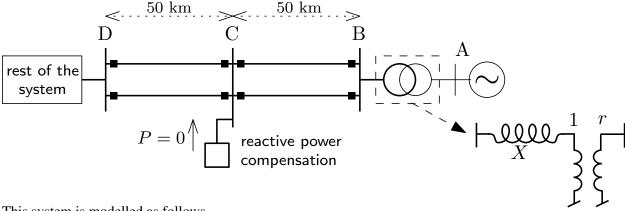
# **Course ELEC0014 - Introduction to electric power and energy systems**

# Additional exercises with answers

### December 2017

#### Exercise A1

Consider the system represented in the figure below. The four transmission lines have a nominal voltage of 220 kV. The network is equipped with a compensator which adjusts its reactive power production in order to keep the voltage at bus C equal to 226.6 kV. The rest of the system will be represented by a Thévenin equivalent.



This system is modelled as follows.

- lines: series resistance and shunt capacitances neglected, series reactance:  $0.3 \Omega/km$ ;
- short-circuit capacity of the rest of the system: 5 000 MVA;
- generator step-up transformer: nominal apparent power: 500 MVA, modeled as shown in the figure with X = 0.15 pu and r = 0.97 pu/pu, on the base (500 MVA, 220 kV, 15 kV);
- generator: round-rotor, stator resistance and saturation neglected, nominal apparent power: 500 MVA, nominal voltage: 15 kV, maximum power of the turbine: 450 MW, synchronous reactance: 2.2 pu (on the generator base). In the capability diagram, under nominal voltage, the curves relative to the turbine maximum power, the stator current and the rotor current cross each other at the same point.

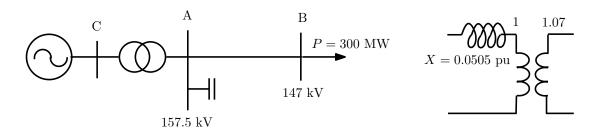
Consider the following operating point: generator at bus A producing its maximum active power, voltage at bus A: 15.3 kV, voltage at bus D: 220 kV.

- 1. Compute the magnitude of the Thévenin equivalent voltage. Answer: 0.96852 pu.
- 2. Compute the generator reactive power production: *Answer:* 92.7 *Mvar.*
- 3. Check that the generator operates below its stator and its rotor current limits. Answer: stator limit= 240 Mvar, rotor limit= 226.6 Mvar

- 4. Determine the reactive power (in Mvar) that the compensator must be able to produce, so that it can keep the voltage at bus C to 226.6 kV after the tripping of any one of the four transmission lines. It is assumed that the magnitude of the Thévenin equivalent voltage remains constant, while the generator keeps constant the voltage at bus A.
  - Answer: 236.2 Mvar, which the compensator must produce after the tripping of one circuit between C and B.
- 5. For each line tripping, compute the currents (in A) in the transmission lines which remain in operation. Answer: after the tripping of one circuit C-B: 1147.9 A in the other circuit C-B, 635.2 A in each circuit C-D; after the tripping of one circuit C-D: 1231.7 A in the remaining circuit C-D, 573.3 A in each circuit C-B.
- 6. Consider the step-up transformer. Its nominal voltages are 220 and 15 kV, respectively. In the factory short-circuit test, the transformer was short-circuited on the 220-kV side, while a reduced voltage  $U_{sc}$  was applied on the 15-kV side, so that the nominal current flows in the short-circuit. What was the value of  $U_{sc}$ , in kV? Answer: 2.183 kV

Perform all computations in per unit with a three-phase base power of 100 MVA and the base voltages of 220 and 15 kV, respectively.

#### Exercise B1



The data of the system shown in the left figure are as follows:

- line: length: 60 km, nominal voltage: 150 kV, series resistance neglected, series reactance: 0.35  $\Omega$ /km, half shunt susceptance:  $\omega C/2 = 2.0~\mu$ S/km
- shunt capacitor bank at bus A: produces 65 Myar under the nominal voltage of 150 kV
- generator: nominal voltage: 15 kV, nominal apparent power: 330 MVA, nominal power of turbine: 313.5 MW, synchronous reactance X=2.2 pu, stator resistance neglected. Under the nominal voltage, the capability curves relative to respectively the stator limit, the rotor limit and the turbine nominal power cross each other at the same point
- transformer: 330 MVA, primary nominal voltage: 15 kV, secondary nominal voltage: 160.5 kV. The factory tests give the following information:
  - open-circuit test: voltage applied to primary side: 15 kV, voltage measured at the opened secondary side: 160.5 kV
  - short-circuit test: secondary side short-circuited and carrying the nominal current, voltage applied to primary: 2.50 kV.

The operating point is given by the two voltages and the load active power shown on the figure.

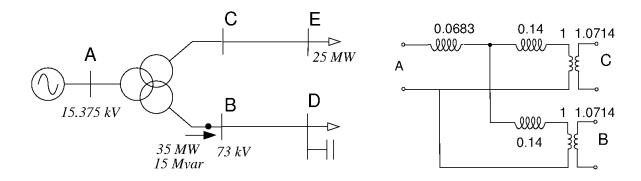
# Questions:

- 1. Show that the transformer can be represented by the equivalent circuit in the right figure. The copper losses and the magnetizing susceptance are neglected. The leakage reactance and the transformer ratio are in per unit on the base specified below.
- 2. The capacitor bank at bus A is made up of three capacitors connected in triangle. What is the capacitance of each of them ? Answer:  $3.537 \mu F$
- 3. Compute the power factor of the load at bus B. Answer: 0.9935
- 4. Compute the phase-to-neutral voltage (in kV) at generator bus C. Answer: 8.756 kV
- 5. Compute the reactive power (in Mvar) produced by the generator. Answer: 82.5 Mvar
- 6. Compute the reactive power reserve (in Mvar) available in the generator without exceeding the stator or the rotor limit. *Answer: rotor limit:* 122.2 *Mvar, stator limit:* 146.0 *Mvar, reserve:* 39.7 *Mvar.*

Questions 3 to 6: perform the computations in per unit on the bases (15 kV, 100 MVA) and (150 kV, 100 MVA), then convert the answers into the requested units.

These bases were used to compute the equivalent circuit of the transformer shown in the right figure.

# **Exercise C1**



The system shown in the left figure has the following characteristics:

## • generator:

- nominal apparent power: 80 MVA

- nominal voltage: 15.0 kV

- stator resistance neglected

- synchronous reactance X = 1.9 pu

- equipped with an automatic voltage regulator which keeps the voltage at bus A at 15.375 kV
- when the generator rotates at nominal speed, with its stator opened and its terminal voltage at 15.0 kV, its excitation voltage  $V_f$  is 350 V.
- three-winding transformer:
  - windings B and C are identical
  - nominal apparent powers of windings: A: 80 MVA, B: 40 MVA, C: 40 MVA

- the copper losses and the magnetizing susceptance are neglected
- the measurements from open- and short-circuit tests are given in the following table.

test No	winding A	winding B	winding C
1	applied voltage:	open	open
	15000 V	measured voltage: 75000 V	measured voltage: 75000 V
2	applied voltage:	short-circuited	open
	1250 V	nominal current flowing	
3	open	applied voltage:	short-circuited
		8400 V	nominal current flowing

• lines B-D and C-E: 70 kV, 20 km,  $X=0.4~\Omega/\mathrm{km}$ . The series resistances and the shunt capacitances are neglected.

The operating point is specified by the voltages and the powers shown in the left figure.

- 1. Show that the transformer equivalent circuit is the one shown in the right figure. The reactances and the transformer ratios are in per unit on the base specified below.
- 2. Compute the reactive power produced by the generator. Suggestion: first, compute the voltage at the internal node of the transformer equivalent. Answer: 44.4 Mvar.
- 3. Compute the generator excitation voltage  $V_f$  (in V). Answer: 868.2 V
- 4. Compute the reactive power (in Mvar) consumed by the load in D. Answer: 21.7 Mvar
- 5. To improve the power factor of the load, a capacitor bank has been installed at bus D. Without this compensation the power factor would be 0.85. The capacitors are connected in triangle. What is the capacitance (in  $\mu$ F) of each of them ? Answer: 1.843  $\mu$ F

Perform all computations in per unit on the bases (15 kV, 100 MVA) and (70 kV, 100 MVA), then convert the answers into the requested units.

Those bases were used to compute the equivalent circuit of the transformer shown in the right figure.

## Exercise D1

Consider the following 50-Hz system, with the shown (phase-to-phase) voltages and (three-phase) powers.

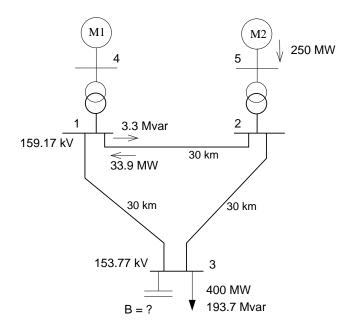
Buses 1, 2 and 3 have a nominal voltage of 150 kV, and buses 4 and 5 a nominal voltage of 15 kV.

Each line has the following parameters:  $x=0.4~\Omega/\mathrm{km},~\omega c/2=1.5~\mu\mathrm{S/km}.$  The series resistances are neglected.

Each transformer has a leakage reactance of 0.18 pu, a nominal apparent power of 300 MVA and a transformer ratio of 165/15 kV/kV. In the equivalent circuit, the copper losses and the magnetizing susceptances are neglected, while the leakage reactance is placed on the 15 kV side.

The capacitor bank at bus 3 is made up of three capacitors connected in star.

The synchronous machine M2 is of the round-rotor type, has a nominal apparent power of 300 MVA and a synchronous reactance X=2.3 pu. When the generator rotates at nominal speed, with its stator opened and its terminal voltage at 15.0 kV, its excitation current  $I_f$  is 800 A. The maximum permanent value of  $I_f$  is 2304 A.

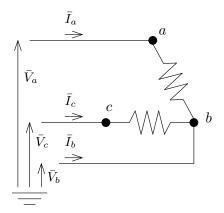


- 1. Compute the capacitance of each of the three capacitors at bus 3. Suggestion: first, compute the phase angle difference between buses 1 and 3. Note that there is no active power lost in the lines. Answer:  $10.368 \ \mu F$
- 2. Compute the phase-to-neutral voltage at bus 5. Answer: 8.834 kV
- 3. Compute the reactive power produced by M2. Answer: 116.6 Mvar.
- 4. Assume that the automatic voltage regulator of M2 keeps the voltage at bus 5 constant. Determine the maximum reactive power that the machine can produce without exceeding the stator nor the rotor current limit. *Answer: stator limit:* 176.5 *Mvar, rotor limit:* 154.7 *Mvar.*

Perform all computations in per unit with a three-phase base power of 100 MVA and the base voltages of 150 and 15 kV, respectively.

# Exercise A2

An imbalanced load consists of two resistors connected in two branches of a triangle, as shown in the figure below. Each resistance consumes 750 kW under a terminal voltage of 6 kV.



Determine the inductor(s) and capacitor(s) to connect between the terminals a,b and c so that, if the voltages  $\bar{V}_a, \bar{V}_b$  and  $\bar{V}_c$  make up a balanced positive sequence, the line currents  $\bar{I}_a, \bar{I}_b$  and  $\bar{I}_c$  also make up a balanced

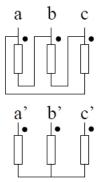
positive sequence.

Suggestion: start from the result of the exercise that involved a single resistor, and use superposition. Answer: connect an inductor of 0.2646 H between a and b, and a capacitor of  $38.29 \mu F$  between b and c

What is the per-phase equivalent of the load and the compensation elements combined ? Answer: a resistance of 24  $\Omega$ 

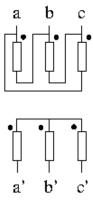
## **Exercise A3**

Find the IEC designation of the three-phase transformer assembled as shown in the figure below. The black dots indicate primary and secondary voltages which are in phase. The terminals a, b and c are on the high voltage side. Answer: Dy11



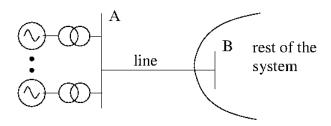
### Exercise B2

Find the IEC designation of the three-phase transformer assembled as shown in the figure below. The black dots indicate primary and secondary voltages which are in phase. The terminals a', b' and c' are on the high voltage side. *Answer: Yd7* 



# **Exercise B3**

In the system shown hereafter, the various machines are identical. Each machine is of the round-rotor type, has a nominal apparent power of 200 MVA, produces 180 MW and has its terminal voltage kept at 1 pu by its automatic voltage regulator. Under this voltage and with this active power production, its reactive power is limited by its stator current.



The line has a nominal voltage of 225 kV and is simply represented by its series reactance  $X_e$ .

The voltage at bus B is kept constant and equal to 225 kV.

All step-up transformers are identical. Each of them has a nominal apparent power of 200 MVA, a leakage reactance of 0.15 pu (on the 200-MVA base) and a unity transformer ratio.

1. Which is the largest number of generators that can be connected if  $X_e=85~\Omega$ ? Answer: there is no operating point with more than two machines.

2. Same question if  $X_e = 75 \Omega$ . Answer: two, otherwise each machine produces too much reactive power.

Suggestion: try with 1, 2, 3, ... machines.

#### Exercise C2

A power plant is connected to a 60-Hz network through a 300-km long transmission line with a nominal voltage of 735 kV. Its parameters are  $x=0.325~\Omega/\mathrm{km}$ ,  $\omega c=5.1~\mu\mathrm{S}/\mathrm{km}$ . The line is assumed lossless. The power plant injects 900 MW and 100 Mvar under 735 kV.

Compute the voltage (in kV) at the receiving end of the line, taking into account its distributed nature.

Answer: 678 kV

#### Exercise F1

In a 60-Hz network, a 735-kV transmission line has the following characteristics: 250 km,  $x=0.332~\Omega$ /km,  $\omega c=5.5~\mu$ S/ km. The line is assumed lossless.

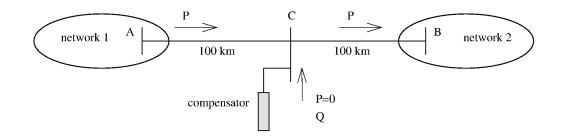
At one extremity, the line is feeding a unity power factor load with a resistance equal to  $0.75 Z_c$  where  $Z_c$  is the surge impedance. Which impedance is seen from the other extremity of the line?

*Answer*:  $193.6 + j \ 35.3 \ \Omega$ 

## Exercise E1

The system shown hereafter includes a 200-km long 380-kV interconnection line with a series reactance  $x=0.3~\Omega/{\rm km}$  and a shunt susceptance  $b=3.0~\mu{\rm S/km}$ . The series resistance is neglected. The nominal apparent power of the line is 1350 MVA.

The voltages at buses A and B are held at 1.05 pu. A reactive power compensator has been installed at the mid-point of the line, with the objective of keeping constant the voltage at bus C. This compensator does neither produce nor consume active power.



Determine in which range of Q values the compensator must operate in order to keep the voltage of bus C at 1.05 pu for all active power flows from P=-1100 to P=+1100 MW ? Answer:  $[-47.8\ 182.8]$  Mvar.

Check that the line does not exceed its nominal current when P=1100 MW. Answer: the current is 1597.2 A while the limit is 2051.1 A.

Treat each section as a "short" line. Perform all computations in per unit on the base (380 kV, 100 MVA) before converting to Mvar.