Homework 7

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Distribution of S_{100}

Expectation

We know that S_{100} is the sum of 100 i.i.d. $\{-1, 1\}$-valued Bernolli random variables, i.e. $S_{100} = \sum_{i=1}^{100} \xi_i$

Thus, $\mathbb{E}(\xi_1 + \xi_2 + ... + \xi_1 00) = \mathbb{E}(\xi_1) + \mathbb{E}(\xi_2) + ... + \mathbb{E}(\xi_1 00) = 100 \cdot \mathbb{E}(\xi_1)$

By conditioning, $100 \cdot \mathbb{E}(\xi_1) = 100 \cdot ((1/2 \cdot 1) + (1/2 \cdot -1)) = 100 \cdot 0 = 0$

Variance

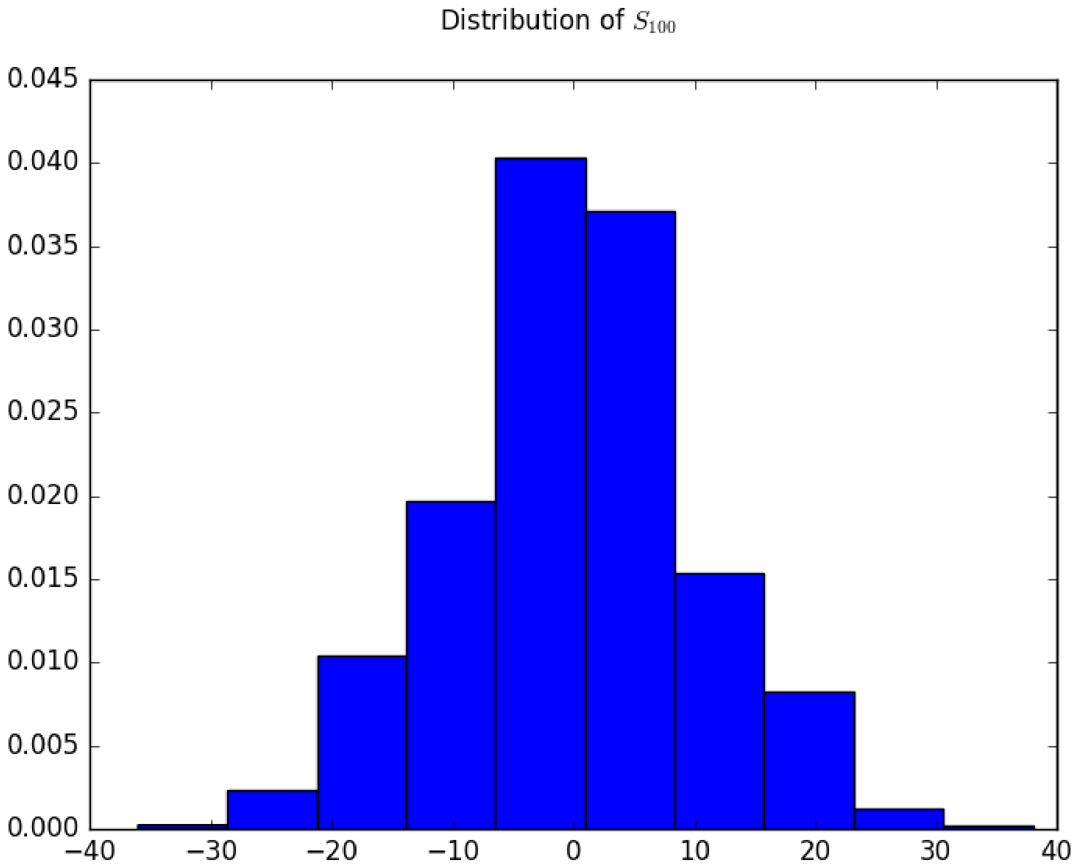
Since the variance of one Bernolli random variable is $p(1-p) = 0.5^2$, the variance of 100 of them should be $100 * 0.5^2 = 25$.

Distribution

Since we're taking 10000 samples, by the Central Limit Theorem S_{100} should be Normally distributed according to the mean and variance described above.

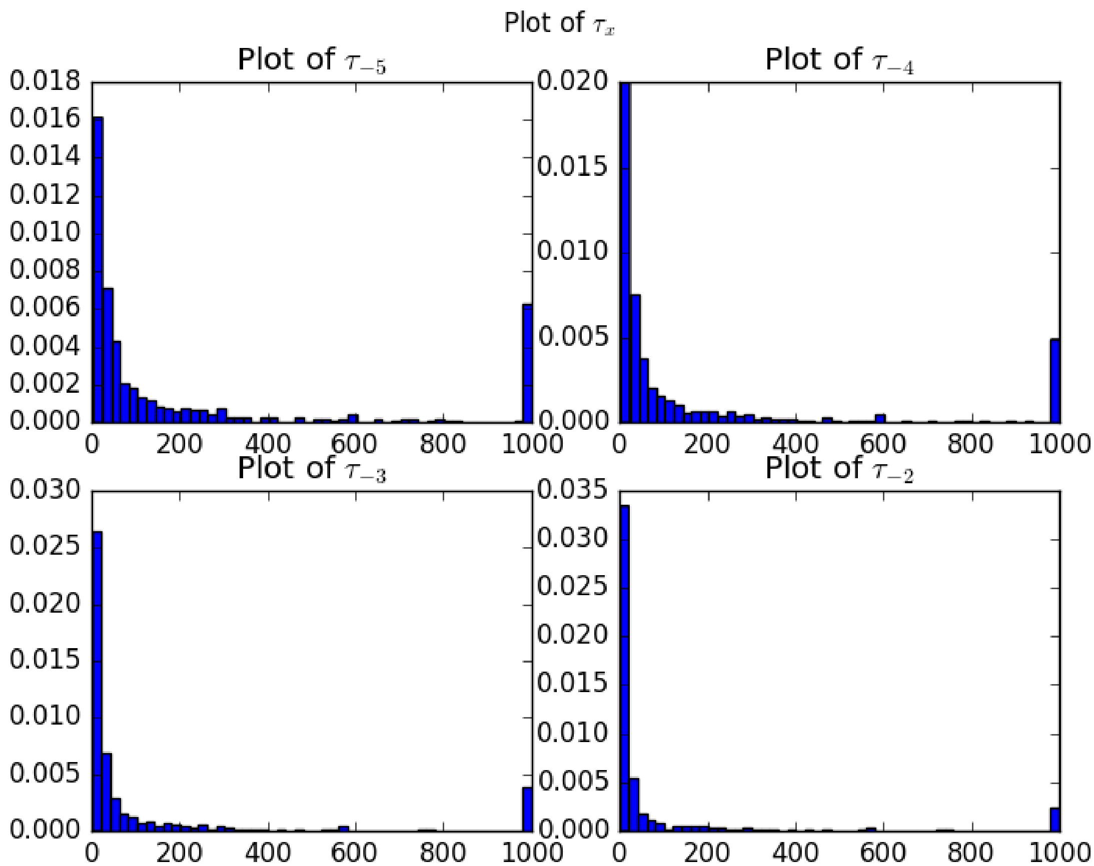
Graph

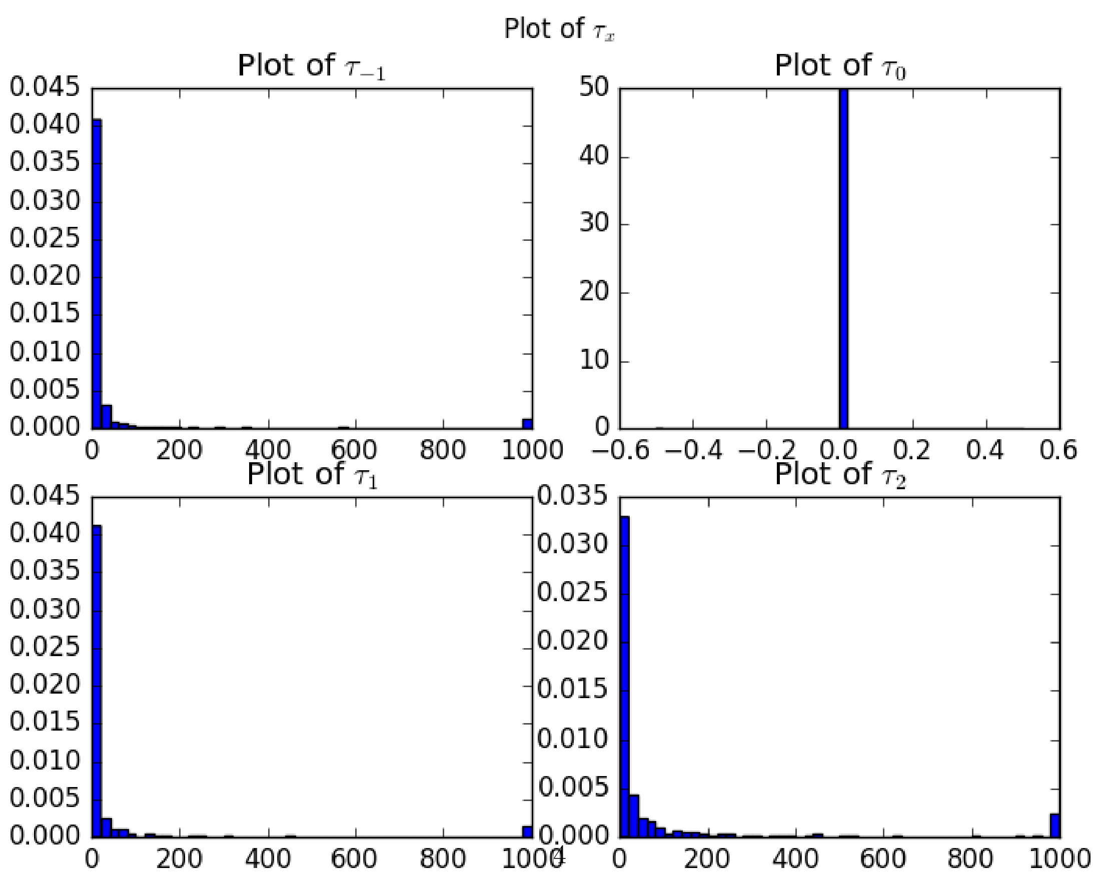
Our calculations can be confirmed in the graph below.

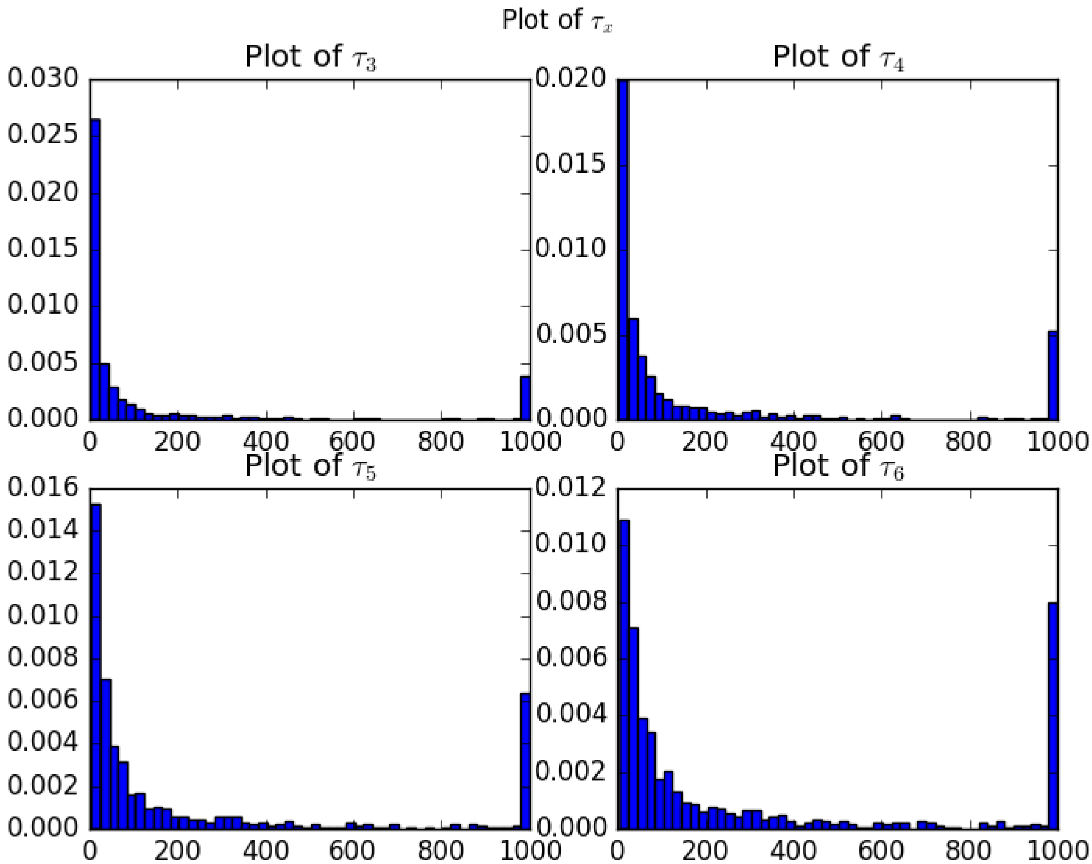


Distribution of the Minimum Hitting Time

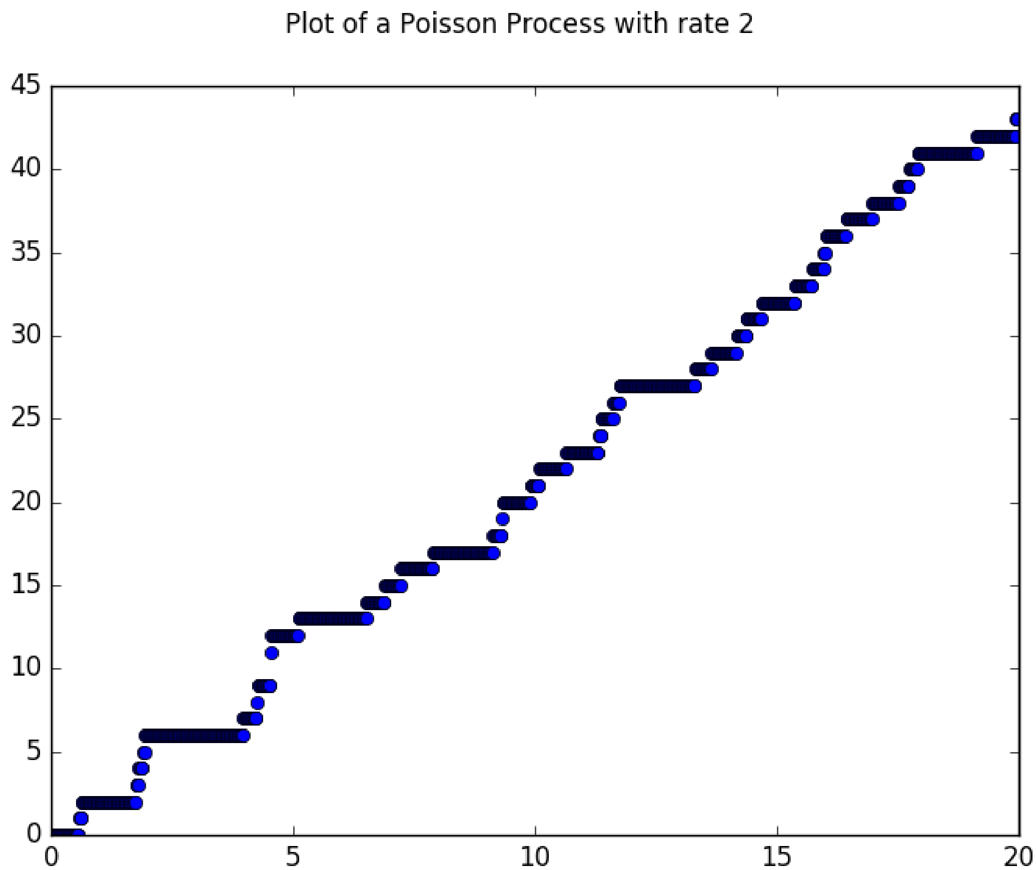
Since the expectation of the minimum hitting time for some $x \in (Z)$ is infinite, I've decided to stop the random walk at 1000 steps.


Plot of τ_x
Plot of τ_{-5}
Plot of τ_{-4}
Plot of τ_{-3}
Plot of τ_{-2}
0.000 0.002 0.004 0.006 0.008 0.010 0.012 0.014 0.016 0.018
0.000 0.005 0.010 0.015 0.020
0.000 0.005 0.010 0.015 0.020 0.025 0.030
0.000 0.005 0.010 0.015 0.020 0.025 0.030 0.035
0 200 400 600 800 1000


Plot of τ_x
Plot of τ_{-1}
Plot of τ_0
Plot of τ_1
Plot of τ_2
0.000 0.005 0.010 0.015 0.020 0.025 0.030 0.035 0.040 0.045
0 10 20 30 40 50
−0.6 −0.4 −0.2 0.0 0.2 0.4 0.6
0.000 0.005 0.010 0.015 0.020 0.025 0.030 0.035
0 200 400 600 800 1000


Plot of τ_x
Plot of τ_3
Plot of τ_4
Plot of τ_5
Plot of τ_6
0.030
0.025
0.020
0.015
0.010
0.005
0.000
0.016
0.014
0.012
0.008
0.006
0.004
0.002
0
200
400
600
800
1000

Graph of a Poisson Process with Rate 2



Expected Value of $N(10)$

Theoreticaly, $\mathbb{E}(N(10)) = 10 \cdot \lambda = 10 \cdot 2 = 20$. My Python calculation is very close to this amount.