Zombies!

PSTAT 160A Project – Vincent La

Summary

This model is a spin-off of the SIR (Susceptible, Infected, Removed) model in epidemiology. The SZR (Susceptible, Zombie, Removed) model splits the population into three groups:

- Susceptible: Humans
- Zombie
- Removed: The dead. This may be humans who died or zombies killed attempting to infect humans.

These models can be described as either a system of differential equations or equivalently as continuous-time Markov Chains (CTMC). However, it is also possible to formulate a discrete approximation with discrete-time Markov Chains (DTMC), which are used in this project. This project was based on the work of two papers. The inspiration and model assumptions were provided by "When zombies attack!: Mathematical modelling of an outbreak of zombie infection" by Munz, et al.[1] However, I found the system of differential equations difficult to translate into a Markov Chain model, so I referred to "An Introduction to Stochastic Epidemic Models" by Allen[2] to implement the model.

To specify an epidemic model, we describe several probabilistic parameters:

$\alpha =$ rate of defeating a zombie
$\beta =$ transmission rate
$\xi =$ resurrection rate (removed $\rightarrow$ zombie)
$\delta =$ death rate (for humans)
$\Pi =$ birth rate (assumed to be 0 for a short-term epidemic)
$N = S(t) + Z(t) =$ population size
$z =$ number of infections (zombies)

Motivation

At first I found the concept of Markov Chains very interesting, but also thought that they were of limited use. After seeing a Markov Chain model for car insurance premiums in the textbook (the Bonus Malus system), I was pleasantly surprised and interested in what else could be simulated using Markov Chain techniques. After some searching, I found that there was an entire literature on modeling infectious outbreaks as stochastic processes. This was a very interesting topic that I could not pass up.

Methods

The model I am attempting to simulate is like other stochastic processes we have discussed in class. However, one difference is that this process keeps track of two variables at once: the number of susceptibles S(t) and the number of zombies Z(t). In other words, this epidemic can be described as $\{(S(t),\, Z(t))\}_{t=0}^{\infty}$.

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[1] http://cdm.yorku.ca/sc13/StochEpidModels.pdf
[2] http://loe.org/images/content/091023/Zombie%20Publication.pdf

Simplifying Assumptions and Modifications

- Birth rate = 0 (because this epidemic takes place over a short time frame)
- Δt is chosen so that at most only one event (infection, death, resurrection, etc.) can happen
  - While it is possible to formulate a DTMC model where multiple events happen with every step, this would require a more complicated transition matrix

Markov Chain Formulation

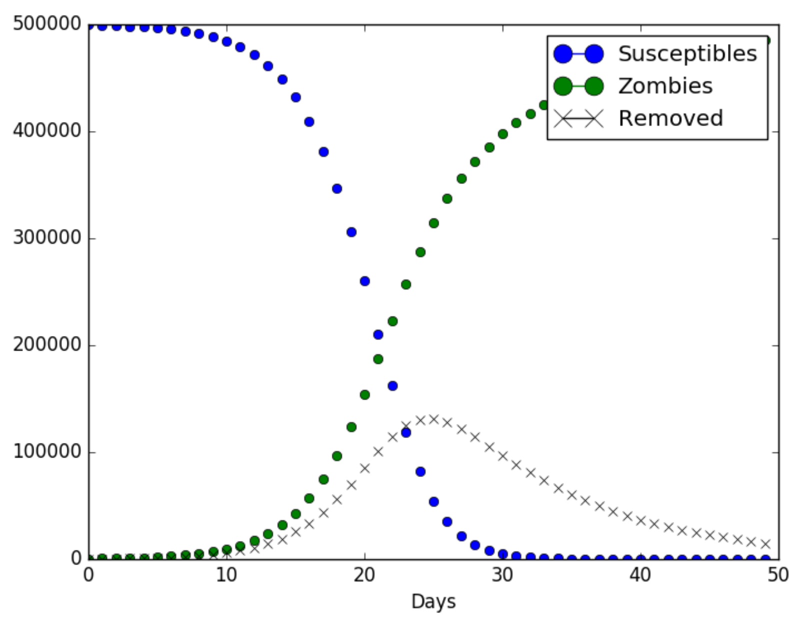
Thus, the state space for this Markov Chain is $\{(0,\ 0), (0,1), (1,0), (1,1), \dots\} \in \mathbb{Z}^2$. However, this seemingly complicated process is a straightforward extension of the ideas used in Markov Chains with integer-valued state spaces. We can describe the transition probabilities as follows:

- **(-1, 1): Infection –** Transmission rate * probability of encountering a susceptible * number of zombies * time change = $\beta z\left(\frac{s}{N}\right)\Delta\mathrm{t}$
- **(0, -1)** Removal – This happens when a zombie loses a fight with a human = $\alpha z\left(\frac{s}{N}\right)\Delta t$
- **(1, 0):** Birth – Doesn't happen (per our assumptions)
- **(-1, 0):** Death of a human – not part of previous SIR model because they wanted to have constant population = $\delta s \Delta t$
- **(0, 1): Resurrection** (Removed --> Zombie) = $\xi r \Delta t$
- **(0, 0): No change** = $1\ - \left[(\beta + \alpha)\left(z\frac{S}{N}\right) + \delta s + \xi z\right]\Delta t$

Results

Scenario 1: Medium-Sized City

$Z = 500, \quad Total\ =\ 500000, \quad \alpha =\ 0.25, \quad \beta = 0.5, \quad \xi = 0.1,$
$\delta = 0.00002, \Delta t\ =\ 0.00001)$

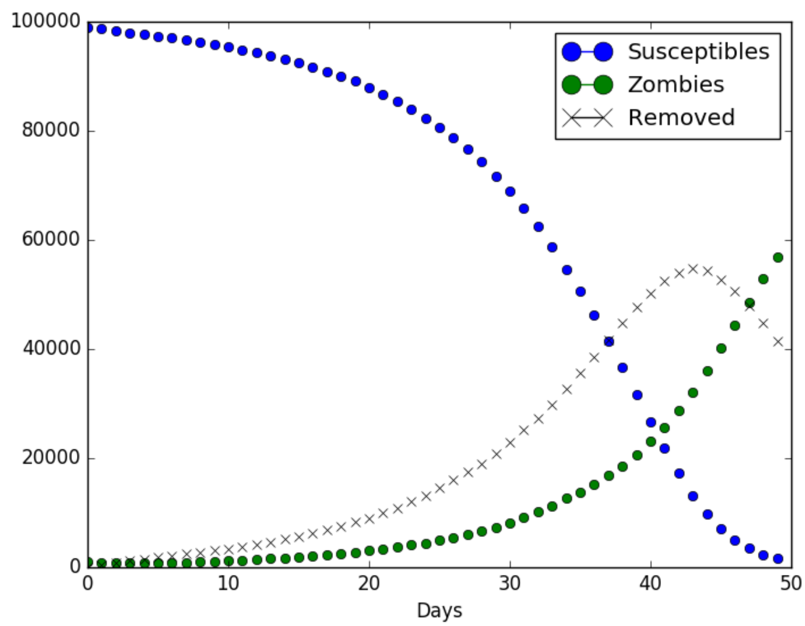


Scenario 2: More Prepared Humans

$Z = 1000, \quad Total = 100000, \quad \alpha = 0.6, \quad \beta = 0.4, \quad \Delta t = 0.00005)$

For this scenario, I envisioned a smaller town but with a population that was much more vigilant and well-armed. I represented this in this model by giving humans (susceptibles) a lower chance of becoming a zombie (β) and a higher chance of destroying a zombie in an encounter (α).

The results came as a surprise to even me. I thought that because $\alpha > \beta$, the humans would actually survive and wipe out the zombie epidemic. However, I was wrong. While the humans were able to last longer than in the first scenario, and killed more zombies in the process, they were eventually wiped out.



Discussion

Future Research Questions

This model seems to assume that everybody has a uniform probability of fighting off a zombie throughout time. But from watching movies and playing video games, we know this is not the case.

1. What if the probability of fighting off a zombie was also a random variable?
   a. Maybe it would be normally distributed, where most people are of "average" fighting ability with weaker and stronger people.
   b. Since humans are an adaptive species, perhaps we could have a model where humans gain a greater ability to fight off zombies and a lower chance of dying in an encounter. That is, as t increases, α increases and β decreases.

In many depictions of zombie outbreaks, eventually humans start to band together and fight zombies from fortified positions. This type of situation doesn't seem to be reflect in the assumptions of a standard SIR model.

2. How could the SIR model be modified to convincingly depict such situations?