Homework 2

Vincent La
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10. (a) See variables X and Y in the .py file.
    (b) See variables U and V in the .py file.
    (c) Distribution of U_1 and V_1.
        i. First we identify the distribution of V_1. First, notice that because X and Y $\sim N(0,1)$, it is the case that X_1^2 and $Y_1^2 \sim \chi_1^2$. Now, we also know that the sum of two chi-square random variables with n and m degrees of freedom respectively is also distributed as a chi-squared random variable with $n+m$ degrees of freedom. Thus, $X_1^2 + Y_1^2 \sim \chi_2^2$. Now, we use the method of moment generating functions to find the distribution of $V := (X_1^2 + Y_1^2)/2$. Let $W = X_1^2 + Y_1^2 \sim \chi_2^2$. Then,

$$M_{V_1}(t) = \mathbb{E}[\exp(tW/2)]$$
$$= M_W(t/2)$$
$$= (1 - 2(\frac{t}{2}))^{-2/2}$$
$$= (1-t)^{-1}$$
$$= \frac{1}{1-t}$$

        Which is also the moment generating function of the exponential distribution with $\beta = 1$. Since MGFs are unique, $V_1 \sim Exp(1)$.
        ii. Now, we identify the distribution of U_1.

$$F_{U_1} = \mathbb{P}(U_1 \leq x)$$
$$= \mathbb{P}(1 - \exp(-V_1) \leq x)$$
$$= \mathbb{P}(\exp(-V_1) \geq 1 - x)$$
$$= \mathbb{P}(-V_1 \geq \ln(1-x))$$
$$= \mathbb{P}(V_1 \leq \ln(1-x))$$

$$= F_{V_1}(\ln(1-x))$$
$$= 1 - e^{\ln(1-x)}$$
$$= 1 - (1-x) = x$$

Thus, U $\sim$ Uniform(0,1).

(d) See .py file.

(e) Yes. In HW2.py I plotted $F_n(\cdot)$ and $F_U(\cdot)$ together on the same graph and they resemble each other a lot.