

CS131: Programming Languages

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CS131: Programming Languages

Introduction

- sample problem:
 - *input*: ASCII text
 - *output*: all words (consecutive alphabetic characters) sorted by frequencies
- Knuth, a famous Turing-winning computer scientist, wanted to write a comprehensive programming book
 - created TeX, a language for typesetting math equations and code fragments
 - allows for creating a scholarly paper discussing a program or source file, eg. in C or Pascal
- however, the code (Pascal) does not always equal the documentation (TeX)
 - despite programmers' best interests to keep them up-to-date and consistent
- to solve this, Knuth created a *unified* file with the code *and* the documentation *interleaved*
 - new programming paradigm called **literate programming**
- have another program that split the unified source file into a .tex and .pas file
 - run one way, the source file is compiled into a readable paper
 - the other way, the source file is compiled into a runnable program
- Knuth, in order to market his new paradigm, created a unified file that solves the above sample problem
 - the associated scholarly paper was published
 - the Pascal solution uses *hashed tries*, is very fast, and has error checking through its compiler
- however, the editor of Knuth's paper brought up an *alternative* approach using **pipelines**:
 - `tr -c 'A-Za-z' '[\n]' | sort | uniq -c | sort -rn`
 - more readable, much shorter, and easier to write than the Pascal solution
- this is an issue for the **choice of notation/programming language** between the languages programmers use to implement solutions:
 - advantages and disadvantages of using full-fledged OOP languages vs. scripting solutions vs. other programming language types
- **Sapir-Whorf hypothesis** on the varieties and constraints of natural language:
 1. no limit on structural diversity

- eg. some languages are not *recursive*, ie. they have a limit on their length
- 2. the languages we use to some extent influences how we use the world
- the above ideas also apply to programming languages:
 - there is a great *diversity* in programming languages:
 - * **imperative** languages like C focus on using assignment and iteration
 - variables have current values, and order of execution is critical
 - * **functional** languages like ML focus on using recursion and single-valued variables
 - * **logic programming** languages express programs in terms of rules about logical inference
 - * **object oriented** languages like Java focus on using objects, or a bundle of data and methods
 - * language types can overlap
 - programming languages will also *evolve* and change over time
- *core* of programming languages:
 - principal and limitations of programming **models**
 - **notations** ie. languages for these models (often textual)
 - methods to evaluate strengths and weaknesses of different notations in different contexts
 - * eg. maintainability, reliability, training cost, program development
 - * also execution overhead, licensing fees, build overhead, porting overhead
 - * choice of notation can also be political, eg. a company preference for a language
- language design *issues*:
 - orthogonality of parts
 - * eg. implementation of functions and selection of types should be independent
 - eg. C is not orthogonal in that its functions cannot return an array type
 - efficiency
 - * eg. CPU, RAM, energy, network access
 - simplicity, ie. ease of *implementation*
 - convenience, ie. easy of *usage*
 - * eg. C provides many ways to increment a variable
 - safety
 - * static, compile-time checking vs. dynamic, runtime checking
 - abstraction
 - * a strength of object oriented languages
 - exceptions

- concurrency
- evolution/mutability of a language

Syntax and Grammar

- *translation* stages of a C source file:
 - the file is translated from a series of literal character bytes into a string of **tokens** through **lexing**
 - * comments and whitespace are ignored
 - * some of the tokens are associated with symbols, eg. functions such as `main` and `getchar`
 - * a **lexeme** is the literal token with its associated programming language metadata
 - a **parse tree** is constructed from the string of tokens through **parsing**
 - * the root of the tree represents the entire program
 - * eg. the children of a function in the tree would include its type, ID, subtrees for arguments and statements, and parentheses, brackets, and semicolons
 - * eg. the children of a function call in the tree would include subtrees for the expression and parameters, and parentheses and semicolons
 - * following the *fringes* or leaves of the tree recreates the string of tokens
 - * different compilers lex and compile in one or two passes
 - the parse tree is *checked* for types, names, etc.
 - *intermediate* code is generated from the parse tree
 - * convenient for the compiler writer
 - intermediate code is turned into assembly code
 - the assembly code is turned into object code by the assembler
 - the object code is turned into an executable by a linker
 - the executable is run by the loader
- grammar is concerned with the lexing and parsing stages of translation
 - the **syntax** of a programming language defines the form and structure of programs
 - * ie. form independent of meaning
 - the **semantics** of a programming language dictates the behavior and meaning of programs
 - syntax without semantics: “Colorless green ideas sleep furiously.”
 - semantics without syntax: “Ireland has leprechauns galore.”
 - ambiguity of syntax and semantics: “Time flies.”

- *judging* syntax:
 - inertia, ie. what people are used to
 - * eg. $3 + 4 * 5$ in C vs. $3\ 4\ 5\ * +$ in Forth vs. $(+ 3 (* 4 5))$ in Lisp
 - simplicity and regularity
 - * Lisp (regularly defined using parentheses) and Forth (no need for parentheses for precedence) win out
 - readability
 - * form should reflect meaning
 - * eg. `if (x > 0 && x < n)` vs. `if (0 < x && x < n)`
 - writability and conciseness
 - redundancy
 - * avoiding silly mistakes
 - * eg. in C, you match declaration to use
 - unambiguity
- programming language **grammar** is a set of rules or definitions that describe how to build a **parse tree**
 - the tree grows downward, where the children of each node follows the forms defined by the grammar
 - * the language is the set of all possible strings formed as the *fringes* of the parse trees
 - * **parsing** a language string finds its parse tree
 - * an abbreviated, simplified parse tree is an **abstract syntax tree (AST)**
 - for a simplified definition of English:
 - * `<noun-phrase> ::= <article> <noun>`
 - * `<sentence> ::= <noun-phrase> <verb> <noun-phrase>`
 - for a simple language using expressions with three variables:
 - * `<exp> ::= <exp> + <exp> | <exp> * <exp> | (<exp>) | a | b | c`
 - * this allows for expressions such as $a + b * c$ and $((a+b) * c)$
 - * a **recursive** grammar allowing for an *infinite* language
 - the language itself is a set of certain strings
 - * a sentence is a member of that set
 - * a string is a finite sequence of tokens, with a corresponding parse tree

Backus-Naur Form

- **Backus-Naur form (BNF)** can be used to explicitly describe **context-free grammars (CFGs)**
- *parts* of grammar specified in BNF:

- the **terminal symbols** or **tokens** are the smallest units of syntax (leaves of parse tree)
 - * whitespace and comments are not tokens
 - * includes identifiers, numbers, operators, **keywords** that are part of the language
 - certain keywords are **reserved words** that cannot be treated as identifiers
 - * eg. strings and symbols, if, \neq
- the **non-terminal symbols** are the different kind of language constructs (interior nodes of parse tree)
 - * listed in angle brackets
 - * eg. sentences, statements, expressions
 - * `<empty>` is a special non-terminal symbol
- the **start symbol** is the non-terminal symbol at the root of the parse tree
- a set of **productions** or **rules**
 - * a production consists of a left-hand side, separator `::=`, and a right-hand side
 - left-hand side is a single non-terminal symbol
 - right-hand side is a sequence of tokens or non-terminal symbols
 - can use `|` notation for multiple definitions
- other extended BNF metasyms include `[]` for optional expressions, `{ }` for repeated expressions
- can alternatively use **syntax diagrams** (directional graphs) to express grammars
- some programming languages do not use CFGs:
 - Fortran, predates CFGs
 - typedefs in C allow for change token types
 - indentation rules in Python
 - * whitespace becomes important, affects parsing
- writing a grammar is similar to writing a program:
 - *divide and conquer* the problem
 - eg. making a BNF grammar for Java variable declarations:
 - * eg. `int a=1, b, c=1+2;`
 - * `<var-dec> ::= <type-name> <declarator-list> ;`
 - * `<type-name> ::= boolean | byte | short | int | long | char | float | double`
 - * `<declarator-list> ::= <declarator> | <declarator> , <declarator-list>`
 - * `<declarator> ::= <variable-name> | <variable-name> = <expr>`

- * ignores array declarations
- the previous examples do not consider tokens as individual characters
 - instead, they defined the **phrase structure** by showing how to construct parse trees
 - they do not define the **lexical structure** by showing how to divide program text into these tokens
 - * languages can have a **fixed-format** lexical structure where columns in lines and end-of-line markers are significant to the interpretation of the language
 - * or a **free-format** lexical structure where end-of-line markers are simply whitespace
- illustrates the two distinct parts of syntax definition:
 - the **scanner** or **lexer** scans the input file and converts it to a stream of tokens without whitespace and comments
 - * note that the lexer scans *greedily*
 - eg. a-----b is interpreted as the syntactically incorrect ((a--)--)
 - b instead of the syntactically correct (a--) - (--b)
 - the **parser** then reads the tokens and forms the parse tree

Syntax and Semantics

- different grammars may generate the *same* language ie. they create parse trees with identical fringes
 - however, the internal *structures* of the parse trees may be very different
 - eg. $\langle \text{subexp} \rangle ::= \langle \text{var} \rangle - \langle \text{subexp} \rangle \mid \langle \text{var} \rangle \text{ vs. } \langle \text{subexp} \rangle ::= \langle \text{subexp} \rangle - \langle \text{var} \rangle$
 - * both grammars could create the language $a - b - c$, but represent different computations and results
 - * $a - (b - c)$ vs. $(a - b) - c$
 - thus, when considering semantics, the semantics represented by unique parse trees must be *unambiguous*
- consider the following grammar which has issues with precedence and associativity:
 - $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle \mid \langle \text{exp} \rangle * \langle \text{exp} \rangle \mid (\langle \text{exp} \rangle) \mid a \mid b \mid c$
- dealing with **precedence**:
 - the grammar can generate different parse trees for $a + b * c$, including one where addition has higher precedence than multiplication, ie. $(a + b) * c$

- the grammar must be modified to eliminate this erroneous tree:
 - * $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{exp} \rangle \mid \langle \text{mulexp} \rangle$
 - * $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{mulexp} \rangle \mid (\langle \text{exp} \rangle) \mid a \mid b \mid c$
 - * essentially does not allow lower-precedence operators to occur in the subtrees of higher-precedence ones unless explicitly parenthesized
 - creates a level of precedence for multiplication
- dealing with **associativity**:
 - with subtraction instead of addition, the grammar can generate different parse trees for $a - b - c$
 - the grammar must only generate one parse tree for each expression
 - without parenthesis, most languages are **left-associative** and choose the $(a - b) - c$ tree
 - * examples of a right-associative operators are the assignment operator $=$ and construct operator
 - the grammar must be modified:
 - * $\langle \text{exp} \rangle ::= \langle \text{exp} \rangle + \langle \text{mulexp} \rangle \mid \langle \text{mulexp} \rangle$
 - * $\langle \text{mulexp} \rangle ::= \langle \text{mulexp} \rangle * \langle \text{rootexp} \rangle \mid \langle \text{rootexp} \rangle$
 - * $\langle \text{rootexp} \rangle ::= (\langle \text{exp} \rangle) \mid a \mid b \mid c$
 - * the productions are only recursive on one side of each operator
 - * essentially does not allow left-associative operators to appear in the parse tree as the right child of another operator at the same level of precedence without parentheses
 - forces trees to grow down to the left
- dealing with other ambiguities, eg. the *dangling else* problem:
 - for if-statements with an optional else, multiple parse trees may be generated for the statement $\text{if } e1 \text{ then if } e2 \text{ then } s1 \text{ else } s2$
 - could group as $\text{if } e1 \text{ then (if } e1 \text{ then } s1) \text{ else } s2$ or $\text{if } e1 \text{ then (if } e2 \text{ then } s1 \text{ else } s2)$
 - most languages attach the else with the nearest unmatched if
 - the grammar must be modified:
 - * add a new non-terminal symbol for the full if-else-statement
 - * substitute the new symbol within the grammar:
 - $\langle \text{if-stmt} \rangle ::= \text{if } \langle \text{expr} \rangle \text{ then } \langle \text{full-stmt} \rangle \text{ else } \langle \text{stmt} \rangle$
 - * grammar can only match an else with an if if all of the nearer if parts are already matched

- ML is a popular functional language
 - has a *standard* dialect (SML) and an *object-oriented* dialect (OCaml)
 - this chapter uses the SML-NJ dialect

Syntax

- literals:
 - 1234; int constant
 - 123.4; real constant
 - ~34; int constant of -34 using negation operator ~
 - true;, false; bool constants
 - "fred"; string constant
 - #"H"; char constant
- operators:
 - 12 div 5; integer division
 - 7 mod 5; modulo remainder
 - ~3; negation
 - 12.0 / 5.0; real division
 - "tip" ^ "top"; concatenation
 - < > ≤ ≥ ordering comparison
 - = equality
 - * cannot use equality operator with real numbers
 - <> inequality
 - or else and also not boolean operators
 - * ML supports **short-circuit** evaluation
 - left-associative, typical precedence levels
- conditionals:
 - syntax: <cond-expr> ::= if <expr> then <expr> else <expr>
 - (if 1 < 2 then 34 else 56) + 1; gives int 35
- type conversion:
 - ML does not support mixed-type expressions or automatic type conversions
 - * 1.0 * 2; throws an error, multiplication is not **overloaded** for different operand types
 - real(123); gives 123.0 with type real
 - floor(3.6); gives 3 with type int
 - also ceil round trunc for real types
 - ord chr str for char and string operations
- function application:

- can call functions *without* parentheses
- $f(1)$, $(f)1$, $(f\ 1)$, $f\ 1$ all equivalent
- style is to use $f\ 1$
- function application has the highest precedence, and is left-associative
 - * $f\ a+1$ is the same as $(f\ a) + 1$, $f\ g\ 1$ is not the same as $f(g(1))$
- variable definition:
 - `val x = 1 + 2 * 3;`
 - `x;` gives 7 with type `int`
 - can use `val` to redefine an existing variable (new value or new type)
 - note that this is *not* like an assignment statement in imperative programming:
 - * a new definition does not have side effects on other parts of the program
 - * parts of the program using the old definition before redefinition is still using the old definition
 - the it variable always has the value of the last expression typed
- tuples:
 - `val barney = (1+2, 3.0*4.0, "brown");` gives $(3, 12.0, \text{"brown"})$ with type `int * real * string`
 - `val point = ("red", (100, 200));` gives $(\text{"red"}, (100, 200))$ with type `string * (int * int)`
 - * `*` is a **type constructor** for tuples
 - * the type `string * (int * int)` is a different type from `(string * int) * int`
 - `#2 barney;` gives 12.0 with type `real` (1-indexed)
 - `#1 (#2 point);` gives 100 with type `int`
 - note that a tuple of size one does not exist
- lists:
 - all elements are the same type
 - `[1, 2, 3];` gives $[1, 2, 3]$ with type `int list`
 - * `list` is a type constructor
 - `[true];` gives $[true]$ with type `bool list`
 - `[(1,2), (1,3)]` gives $[(1,2), (1,3)]$ with type `(int * int) list`
 - `[[1,2,3], [1,2]]` gives $[[1,2,3], [1,2]]$ with type `int list list`
 - `nil` or `[]` is an empty list
 - * has type `'a list`
 - * names beginning with an apostrophe are **type variables** (unknown type)
 - `null` function checks whether a list is empty
 - `hd` function returns first element, `tl` function returns rest of list after first

- element
 - * error on empty lists
 - explode function converts a string into a char list, implode function performs the opposite
 - @ operator concatenates two lists of the same type
 - * [1,2] @ [3,4]; gives [1,2,3,4] with type int list
 - :: operator pushes an element into the front of a list (*cons* or construct operator)
 - * 1::[2,3]; gives [1,2,3] with type int list
 - * used often for natural recursive constructions
 - * right-associative, 1::2::3::[]; gives [1,2,3] with type int list
- function definitions:
 - syntax: <fun-def> ::= fun <fun-name> <parameter> = <expression> ;
 - fun firstChar s = hd (explode s); gives firstChar with type fn : string -> char
 - * -> is a type constructor for functions
 - * domain and range types are automatically determined
 - firstChar "abc" gives #"a" with type char
 - for multiple parameters, use tuples:
 - * fun quot (a, b) = a div b; gives quot with type fn : int * int -> int
 - * quot (6, 2); gives 3 with type int
 - val pair = (6, 2);, quot pair gives the same result
- using recursion:
 - recursion is used heavily in ML
 - fun fact n = if n = 0 then 1 else n * fact(n - 1);
 - fun listsum x = if null x then 0 else hd x + listsum(tl x);
 - fun length x = if null x then 0 else 1 + length(tl x);
 - * function length has type fn : 'a list -> int
 - * indicates input is a list of elements with unknown type
 - * this is a **polymorphic** function that allows parameters of different types
 - fun badlength x = if x = [] then 0 else 1 + badlength(tl x);
 - * function badlength has type fn : 'a list -> int
 - * indicates input is restricted to *equality-testable* types
 - * function does not work on lists of reals, since reals cannot be tested for equality
 - due to x = [] check
 - fun reverse L = if null L then nil else reverse(tl L) @ [hd L]
- types and type annotations:
 - type constructors include ‘* list ->‘

- list has the highest precedence, \rightarrow has the lowest precedence
 - * `int * int list` is the same type as `int * (int list)`
- for the function `fun prod(a, b) = a * b;`, ML decides on the type `fn : int * int \rightarrow int`
 - * ML uses the *default type* for the multiplication operator
 - * to use with reals, have to include a **type annotation**
 - type annotations can be placed after any variable or expression, but best to keep it as readable as possible
 - * `fun prod(a:real, b:real) : real = a * b;` has type `fn : real * real \rightarrow real`

Patterns

- ML automatically tries to match values to certain **patterns**
 - patterns also introduce new variables
 - eg. patterns appear in function parameters:
 - * `fun f n = n * n;`
 - the pattern `n` matches any parameter and introduces a variable `n`
 - * `fun f (a, b) = a * b;`
 - the pattern `(a, b)` matches any tuple of two items and introduces two variables `a` and `b`
- more patterns:
 - `_` in ML matches anything and does not introduce any variables:
 - * `fun f _ = "yes";` has type `fn : 'a \rightarrow string`
 - can match only a single constant:
 - * `fun f 0 = "yes";` has type `fn : 'int \rightarrow string'` but with a warning for *non-exhaustive* matching
 - throws an error if called on an integer value that isn't 0
 - matching a list of patterns:
 - * `fun f [a, _] = a;` has type `fn : 'a list \rightarrow 'a` but with a non-exhaustive matching warning
 - only matches lists with exactly two elements
 - matching a cons of patterns:
 - * `fun f (x :: xs) = x;` has type `fn : 'a list \rightarrow 'a` but with a non-exhaustive matching warning
 - matches any non-empty list and introduces `x` bound to the head element and `xs` bound to the tail
 - almost exhaustive, but fails on the empty list
- the grammar for multiple pattern function definitions:

- `<fun-def> ::= fun <fun-bodies> ;`
- `<fun-bodies> ::= <fun-body> | <fun-body> '|' <fun-bodies>`
- `<fun-body> ::= <fun-name> <pattern> = <expression>`

Using multiple function patterns:

```
(* type int -> string, non-exhaustive *)
fun f 0 = "zero"
    | f 1 = "one";
```

For overlapping patterns, ML tries patterns in order:

```
(* type int -> string, exhaustive *)
fun f 0 = "zero"
    | f _ = "non-zero";
```

Equivalently, in non pattern-matching style:

```
fun f n =
  if n = 0 then "zero"
  else "non-zero";
```

Rewriting functions in this style clearly separates base case from the recursive case:

```
fun fact 0 = 123
    | fact n = n * fact(n - 1);

fun reverse nil = nil
    | reverse (first :: rest) = reverse rest @ [first];

fun sum nil = 0
    | sum (first :: rest) = first + sum rest;

fun countTrue nil = 0
    | countTrue (true :: rest) = 1 + count_true rest
```

```
| countTrue (false :: rest) = count_true rest;

fun incrAll nil = nil
  | incrAll (first :: rest) = first + 1 :: incr_all rest;
```

Restrictions of pattern-matching style:

```
(* the same variable cannot be used more than once in a pattern
* fun f (a, a) = ...
* | f (a, b) = ...;
*)

(* cannot use pattern-matching *)
fun f (a, b) =
  if (a = b) then ...
  else ...;
```

Pattern-matching in variable definitions:

```
val (a, b) = (1,2.3);
val a :: b = [1,2,3,4,5];
```

Local Variable Definitions

- the let expression allows for local variable definitions
 - syntax: <let-exp> ::= let <definitions> in <expression> end
 - definitions cannot be accessed outside the environment of the let
 - the value of the evaluated expression is the value of the entire let expression

Using let:

```
let val x = 1 val y = 2 in x + y end;
(* it has value 3, x and y are unbound *)
```

Alternatively:

```
let
  val x = 123
  val y = 2cm
in
  x + y
end;
```

More practical example with let:

```
fun days2ms days =
  let
    val hours = days * 24.0
    val minutes = hours * 60.0
    val seconds = minutes * 60.0
  in
    seconds * 1000.0
  end;
```

let with function pattern-matching:

```
fun halve nil = (nil, nil)
  | halve [a] = ([a], nil)
  | halve (a :: b :: cs) =
    let
      val (x, y) = halve cs
    in
      (a :: x, b :: y)
    end;

fun merge (nil, ys) = ys
  | merge (xs, nil) = xs
  | merge (x :: xs, y :: ys) =
    if (x < y) then x :: merge(xs, y :: ys)
```

```
    else y :: merge(x :: xs, ys);

fun mergeSort nil = nil
  | mergeSort [e] = [e]
  | mergeSort theList =
    let
      val (x, y) = halve theList
    in
      merge(mergeSort x, mergeSort y)
    end;
```

Case Expression

- syntax for a case expression:
 - `<rule> ::= <pattern> => <expression>`
 - `<match> ::= <rule> | <rule> '|' <match>`
 - `<case-exp> ::= case <expression> of <match>`

Although many languages have a case construct, ML's case allows for powerful general pattern matching:

```
(* returns the third element if 3+ elements, second if 2, first if 1, 0 if empty *)
case x of
  _ :: _ :: c :: _ => c |
  _ :: b :: _ => b |
  a :: _ => a |
  nil => 0
```

Higher-Order Functions

- function names are variables just like any others in ML
 - they are just initially bound to a function
 - functions themselves do not *have* names
 - eg. can rebind the negation operator:

- * `val x = ~;`
 - * `x 3;` gives -3 with type `int`
- can *extract* the function itself from a builtin operator such as `>` using `op`
 - * `quicksort([1,2,3,4,5], op >)` gives `[5,4,3,2,1]` if the `quicksort` function takes a list and a comparison function
- can create **anonymous** functions using the keyword `fn` followed by a match instead of `fun`:
 - `fun f x = x + 2;` has the same effect as `val f = fn x => + 2;`
 - * except that only the `fun` definition has a scope including the function body, so only the `fun` version can be recursive
 - `(fn x => x + 2) 1;` gives 3 with type `int`
- **higher-order functions (HOFs)** are functions that take another function as a parameter or returns a function
 - functions that do not involve other functions have order 1 and are not higher-order
 - HOFs provide an alternative for squeezing multiple parameters into a single tuple:
 - * using **currying** to write a function that takes the first parameter, and returns another function that takes the second parameter, etc., until the ultimate result is returned

Using currying:

```
fun f (a, b) = a + b;
f (2, 3);

fun g a = fn b => a + b;
g 2 3; (* same as (g 2) 3 *)
```

Calling curried functions with only some of their parameters:

```
val add2 = g 2;
val add3 = g 3;
add2 3; (* gives 5 *)
add3 3; (* gives 6 *)

(* defining quicksort as a curried function with type:
   * ('a * 'a -> bool) -> 'a list -> 'a list
```

```
* )
quicksort (op <) [1,4,3,2,5]; (* gives [1,2,3,4,5] *)
val sortBackward = quicksort (op >);
sortBackward [1,4,3,2,5]; (* gives [5,4,3,2,1] *)
```

Extending parameters:

```
fun f (a,b,c) = a + b + c;
f (1,2,3);

fun g a = fn b => fn c => a + b + c;
g 1 2 3;

fun g a b c = a + b + c; (* equivalent abbreviation *)
```

Predefined Higher Order Functions

- the `map` function has the type `('a -> 'b) -> 'a list -> 'b list`
 - applies some function to every element of a list, creating a list with the same size
 - `map ~ [1,2,3,4]; gives [-1,-2,-3,-4]`
 - `map (fn x => x+1) [1,2,3,4]; gives [2,3,4,5]`
 - `map (fn x => x mod 2 = 0) [1,2,3,4]; gives [false,true,false,true]`
 - `map (op +) [(1,2),(3,4),(5,6)]; gives [3,7,11]`
- the `foldr` function has the type `('a * 'b -> 'b) -> 'b -> 'a list -> 'b -> 'a list -> 'b`
 - combines all the elements of a list into one value, starting from the rightmost element
 - takes a function, a starting value, and a list of elements
 - * `foldr (fn (a,b) => ...) c x`
 - * first call of anonymous function starts with `a` as rightmost element and `b` as `c`
 - * then, `b` will hold the result accumulated so far
 - * `b`, `c`, and the return value of `foldr` and the anonymous function are all the same type
 - * `a` and the type of elements of `x` are the same type
 - * `c` is returned when the list is empty
 - `foldr (op +) 0 [1,2,3,4]; gives 10`

- foldr (op *) 1 [1,2,3,4]; gives 24
 - * need extra space to avoid comment delimiting
- foldr (op ^) "" ["abc","def","ghi"]; gives "abcdefghi"
- foldr (op ::) [5] [1,2,3,4]; gives [1,2,3,4,5]
- fun filterPositive L = foldr (fn (a,b) => if a < 0 then b else a::b) [] L;
- the foldl function has the same type as foldr, but starts from the leftmost elements
 - same result as foldr for associative and commutative operations
 - foldl (op ^) "" ["abc","def","ghi"]; gives "ghidefabcd"
 - foldl (op -) 0 [1,2,3,4]; gives 2 as opposed to -2 called with foldr

Type and Data Constructors

-
- the datatype definition creates an enumerated type:
 - datatype day = Mon | Tue | Wed | Thu | Fri | Sat | Sun;
 - fun isWeekDay x = not (x = Sat or else x = Sun);
 - the name of the type is a **type constructor** and the member names are **data constructors**
 - * data constructors here act as *constants* in a pattern
 - the only permitted operators are comparisons for equality
 - the actual ML definition for booleans is datatype bool = true | false;
 - a parameter to a data constructor can be added with the keyword of:
 - datatype exint = Value of int | PlusInf | MinusInf;
 - * each Value will contain an int, Value itself is a function that takes an int and returns exint
 - * Value 3; gives Value 3 with type exint
 - * however, cannot treat as an int and perform operations
 - * have to extract using pattern matching:
 - val x = Value 5;; val (Value y) = x; gives 5 with type int

Pattern matching with data constructors:

```
(* exhaustive matching *)
val s = case x of
  PlusInf => "infinity" |
  MinusInf => "-infinity" |
  Value y => Int.toString y;
```

```
fun square PlusInf = PlusInf
  | square MinusInf = PlusInf
  | square (Value x) = Value (x * x);
```

- a type constructor can have parameters too, allowing for *polymorphic* type parameters:
 - datatype 'a option = NONE | SOME of 'a;
 - the type constructor is named option and takes type 'a as a parameter
 - SOME 4; gives the type int option
 - SOME 1.2; gives the type real option
 - SOME "pig"; gives the type string option

Polymorphic type parameter examples:

```
fun optdiv a b = if b = 0 then NONE else SOME (a div b);

datatype 'x bunch = One of 'x | Group of 'x list;
One 1.0; (* type real bunch *)
Group [true,false]; (* type bool bunch *)

fun size (One _) = 1
  | size (Group x) = length x;

(* here, ML resolves the returned type to int *)
fun sum (One x) = x
  | sum (Group xlist) = foldr (op +) 0 xlist;
```

Recursion with Constructors

- type constructors can also be used *recursively*
 - eg. the actual list type definition in ML is recursive
 - datatype 'element list = nil | :: of 'element * element list;

Defining type constructors recursively:

```
datatype intlist = INTNIL | INTCONS of int * intlist;
```

```
INTNIL; (* represents an empty list *)
INTCONS (1, INTNIL); (* represents a list of just 1 *)
INTCONS (1, INTCONS (2, INTNIL)); (* represents a list of 1 and 2 *)

fun intlistLength INTNIL = 0
  | intlistLength (INTCONS (_,tail)) =
    1 + (intlistLength tail);
```

Creating a parameterized list type:

```
datatype 'element myList = NIL | CONS of 'element * element myList;

fun myfoldr f c NIL = c
  | myfoldr f c (CONS(a,b)) =
    f(a, myfoldr f c b);
```

Defining polymorphic binary trees:

```
datatype 'data tree = Empty | Node of 'data tree * 'data * 'data tree;
val treeEmpty = Empty;
val tree2 = Node(Empty, 2, Empty);
val tree 123 = Node(Node(Empty,1,Empty), 2, Node(Empty,3,Empty));
```

Binary tree operations:

```
fun sumall Empty = 0
  | sumall (Node(x,y,z)) =
    sumall x + y + sumall z;

fun isintree x Empty = false
  | isintree x (Node(l,y,r)) =
    x = y
    orelse isintree x l
    orelse isintree x r;
```

OCaml Syntax

- in OCaml, statements are ended by the double semicolon `;;` rather than the single `;`;
- literals:
 - `3.141;;` has type `float`
 - `'j';;` has type `char`
 - `(3, true, "hi");;` has type `int * bool * string`
 - `[1; 2; 3];;` has type `int list`
- operators:
 - negation operator is `-` instead of `~`
 - division operator for int is `/` instead of `div`
 - `-3 * (1+7) / 2 mod 3`
 - `-1.0 /. 2.0 +. 1.9 *. 2`, float operations have an extra `.`
 - `a || b && c`
- variable definition:
 - uses `let` instead of `val`
 - `let name = ...`
- functions:
 - instead of `fun f x y = ...`, `let f x y = ...`
 - can use function syntactical sugar for pattern matching on a single parameter
 - can omit `fun` altogether
 - the `rec` allows for recursive variable definition

fib in SML:

```
fun fib 0 = 0
  | fib 1 = 1
  | n = fib (n-1) + fib (n-2)
```

fib in OCaml with full fun:

```
let rec fib = fun n ->
  if n < 2
  then n
  else fib (n-1) + fib (n-2)
;;
```

fib in OCaml with function syntactic sugar for matching:

```
let rec fib = function
  0 -> 0
| 1 -> 1
| n -> fib (n-1) + fib (n-2)
;;
```

fib in OCaml with fully abbreviated syntactic sugar:

```
let rec fib n =
  if n < 2
  then n
  else fib (n-1) + fib (n-2)
;;
```

- type declarations:
 - uses type instead of datatype
 - type 'a option = None | Some of 'a
- pattern matching:
 - uses match instead of case
 - match opt with None -> ... | Some x -> x
- local declarations:
 - let x = 123 in let y = 321 in x + y gives 444 with type int
- tuples:
 - cannot use # to index into tuple, instead use pattern matching
- lists:
 - uses List.fold_left and List.fold_right instead of foldl and foldr