Principal Component Analysis

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UNDERSTANDING PCA (work reproduced from this web-page:https://goo.gl/Wgeieb)

While performing principal components analysis the original n-dimensional data get's projected onto several planes and the plan which has captured the maximum variance of the data becomes the first pricipal component. We select as many principal components as possible so that the maximum variance of the data is captured in minimal number of principal components.

Maximum variance is captured when the mean squared error between the original dataset and it's projection on to a plane is is minimum.

Reading data

```
data = read.csv('diamonds.csv')
```

Viewing all the variable names in the dataset

```
colnames(data)
```

```
##  [1] "X"       "carat"   "cut"     "color"   "clarity" "depth"   "table"
##  [8] "price"   "x"       "y"       "z"
```

Taking only the numeric variables so we could use it in our analysis

```
library(dplyr)
```

```
##
## Attaching package: 'dplyr'
```

```
## The following objects are masked from 'package:stats':
##
##     filter, lag
```

```
## The following objects are masked from 'package:base':
##
##     intersect, setdiff, setequal, union
```

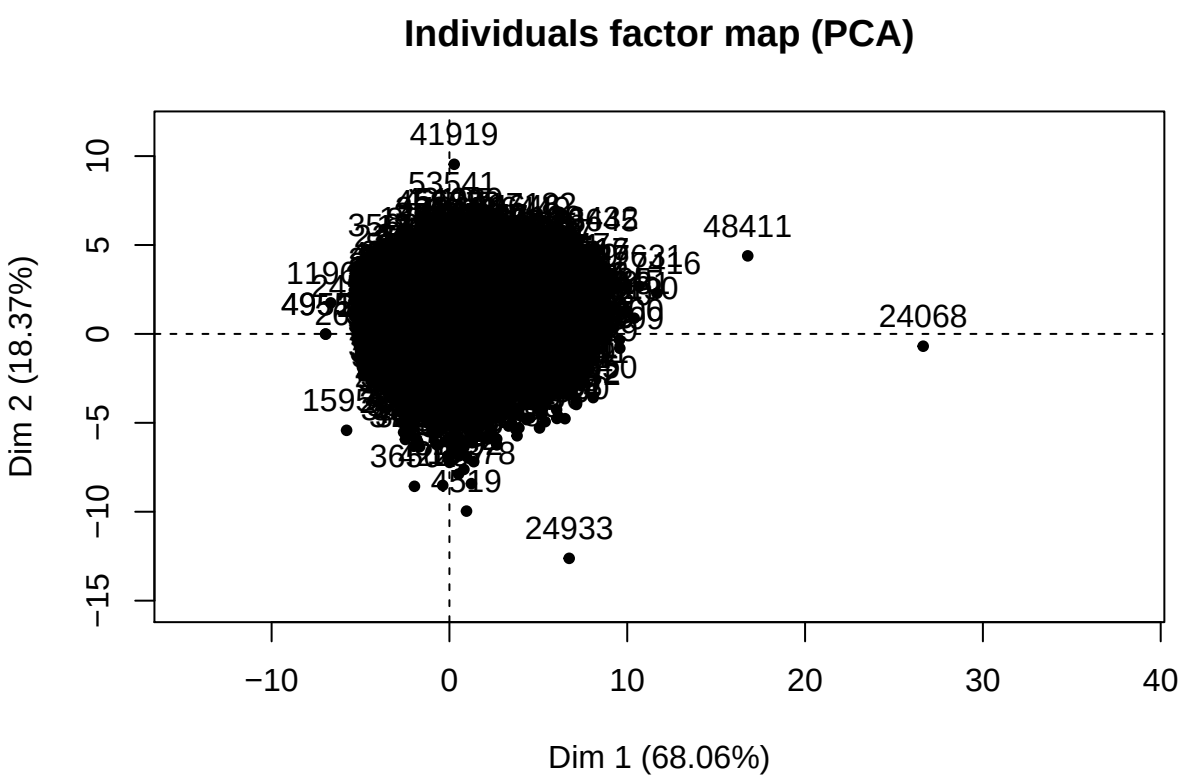
```
data_for_pca <- select(data, -X, -cut, -color, -clarity)
```

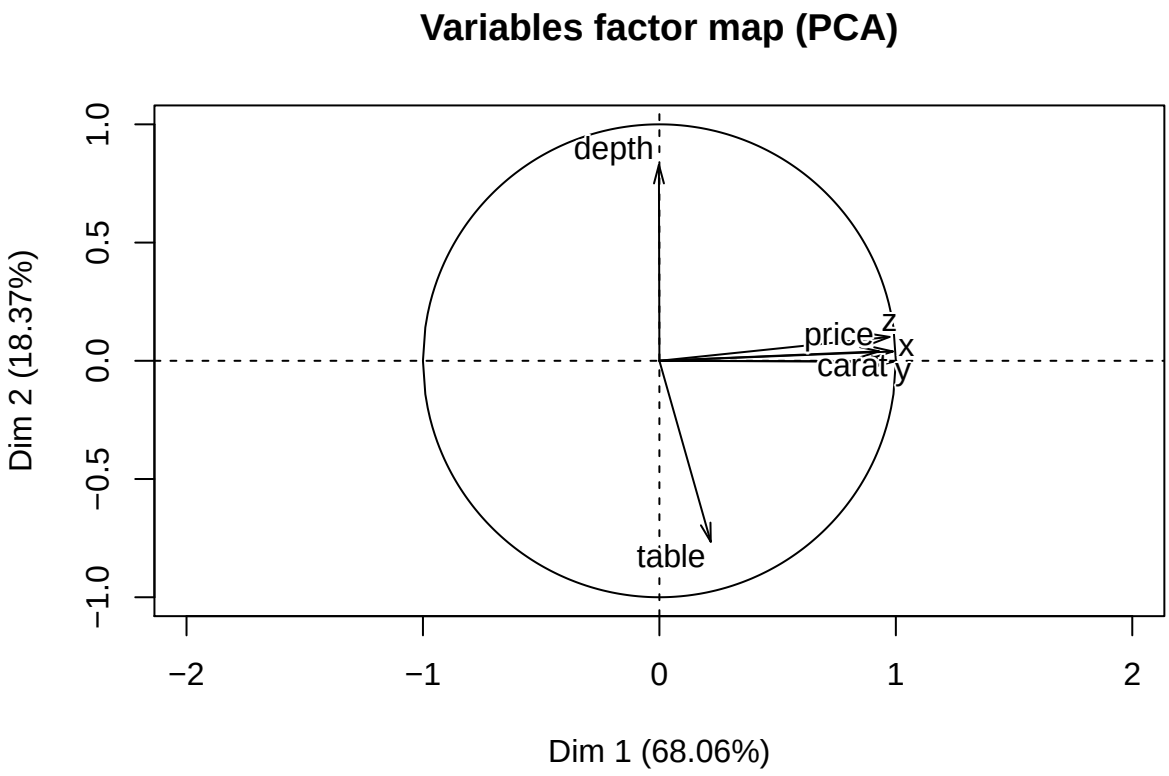
Principal components

Installing the factominer package for PCA

```
library(FactoMineR)
```

```
pca = PCA(data_for_pca)
```


Individuals factor map (PCA)
41919
48411
24068
24933
Dim 1 (68.06%)
Dim 2 (18.37%)
−10
0
10
20
30
40
10
5
0
−5
−10
−15



Here,the variables, 'price', 'carat', 'x', 'y', and 'z' form a composite variable called the Principal component 1 or Dim 1 which explains 68.06% of the variance in the data. Variable 'depth' explains 18.37% of the variance in the data along the second dimension. The variable 'table' is in the third dimension.

```
pca$eig
```

```
##        eigenvalue percentage of variance cumulative percentage of variance
## comp 1 4.76391480             68.0559258                           68.05593
## comp 2 1.28586808             18.3695440                           86.42547
## comp 3 0.69081126              9.8687323                           96.29420
## comp 4 0.17375333              2.4821905                           98.77639
## comp 5 0.04030722              0.5758174                           99.35221
## comp 6 0.03294659              0.4706656                           99.82288
## comp 7 0.01239871              0.1771245                          100.00000
```

Eigen values

In the table above, eigen values indicate how much variance each component explains. For example if we divide the eigen value 4.763 of the first principal component by the total of the eigen values of all the components then we will get a percentage of variance of 68.055. Likewise, the same for all other components also.

Eigen Vectors

Eigen vectors are the vector locations of these principal components. MAtrix multiplication of our original dataset with eigen vector number 1 will generate data for principal component 1. Each of these components

are projected in a different direction in the 3-D space.

How much of each variable is represented in each principal component

Now, let's see how much variance of each variable is explained by each principal component.

```
Correlation_Matrix = as.data.frame(round(cor(data_for_pca,pca$ind$coord)^2*100,0))
Correlation_Matrix[with(Correlation_Matrix, order(-Correlation_Matrix[,1])),]
```

```
##       Dim.1 Dim.2 Dim.3 Dim.4 Dim.5
## carat    98     0     0     0     0
## x        98     0     0     1     0
## y        95     0     0     2     2
## z        95     1     0     2     1
## price    86     0     1    13     0
## table     5    59    37     0     0
## depth     0    69    31     0     0
```

This correlation matric tells us that 98% of the information in carat and X variables are loaded in the first dimension, 95% of the information from y and z variables are loaded in the first dimension. Information from the variables table and depth is spread between dimension 2 and 3. Information of the variable price is spread between 1st and 4th principal components.

If we discard the 5th principal component we will only loose 3% of the information from only 2 variables. Therefore it is safe is discard this dimension and only keep the 4 remaining dimensions.

COMPUTING PRINCIPAL COMPONENT ANALYSIS STEP BY STEP

Let's use the iris data set to perform pricipal component analysis

```
df <- iris[, -5]
```

Let's centre and scale the data

```
df.scaled <- scale(df, center = TRUE, scale = TRUE)
```

Computing the correlation matrix

```
res.cor = cor(df.scaled)
round(res.cor, 2)
```

```
##              Sepal.Length Sepal.Width Petal.Length Petal.Width
## Sepal.Length         1.00       -0.12         0.87        0.82
## Sepal.Width         -0.12        1.00        -0.43       -0.37
## Petal.Length         0.87       -0.43         1.00        0.96
## Petal.Width          0.82       -0.37         0.96        1.00
```

Calculating Eigen Values and Eigen Vectors

```
res.eig = eigen(res.cor)

res.eig
```

```
## eigen() decomposition
## $values
## [1] 2.91849782 0.91403047 0.14675688 0.02071484
##
## $vectors
##            [,1]        [,2]       [,3]       [,4]
## [1,]  0.5210659 -0.37741762  0.7195664  0.2612863
## [2,] -0.2693474 -0.92329566 -0.2443818 -0.1235096
## [3,]  0.5804131 -0.02449161 -0.1421264 -0.8014492
## [4,]  0.5648565 -0.06694199 -0.6342727  0.5235971
```

Calculating principal components

As mentioned under the heading 'Eigen Vector' above, to calculate principal components, we need to multiply the scaled and centred data set with eigen vectors. Multiplying the first eigen vector with the data matrix gives us the first principal component and multiplying the data matrix with the second eigen vector gives us the second principal component and so on and so forth.

Transposing the eigen vectors

```
eigenvectors.t = t(res.eig$vectors)
```

Transposing the adjusted data

```
df.scaled.t = t(df.scaled)
```

Calculating the new principal components

```
pc = eigenvectors.t %*% df.scaled.t
pc = t(pc)
colnames(pc) <- c("PC1", "PC2", "PC3", "PC4")
head(pc)
```

```
##            PC1        PC2         PC3          PC4
## [1,] -2.257141 -0.4784238  0.12727962  0.024087508
## [2,] -2.074013  0.6718827  0.23382552  0.102662845
## [3,] -2.356335  0.3407664 -0.04405390  0.028282305
## [4,] -2.291707  0.5953999 -0.09098530 -0.065735340
## [5,] -2.381863 -0.6446757 -0.01568565 -0.035802870
## [6,] -2.068701 -1.4842053 -0.02687825  0.006586116
```