NYU Tandon Bridge Winter 2023

Assignment 3

NAMES: PATRICK CORMIER, LEAH DEL GIUDICE, TED HILGER, CHELSEA HUDSON

Question 7

a) EXERCISE 3.1.1

a	27 ∈ A	TRUE	27 % 3 = 0 therefore 27 is an element of A.
b	27 ∈ B	FALSE	There is no integer y where $y^2 = 27$ therefore 27 is not an element of B.
c	100 ∈ B	TRUE	$10^2 = 100$ therefore 10 is a perfect square and 10 is an element of B.
d	E ⊆ C or C ⊆ E	FALSE	Not every element of E is in C and not every element of C is in E therefore neither of the two are subsets of the other. For example, $3 \in E$ but $3 \notin C$ and $5 \in C$ but $5 \notin E$.
e	E ⊆ A	TRUE	A is a set of integers where each are multiples of 3. Each element of E is a multiple of 3 therefore E is a subset of A.
f	A ⊂ E	FALSE	A is not a proper subset of E because A contains elements that are not in E. For example, 12 would be an element of A but it is not an element of E.
g	E ∈ A	FALSE	Elements of set A are integers. The set E is not an integer therefore it is not an element of set A.

b) EXERCISE 3.1.2

a	15 ⊂ A	FALSE	15 is not a set therefore cannot be a subset of A.
b	{15} ⊂ A	TRUE	{15} is a set of A.
c	∅ ⊂ A	TRUE	The null set is a subset of every set therefore the null set is a subset of A.
d	A ⊆ A	TRUE	A = A therefore A is a subset of A.
e	∅ ∈ B	FALSE	Elements of B are integers. The empty set is a set and therefore cannot be an element of B.

c) **EXERCISE 3.1.5**

b	{ 3, 6, 9, 12, ... }	Infinite set. { $x \in$ **N** : x is a multiple of 3 } All x from the set of Natural Numbers such that x is a multiple of 3.
d	{ 0, 10, 20, 30, ..., 1000 }	Finite set. { $x \in$ **N** : $x \geq 1000$ and x is a multiple of 10 } | { $x \in$ **N** : $x \geq 1000$ and x is a multiple of 10 } | = 101 All x from a set of Natural Numbers less than 1000 such that x is a multiple of 10. The cardinality of the set is 101.

d) EXERCISE 3.2.1

a	$2 \in X$	TRUE	2 is an element of X
b	$\{2\} \subseteq X$	TRUE	{2} is a subset of X
c	$\{2\} \in X$	FALSE	Although 2 is an element of X the set {2} is not listed as an element of X
d	$3 \in X$	FALSE	Although the set {3} is an element of X, the element 3 is not listed as an element of X
e	$\{1, 2\} \in X$	TRUE	The set {1,2} is an element of X
f	$\{1, 2\} \subseteq X$	TRUE	The set {1,2} is a subset of X since both elements are elements of X
g	$\{2, 4\} \subseteq X$	TRUE	The set {2,4} is a subset of X since 2 and 4 are elements of X
h	$\{2, 4\} \in X$	FALSE	{2,4} is not a set listed as an element of X
i	$\{2, 3\} \subseteq X$	FALSE	{2,3} is not a subset of X since 3 is not an element of X
j	$\{2, 3\} \in X$	FALSE	{2,3} is not a set listed as an element of X
k	$\|X\| = 7$	FALSE	The cardinality of x is $\|X\| = 6$

Question 8
EXERCISE 3.2.4

b	Let $A = \{1, 2, 3\}$ What is $\{X \in P(A): 2 \in X\}$?
	$\|P(A)\| = 2^3$ or 8 $P(A) = \{\varnothing, \{1\}, \{2\}, \{3\}, \{1,2\}, \{1,3\}, \{2,3\}, \{1,2,3\}$
	$\{X \in P(A): 2 \in X\} = \{\{2\}, \{1, 2\}, \{2, 3\}, \{1, 2, 3\}\}$

Question 9:

a) EXERCISE 3.3.1

c	A ∩ C	A = {-3, 0, 1, 4, 17} C = {x ∈ Z: x is odd}	{-3, 1, 17}
d	A ∪ (B ∩ C)	A = {-3, 0, 1, 4, 17} B = {-12, -5, 1, 4, 6} C = {x ∈ Z: x is odd}	{-5, -3, 0, 1, 4, 17}
e	A ∩ B ∩ C	A = {-3, 0, 1, 4, 17} B = {-12, -5, 1, 4, 6} C = {x ∈ Z: x is odd}	{1}

b) EXERCISE 3.3.3

a	$\bigcap_{i=2}^{5} = A_i$	= {1,2,4} ∩ {1,3,9} ∩ {1,4,16} ∩ {1,5,25} = {1}
b	$\bigcup_{i=2}^{5} = A_i$	= {1,2,4} ∪ {1,3,9} ∪ {1,4,16} ∪ {1,5,25} = {1,2,3,4,5,9,16, 25}
e	$\bigcap_{i=1}^{100} = C_i$	$C_i \supseteq C_j fori \leq j$ = $\{x \in R : (-1/100) <= x <= 1/100\}$
f	$\bigcup_{i=1}^{100} = C_i$	$C_i \supseteq C_j fori \leq j$ = $\{x \in R : -1 \leq x \leq 1\}$

c) EXERCISE 3.3.4

b	P(A ∪ B) A = {a, b} B = {b, c}	P(A ∪ B) = {∅, {a}, {b}, {c}, {a, b} {a, c} {b, c} {a, b, c}}
d	P(A) ∪ P(B) P(A) = {{}, {a}, {b}, {a, b}} P(B) = {{}, {b}, {c}, {b, c}}	P(A) ∪ P(B) = {∅, {a}, {b}, {c}, {a, b}, {b, c}}

Question 10

a) EXERCISE 3.5.1

b	An element from the set B × A × C A = {tall, grande, venti} B = {foam, no-foam} C = {non-fat, whole}	Solution: (foam, tall, non-fat)

c	Write the set B × C using roster notation.
	
	B = {foam, no-foam}
	C = {non-fat, whole}
		Solution:
		
		{
		(foam, non-fat),
		(foam, whole),
		(no-foam, non-fat),
		(no-foam, whole)
		}

b) EXERCISE 3.5.3

b	$Z^2 \subseteq R^2$	TRUE	Since Z = integers and integers are Real Numbers then the cartesian product of Z^2 would be a subset of R^2
c	$Z^2 \cap Z^3 = \varnothing$	TRUE	Z^2 contains ordered pairs and Z^3 contains triples therefore the intersection of the two sets would be ∅
e	For any three sets, A, B, and C, if A ⊆ B, then A × C ⊆ B × C.	TRUE	The elements of A are present in B therefore the cartesian product of A x C would be a subset of B x C.

c) EXERCISE 3.5.6

d	{ xy: where x ∈ {0} ∪ {0}2 and y ∈ {1} ∪ {1}2 }	Solution: { 01, 011, 001, 0011 }
e	{ xy: x ∈ {aa, ab} and y ∈ {a} ∪ {a}2 }	Solution: { aaa, aaaa, aba, abaa }

d) EXERCISE 3.5.7

	A = {a} B = {b, c} C = {a, b, d}	Solutions:
c	$(A \times B) \cup (A \times C)$	{aa, ab, ac, ad}
f	$P(A \times B)$	{∅, {ab}, {ac}, {ab, ac}}
g	$P(A) \times P(B)$ Use ordered pair notation for elements of the Cartesian product.	{(∅, ∅), (∅, {b}), (∅, {c}), (∅, {b, c}), ({a}, ∅)), ({a}, {b}), ({a}, {c}), ({a}, {b, c})}

Question 11

a) EXERCISE 3.6.2

b	
$(B \cup A) \cap (\overline{B} \cup A)$	
$(B \cap \overline{B}) \cup A$	Distributive Law
$\varnothing \cup A$	Compliment Law
A	Identity Law

c	
$\overline{A \cap \overline{B}}$	
$\overline{A} \cup \overline{\overline{B}}$	De Morgan's Law
$\overline{A} \cup B$	Double Compliment

b) EXERCISE 3.6.3

		Solution:	
b	A - (B ∩ A) = A A = {1,2} B = {2,3}	A – (B ∩ A) = {1} {1, 2} - ({2, 3} ∩ {1, 2}) {1, 2} - ({2}) {1}	Answer: {1}

		Solution:	
d	$(B - A) \cup A = A$ A = {1, 2} B = {2, 3}	$(B - A) \cup A = A$ ({2, 3} - {1, 2}) ∪ {1, 2} ({3}) ∪ {1, 2} {1, 2, 3}	Answer: {1, 2, 3}

c) EXERCISE 3.6.4

b	A ∩ (B - A) = ∅	
	$A \cap (B - A)$	
	$A \cap (B - \overline{A})$	Set Subtraction Law
	$A \cap (\overline{A} - B)$	Commutative Law
	$(A \cap \overline{A}) \cap B)$	Associative Law
	$\varnothing \cap B$	Complement Law
	$\varnothing$	Domination Law

c	$A \cup (B - A)$	
	$A \cup (B - \overline{A})$	Set Subtraction Law
	$(A \cup B) \cap (A \cap \overline{A})$	Distributive Law
	$(A \cup B) \cap U$	Complement Law
	$A \cup B$	Identity Law