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Question 1:

A. Convert the following numbers to their decimal representation. Show your work.

1. $10011011_2 =$

Solution: Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation.

$10011011_2 = (1x2^0) + (1x2^1) + (0x2^2) + (1x2^3) + (1x2^4) + (0x2^5) + (0x2^6) + (1x2^7)$

$10011011_2 = (1x1) + (1x2) + (0x4) + (1x8) + (1x16) + (0x32) + (0x64) + (1x128)$

$10011011_2 = (1) + (2) + (0) + (8) + (16) + (0) + (0) + (128)$

$10011011_2 = (155)_{10}$

Therefore the decimal representation of 10011011_2 is $\mathbf{(155)_{10}}$

2. $456_7 =$

Solution: Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation.

$456_7 = (6x7^0) + (5x7^1) + (4x7^2)$

$456_7 = (6x1) + (5x7) + (4x49)$

$456_7 = (6) + (35) + (196)$

$456_7 = (237)_{10}$

Therefore the decimal representation of 456_7 is $\mathbf{(237)_{10}}$

3. $38A_{16} =$

Solution: Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation.

$38A_{16} = (Ax16^0) + (8x16^1) + (3x16^2)$

$38A_{16} = (Ax1) + (8x16) + (3x256)$

$38A_{16} = (10) + (128) + (768)$

$38A_{16} = (906)_{10}$

Therefore the decimal representation of $38A_{16}$ is $\mathbf{(906)_{10}}$

4. $2214_5 =$

Solution: Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation.

$2214_5 = (4x5^0) + (1x5^1) + (2x5^2) + (2x5^3)$

$2214_5 = (4x1) + (1x5) + (2x25) + (2x125)$

$2214_5 = (4) + (5) + (50) + (250)$

$2214_5 = (309)_{10}$

Therefore the decimal representation of 2214_5 is $\mathbf{(309)_{10}}$

B. Convert the following numbers to their binary representation:

1. $69_{10} =$

Solution: Use successive division to recursively divide the quotients beginning with the initial number. Each quotient will be divided by 2 since the initial number is in base 10 and the desired representation is base two. The remainders of each quotient divided by 2 will represent the Least Significant Bit to the Most Significant Bit of the binary representation.

$69_{10} =$

$69 \div 2 = 34 \ \text{ mod } 1$ Least Significant Bit

$34 \div 2 = 17 \ \text{ mod } 0$

$17 \div 2 = 8 \ \text{ mod } 1$

$8 \div 2 = 4 \ \text{ mod } 0$ $\Big\}$ $= 1000101_2$

$4 \div 2 = 2 \ \text{ mod } 0$

$2 \div 2 = 1 \ \text{ mod } 0$

$1 \div 2 = 0 \ \text{ mod } 1$ Most Significant Bit

Therefore the binary representation of 69_{10} is $(\mathbf{1000101})_{\mathbf{2}}$

2. $485_{10} =$

Solution: Use successive division to recursively divide the quotients beginning with the initial number. Each quotient will be divided by 2 since the initial number is in base 10 and the desired representation is base two. The remainders of each quotient divided by 2 will represent the Least Significant Bit to the Most Significant Bit of the binary representation.

$485_{10} =$

$485 \div 2 = 242 \ \text{ mod } 1$ Least Significant Bit

$242 \div 2 = 121 \ \text{ mod } 0$

$121 \div 2 = 60 \ \text{ mod } 1$

$60 \div 2 = 30 \ \text{ mod } 0$

$30 \div 2 = 15 \ \text{ mod } 0$ $\Big\}$ $= 111100101_2$

$15 \div 2 = 7 \ \text{ mod } 1$

$7 \div 2 = 3 \ \text{ mod } 1$

$3 \div 2 = 1 \ \text{ mod } 1$

$1 \div 2 = 0 \ \text{ mod } 1$ Most Significant Bit

Therefore the binary representation of 485_{10} is $(\mathbf{111100101})_{\mathbf{2}}$

3. $6D1A_{16} =$

Solution: Convert each hexadecimal digit to binary.

$6D1A_{16} =$

$$\left.\begin{array}{ll} A_{16} = 1010 & \text{Least Significant Bit} \\ 1_{16} = 0001 & \\ D_{16} = 1101 & \\ 6_{16} = 0110 & \text{Most Significant Bit} \end{array}\right\} = 0110110100011010_2$$

Therefore the binary representation of $6D1A_{16}$ is $(\mathbf{0110110100011010})_\mathbf{2}$

C. Convert the following numbers to their hexadecimal representation:

1. 1101011_2

Solution: Each hexadecimal value can be converted from the binary value by grouping the binary digits into groups of four starting with the Least Significant Bit and appending additional 0's after the Most Significant Bit when necessary.

$$\left.\begin{array}{ll} 1101011_2 = & \\ 1011_2 = B_{16} & \text{Least Significant Bit} \\ \text{Additional } 0_2 + 110_2 = 0110_2 = 6_{16} & \text{Most Significant Bit} \end{array}\right\} = 6B_{16}$$

Therefore the hexadecimal representation of 1101011_2 is $\mathbf{(6B)_{16}}$

2. $895_{10} =$

Solution: Use successive division to recursively divide the quotients by 16. The remainders of each division will represent the Least Significant Bit to the Most Significant Bit of the hexadecimal representation.

$$\left.\begin{array}{ll} 895_{10} = & \\ 895_{10} \div 16 = 55 \mod 15 & \text{Least Significant Bit} \\ 55_{10} \div 16 = 3 \mod 7 & \\ 3_{10} \div 16 = 0 \mod 3 & \text{Most Significant Bit} \end{array}\right\} = 37F_{16}$$

Therefore the hexadecimal representation of 895_{10} is $\mathbf{(37F)_{16}}$

Question 2:
Solve the following, do all calculation in the given base. Show your work.

1. $7566_8 + 4515_8 = 14303_8$

	1	1	1	1	
		7	5	6	6_8
+		4	5	1	5_8
	1	4	3	0	3_8

2. $10110011_2 + 1101_2 = 11000000_2$

		1	1	1	1	1	1	
	1	0	1	1	0	0	1	1_2
+					1	1	0	1_2
	1	1	0	0	0	0	0	0_2

3. $7A66_{16} + 45C5_{16} = C02B_{16}$

	1	1		
	7	A	6	6_{16}
+	4	5	C	5_{16}
	C	0	2	B_{16}

4. $3022_5 - 2433_5 = 11010_5$

	1	1	1	1	
		3	0	2	2_5
+		2	4	3	3_5
	1	1	0	1	0_5

Question 3:

A. Convert the following numbers to their 8-bits two's complement representation. Show your work.

1. $124_{10} =$

Solution: Since 124_{10} is even the binary value for the 8-bit two's compliment can be found through successive division. Recursively divide the quotients beginning with the initial number. Each quotient will be divided by 2 since the initial number is in base 10 and the desired representation is base two. The remainders of each quotient divided by 2 will represent the Least Significant Bit to the Most Significant Bit of the binary representation. The result of the successive division is a 7-digit binary number. Since the objective is to find the 8-bit two's compliment a 0 must be prepended to the left-most bit to provide an 8-digit binary number.

$124 \div 2 = 62 \;\; \text{mod } 0$ Least Significant Bit

$62 \div 2 = 31 \;\; \text{mod } 0$

$31 \div 2 = 15 \;\; \text{mod } 1$

$15 \div 2 = 7 \;\; \text{mod } 1$ $\quad = 01111100_2$

$7 \div 2 = 3 \;\; \text{mod } 1$

$3 \div 2 = 1 \;\; \text{mod } 1$

$1 \div 2 = 0 \;\; \text{mod } 1$ Most Significant Bit

Additional 0 to prepend to the Most Significant Bit.

Therefore the 8-bit two's compliment of 124_{10} is $(\mathbf{01111100})_2$

2. $-124_{10} =$

Solution: Since -124_{10} is negative the binary value for the 8-bit two's compliment can be found through successive division of its additive inverse 124_{10}. Recursively divide the quotients beginning with the initial number. Each quotient will be divided by 2 since the initial number is in base 10 and the desired representation is base two. The remainders of each quotient divided by 2 will represent the Least Significant Bit to the Most Significant Bit of the binary representation. The result of the successive division is a 7-digit binary number. Since the objective is to find the 8-bit two's compliment the negative integer then each binary integer will be inverted and a 1 must be prepended to the left-most bit to provide an 8-digit binary number.

$124 \div 2 = 62 \;\; \text{mod } 0 \quad \rightarrow 1$ Least Significant Bit

$62 \div 2 = 31 \;\; \text{mod } 0 \quad \rightarrow 1$

$31 \div 2 = 15 \;\; \text{mod } 1 \quad \rightarrow 0$

$15 \div 2 = 7 \;\; \text{mod } 1 \quad \rightarrow 0$ $\quad = 10000011_2$

$7 \div 2 = 3 \;\; \text{mod } 1 \quad \rightarrow 0$

$3 \div 2 = 1 \;\; \text{mod } 1 \quad \rightarrow 0$

$1 \div 2 = 0 \;\; \text{mod } 1 \quad \rightarrow 0$ Most Significant Bit

Additional 1 to prepend to the Most Significant Bit since the number is negative.

Therefore the 8-bit two's compliment of -124_{10} is $(\mathbf{10000011})_2$

3. $109_{10} =$

Solution: Since 109_{10} is even the binary value for the 8-bit two's compliment can be found through successive division. Recursively divide the quotients beginning with the initial number. Each quotient will be divided by 2 since the initial number is in base 10 and the desired representation is base two. The remainders of each quotient divided by 2 will represent the Least Significant Bit to the Most Significant Bit of the binary representation. The result of the successive division is a 7-digit binary number. Since the objective is to find the 8-bit two's compliment a 0 must be prepended to the left-most bit to provide an 8-digit binary number.

$109 \div 2 = 54 \mod 1$ Least Significant Bit

$54 \div 2 = 27 \mod 0$

$27 \div 2 = 13 \mod 1$

$13 \div 2 = 6 \mod 1$ $= 01101101_2$

$6 \div 2 = 3 \mod 0$

$3 \div 2 = 1 \mod 1$

$1 \div 2 = 0 \mod 1$ Most Significant Bit

Additional 0 to prepend to the Most Significant Bit.

Therefore the 8-bit two's compliment of 124_{10} is $(\mathbf{01101101})_{\mathbf{2}}$

4. $-79_{10} =$

Solution: Since -79_{10} is negative the binary value for the 8-bit two's compliment can be found through successive division of its additive inverse 79_{10}. Recursively divide the quotients beginning with the initial number. Each quotient will be divided by 2 since the initial number is in base 10 and the desired representation is base two. The remainders of each quotient divided by 2 will represent the Least Significant Bit to the Most Significant Bit of the binary representation. The result of the successive division is a 7-digit binary number. Since the objective is to find the 8-bit two's compliment the negative integer then each binary integer will be inverted and a 1 must be prepended to the left-most bit to provide an 8-digit binary number.

$79 \div 2 = 39 \mod 1$ $\rightarrow 0$ Least Significant Bit

$39 \div 2 = 19 \mod 1$ $\rightarrow 0$

$19 \div 2 = 9 \mod 1$ $\rightarrow 0$

$9 \div 2 = 4 \mod 1$ $\rightarrow 0$ $= 10110000_2$

$4 \div 2 = 2 \mod 0$ $\rightarrow 1$

$2 \div 2 = 1 \mod 0$ $\rightarrow 1$

$1 \div 2 = 0 \mod 1$ $\rightarrow 0$ Most Significant Bit

Additional 1 to prepend to the Most Significant Bit since the number is negative.

Therefore the 8-bit two's compliment of -79_{10} is $(\mathbf{10110000})_{\mathbf{2}}$

B. Convert the following numbers (represented as 8-bit two's complement) to their decimal representation. Show your work.

1. 00011110 8 bit 2's comp =

Solution: Since the 8 bit 2's complement begins with a 0 the integer is positive and can be directly converted to its decimal representation. Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation of the 8 bit 2'c compliment.

$00011110_2 = (0x\,2^0) + (1x\,2^1) + (1x\,2^2) + (1x\,2^3) + (1x\,2^4) + (0x\,2^5) + (0x\,2^6) + (0x\,2^7)$

$00011110_2 = (0x1) + (1x\,2) + (1x\,4) + (1x\,8) + (1x16) + (0x\,32) + (0x\,64) + (0x128)$

$00011110_2 = (0) + (2) + (4) + (8) + (16) + (0) + (0) + (0)$

$00011110_2 = (30)_{10}$

Therefore the decimal representation of 00011110_2 is $(\mathbf{30})_{\mathbf{10}}$

2. 11100110 8 bit 2's comp =

Solution: Since the 8 bit 2's complement begins with a 1 the integer is negative its additive inverse must be calculated by subtracting 1_2 and inverting the binary digits. The additive inverse can be converted from binary to its decimal representation. Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation of the 8 bit 2'c compliment.

Subtract 1_2: $11100110_2 - 1_2 = 11100101_2$

Invert the binary integers: $11100101_2 \rightarrow 00011010_2$

$00011010_2 = (0x\,2^0) + (1x\,2^1) + (0x\,2^2) + (1x\,2^3) + (1x\,2^4) + (0x\,2^5) + (0x\,2^6) + (0x\,2^7)$

$00011010_2 = (0x1) + (1x\,2) + (0x\,4) + (1x\,8) + (1x16) + (0x\,32) + (0x\,64) + (0x128)$

$00011010_2 = (0) + (2) + (0) + (8) + (16) + (0) + (0) + (0)$

$00011010_2 = (26)_{10}$

Therefore the decimal representation of 11100110_2 is $(\mathbf{-26})_{\mathbf{10}}$

3. 00101101 8 bit 2's comp =

Solution: Since the 8 bit 2's complement begins with a 0 the integer is positive and can be directly converted to its decimal representation. Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation of the 8 bit 2'c compliment.

$00101101_2 = (1x\,2^0) + (0x\,2^1) + (1x\,2^2) + (1x\,2^3) + (0x\,2^4) + (1x\,2^5) + (0x\,2^6) + (0x\,2^7)$

$00101101_2 = (1x1) + (0x\,2) + (1x\,4) + (1x\,8) + (0x16) + (1x\,32) + (0x\,64) + (0x128)$

$00101101_2 = (1) + (0) + (4) + (8) + (0) + (32) + (0) + (0)$

$00101101_2 = (45)_{10}$

Therefore the decimal representation of 00101101_2 is $(\mathbf{45})_{\mathbf{10}}$

4. 10011110 8 bit 2's comp =

Solution: Since the 8 bit 2's complement begins with a 1 the integer is negative its additive inverse must be calculated by subtracting 1_2 and inverting the binary digits. The additive inverse can be converted from binary to its decimal representation. Multiply each binary digit from least significant bit to most significant bit by powers of two beginning with zero. Add each product to calculate the decimal representation of the 8 bit 2'c compliment.

Subtract 1_2: $10011110_2 - 1_2 = 10011101_2$

Invert the binary integers: $10011101_2 \rightarrow 01100010_2$

$01100010_2 = (0x\,2^0) + (1x\,2^1) + (0x\,2^2) + (0x\,2^3) + (0x\,2^4) + (1x\,2^5) + (1x\,2^6) + (0x\,2^7)$

$01100010_2 = (0x1) + (1x\,2) + (0x\,4) + (0x\,8) + (0x16) + (1x\,32) + (1x\,64) + (0x128)$

$01100010_2 = (0) + (2) + (0) + (0) + (0) + (32) + (64) + (0)$

$01100010_2 = (98)_{10}$

Therefore the decimal representation of 10011110_2 is $\mathbf{(-98)_{10}}$

Question 4:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.4

(b).

p	q	$(p \vee q)$	$\neg(p \vee q)$
T	T	T	F
T	F	T	F
F	T	T	F
F	F	F	T

(c).

p	q	r	$(p \wedge \neg q)$	$r \vee (p \wedge \neg q)$
T	T	T	F	T
T	T	F	F	F
T	F	T	T	T
T	F	F	T	T
F	T	T	F	T
F	T	F	F	F
F	F	T	F	T
F	F	F	F	F

2. Exercise 1.3.4

(b).

p	q	$(p \rightarrow q) \rightarrow (q \rightarrow p)$
T	T	T
T	F	T
F	T	F
F	F	T

(d).

p	q	$(p \leftrightarrow q) \oplus (p \leftrightarrow q)$
T	T	T
T	F	T
F	T	F
F	F	T

Question 5:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.2.7

    (b). Solution:

$$(B \wedge D) \vee (B \wedge M) \vee (D \wedge M)$$

    (c). Solution:

$$B \vee (D \wedge M)$$

2. Exercise 1.3.7

    (b). Solution:

$$(s \vee v) \rightarrow p$$

    (c). Solution:

$$p \rightarrow y$$

    (d). Solution:

$$p \leftrightarrow (s \wedge y)$$

    (e). Solution:

$$p \rightarrow (s \vee y)$$

3. Exercise 1.3.9

    (c). Solution:

$$c \rightarrow p$$

    (d). Solution:

$$c \rightarrow p$$

Question 6:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.3.6

(b). Solution: **If Joe is eligible for the honors program, the he has maintained a B average.**

(c). Solution: **If Rajiv can go on the rollercoaster, then he is at least four feet tall.**

(d). Solution: **If Rajiv is at least four feet tall, he can go on the roller coaster.**

2. Exercise 1.3.10

(c). $(p \vee r) \leftrightarrow (q \wedge p)$

Solution: **False**. Justification: $(p \vee r) = True$ since $p = True$ and $(q \wedge p) = False$ since $q = false$. Since the statement is a biconditional proposition and $q = false$ the statement is false regardless of the value of r.

(d). $(p \wedge r) \leftrightarrow (q \wedge r)$

Solution: **Unknown**. Justification: The value of $(q \wedge r) = false$ since $q = false$ however, the value of $r = unknown$ therefore the value of $(p \wedge r) = unknown$.

(e). $p \rightarrow (r \vee q)$

Solution: **Unknown**. Justification: The hypothesis $p = True$ however, the truth value of the conclusion is dependent on the value of r.

(f). $(p \wedge q) \rightarrow r$

Solution: **True**. Justification: Since $q = False$ the hypothesis $(p \wedge q) = False$. Since the hypothesis is false the conditional statement is true regardless of the truth value of the conclusion.

Question 7:

Solve Exercise 1.4.5

(b). Solution:

"If Sally did not get the job, then she was late for her interview or did not update her resume."	$\neg j \rightarrow (l \vee \neg r)$
"If Sally updated her resume and was not late for her interview, then she got the job."	$(r \wedge \neg l) \rightarrow j$

j	l	r	$\neg j \rightarrow (l \vee \neg r)$	$(r \wedge \neg l) \rightarrow j$
T	T	T	T	T
T	T	F	T	T
T	F	T	T	T
T	F	F	T	T
F	T	T	T	T
F	T	F	T	T
F	F	T	F	F
F	F	F	T	T

Therefore, the two statements are logically equivalent.

(c). Solution:

"If Sally got the job then she was not late for her interview."	$j \rightarrow \neg l$
"If Sally did not get the job, then she was late for her interview."	$\neg j \rightarrow l$

j	l	$j \rightarrow \neg l$	$\neg j \rightarrow l$
T	T	F	T
T	F	T	T
F	T	T	T
F	F	T	F

Therefore, the two statements are not logically equivalent.

(d). Solution:

If Sally updated her resume or she was not late for her interview, then she got the job."	$(r \vee \neg l) \rightarrow j$
"If Sally got the job, then she updated her resume and was not late for her interview."	$j \rightarrow (r \wedge \neg l)$

j	l	r	$(r \vee \neg l) \rightarrow j$	$j \rightarrow (r \wedge \neg l)$
T	T	T	T	F
T	T	F	T	F
T	F	T	T	T
T	F	F	T	F
F	T	T	F	T
F	T	F	T	T
F	F	T	F	T
F	F	F	F	T

Therefore, the two statements are not logically equivalent.

Question 8:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.5.2

(c).

$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$	
$(\neg p \vee q) \wedge (p \to r)$	Conditional
$(\neg p \vee q) \wedge (\neg p \vee r)$	Conditional
$\neg p \vee (q \wedge r)$	Distributive
$p \to (q \wedge r)$	Conditional

(f).

$\neg(p \vee (\neg p \wedge q)) \equiv \neg((p \vee \neg p) \wedge (p \vee q))$	
$\neg(T \wedge (p \vee q))$	Complement
$\neg(p \vee q)$	Identity
$\neg p \wedge \neg q$	De Morgan

(i).

$(p \wedge q) \to r \equiv \neg(p \wedge q) \vee r$	
$(\neg p \vee \neg q) \vee r$	De Morgan's
$\neg p \vee r \vee \neg q$	Associative
$\neg(p \wedge \neg r) \vee \neg q$	De Morgan's
$(p \wedge \neg r) \to \neg q$	Conditional

2. Exercise 1.5.3

(c).

$\neg r \wedge (\neg r \to p) \equiv \neg r \vee (\neg(\neg r) \vee p)$	
$\neg r \vee (r \vee p)$	Double negation
$(\neg r \vee r) \vee p$	Associative
$T \vee p$	Complement
T	Domination

(d).

$\neg(p \to q) \to \neg q \equiv \neg(\neg p \vee q) \to \neg q$	
$\neg\neg(\neg p \vee q) \vee \neg q$	Conditional
$\neg p \vee q \vee \neg q$	Double negation
$\neg p \vee T$	Complement
T	Domination

Question 9:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.6.3
   (c). $\exists x(x = x2)$
   (d). $\forall x(x \leq x2 + 1)$

2. Exercise 1.7.4
   (b). $\forall x(\neg S(x) \land W(x))$
   (c). $\forall x(S(x) \rightarrow \neg W(x))$
   (d). $\exists x(S(x) \land W(x))$

Question 10:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.7.9
   - (c). $True, x = b$
   - (d). $True, x = e$
   - (e). $True, Q(a) \wedge P(d) = (T) \wedge (T)$
   - (f). $True$.
   - (g). $False, x = c$
   - (h). $True$.
   - (i). $True, x = e$

2. Exercise 1.9.2
   - (b). $True, x = 2$
   - (c). $True, y = 1$
   - (d). $False, allFalseforS(x, y)$.
   - (e). $False, x = 1$
   - (f). $True, P(1,1) = T, P(2,1) = T, P(3,1) = T$
   - (g). $False, P(1,2) = F$.
   - (h). $True, Q(2,2) = T$.
   - (i). $True, all \neg S(x, y) = T$.

Question 11:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.10.4
   - (c). $\exists x \exists y (x + y = x * y)$
   - (d). $\forall x \forall y (((x > 0) \wedge (y > 0)) \rightarrow (x / y > 0))$
   - (e). $\forall x ((x > 0) \wedge (x < 1) \rightarrow (1/x > 1))$
   - (f). $\forall x \exists y (x > y)$
   - (g). $\forall x \exists y ((x = / = 0) \rightarrow (x y = 1))$

2. Exercise 1.10.7
   - (c). $\exists x (N(x) \wedge D(x))$
   - (d). $\forall x (D(x) \rightarrow P(Sam, x))$
   - (e). $\exists x \forall y (N(x) \wedge P(x, y))$
   - (f). $\exists x \forall y ((N(x) \wedge D(x)) \wedge (((y = / = x) \wedge N(y)) \rightarrow \neg D(y)))$

3. Exercise 1.10.10, sections c – f
   - (c). $\forall x \exists y ((y = / =' Math101') \wedge T(x, y))$
   - (d). $\exists x \forall y ((y = / =' Math101') \rightarrow T(x, y))$
   - (e). $\forall x \exists y \exists z ((x = / = Sam) \rightarrow ((y = / = z) \wedge T(x, y) \wedge T(x, z)))$
   - (f). $\exists x \exists y \forall z ((x = / = y) \wedge T(Sam, x) \wedge T(Sam, y) \wedge (((z = / = x) \wedge (z = / = y)) \rightarrow \neg T(Sam, z)))$

Question 12:

Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.8.2

(b).
$\forall x\, (D(x) \vee P(x))$
$\neg\forall x\, (D(x) \vee P(x))$
$\exists x\, \neg(D(x) \vee P(x))$
$\exists x\, (\neg D(x) \wedge \neg P(x))$ – “There exists a patient was not given the medication and not given the placebo”

(c).
$\exists x\, (D(x) \wedge M(x))$
$\neg\exists x\, (D(x) \wedge M(x))$
$\forall x\, \neg(D(x) \wedge M(x))$
$\forall x\, (\neg D(x) \vee \neg M(x))$
“Every patient either did not take the medication or did not have migraines or both”

(d).
$\forall x\, (P(x) \rightarrow M(x))$
$\neg\forall x\, (P(x) \rightarrow M(x))$
$\exists x\, \neg(P(x) \rightarrow M(x))$
$\exists x\, \neg\, (\neg P(x) \vee M(x))$
$\exists x\, (P(x) \wedge \neg M(x))$ – “There exists a patient that took the placebo and did not have migraines”

(e).
$x\, (M(x) \wedge P(x))$
$\neg\exists x\, (M(x) \wedge P(x))$
$\forall x\, \neg(M(x) \wedge P(x))$
$\forall x\, (\neg M(x) \vee \neg P(x))$ – “Every patient either did not have migraines or did not take the placebo or both”

2. Exercise 1.9.4

(c).
$\exists x\, \forall y\, (P(x, y) \rightarrow Q(x, y))$
$\neg\exists x\, \forall y\, (P(x, y) \rightarrow Q(x, y))$
$\forall x\, \exists y\, \neg(P(x, y) \rightarrow Q(x, y))$
$\forall x\, \exists y\, \neg(\neg P(x, y) \vee Q(x, y))$
$\forall x\, \exists y\, (P(x, y) \wedge \neg Q(x, y))$

(d).
$\exists x\, \forall y\, (P(x, y) \leftrightarrow P(y, x))$
$\neg\exists x\, \forall y\, (P(x, y) \leftrightarrow P(y, x))$
$\forall x\, \exists y\, \neg(P(x, y) \leftrightarrow P(y, x))$
$\forall x\, \exists y\, \neg((P(x, y) \rightarrow P(y, x)) \wedge (P(y, x) \rightarrow P(x, y)))$
$\forall x\, \exists y\, \neg((\neg P(x, y) \vee P(y, x)) \wedge (\neg P(y, x) \vee P(x, y)))$
$\forall x\, \exists y\, (\neg\, (\neg P(x, y) \vee P(y, x)) \vee \neg\, (\neg P(y, x) \vee P(x, y)))$
$\forall x\, \exists y\, ((\neg\neg P(x, y) \wedge \neg P(y, x)) \vee (\neg\neg P(y, x) \wedge \neg P(x, y)))$
$\forall x\, \exists y\, ((P(x, y) \wedge \neg P(y, x)) \vee (P(y, x) \wedge \neg P(x, y)))$

(e).
$\exists x\, \exists y\, P(x, y) \wedge \forall x\, \forall y\, Q(x, y)$
$\neg\, (\exists x\, \exists y\, P(x, y) \wedge \forall x\, \forall y\, Q(x, y))$
$\neg\exists x\, \exists y\, P(x, y) \vee \neg\forall x\, \forall y\, Q(x, y)$
$\forall x\, \forall y\, \neg P(x, y) \vee \exists x\, \exists y\, \neg Q(x, y)$