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DEPARTMENT OF MECHANICAL ENGINEERING
FACULTY OF ENGINEERING

MODULE OUTLINE

Module Name	**Solid Mechanics and Mechanical Design**		
Module Code	ME3531	Version No.	2017-1
Year/Level	3	Semester	1
Credit Points	3		
Pre-requisites	CE101 Engineering Mechanics MT101 Engineering Materials MA130 Engineering Mathematics I		
Co-requisites	None		
Methodsof Delivery	Lectures (Face-to-face) Tutorial Laboratory work	2 Hours/Week 1 Hour/Week 2 Hours/Fortnight	
Course Web Site	http://courseweb.sliit.lk		
Date of Original Approval	September 2012		
Date of Next Review	September 2017		

MODULE DESCRIPTION

Introduction	This module presents basics of solid mechanics required for the design of machine components, and introduces students to the design of machine elements.		
Learning Outcomes	On successful completion of this unit you will be able to: **LO1:** Quantify the principal stresses, and stress distributions using analytical, graphical, simulation, and experimental methods. **LO2:**Analyse the modes of failure of columns and beams **LO3:** Discuss Stress Concentration Factors, Time Varying Stresses & and Fatigue Failure **LO4:** Discuss Reliability of mechanical components **LO5:** Design Joining Mechanisms in mechanical systems **LO6:** Design Power Transmission system elements		
Assessment Criteria	• Continuous assessments will carry 50% of the total marks. Continuous assessments include laboratory practical sessions, assignments, and midterm test. Number of practical sessions and assignments required will be decided by the lecturer in charge. Closed book mid semester test will carry 20% to the final total marks. The mid semester exam can be conducted at the completion of 7th week lecture. • Final examination is for 50% marks and the final exam will be a closed book end of semester exam.		
	Continuous Assessment • Labs • Assignments • Midterm Examination	 24% 6% 20%	 LO1 – LO6 LO1 – LO6 LO1 – LO3
	End Semester Assessment • Final Examination	 50%	 LO1 – LO6
	TOTAL	100%	
Estimated Student Workload	Contact Hours		
	• Lecture	24	
	• Tutorial	12	
	• Laboratory	12	

	Time Allocated for Assessments	
	• Continuous Assessments	10
	• Final Examination	2
	Reading and Independent Study	90
	TOTAL	150
Module Requirement	"To pass this module students are required to obtain a pass mark for continuous assessment and final examination respectively and overall mark that would qualify for a "C" grade or above."	
Learning Resources	Recommended Texts: 1. A. Pytel and J. Kiusalaas, *Mechanics of Materials*, CENGAGE Learning USA, 2011. 2. R. S. Khurmi and J. K. Gupta, *A Textbook of Machine Design*, 14th ed, S. Chand & Co Ltd, India, 2005. 3. R. C. Juvinall and K. M. Marshek, *Fundamentals of Machine Component Design*, 5th ed. John Wiley & Sons, USA, 2011.	

MODULE ADMINISTRATION PROCEDURE

Contact Information

Lecturers-in-charge	Munidasa Ranaweera & Thilina Weerakkody		
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Location	5th Floor		
Consultation Time			

CONTENTS OF THE MODULE

1. Review of elementary Mechanics of Materials. Stresses due to Torsion & Bending.
2. Two and three dimensional stress.
3. Transformation of stresses and principal stresses.
4. Mohr's circle representation.
5. Elastic strain measurement using strain gauges.
6. Stress Concentration Factors, Time Dependent Stresses, and Fatigue Failure.
7. Deflections and stiffness for various loadings; particularly beam deflection.
8. Column buckling, Johnson, Euler and Secant formula methods.
9. Reliability and Statistics.
10. Design of Screwed Joints
11. Design of Power Screws.
12. Design of Welded joints
13. Torsional and Bending stresses in machine parts
14. Design of Shafts

GENERIC INFORMATION

Any type of plagiarism is not allowed.

Plagiarism: Academic honesty is crucial to a student's credibility and self-esteem, and ultimately reflects the values and morals of the Institute as a whole. A student may work together with one or a group of students discussing assignment content, identifying relevant references, and debating issues relevant to the subject. Plagiarism occurs when the work of another person, or persons, is used and presented as one's own.

End of Module Outline

Unit Study Calendar

ME3531−<Solid Mechanics and Mechanical Design>

Year 2019, Semester 1: *5th February −24th May*

Week	Dates	Lecture	Date	Tutorial	Due	Remarks
1	Feb 04	**Independence Day**	Feb 06	Introduction & review		**Lecture on Wednesday*
2	Feb 11	Stresses due to torsion & bending	Feb 13	Tutorial 1	Week 2	
3	Feb 18	2D & 3D stress, Transformation of stress and Mohr circle	Feb 20	Tutorial 2	Week 3	
4	Feb 25	2D Strains, Transformation of strain, Strain measurement & strain rosettes	Feb 27	Tutorial 3	Week 4	
5	Mar 04	**Mahashiwa Rathree**	Mar 06	Stress concentration, cyclic stress and fatigue failure	Week 5	**Lecture on Wednesday*
6	Mar 11	Deflections in machine elements, especially beams	Mar 13	Tutorial 4	Week 6	
7	Mar 18	Column buckling & column design	Mar 20	**Poya Day**		**No tutorial*
8	Mar 25	**Mid Semester Exams**				
9	Apr 01	Design of Screwed Joints	Apr 03	Tutorial 5	Week 9	
10	Apr 08		Apr 10			**Rescheduled*
11	Apr 15	**New Year Holidays**				
12	Apr 22	**Academic Holiday**	Apr 24	**Academic Holiday**		**Rescheduled*
13	Apr 29	**Academic Holiday**	May 01	***"May Day"***	Week 13	**Rescheduled*
14	May 06 May 08	Design of Power Screws Design of Welded Joints	May 08 May 08	Tutorial 6 Tutorial 7	Week 14	
15	May 13 May 13	Design of Shafts Reliability and Statistics	May 15 May 15	Tutorial 8 Tutorial 9	Week 15	
16	May 20	Tutorial 10 −Review	May 22			Extended week

ME3531: Solid Mechanics and Mechanical Design – S1:2019
Lecture 8–01 April 2019   Thilina H. Weerakkody
Design of Screwed Joints

Material Covered

1. Introduction
2. Advantages and disadvantages of screw joints.
3. Basic definitions of screw threads.
4. Types of screw threads.
5. Types of screw fastenings
6. Locking devices for screwed joints.
7. Screwing–up torque, Screwing–off torque and Self–locking condition.
8. Strength calculation of screw threads.
   - Initial stresses due to screwing–up forces.
   - Stresses due to external loads.
   - Stresses due to combined forces.
   - Optimum bolt load

Course Text:

R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd. – Chapter 11. Screwed Joints (pp. 377-430).

ME3531: Solid Mechanics and Mechanical Design
Design of Screwed Joints

Thilina H. Weerakkody
Department of Mechanical Engineering
SLIIT-Sri Lanka

Year 3 −Semester 1, 2019

Outline

- Introduction
- Advantages and disadvantages of screw joints.
- Basic definitions of screw threads.
- Types of screw threads.
- Types of screw fastenings
- Locking devices for screwed joints.
- Screwing−up torque, Screwing−off torque and Self−locking condition.
- Strength calculation of screw threads.
  - Initial stresses due to screwing−up forces.
  - Stresses due to external loads.
  - Stresses due to combined forces.
  - Optimum bolt load

Intended Learning Outcomes

Upon completing of lecture, the student should be able to:

- Select appropriate screws for a given application.
- Determine the correct size of nuts and bolts for a given loading condition.
- Use standard charts to select nuts and bolts
- Design screw joints for given loading conditions

Joint Classification

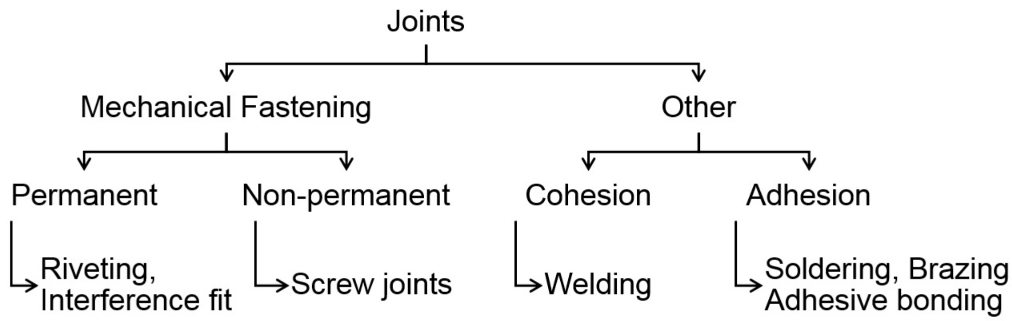


Figure 1: Joint Classification

Importance of Joints

- Large components cannot be produced as a single piece.
- Mechanical systems consist of many components made of different materials.
- Majority of systems fail at joints.
- To facilitate operation, maintenance and transportation.

Introduction

Screwed or threaded joints are detachable joints fitted together by bolts, screws, studs and nuts. Screwed joints are widely used for fastening purposes. Screws are also used for power transmission and to obtain fine adjustments and movement in measuring instruments. The screw and nut pair is a force amplifier.

Advantages

- Screwed joints are highly reliable in operation.
- Screwed joints are convenient to assemble and disassemble.
- A wide range of screwed joints may be adopted to various operating conditions.
- Screws are relatively cheap to produce due to standardized and highly efficient manufacturing processes.

Disadvantages

- Presence of numerous points of stress concentration on the threaded surfaces, which reduces the strength of the joints under varying loads
- May reduce the strength of the parts at the joint.
- Increases the weight of the joined system processes.

Basic dimensions of screw threads

A screwed joint is illustrated in the Figure 2. The basic parameters of a screw thread are defined below.

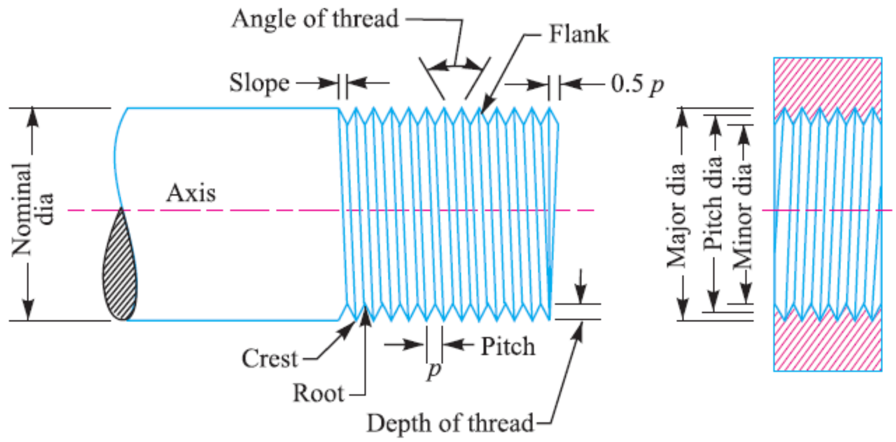


Figure 2: Basic dimensions of a screwed threads

1. Major diameter (d)

The largest diameter of an external or internal screw thread. The screw is specified by this diameter. It is also known as **outside** or **nominal diameter**.

2. Minor diameter (d_m)

Tthe smallest diameter of an external or internal screw thread. It is also known as **core** or **root diameter**.

3. Pitch diameter (d_p)

The diameter of an imaginary cylinder, on a cylindrical screw thread, the surface of which would pass through the thread at such points as to make equal the width of the thread and the width of the spaces between the threads. It is also called an **effective diameter**.

4. Pitch (p)

The distance from a point on one thread to the corresponding point on the next. This is measured in an axial direction between corresponding points in the same axial plane. Mathematically,

$$Pitch\,(p) = \frac{1}{No.\ of\ threads\ per\ unit\ length\ of\ screw}$$

5. Lead (L)

The distance between two corresponding points on the same helix. It may also be defined as the distance which a screw thread advances axially in one rotation of the nut. Lead is equal to the pitch in case of single start threads; twice the pitch in double start; thrice the pitch in triple start.

6. Thread Angle (μ)

The angle included between the sides of the threads measured in an axial plane

7. Helix Angle (ψ)

The angle between any helix and an axial line on its right, circular cylinder or cone.

Types of Screwed Threads

1. Metric V Thread

The basic profile of the metric V - thread is shown in the Figure 3. The thread angle $\alpha = 60°$. The root of the thread on the screw is either flattened or rounded. Metric threads are subdivided into coarse and fine series.

The coarse thread has been established as the basic series. The fine pitch threads are used for fine adjustments and where the connected parts are subjected to the dynamic loads. Coarse pitch metric thread is specified by the capital letter M followed by the nominal diameter in mm. Example, M 20 denotes a metric V - thread with 20 mm nominal diameter. The pitch is additionally indicated for fine pitch thread, for example M 20 $\times$ 1.5 where, the pitch specified is 1.5 mm.

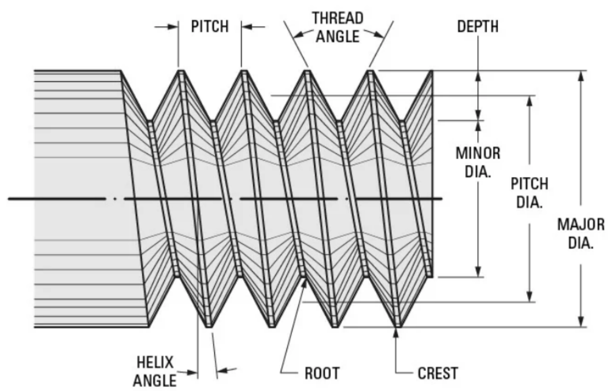


Figure 3: Metric V Thread definition

2. British Standard Whitworth (B.S.W.) Thread (Inch Thread)

This thread is geometrically similar to the metric thread profile, but the thread angle is 55°. This thread is also used mainly for fastening purposes. The British Standard threads with fine pitches (B.S.F.) are used as same as fine pitch metric threads. Inch threads are specified by the number of threads per inch (tpi).

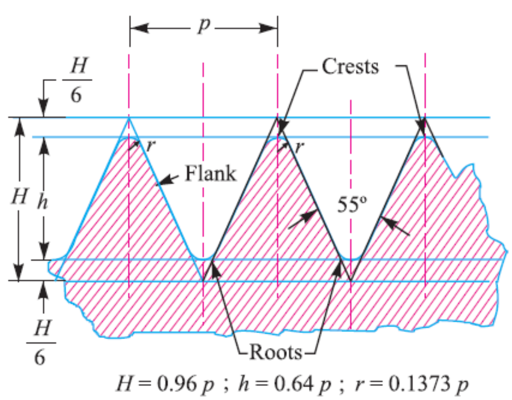


Figure 4: British Standard Whitworth (B.S.W.) Thread (Inch Thread)

3. Square Thread

Square thread (Figure 5) is widely used for transmission of power in either directions, because of their high efficiency. Square threads are not as strong as V-threads but they offer less frictional resistance to motion than V-threads. They are used in machine tools, screw jacks etc.

4. Trapezoidal Threads

Trapezoidal thread (Figure 6) is a modification of square thread. It is much stronger than square thread and can be easily produced. These threads are frequently used on screw cutting lathes, bench vices etc.

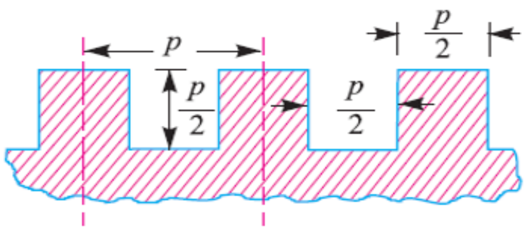


Figure 5: Square Thread

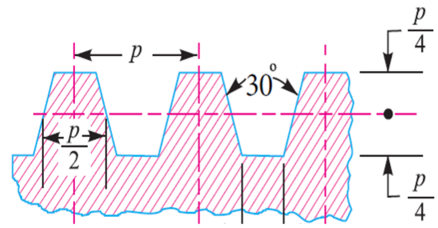
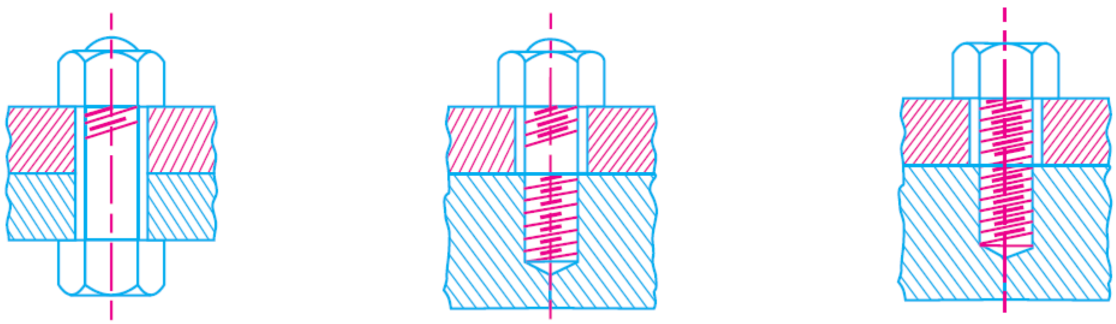


Figure 6: Trapezoidal Thread

Types of Screw Fastenings

Depending upon the type of screw joint involved, screw fastenings are classified as follows.



(a) Through bolts (Nut and Bolt) (b) Tap bolts (c) Studs

Figure 7: Screw Fastenings (a)Through bolts, (b)Tap bolts and (c) Studs

1. Through bolt (Nut and Bolt)

A through bolt (or simply a bolt) is shown in Figure 7 (a). It is **a cylindrical bar with threads <u>for the nut</u>** at one end, and **<u>head</u>** at the other end.

The cylindrical part of the bolt is known as **shank**. It is passed through drilled holes in the two parts to be fastened together and clamped them securely to each other as the nut is screwed on to the threaded end. The through bolts may or may not have a machined finish and are made with either hexagonal or square heads. A through bolt should pass easily in the holes, when put under tension by a load along its axis. If the load acts perpendicular to the axis, tending to slide one of the connected parts along the other end thus subjecting it to shear, the holes should be reamed so that the bolt shank fits snugly there in. The through bolts according to their usage may be known as machine bolts, carriage bolts, automobile bolts, eye bolts etc.

2. Tap Bolt

A tap bolt or screw differs from a bolt. It is screwed into a tapped hole of one of the parts to be fastened without the nut, as shown in Figure 7 (b).

3. Studs

A stud is a round bar threaded at both ends. One end of the stud is screwed into a tapped hole of the parts to be fastened, while the other end receives a nut on it, as shown in Figure 7 (c). Studs are chiefly used instead of tap bolts for securing various kinds of covers e.g. covers of engine and pump cylinders, valves, chests etc.
This is due to the fact that when tap bolts are unscrewed or replaced, they have a tendency to break the threads in the hole. This disadvantage is overcome by the use of studs.

4. Cap Screw

The cap screws are similar to tap bolts except that they are of small size and a variety of shapes of heads are available as shown in Figure 8.

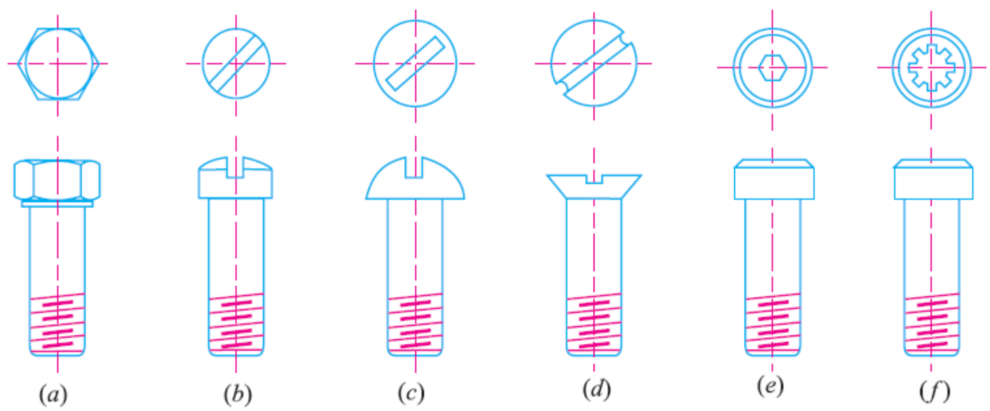


Figure 8: Types of cap screws: (a) Hexagonal head; (b) Fillister head; (c) Round head; (d) Flat head; (e) Hexagonal socket; (f) Fluted socket.

5. Set Screws

The set screws are shown in Figure 9. These are used to prevent relative motion between the two parts. A set screw is screwed through a threaded hole in one part so that its point (i.e. end of the screw) presses against the other part. This resists the relative motion between the two parts by means of friction between the point of the screw and one of the parts. They may be used instead of key to prevent relative motion between a hub and a shaft in light power transmission members. They may also be used in connection with a key, where they prevent relative axial motion of the shaft, key and hub assembly.

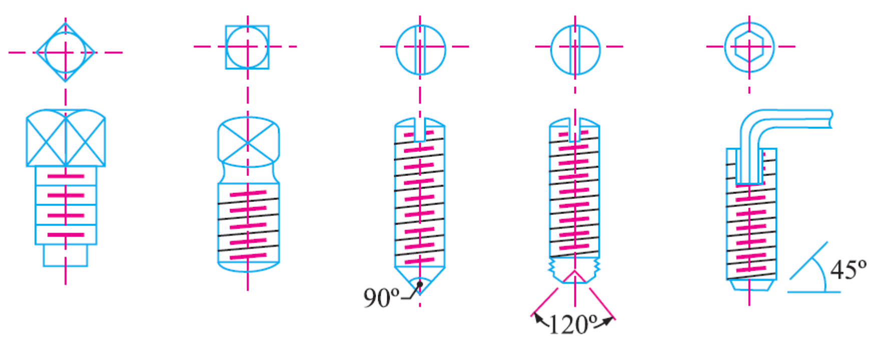


Figure 9: Set Screws.

Locking devices for Screw Joints

Ordinary thread fastenings, generally, remain tight under static loads, but many of these fastenings become loose under the action of variable loads or when machine is subjected to vibrations. The loosening of fastening is very dangerous and must be prevented. In order to prevent this, a large number of locking devices are available, some of which are discussed below:

1. Jam nut or lock nut.

Most common locking device is a **jam, lock or check nut**. It has about one-half to two-third thickness of the standard nut. The thin lock nut is first tightened down with ordinary force, and then the upper nut (i.e. thicker nut) is tightened down upon it, as shown in Figure 10 (a). The upper nut is then held tightly while the lower one is slackened back against it.
In slackening back the lock nut, a thin spanner is required which is difficult to find in many shops. Therefore to overcome this difficulty, a thin nut is placed on the top as shown in Figure 10 (b).
If the nuts are really tightened down as they should be, the upper nut carries a greater tensile load than the bottom one. Therefore, the top nut should be thicker one with a thin nut below it because it is desirable to put whole of the load on the thin nut. In order to overcome both the difficulties, both the nuts are made of the same thickness as shown in Figure 10 (c).

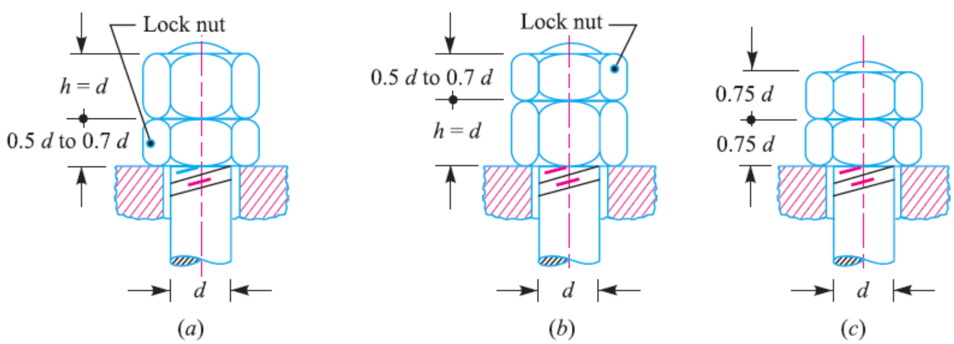


Figure 10: Jam Nut or Lock Nut

2. Castle nut

It consists of a hexagonal portion with a cylindrical upper part which is slotted in line with the centre of each face, as shown in Figure 11. The split pin passes through two slots in the nut and a hole in the bolt, so that a positive lock is obtained unless the pin shears. It is extensively used on jobs subjected to sudden shocks and considerable vibration such as in automobile industry.

3. Sawn nut

It has a slot sawed about half way through, as shown in Figure 12. After the nut is screwed down, the small screw is tightened which produces more friction between the nut and the bolt. This prevents the loosening of nut.

4. Penn, ring or grooved nut

It has a upper portion hexagonal and a lower part cylindrical as shown in Figure 13. It is largely used where bolts pass through connected pieces reasonably near their edges such as in marine type connecting rod ends. The bottom portion is cylindrical and is recessed to receive the tip of the locking set screw. The bolt hole requires counter-boring to receive the cylindrical portion of the nut. In order to prevent bruising of the latter by the case hardened tip of the set screw, it is recessed.

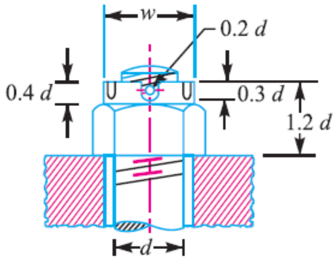


Figure 11: Castle nut

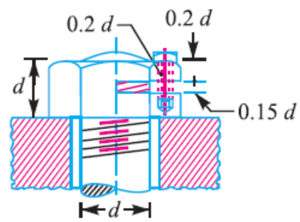


Figure 12: Sawn nut

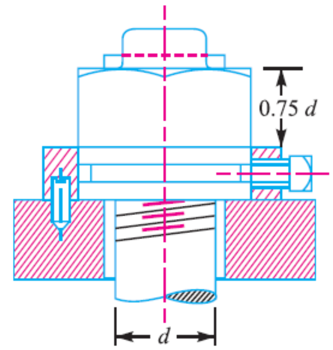


Figure 13: Penn or ring nut

5. Locking with pin

The nuts may be locked by means of a taper pin or cotter pin passing through the middle of the nut as shown in Figure 14 (a). But a split pin is often driven through the bolt above the nut, as shown in Figure 14 (b).

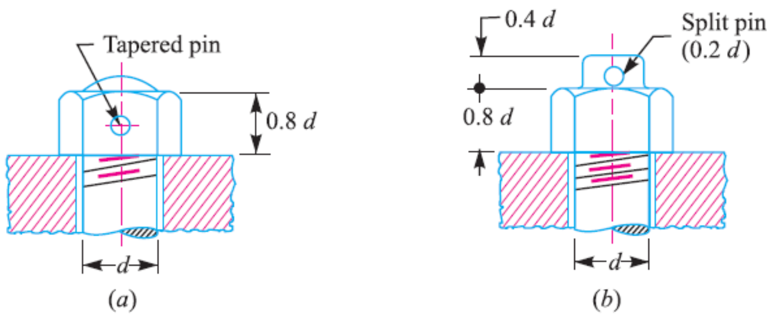


Figure 14: Locking with pin

6. Locking with plate

A form of stop plate or locking plate is shown in Figure 15. The nut can be adjusted and subsequently locked through angular intervals of 30° by using these plates.

7. Spring lock washer

A spring lock washer is shown in Figure 16. As the nut tightens the washer against the piece below, one edge of the washer is caused to dig itself into that piece, thus increasing the resistance so that the nut will not loosen so easily. There are many kinds of spring lock washers manufactured, some of which are fairly effective.

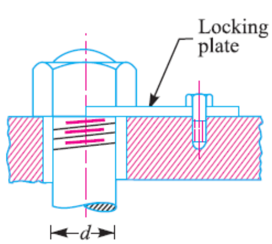


Figure 15: Locking with plate

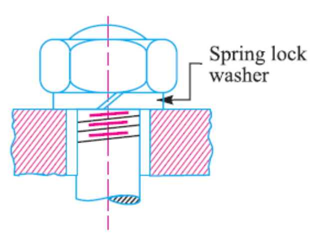


Figure 16: Locking with washer

Formation of Screwed Joint

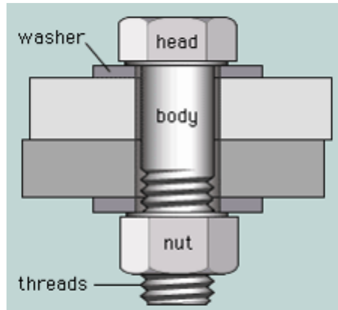


Figure 17: Formation of Screwed Joint

Screwing–up Torque

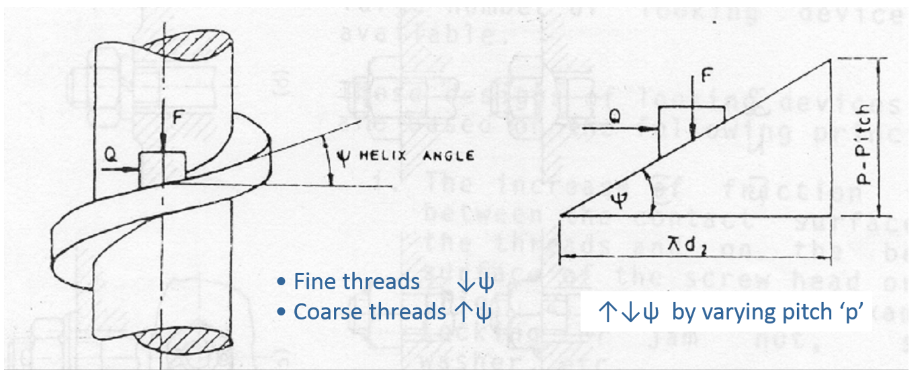


Figure 18: Basic Definition

The total torque required in screwing up a bolt or tightening a nut is the sum of the torque required to overcome the friction in the threads and the torque required to overcome the friction at the bearing surface of the screw head or nut.

$$T_{su} = T_{th} + T_{cf}$$

$$T_{su} = F\frac{d_p}{2}\Big[\tan(\lambda + \psi) + \mu\frac{d_m}{d_p}\Big]$$

where, $d_p =$ Pitch diameter
$\lambda =$ Angle of friction
$\psi =$ Helix angle
$\mu =$ Coefficient of friction
$d_m = \frac{(D+d)}{2} =$ Mean diameter of the bearing surface of the nut of screw head.

Screwing–off Torque

The torque required to unscrew a cap screw or a nut is found in a way similar to the screwing up torque, but the directions of the torque and friction are reversed.

$$T_{un} = F\frac{d_p}{2}\Big[\tan(\lambda - \psi) + \mu\frac{d_m}{d_p}\Big]$$

Self-locking condition

In the case of unscrewing a cap screw or a nut, the torque required when the collar friction is neglected:

$$T_{un} = F\frac{d_p}{2}\tan(\lambda - \psi)$$

When, $\lambda < \psi$ then torque T will be negative. In other words the nut will start moving without the application of any torque. When, $\lambda > \psi$ then the torque T will be positive. That means, an external torque should be applied to unscrew a nut such a condition is known as self-locking condition of screw threads.

Efficiency of Screw Threads

It is important to consider the efficiency of screw threads when the screws used for power transmission are considered. The efficiency of a thread is the ratio of the useful work done by the screw to the work input.

$$\eta = \frac{Work\ Output}{Work\ Input} = \frac{\tan\psi}{tan(\lambda + \psi) + \mu\frac{d_m}{d_p}}$$

Strength Calculations of Screw Joints under Static Loads

Under static loading conditions following stresses may be induced in screw fastenings.

1. Initial stresses due to screwing up forces.
2. Stresses due to external forces.
3. Stresses due to combination of screwing up and external forces.

Initial Stresses Due to Screwing Up Forces

Accurate determination of the initial stress components is extremely difficult and complicated. Therefore, following estimations have been obtained through experiments.

For fluid tight joints such as cylinder heads, initial tension:

$$F_i = 2840 * d$$

For other joints: ex. steam engine cylinder cover joints etc.

$$F_i = 1420 * d$$

Where, F_i is initial bolt tension in N
d is nominal diamter in mm

The following stresses are induced in a bolt, screw or stud when it is screwed up tightly.

1. Tensile stress due to stretching of bolt

Note: The small diameter bolts may fail during tightening, therefore bolts of smaller diameter (less than M 16 or M 18) are not permitted in making fluid tight joints.

If the bolt is not initially stressed, then the maximum safe axial load which may be applied to it is given by;

$$Permissible\ \ Stress(\sigma) = \frac{F}{\frac{\pi}{4}\left(\frac{d_p+d_m}{2}\right)^2}$$

d_p = Pitch diameter
d_m = Minor or Core diameter

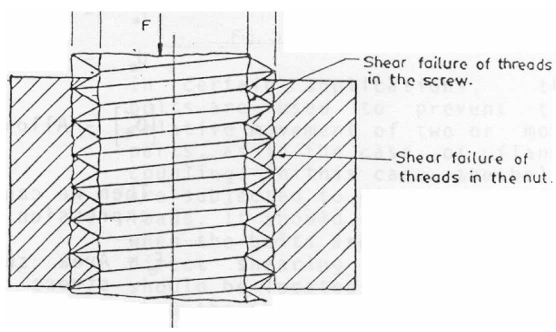


Figure 19: Tensile Failure of the Bolt

2. Torsional shear stress caused by the frictional resistance of the threads during its tightening

The torsional shear stress caused by the frictional resistance of the threads during its tightening may be obtained by using the torsion equation.

$$\frac{T}{J} = \frac{\tau}{r} \qquad \Longrightarrow \qquad \tau = \frac{T}{J} \times r = \frac{T}{\frac{\pi}{32}(d_m)^4} \times \frac{d_m}{2} = \frac{16T}{\pi (d_c)^3}$$

τ = Torsional shear stress
T = Torque applied
d_m = Minor or core diameter of the thread

It has been shown during experiments that due to repeated unscrewing and tightening of the nut, there is a gradual scoring of the threads, which increases the torsional twisting moment (T).

3. Shear stress across the threads

The average thread shearing stress for the **Screw** (τ_s) is obtained by using the relation:

$$\tau_s = \frac{F}{\pi d_m \times b \times n}$$

The average thread shearing stress for the **Nut**(τ_n) is:

$$\tau_n = \frac{F}{\pi d \times b \times n}$$

where $d =$ Major diameter
$b =$ Width of the thread section at the root

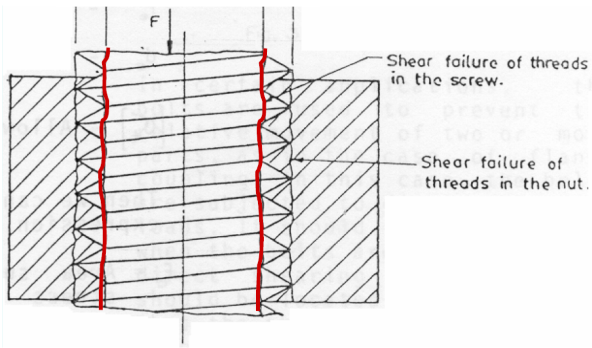


Figure 20: Average thread shearing stress for the **Screw** (τ_s)

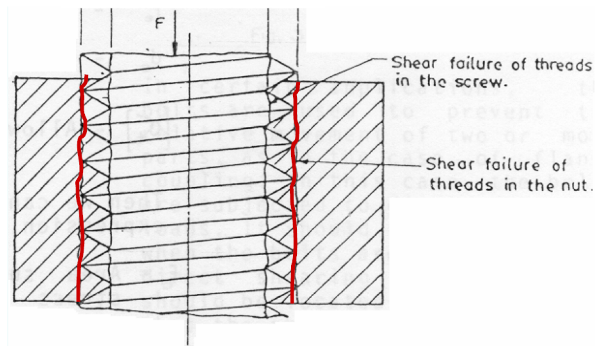


Figure 21: Average thread shearing stress for the **Nut** (τ_n)

4. Compression or crushing stress on threads

The compression or crushing stress between the threads (σ_c) may be obtained by using the relation:

$$\sigma_c = \frac{F}{\pi[d^2 - (d_m)^2]n}$$

where $d =$ Major diameter
$d_m =$ Minor diameter
$n =$ Number of threads in engagement

5. Bending stress (if the surfaces under the head or nut are not perfectly parallel to the bolt axis)

When the outside surfaces of the parts to be connected are not parallel to each other, then the bolt will be subjected to bending action. The bending stress (σ_b) induced in the shank of the bolt is given by:

$$\sigma_b = \frac{x.E}{2l}$$

where $x =$ Difference in height between the extreme corners of the nut or head
$l =$ Length of the shank of the bolt
$E =$ Young's modulus for the material of the bolt

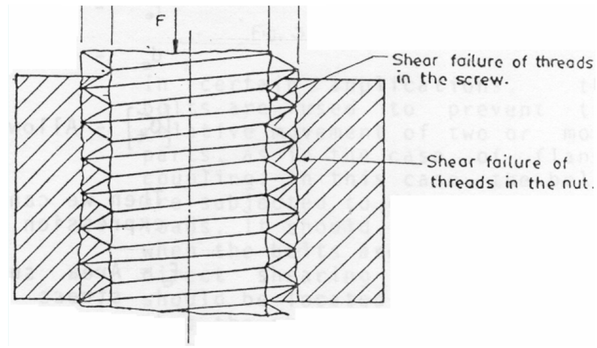


Figure 22: Crushing Failure of the Thread

Examples:

1. Determine the safe tensile load for a bolt of M 30, assuming a safe tensile stress of 42 MPa.
   Answer
   Given: $d = 30$ mm; $\sigma_t = 42$ MPa $= 42$ N/mm^2

   From Table 1 (coarse series), we find that the stress area i.e. cross-sectional area at the bottom of the thread corresponding to M 30 is 561 mm^2.
   $\therefore$ Safe tensile load = Stress area $\times \sigma_t = 561 \times 42 = 23\ 562$N $= 23.562$kN

   Note: In the above example, we have assumed that the bolt is not initially stressed.

2. Two machine parts are fastened together tightly by means of a 24 mm tap bolt. If the load tending to separate these parts is neglected, find the stress that is set up in the bolt by the initial tightening. **Answer**
   Given: $d = 24$mm
   From Table 1 (coarse series), we find that the core diameter of the thread corresponding to M 24 is $d_c = 20.32$mm

   Let σ_t = Stress set up in the bolt. We know that initial tension in the bolt, $P = 2840\ d = 2840 \times 24 = 68\ 160$N We also know that initial tension in the bolt (P),

$$68\ 160 = \frac{\pi}{4}(d_c)^2\sigma_t = \frac{\pi}{4}(20.30)^2\sigma_t = 324\sigma_t$$

$$\therefore \sigma_t = \frac{68\ 160}{324} = 210\text{N/mm}^2 = 210\text{ MPa}$$

Stresses Due to External Loads

1. Tensile stress

The bolts, studs and screws usually carry a load in the direction of the bolt axis which induces a tensile stress in the bolt.

Let d_m = Minor or core diameter of the thread.
σ_t = Permissible tensile stress for the bolt material.

We know that external load applied,

$$F = \frac{\pi}{4}(d_m)^4\sigma_t \qquad or \qquad d_m = \sqrt{\frac{4F}{\pi\sigma_t}}$$

<u>**Notes:**</u> **(a)** If the external load is taken up by a n number of bolts, then $F = \frac{\pi}{4}(d_m)^4\sigma_t \times n$

(b) In case the standard table is not available, then for coarse threads, $d_c = 0.84\ d$ where d is the nominal diameter of the bolt.

2. Shear stress

Sometimes, the bolts are used to prevent the relative movement of two or more parts, as in case of **flange coupling**, then the shear stress is induced in the bolts.
The shear stresses should be avoided as far as possible. It should be noted that when the bolts are subjected to direct shearing loads, they should be located in such a way that **the shearing load comes upon the body (i.e.shank) of the bolt** and <u>not upon the threaded portion</u>.
In some cases, the bolts may be relieved of shear load by using shear pins. When a number of bolts are used to share the shearing load, the finished bolts should be fitted to the reamed holes.

Let d = Nominal diameter of the bolt.
n = Number of bolts.

We know that external load applied,

$$F = \frac{\pi}{4}(d_m)^4\sigma_t \qquad or \qquad d_m = \sqrt{\frac{4F}{\pi\sigma_t}}$$

3. Combined tension and shear stress.

When the bolt is subjected to both tension and shear loads, as in case of coupling bolts or bearing, then the diameter of the shank of the bolt is obtained from the shear load and that of threaded part from the tensile load. A diameter slightly larger than that required for either shear or tension may be assumed and stresses due to combined load should be checked for the following principal stresses.

Maximum principle shear stress (τ_{max}),

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

Maximum principal tensile stress ($\sigma_{t(max)}$),

$$\sigma_{t(max)} = \frac{\sigma_t}{2} + \frac{1}{2}\sqrt{(\sigma_t)^2 + 4\tau^2}$$

These stresses should not exceed the safe permissible values of stresses.

Examples:

1. Two shafts are connected by means of a flange coupling to transmit torque of 25 Nm. The flanges of the coupling are fastened by four bolts of the same material at a radius of 30 mm. Find the size of the bolts if the allowable shear stress for the bolt material is 30 MPa.
   Answer
   Given: $T = 25\text{ Nm} = 25 \times 103\text{ N} - \text{mm}; n = 4;\ R_p = 30\text{ mm};\ \tau = 30\text{ MPa} = 30\text{ N/mm}^2$
   We know that the shearing load carried by flange coupling.
   $$P_s = \frac{T}{R_p} = \frac{25 \times 10^3}{30} = 833.3\text{N}$$
   Let d_c = Core diameter of the bolt.
   $\therefore$ Resisting load on the bolts,
   $$= \frac{\pi}{4}(d_c)^2\ \tau \times n = \frac{\pi}{4}(d_c)^2\ 30 \times 4\ =\ 94.25(d_c)^2$$
   By equating two equations, we get
   $$(d_c)^2 = \frac{833.3\text{ N}}{94.26\text{N/mm}^2} = 8.84\ mm^2 \qquad d_c = 2.97\text{ mm}$$
   From Table 1 (coarse series), we find that the standard core diameter of the bolt is 3.141 mm and the corresponding size of the bolt is M 4.

2. lever loaded safety valve has a diameter of 100 mm and the blow off pressure is 1.6 N/mm^2. The fulcrum of the lever is screwed into the cast iron body of the cover. Find the diameter of the threaded part of the fulcrum if the permissible tensile stress is limited to 50 MPa and the leverage ratio is 8.
   Answer
   Given: $D = 100$ mm; $p = 1.6\text{ N/mm}^2$; $\sigma_t = 50\text{ MPa} = 50\text{ N/mm}^2$ We know that the load acting on the valve,
   $$F = Area \times Pressure = \frac{\pi}{4} \times D^2 \times p = \frac{\pi}{4} \times 100^2 \times 1.6 = 12\,568\text{ N}$$
   Since the leverage is 8, therefore load at the end of the lever,
   $$W = \frac{12\,568}{8} = 1\,571\text{ N}$$
   $\therefore$ Load on the fulcrum,
   $$P = F - W = 12\,568 - 1\,571 = 10\,997\text{ N}$$
   Let d_c = Core diameter of the threaded part.
   $\therefore$ Resisting load on the threaded part of the fulcrum,
   $$P = \frac{\pi}{4}(d_c)^2\sigma_t = \frac{\pi}{4}(d_c)^2 50 = 39.3(d_c)^2$$
   By equating two equations, we get
   $$(d_c)^2 = \frac{10\,997\text{ N}}{39.3\text{ N/mm}^2} = 280\ mm^2 \qquad d_c = 16.7\text{ mm}$$
   From Table 1 (fine series), we find that the standard core diameter of the bolt is 18.376 mm and the corresponding size of the bolt is M 20 × 1.5.

Stresses Due to Combined Forces

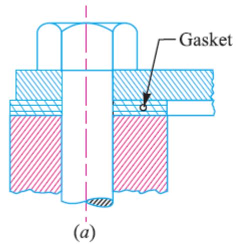


Figure 23: Stress due to Combined Forces

The resultant axial load on a bolt depends upon the following factors:

1. The initial tension due to tightening of the bolt.
2. The external load.
3. The relative elastic yielding (springiness) of the bolt and the connected members.

When the connected members are very yielding as compared with the bolt, which is a soft gasket, as shown in Figure 23(a), then the resultant load on the bolt is approximately equal to the sum of the initial tension and the external load.
On the other hand, if the bolt is very yielding as compared with the connected members, as shown in Figure 23(b), then the resultant load will be either the initial tension or the external load, whichever is greater.
The actual conditions usually lie between the two extremes. In order to determine the resultant axial load (P) on the bolt, the following equation may be used:

$$P = P_1 + \frac{1}{1+a} \times P_2 = P_1 + K.P_2 \qquad \ldots \left(Subtiuting \quad \frac{a}{1+a} = K \right)$$

where P_1 = Initial tension due to tightening of the bolt
P_2 = External load on the bolt
a = Ratio of elasticity of connected parts to the elasticity of bolt

For soft gaskets and large bolts, the value of a is high and the value of $\frac{a}{1+a}$ is approximately equal to unity, so that the resultant load is equal to the sum of the initial tension and the external load.
For hard gaskets or metal to metal contact surfaces and with small bolts, the value of a is small and the resultant load is mainly due to the initial tension (or external load, in rare case it is greater than initial tension).
The value of a may be estimated by the designer to obtain an approximate value for the resultant load. The values of $\frac{a}{1+a}$ (i.e. K) for various type of joints are shown in Table 1. The designer thus has control over the influence on the resultant load on a bolt by proportioning the sizes of the connected parts and bolts and by specifying initial tension in the bolt.

Table 1: Values of K for various types of joints.

Type of joint	$K = \frac{a}{1+a}$
Metal to metal joint with through bolts	0.00 to 0.10
Hard copper gasket with long through bolts	0.25 to 0.50
Soft copper gasket with long through bolts	0.50 to 0.75
Soft packing with through bolts	0.75 to 1.00
Soft packing with studs	1.00

Pre-loaded joints – closing load

Tightening torque related to pre−load and bolt diameter.The constant value, 0.2, remains approximately the same regardless of the bolt size.

$$T = 0.2F_1 d$$

For critical applications a torque wrench should be used to apply the proper pre−load.

1. Bolt force:

The load in the bolt is reduced by an amount which is a fraction of the total joint displacement δ and the bolt stiffness k_b.

2. Clamped Member Force:

The magnitude of the load in the clamped members is increased by an amount which is a function of the total joint displacement δ and the clamped member stiffness k_c.

3. Physical representation:

The load-deflection behaviour of the joint can be plotted on the following load-deflection graph. The graph is based on two fundamental principles of mechanics, namely compatibility and equilibrium.

Pre-loaded Joints: Opening Loads

1. Bolt force:

The load in the bolt is increased by an amount which is a function of the total joint displacement δ and the bolt stiffness k_b.

2. Clamped Member Force:

The magnitude of the load in the clamped members is decreased by an amount which is a function of the total joint displacement δ and the clamped member stiffness k_c.

3. Physical representation:

The load displacement behaviour of the joint can be plotted on the following load-deflection graph. Bolt joints can be failed by;

1. Bolt yielding.
2. Gapping the joint ($F_c = 0$).

Optimum Bolt Load

Higher F_i-Joint will fail by bolt yielding.
Small F_i-Joint will fail by gapping.

The optimum bolt pre−load can be considered to be that which gives an equal margin of safety between bolt yield and gapping of the joint. That is:

Where, $F_{i(yield)}$ is the bolt pre−load level at which the bolts yields under the maximum external opening load, F_e and F_i(just gap) is the bolt pre−load level at which the joint is on the point of gapping under the maximum external opening load F_e. The force at which the bolt will yield is given by: Where, A_t is the tensile stress area.

$F_{i(yield)}$ can be calculated by equating the bolt load, F_b to $F_{b(yield)}$. The preload force F_i(just gap) at which the joint is on the point of gapping under the maximum external force can be calculated by equating the load in the clamped members to zero, i.e. $F_c = 0$.

Design of a Nut

When a bolt and nut is made of **mild steel**, then **the effective height of nut** is **equal** to the **nominal diameter of the bolt**. If the nut is made of **weaker material than the bolt,** then the **height of nut should be larger**, such as 1.5 d for gun metal, 2 d for cast iron and 2.5 d for aluminium alloys (where d is the nominal diameter of the bolt).

In case cast iron or aluminium nut is used, then V-threads are permissible only for permanent fastenings, because **threads in these materials are damaged due to repeated screwing and unscrewing**. When these materials are to be used for parts frequently removed and fastened, a screw in steel bushing for cast iron and cast-in-bronze or Monel metal insert should be used for aluminium and should be drilled and tapped in place.
Monel metal is *a nickel-copper alloy with high tensile strength and resistance to corrosion.*

Bolted Joints under Eccentric Loading

There are many applications of the bolted joints which are subjected to eccentric loading such as a wall bracket, pillar crane, etc. The eccentric load may be

1. Parallel to the axis of the bolts.
2. Perpendicular to the axis of the bolts.
3. In the plane containing the bolts.

We will not discuss the above cases in detail. In case if you need it at some point, please refer the reference book.

Reference

1. R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd. − Chapter 11. Screwed Joints (pp. 377-430).

1. Standard Dimensions of Screw Threads

The design dimensions of I.S.O. screw threads for screws, bolts and nuts of coarse and fine series are shown in Table 11.1.

Table 1. Design dimensions of screw threads, bolts and nuts according to IS : 4218 (Part III) 1976 (Reaffirmed 1996)

Designation	*Pitch mm*	*Major or nominal diameter Nut and Bolt (d = D) mm*	*Effective or pitch diameter Nut and Bolt (d_p) mm*	*Minor or core diameter (d_c) mm*		*Depth of thread (bolt) mm*	*Stress area mm²*
				Bolt	*Nut*		
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
Coarse series							
M 0.4	0.1	0.400	0.335	0.277	0.292	0.061	0.074
M 0.6	0.15	0.600	0.503	0.416	0.438	0.092	0.166
M 0.8	0.2	0.800	0.670	0.555	0.584	0.123	0.295
M 1	0.25	1.000	0.838	0.693	0.729	0.153	0.460
M 1.2	0.25	1.200	1.038	0.893	0.929	0.158	0.732
M 1.4	0.3	1.400	1.205	1.032	1.075	0.184	0.983
M 1.6	0.35	1.600	1.373	1.171	1.221	0.215	1.27
M 1.8	0.35	1.800	1.573	1.371	1.421	0.215	1.70
M 2	0.4	2.000	1.740	1.509	1.567	0.245	2.07
M 2.2	0.45	2.200	1.908	1.648	1.713	0.276	2.48
M 2.5	0.45	2.500	2.208	1.948	2.013	0.276	3.39
M 3	0.5	3.000	2.675	2.387	2.459	0.307	5.03
M 3.5	0.6	3.500	3.110	2.764	2.850	0.368	6.78
M 4	0.7	4.000	3.545	3.141	3.242	0.429	8.78
M 4.5	0.75	4.500	4.013	3.580	3.688	0.460	11.3
M 5	0.8	5.000	4.480	4.019	4.134	0.491	14.2
M 6	1	6.000	5.350	4.773	4.918	0.613	20.1

Designation	Pitch (mm)	Major or nominal dia: ($d = D$)	Pitch dia: (d_p) mm	Minor or core (d_c) mm Bolt	Nut	Depth of thread (bolt) mm	Stress Area mm^2
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
M 7	1	7.000	6.350	5.773	5.918	0.613	28.9
M 8	1.25	8.000	7.188	6.466	6.647	0.767	36.6
M 10	1.5	10.000	9.026	8.160	8.876	0.920	58.3
M 12	1.75	12.000	10.863	9.858	10.106	1.074	84.0
M 14	2	14.000	12.701	11.546	11.835	1.227	115
M 16	2	16.000	14.701	13.546	13.835	1.227	157
M 18	2.5	18.000	16.376	14.933	15.294	1.534	192
M 20	2.5	20.000	18.376	16.933	17.294	1.534	245
M 22	2.5	22.000	20.376	18.933	19.294	1.534	303
M 24	3	24.000	22.051	20.320	20.752	1.840	353
M 27	3	27.000	25.051	23.320	23.752	1.840	459
M 30	3.5	30.000	27.727	25.706	26.211	2.147	561
M 33	3.5	33.000	30.727	28.706	29.211	2.147	694
M 36	4	36.000	33.402	31.093	31.670	2.454	817
M 39	4	39.000	36.402	34.093	34.670	2.454	976
M 42	4.5	42.000	39.077	36.416	37.129	2.760	1104
M 45	4.5	45.000	42.077	39.416	40.129	2.760	1300
M 48	5	48.000	44.752	41.795	42.587	3.067	1465
M 52	5	52.000	48.752	45.795	46.587	3.067	1755
M 56	5.5	56.000	52.428	49.177	50.046	3.067	2022
M 60	5.5	60.000	56.428	53.177	54.046	3.374	2360
Fine series							
M 8 × 1	1	8.000	7.350	6.773	6.918	0.613	39.2
M 10 × 1.25	1.25	10.000	9.188	8.466	8.647	0.767	61.6
M 12 × 1.25	1.25	12.000	11.184	10.466	10.647	0.767	92.1
M 14 × 1.5	1.5	14.000	13.026	12.160	12.376	0.920	125
M 16 × 1.5	1.5	16.000	15.026	14.160	14.376	0.920	167
M 18 × 1.5	1.5	18.000	17.026	16.160	16.376	0.920	216
M 20 × 1.5	1.5	20.000	19.026	18.160	18.376	0.920	272
M 22 × 1.5	1.5	22.000	21.026	20.160	20.376	0.920	333
M 24 × 2	2	24.000	22.701	21.546	21.835	1.227	384
M 27 × 2	2	27.000	25.701	24.546	24.835	1.227	496
M 30 × 2	2	30.000	28.701	27.546	27.835	1.227	621
M 33 × 2	2	33.000	31.701	30.546	30.835	1.227	761
M 36 × 3	3	36.000	34.051	32.319	32.752	1.840	865
M 39 × 3	3	39.000	37.051	35.319	35.752	1.840	1028

Note : In case the table is not available, then the core diameter (d_c) may be taken as 0.84 d, where d is the major diameter.

Fastener Type Chart

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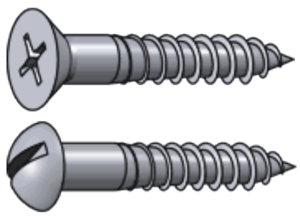
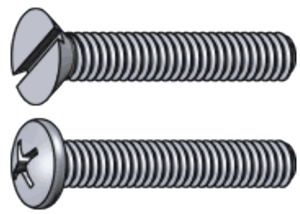
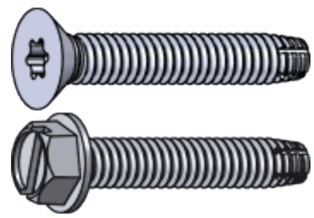
Fastener Categories



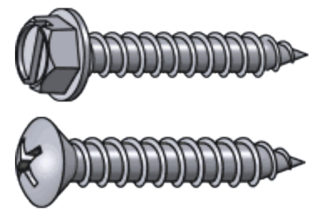
Wood Screws
Screws with a smooth shank and tapered point for use in wood. Abbreviated WS.



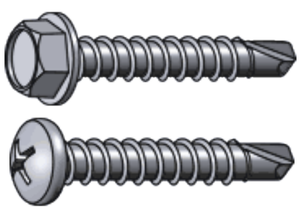
Machine Screws
Screws with threads for use with a nut or tapped hole. Abbreviated MS.



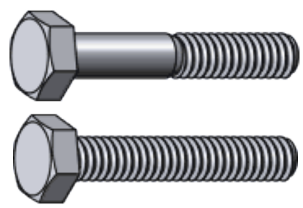
Thread Cutting Machine Screws
Machine screws with a thread cutting (self tapping) point.



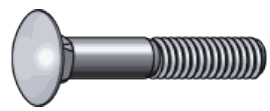
Sheet Metal Screws
Fully threaded screws with a point for use in sheet metal. Abbreviated SMS.



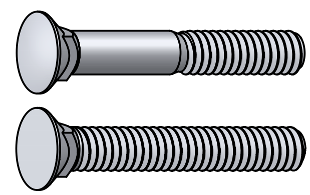
Self Drilling SMS
A sheet metal screw with a self drilling point.



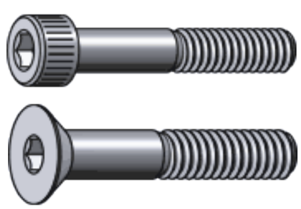
Hex Bolts
Bolts with a hexagonal head with threads for use with a nut or tapped hole. Abbreviated HHMB or HXBT.



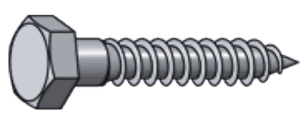
Carriage Bolts
Bolts with a smooth rounded head that has a small square section underneath.



Plow Bolts
Similar to carriage bolts but used for attaching the cutting edge of a plow to the plow blade.



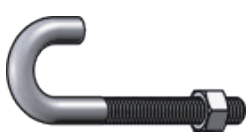
Socket Screws
Socket screws, also known as Allen Head, are fastened with a hex Allen wrench.



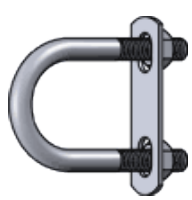
Lag Bolts
Bolts with a wood thread and pointed tip. Abbreviated Lag.

Eye Bolts
A bolt with a circular ring on the head end. Used for attaching a rope or chain.

Eye Lags
Similar to an eye bolt but with wood threads instead of machine thread.



J-Bolts
J shaped bolts are used for tie-downs or as an open eye bolt.



U-Bolts
Bolts in U shape for attaching to pipe or other round surfaces. Also available with a square bend.

Shoulder Bolts
Shoulder bolts (also known as stripper bolts) are used to create a pivot point.

Elevator Bolts
Elevator bolts are often used in conveyor systems. They have a large, flat head.

Fastener Type Chart

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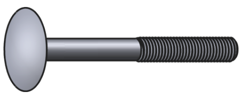
Fastener Categories (continued)

Cotter Pins
Cotter or split pins have two tines which are bent apart to hold them in place.

Rivets
Used to join sheets of metal. During installation the rivet body is deformed to permanently lock in place. Blind rivets can be installed without access to the back side of the material.

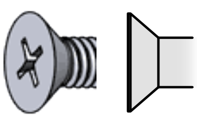
Hanger Bolts
Hanger bolts have wood thread on one end and machine thread on the other end

Set Screws
Machine screws with no head for screwing all the way into threaded holes.

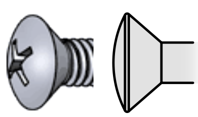


Timber Bolts
Machine threaded fasteners with a wide domed head. The head has fins underneath that prevent the bolt from spinning during installation. Typically used in wood.

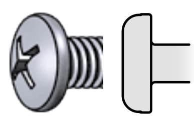
Head Styles



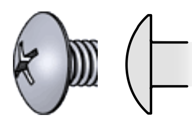
Flat
A countersunk head with a flat top.
Abbreviated FH



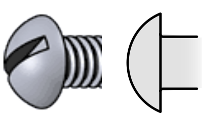
Oval
A countersunk head with a rounded top.
Abbreviated OH or OV



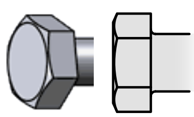
Pan
A slightly rounded head with short vertical sides.
Abbreviated PN



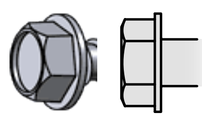
Truss
An extra wide head with a rounded top.



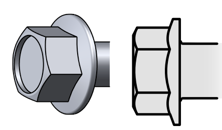
Round
A domed head.
Abbreviated RH



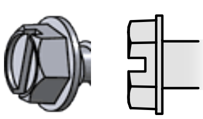
Hex
A hexagonal head
Abbreviated HH or HX



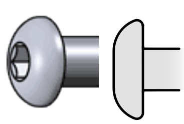
Hex Washer
A hex head with built in washer.



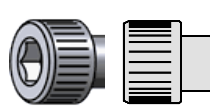
Hex Flange
A hex head with built in flange.



Slotted Hex Washer
A hex head with built in washer and a slot.



Button
A low-profile rounded head using a socket drive.



Socket Cap
A small cylindrical head using a socket drive.

Fastener Type Chart

Bolt Depot.com
fastener shopping made easy

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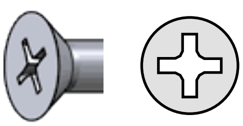
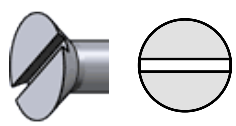
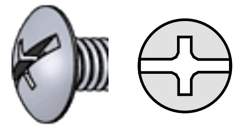
Drive Types



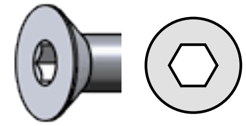
Phillips and Frearson
An X-shaped drive.
Abbreviated PH.



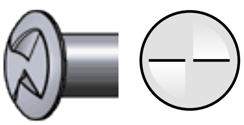
Slotted
A slot in the head.
Abbreviated SL.



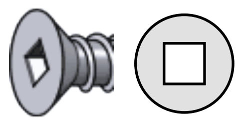
Combination
A combination of slotted and Phillips drives.
Abbreviated combo.



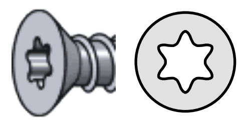
Socket, Hex or Allen
A hexagonal hole for use with an Allen wrench.



One Way
Installs with a normal slotted driver but can not be removed without special tools.

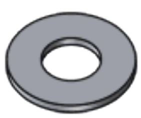


Square
Also known as Robertson drive.
Abbreviated SQ or SD.

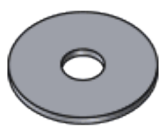


Star
A six-pointed star pattern, specifically designed to prevent cam-out and stripped heads.

Washer Types



Flat
A flat washer, used to distribute load. Available in SAE, USS and other patterns.



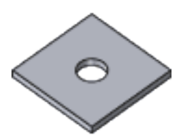
Fender
An oversize flat washer used to further distribute load especially on soft materials.

Finishing
A washer used to obtain a 'finished' look. Usually used with oval head screws.

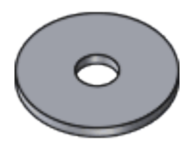
Split Lock
The most common style of washer used to prevent nuts and bolts from backing out.

External Tooth Lock
A washer with external 'teeth'. Used to prevent nuts and bolts from backing out.

Internal Tooth Lock
A washer with internal 'teeth'. Used to prevent nuts and bolts from backing out.



Square
A square shaped washer.



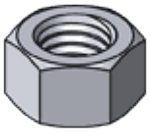
Dock
Dock washers have a larger outside diameter and are thicker than standard.

Ogee
Thick, large diameter, cast iron washers with a curved or sculpted appearance. Typically used in dock and wood construction.

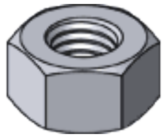
Fastener Type Chart

www.boltdepot.com/tools

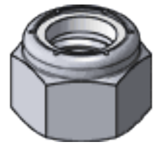
Nut Types



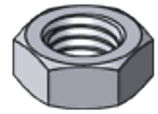
Hex
A six sided nut. Also referred to as a Finished Hex Nut.



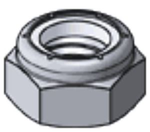
Heavy Hex
A heavier pattern version of a standard hex nut.



Nylon Insert Lock
A nut with a nylon insert to prevent backing off. Also referred to as a Nylock.

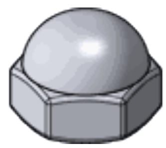


Jam
A hex nut with a reduced height.



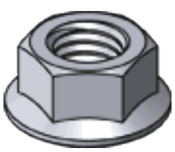
Nylon Insert Jam Lock
A nylock nut with a reduced height.

Wing
A nut with ‘wings’ for hand tightening.

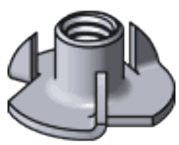


Cap
A nut with a domed top over the end of the fastener.

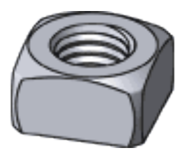
Acorn
Acorn nuts are a high crown type of cap nut, used for appearance.



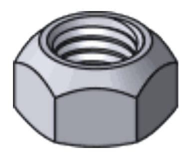
Flange
A nut with a built in washer like flange.



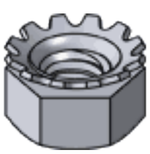
Tee
A nut designed to be driven into wood to create a threaded hole.



Square
A four sided nut.

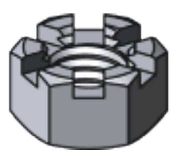


Prevailing Torque Lock
A non-reversible lock nut used for high temperature applications.

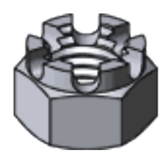


K-Lock or Kep
A nut with an attached free-spinning external tooth lock washer.

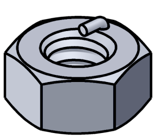
Coupling
Coupling nuts are long nuts used to connect pieces of threaded rod or other male fasteners.



Slotted
Slotted nuts are used in conjunction with a cotter pin on drilled shank fasteners to prevent loosening.



Castle
Castle nuts are used in conjunction with a cotter pin on drilled shank fasteners to prevent loosening.



Pin Lock
A nut that does not require an high installation torque and can be installed and removed without thread damage.

Fastener Type Chart

Bolt Depot.com fastener shopping made easy

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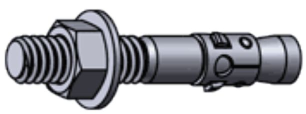
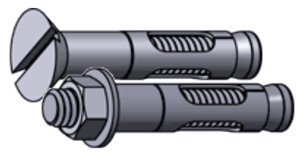
Anchoring Products



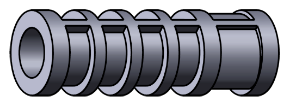
Stud Anchors

A.k.a. Wedge Anchors. One piece expansion bolts for heavy duty fastening into stone or solid concrete.



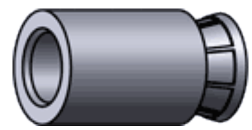
Sleeve Anchors

Heavy duty masonry anchors. Does not require a solid base material for installation.



Lag Shields

Medium dury anchors for use in concrete, brick or mortar. Use with a lag bolt.

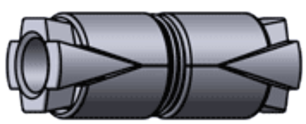


Machine Screw Anchors

A two-piece machine thread anchor for use in stone, brick, or concrete.

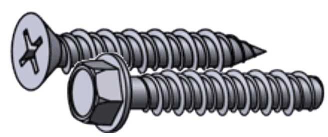
Drop-in Anchors

A heavy duty machine thread anchor for concrete or stone.



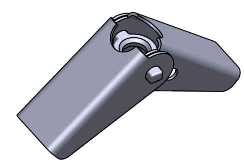
Double Expansion Sleeves

Expansion anchor for masonry that ensures contact along the length of the hole.



Concrete Screws

Used in concrete, brick or block. A quick and easy way to fasten in light to medium duty applications

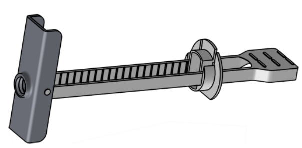


Spring Toggle Wings

Non-removable fasteners that expand behind the material, e.g. inside a wall, for a secure grip.

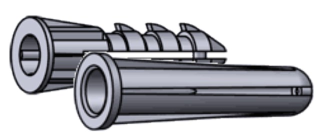
Plastic Toggle

When these anchors are driven in they expand inside the hole for a secure grip. Drill hole the same size as the anchor. Non-removable.



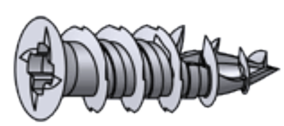
Kaptoggle®

A non removable anchor commonly used for hollow spaces such as drywall and masonry block.



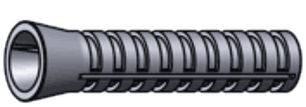
Conical Anchors

Plastic anchors used with sheet metal screws. Can be used in most materials.



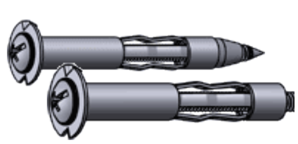
Self Drilling Drywall Anchors

Quick-install plastic anchors used in drywall with sheet metal screws.



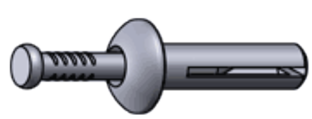
Wood Screw Anchors

This anchor is made of lead and can be used with wood screws or sheet metal screws.



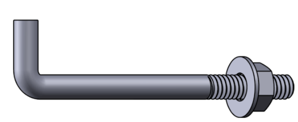
Hollow Wall Anchors

A.k.a. Molly Bolts. Used for light duty anchoring in drywall or other hollow walls.



Nail Drive Anchors

Non removable anchors that expand inside the hole when the nail like pin is driven.



Anchor Bolts

L shaped, machine threaded anchors. Typically embedded in concrete when it is poured.

Fastener Type Chart

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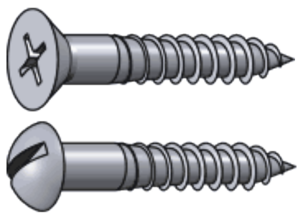
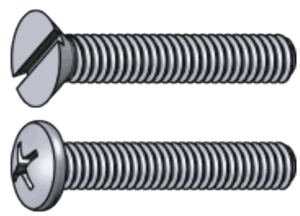
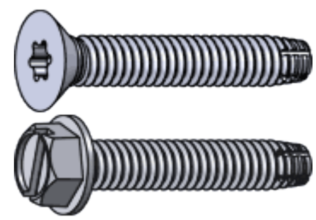
Fastener Categories



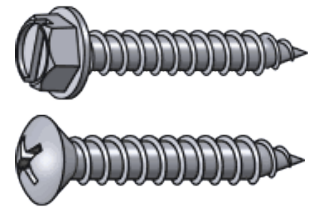
Wood Screws
Screws with a smooth shank and tapered point for use in wood. Abbreviated WS.



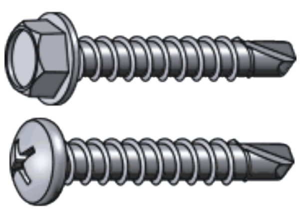
Machine Screws
Screws with threads for use with a nut or tapped hole. Abbreviated MS.



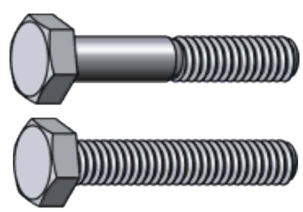
Thread Cutting Machine Screws
Machine screws with a thread cutting (self tapping) point.



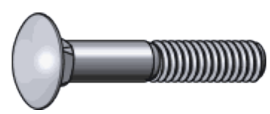
Sheet Metal Screws
Fully threaded screws with a point for use in sheet metal. Abbreviated SMS.



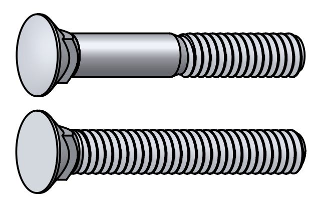
Self Drilling SMS
A sheet metal screw with a self drilling point.



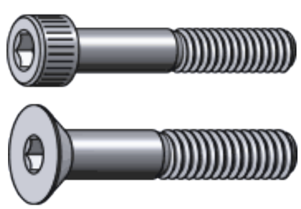
Hex Bolts
Bolts with a hexagonal head with threads for use with a nut or tapped hole. Abbreviated HHMB or HXBT.



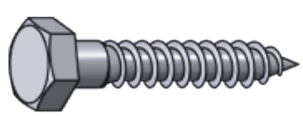
Carriage Bolts
Bolts with a smooth rounded head that has a small square section underneath.



Plow Bolts
Similar to carriage bolts but used for attaching the cutting edge of a plow to the plow blade.



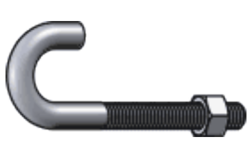
Socket Screws
Socket screws, also known as Allen Head, are fastened with a hex Allen wrench.



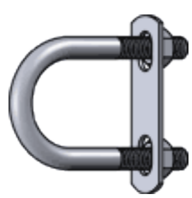
Lag Bolts
Bolts with a wood thread and pointed tip. Abbreviated Lag.

Eye Bolts
A bolt with a circular ring on the head end. Used for attaching a rope or chain.

Eye Lags
Similar to an eye bolt but with wood threads instead of machine thread.



J-Bolts
J shaped bolts are used for tie-downs or as an open eye bolt.



U-Bolts
Bolts in U shape for attaching to pipe or other round surfaces. Also available with a square bend.

Shoulder Bolts
Shoulder bolts (also known as stripper bolts) are used to create a pivot point.

Elevator Bolts
Elevator bolts are often used in conveyor systems. They have a large, flat head.

Fastener Type Chart

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Fastener Categories (continued)

Cotter Pins

Cotter or split pins have two tines which are bent apart to hold them in place.

Rivets

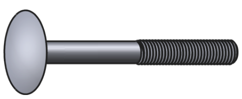
Used to join sheets of metal. During installation the rivet body is deformed to permanently lock in place. Blind rivets can be installed without access to the back side of the material.

Hanger Bolts

Hanger bolts have wood thread on one end and machine thread on the other end

Set Screws

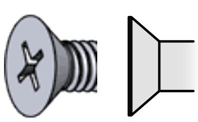
Machine screws with no head for screwing all the way into threaded holes.



Timber Bolts

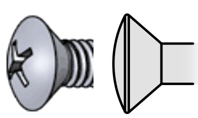
Machine threaded fasteners with a wide domed head. The head has fins underneath that prevent the bolt from spinning during installation. Typically used in wood.

Head Styles



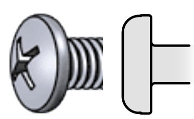
Flat

A countersunk head with a flat top.
Abbreviated FH



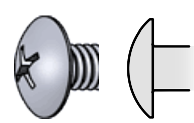
Oval

A countersunk head with a rounded top.
Abbreviated OH or OV



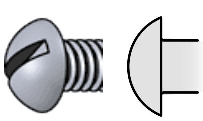
Pan

A slightly rounded head with short vertical sides.
Abbreviated PN



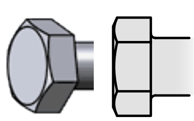
Truss

An extra wide head with a rounded top.



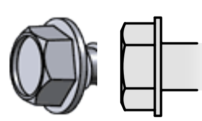
Round

A domed head.
Abbreviated RH



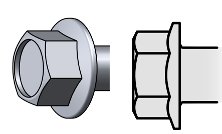
Hex

A hexagonal head
Abbreviated HH or HX



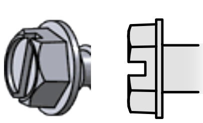
Hex Washer

A hex head with built in washer.



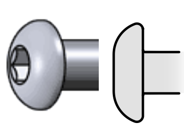
Hex Flange

A hex head with built in flange.



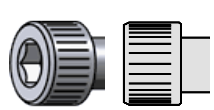
Slotted Hex Washer

A hex head with built in washer and a slot.



Button

A low-profile rounded head using a socket drive.



Socket Cap

A small cylindrical head using a socket drive.

Fastener Type Chart

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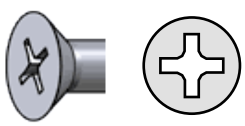
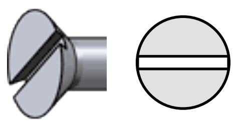
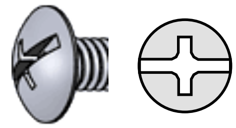
Drive Types



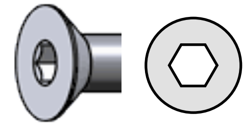
Phillips and Frearson
An X-shaped drive.
Abbreviated PH.



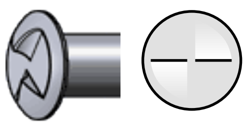
Slotted
A slot in the head.
Abbreviated SL.



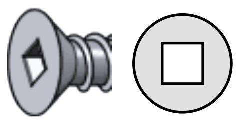
Combination
A combination of slotted and Phillips drives.
Abbreviated combo.



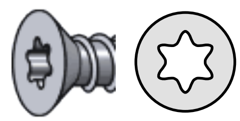
Socket, Hex or Allen
A hexagonal hole for use with an Allen wrench.



One Way
Installs with a normal slotted driver but can not be removed without special tools.

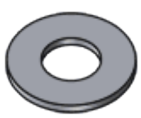


Square
Also known as Robertson drive.
Abbreviated SQ or SD.

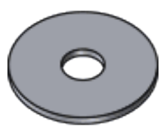


Star
A six-pointed star pattern, specifically designed to prevent cam-out and stripped heads.

Washer Types



Flat
A flat washer, used to distribute load. Available in SAE, USS and other patterns.



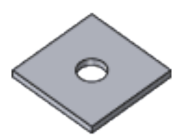
Fender
An oversize flat washer used to further distribute load especially on soft materials.

Finishing
A washer used to obtain a 'finished' look. Usually used with oval head screws.

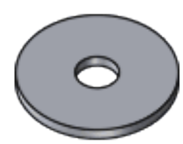
Split Lock
The most common style of washer used to prevent nuts and bolts from backing out.

External Tooth Lock
A washer with external 'teeth'. Used to prevent nuts and bolts from backing out.

Internal Tooth Lock
A washer with internal 'teeth'. Used to prevent nuts and bolts from backing out.



Square
A square shaped washer.



Dock
Dock washers have a larger outside diameter and are thicker than standard.

Ogee
Thick, large diameter, cast iron washers with a curved or sculpted appearance. Typically used in dock and wood construction.

Fastener Type Chart

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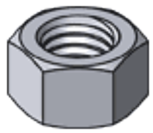
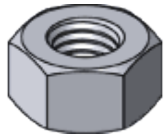
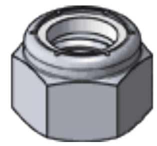
Nut Types



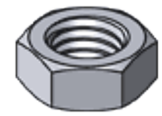
Hex
A six sided nut. Also referred to as a Finished Hex Nut.



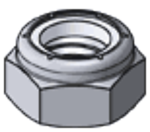
Heavy Hex
A heavier pattern version of a standard hex nut.



Nylon Insert Lock
A nut with a nylon insert to prevent backing off. Also referred to as a Nylock.

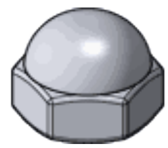


Jam
A hex nut with a reduced height.



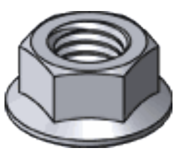
Nylon Insert Jam Lock
A nylock nut with a reduced height.

Wing
A nut with 'wings' for hand tightening.

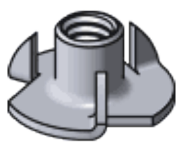


Cap
A nut with a domed top over the end of the fastener.

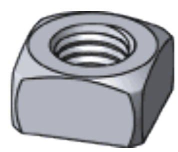
Acorn
Acorn nuts are a high crown type of cap nut, used for appearance.



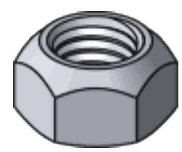
Flange
A nut with a built in washer like flange.



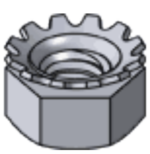
Tee
A nut designed to be driven into wood to create a threaded hole.



Square
A four sided nut.

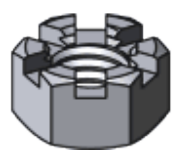


Prevailing Torque Lock
A non-reversible lock nut used for high temperature applications.

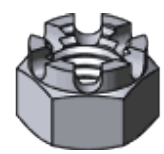


K-Lock or Kep
A nut with an attached free-spinning external tooth lock washer.

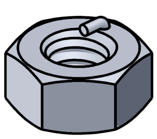
Coupling
Coupling nuts are long nuts used to connect pieces of threaded rod or other male fasteners.



Slotted
Slotted nuts are used in conjunction with a cotter pin on drilled shank fasteners to prevent loosening.



Castle
Castle nuts are used in conjunction with a cotter pin on drilled shank fasteners to prevent loosening.



Pin Lock
A nut that does not require an high installation torque and can be installed and removed without thread damage.

Fastener Type Chart

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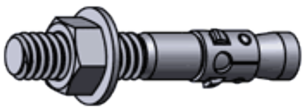
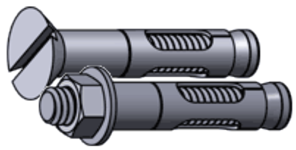
Anchoring Products



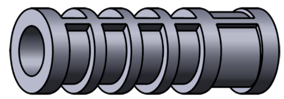
Stud Anchors

A.k.a. Wedge Anchors. One piece expansion bolts for heavy duty fastening into stone or solid concrete.



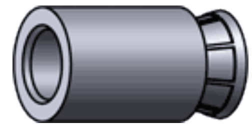
Sleeve Anchors

Heavy duty masonry anchors. Does not require a solid base material for installation.



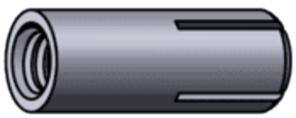
Lag Shields

Medium dury anchors for use in concrete, brick or mortar. Use with a lag bolt.



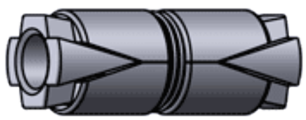
Machine Screw Anchors

A two-piece machine thread anchor for use in stone, brick, or concrete.



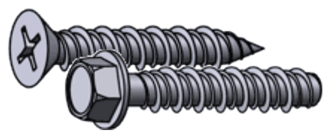
Drop-in Anchors

A heavy duty machine thread anchor for concrete or stone.



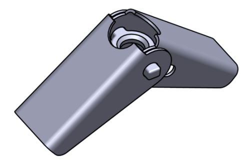
Double Expansion Sleeves

Expansion anchor for masonry that ensures contact along the length of the hole.



Concrete Screws

Used in concrete, brick or block. A quick and easy way to fasten in light to medium duty applications

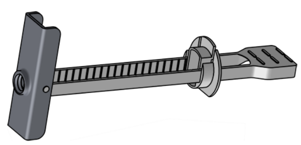


Spring Toggle Wings

Non-removable fasteners that expand behind the material, e.g. inside a wall, for a secure grip.

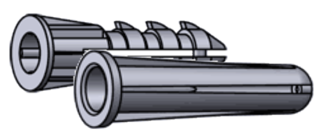
Plastic Toggle

When these anchors are driven in they expand inside the hole for a secure grip. Drill hole the same size as the anchor. Non-removable.



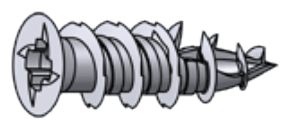
Kaptoggle®

A non removable anchor commonly used for hollow spaces such as drywall and masonry block.



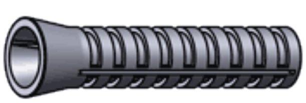
Conical Anchors

Plastic anchors used with sheet metal screws. Can be used in most materials.



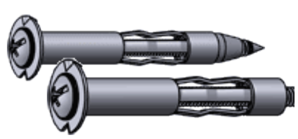
Self Drilling Drywall Anchors

Quick-install plastic anchors used in drywall with sheet metal screws.



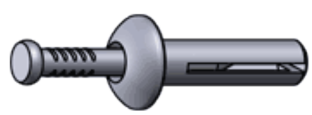
Wood Screw Anchors

This anchor is made of lead and can be used with wood screws or sheet metal screws.



Hollow Wall Anchors

A.k.a. Molly Bolts. Used for light duty anchoring in drywall or other hollow walls.



Nail Drive Anchors

Non removable anchors that expand inside the hole when the nail like pin is driven.

Anchor Bolts

L shaped, machine threaded anchors. Typically embedded in concrete when it is poured.

ME3531: Solid Mechanics and Mechanical Design – S1:2019
Lecture 9–06 May 2019 Thilina H. Weerakkody
Design of Power Screws

Material Covered

1. Introduction
2. Types of Screw Threads used for Power Screws.
3. Multiple Threads.
4. Torque Required to Raise Load by Square Threaded Screws.
5. Torque Required to Lower Load by Square Threaded Screws.
6. Efficiency of Square Threaded Screws.
7. Maximum Efficiency of Square Threaded Screws.
8. Efficiency vs. Helix Angle.
9. Overhauling and Selflocking Screws.
10. Efficiency of Self Locking Screws.
11. Stresses in Power Screws.
12. Design of Screw Jack and other applications of power screw threads

Course Text:

R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd. – Chapter 17. Power Screws (pp. 624-676).

ME3531: Solid Mechanics and Mechanical Design
Design of Powered Screws

Thilina H. Weerakkody
Department of Mechanical Engineering
SLIIT-Sri Lanka

Year 3 −Semester 1, 2019

Outline

- Introduction
- Types of Screw Threads used for Power Screws.
- Multiple Threads.
- Torque Required to Raise Load by Square Threaded Screws.
- Torque Required to Lower Load by Square Threaded Screws.
- Efficiency of Square Threaded Screws.
- Maximum Efficiency of Square Threaded Screws.
- Efficiency vs. Helix Angle.
- Overhauling and Selflocking Screws.
- Efficiency of Self Locking Screws.
- Stresses in Power Screws.
- Design of Screw Jack.

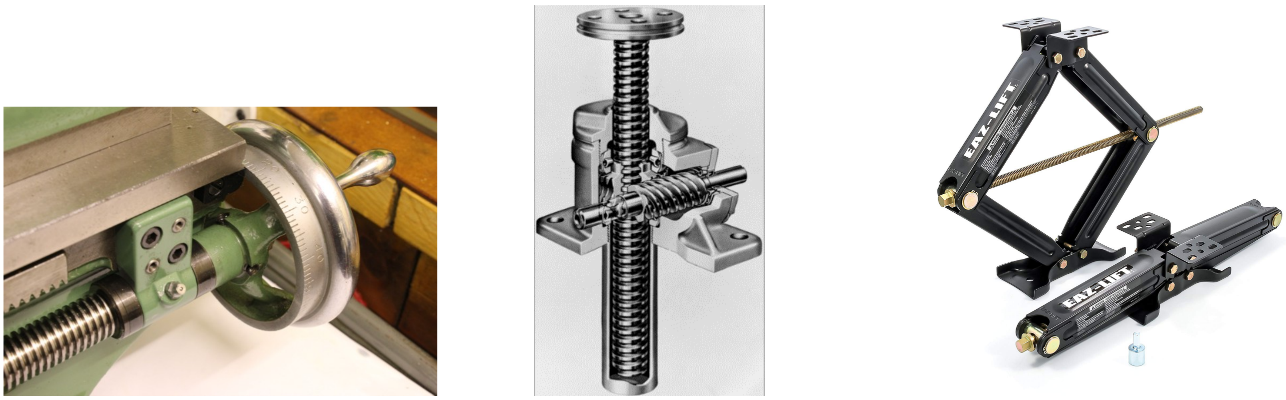
Intended Learning Outcomes

Upon completing of lecture, the student should be able to:

- Differentiate the Square, Acme and Buttress threads.
- calculate torque and efficiency of a square thread power screw.
- Identify the stresses affecting on power screws.
- Design a screw jack based on given data.

1 Introduction

The power screws *(also known as **translation screws**)* are used to convert rotary motion into translatory motion. For example, in the case of the ***lead screw of lathe,*** the rotary motion is available but the tool has to be advanced in the direction of the cut against the cutting resistance of the material (refer Figure 1a). In case of screw jack, a small force applied in the horizontal plane is used to raise or lower a large load (refer Figure 1b and Figure 1c). Power screws are also used in vices, testing machines, presses, etc.



(a) Lead Screw of the Lathe" (b) Screw Jack (c) Lifting screw jack

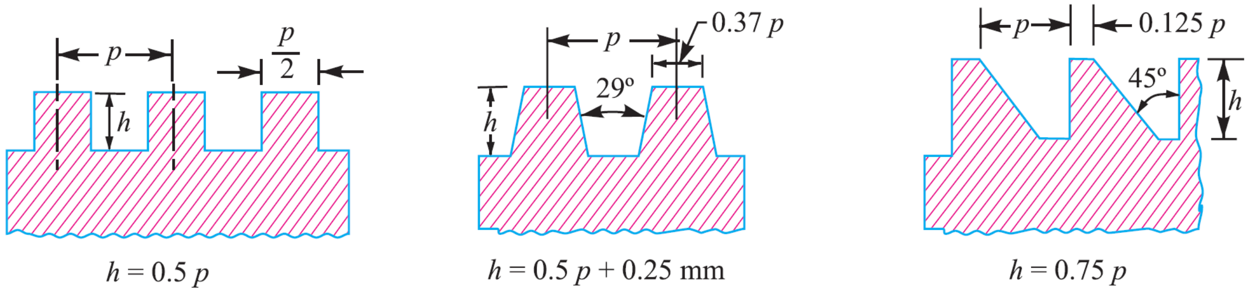
Figure 1: Power screw applications

In most of the power screws, the **nut has axial motion** against the resisting axial force **while the screw rotates in its bearings**. In some screws, **the screw rotates and moves axially** against the resisting force **while the nut is stationary** and in others **the nut rotates** while **the screw moves axially with no rotation**.

2 Types of Screw Threads used for Power Screws

1. Square Thread

A square thread, as shown in Figure 2a, is adapted for the transmission of power in either direction. This thread results in maximum efficiency and minimum radial or bursting pressure on the nut. It is difficult to cut with taps and dies. It is usually cut on a lathe with a single point tool and it can not be easily compensated for wear. The square threads are employed in screw jacks, presses and clamping devices.



(a) Square Thread (b) ACME Thread (c) Buttress Thread

Figure 2: Types of Screw Threads used for Power Screws

2. Acme or trapezoidal thread

An acme or trapezoidal thread, as shown in Figure 2b, is a modification of square thread. The slight slope given to its sides lowers the efficiency slightly than square thread and it also introduce some bursting pressure on the nut, but increases its area in shear. It is used where a split nut is required and where provision is made to take up wear as in the lead screw of a lathe. Wear may be taken up by means of an adjustable split nut. An acme thread may be cut by means of dies and hence it is more easily manufactured than square thread.

3. Buttress thread

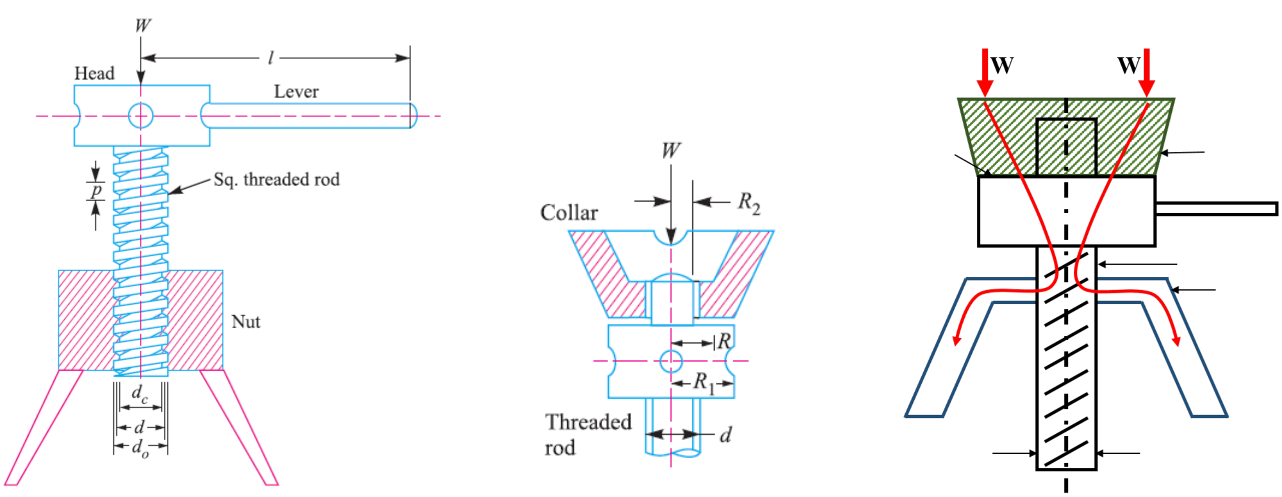
A buttress thread, as shown in Figure 2c , is used when large forces act along the screw axis in one direction only. This thread combines the higher efficiency of square thread and the ease of cutting and the adaptability to a split nut of acme thread. It is stronger than other threads because of greater thickness at the base of the thread. The buttress thread has limited use for power transmission. It is employed as the thread for light jack screws and vices.

3 Multiple Threads

The power screws with multiple threads such as double, triple etc. are employed when it is desired to secure a large lead with fine threads or high efficiency. Such type of threads are usually found in **high speed actuators**.

4 Torque Required to Raise Load by Square Threaded Screws

The torque required to raise a load by means of square threaded screw may be determined by considering a screw jack as shown in Figure 3a. The load to be raised or lowered is placed on the head of the square threaded rod which is rotated by the application of an effort at the end of lever for lifting or lowering the load.



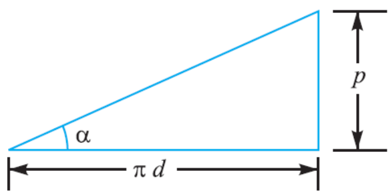
(a) Screw Jack Section view

(b) Thrust bearing collar

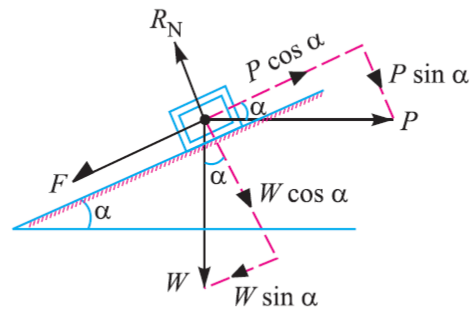
(c) Thrust bearing collar section

Figure 3: Screw Jack and Thrust bearing

A little consideration will show that if one complete turn of a screw thread be imagined to be unwound, from the body of the screw and developed, it will form an inclined plane as shown in Figure 4a.



(a) Development of a Screw



(b) Forces acting on the screw

Figure 4

Let $p =$ Pitch of the screw

$d =$ Mean diameter of the screw

$\alpha =$ Helix angle

$P =$ Effort applied at the circumference of the screw to lift the load

$W =$ Load to be lifted

$\mu =$ Coefficient of friction, between the screw and nut

$\mu = \tan\phi$, where$\phi$is the friction angle

From the geometry of the Figure 4a, we find that

$$\tan\alpha = \frac{p}{\pi d}$$

Since the principle, on which a screw jack works is similar to that of an inclined plane, therefore the force applied on the circumference of a screw jack may be considered to be horizontal as shown in Figure 4b.

Since the load is being lifted, therefore the force of friction ($F = \mu R_N$) will act downwards. All the forces acting on the body are shown in Figure 4b.

Resolving the forces along the plane,

$$P\cos\alpha = W\sin\alpha + F = W\sin\alpha + \mu R_N \tag{1}$$

and resolving the forces perpendicular to the plane,

$$R_N = P\sin\alpha + W\cos\alpha \tag{2}$$

Substituting this value of R_N in equation (1), we have

$$\begin{aligned} P\cos\alpha &= W\sin\alpha + \mu(P\sin\alpha + W\cos\alpha) \\ &= W\sin\alpha + \mu P\sin\alpha + \mu W\cos\alpha \\ P\cos\alpha - \mu P\sin\alpha &= W\sin\alpha + \mu W\cos\alpha \\ P(\cos\alpha - \mu\sin\alpha) &= W(\sin\alpha + \mu\cos\alpha) \\ P &= W \times \frac{(\sin\alpha + \mu\cos\alpha)}{(\cos\alpha - \mu\sin\alpha)} \\ P &= W \times \frac{(\sin\alpha + \tan\phi\cos\alpha)}{(\cos\alpha - \tan\phi\sin\alpha)} \end{aligned}$$

By substituting the value of $\mu = \tan\phi$ in the above equation

$$P = W \times \frac{(\sin\alpha + \tan\phi\cos\alpha)}{(\cos\alpha - \tan\phi\sin\alpha)}$$

Multiplying the numerator and denominator by $\cos\phi$,

$$\begin{aligned} P &= W \times \frac{(\sin\alpha\cos\phi + \sin\phi\cos\alpha)}{(\cos\alpha\cos\phi - \sin\phi\sin\alpha)} \\ &= W \times \frac{\sin(\phi+\alpha)}{\cos(\phi+\alpha)} \\ P &= W\tan(\phi+\alpha) \end{aligned}$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$\begin{aligned} T_{screw} &= P \times \frac{d}{2} \\ T_{screw} &= W\tan(\phi+\alpha)\frac{d}{2} \end{aligned}$$

When the axial load is taken up by a thrust collar as shown in Figure 3b, so that the load does not rotate with the screw, then the torque required to overcome friction at the collar,

$$\begin{aligned} T_{thrustBearing} &= \frac{2}{3} \times \mu_1 \times W\left[\frac{(R_1)^3 - (R_2)^3}{(R_1)^2 - (R_2)^2}\right] \\ &= \mu_1 \times W\left(\frac{R_1+R_2}{2}\right) \\ T_{thrustBearing} &= \mu_1 W R \end{aligned}$$

Where, R_1 and R_2 = Outside and inside radii of collar
R = Mean radius of collar = $\frac{R_1+R_2}{2}$
μ_1 = Coefficient of friction for the collar.

$\therefore$ Total torque required to overcome friction *(i.e. to rotate the screw)*: or
Total torque required to lift the load (T),

$$T = T_{screw} + T_{thrustBearing}$$

If an effort P_1 is applied at the end of a lever of arm length l, then the total torque required to overcome friction must be equal to the torque applied at the end of lever, i.e.

$$T = p \times \frac{d}{2} = P_1 \times l$$

Note When the nominal diameter (d_o) and the core/minor diameter (d_m) or (d_c) of the screw is given, then Mean diameter of screw = d

$$d = \frac{d_o + d_c}{2} = d_0 - \frac{p}{2} = d_c + \frac{p}{2}$$

Since the mechanical advantage is the ratio of the load lifted (W) to the effort applied (P_1) at the end of the lever, therefore mechanical advantage (M.A.),

$$\begin{aligned} M.A. &= \frac{W}{P_1} = \frac{W \times 2l}{P \times d} \qquad \ldots\left(\because P \times \frac{d}{2} = P_1 \times l \ or \ P_1 = \frac{P \times d}{2l}\right) \\ &= \frac{W \times 2l}{W\tan(\alpha+\phi)d} = \frac{2l}{d\tan(\alpha+\phi)} \end{aligned}$$

5 Torque Required to Lower Load by Square Threaded Screws

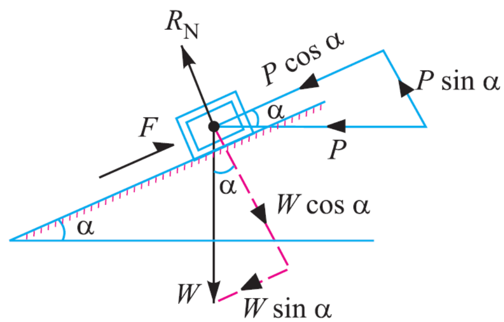


Figure 5: Forces acting on the screw when lowering the load

A little consideration will show that when the load is being lowered, the force of friction ($F = \mu.R_N$) will act upwards. All the forces acting on the body are shown in Figure 5.
Resolving the forces along the plane,

$$\begin{aligned} P \cos \alpha &= F - W \sin \alpha \\ &= \mu R_N - W \sin \alpha \end{aligned} \tag{3}$$

Resolving the forces perpendicular to the plane

$$R_N = W \cos \alpha - P \sin \alpha \tag{4}$$

Substituting this value of R_N in equation (3), we have,

$$\begin{aligned} P \cos \alpha &= \mu(W \cos \alpha - P \sin \alpha) - W \sin \alpha \\ &= \mu W \cos \alpha - \mu P \sin \alpha - W \sin \alpha \\ P \cos \alpha + \mu P \sin \alpha &= \mu W \cos \alpha - W \sin \alpha \\ P(\cos \alpha + \mu \sin \alpha) &= W(\mu \cos \alpha - \sin \alpha) \\ P &= W \times \frac{(\mu \cos \alpha - \sin \alpha)}{(\cos \alpha + \mu \sin \alpha)} \end{aligned}$$

Substituting the value of $\mu = \tan \phi$ in the above equation,

$$P = W \times \frac{(\tan \phi \cos \alpha - \sin \alpha)}{(\cos \alpha + \tan \phi \sin \alpha)}$$

Multiplying the numerator and denominator by $\cos \phi$, we have

$$\begin{aligned} P &= W \times \frac{(\sin \phi \cos \alpha - \cos \phi \sin \alpha)}{(\cos \phi \cos \alpha + \sin \phi \sin \alpha)} \\ P &= W \times \frac{\sin(\phi - \alpha)}{\cos(\phi - \alpha)} \\ p &= W \tan(\phi - \alpha) \end{aligned}$$

$\therefore$ Torque required to overcome friction between the screw and nut,

$$\begin{aligned} T_1 &= P \times \frac{d}{2} \\ T_1 &= W \tan(\phi - \alpha) \frac{d}{2} \end{aligned}$$

Note: When $\alpha > \phi$, then $P = W \tan(\alpha - \phi)$

6 Efficiency of Square Threaded Screws

The efficiency of square threaded screws may be defined as the **ratio between the ideal effort** *(i.e. the effort required to move the load, neglecting friction)* **to the actual effort** *(i.e. the effort required to move the load taking friction into account)*.

In the section **Torque Required to Raise Load by Square Threaded Screws** we have seen that the effort applied at the circumference of the screw to lift the load is

$$P = W\tan(\phi + \alpha)$$

Where $W =$ Load to be lifted
$\alpha =$ Helix angle
$\phi =$ Angle of friction
$\mu =$ Coefficient of friction between the screw and nut
$\mu = \tan\phi$

If there would have been no friction between the screw and the nut, then ϕ will be equal to zero. The value of effort P_0 necessary to raise the load, will then be given by the equation,

$$P_0 = W\tan\alpha \qquad \ldots[\textit{Substituting } \phi = 0 \textit{ in equation } 1]$$

$$\therefore \text{Efficiency}, \eta = \frac{\text{Ideal effort}}{\text{Actual effort}} = \frac{P_0}{P} = \frac{W\tan\alpha}{W\tan(\phi+\alpha)} = \frac{\tan\alpha}{\tan(\phi+\alpha)}$$

This shows that the efficiency of a screw jack, is independent of the load raised. In the above expression for efficiency(η), only the screw friction is considered. However, if the screw friction and collar friction is taken into account, then

$$\begin{aligned}\eta &= \frac{\text{Torque required to move the load, neglecting friction}}{\text{Torque required to move the load, including screw and collar friction}}\\ &= \frac{T_0}{T} = \frac{P_0 \times d/2}{P \times d/2 + \mu_1 W R}\end{aligned}$$

Note: The efficiency may also be defined as the ratio of mechanical advantage to the velocity ratio. We know that mechanical advantage (M.A.),

$$M.A. = \frac{W}{P_1} = \frac{W \times 2l}{P \times d} = \frac{W \times 2l}{W\tan(\alpha+\phi)d}$$

and the velocity ratio,

$$\begin{aligned}V.R. &= \frac{\text{Distance moved by the effort}(P_1)\text{ in one revolution}}{\text{Distance moved by the load }(W)\text{ in one revolution}}\\ &= \frac{2\pi l}{p} = \frac{2\pi l}{\tan\alpha \times \pi d} = \frac{2l}{d\tan\alpha} \qquad (\because \tan\alpha = \frac{p}{\pi d})\end{aligned}$$

$$\begin{aligned}\therefore \text{Efficiency}, \quad \eta &= \frac{\text{M.A.}}{\text{V.R.}} = \frac{2l}{d\tan(\alpha+\phi)} \times \frac{d\tan\alpha}{2l}\\ \eta &= \frac{\tan\alpha}{\tan(\alpha+\phi)}\end{aligned}$$

7 Maximum Efficiency of a Square Threaded Screw

$$\eta = \frac{\tan\alpha}{\tan(\alpha+\phi)} = \frac{\sin\alpha/\cos\alpha}{\sin(\alpha+\phi)/\cos(\alpha+\phi)} = \frac{\sin\alpha\times\cos(\alpha+\phi)}{\cos\alpha\times\sin(\alpha+\phi)}$$

Multiplying the numerator and denominator by 2, we have,

$$\eta = \frac{2\sin\alpha\times\cos(\alpha+\phi)}{2\cos\alpha\times\sin(\alpha+\phi)} = \frac{\sin(2\alpha+\phi)-sin\phi}{\sin(2\alpha+\phi)+sin\phi} \tag{5}$$

$$\because 2\sin A\cos B = \sin(A+B)+\sin(A-B)$$
$$2\sin A\cos B = \sin(A+B)+\sin(A-B)$$

The efficiency given by equation (5) will be maximum when $\sin(2\alpha+\phi)$ is maximum, i.e. when

$$\sin(2\alpha+\phi)=1 \qquad \text{or} \qquad 2\alpha+\phi=90^\circ$$
$$\therefore 2\alpha = 90^\circ-\phi \qquad \text{or} \qquad \alpha = 45^\circ-\frac{\phi}{2}$$

Substituting the value of 2α in equation (5), we have maximum efficiency,

$$\eta_{max} = \frac{\sin(90^\circ-\phi+\phi)-\sin\phi}{\sin(90^\circ-\phi+\phi)+\sin\phi} = \frac{\sin 90^\circ-\sin\phi}{\sin 90^\circ+\sin\phi}$$
$$\eta_{max} = \frac{1-\sin\phi}{1+\sin\phi}$$

8 Efficiency Vs Helix Angle

As it was discussed in the section **"6.Efficiency of Square Threaded Screws"** the efficiency of a square threaded screw depends upon the helix angle α and the friction angle ϕ. The variation of efficiency of a square threaded screw for raising the load with the helix angle α is shown in Figure 6. The efficiency of a square threaded screw increases rapidly up-to helix angle of 20°, after which the increase in efficiency is slow. The efficiency is maximum for helix angle between 40° to 45°.

When the helix angle further increases say 70°, the efficiency drops. This is due to the fact that the normal thread force becomes large and thus the force of friction and the work of friction become large as compared with the useful work. This results in low efficiency.

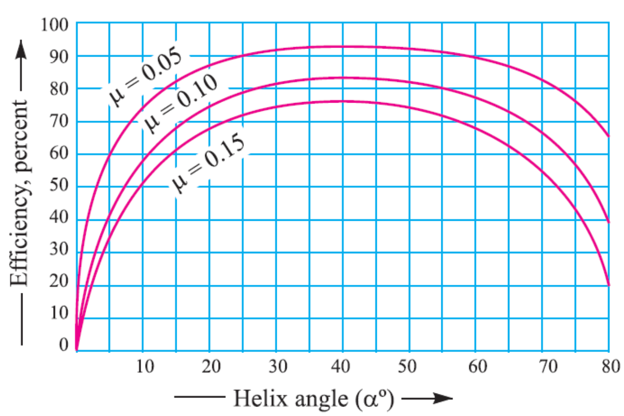


Figure 6: Graph between efficiency(η) and helix angle(α)

9 Over Hauling and Self Locking Screws

In the section **5.Torque Required to Lower Load by Square Threaded Screws** it was discussed that the effort required at the circumference of the screw to lower the load is

$$P = W\tan(\phi - \alpha)$$

and the torque required to lower the load,

$$T = P \times \frac{d}{2}$$
$$T = W\tan(\phi - \alpha)\frac{d}{2}$$

In the above expression, if $\phi < \alpha$, then torque required to lower the load will be negative. In other words, the load will start moving downward without the application of any torque. Such a condition is known as over hauling of screws. If however, $\phi > \alpha$, the torque required to lower the load will be positive, indicating that an effort is applied to lower the load. Such a screw is known as ***self locking screw***. In other words, a screw will be self locking if the friction angle is greater than helix angle or coefficient of friction is greater than tangent of helix angle.

$$\text{i.e. } \mu \text{ or } \tan\phi > \tan\alpha$$

.

10 Efficiency of Self Locking Screws

The efficiency of screw,

$$\eta = \frac{\tan\phi}{\tan(\alpha + \phi)}$$

and for self locking screws, $\phi \geq \alpha$ or $\alpha \leq \phi$.
$\therefore$ Efficiency for self locking screws,

$$\eta \leq \frac{\tan\phi}{\tan(\phi + \phi)} \leq \frac{\tan\phi}{\tan 2\phi} \leq \frac{\tan\phi(1 - \tan^2\phi)}{2\tan\phi} \leq \frac{1}{2} - \frac{\tan^2\phi}{2}$$

$$\left(\because \tan 2\phi = \frac{2\tan\phi}{1 - \tan^2\phi}\right)$$

From this expression we see that efficiency of self locking screws is less than $\frac{1}{2}$ or 50%.
If the efficiency is more than 50%, then the screw is said to be ***overhauling***.

Note: It can be proved as follows:

$$\begin{aligned} \text{Let} \qquad W &= \text{Load to be lifted} \\ h &= \text{Distance through which the load is lifted} \\ \therefore \text{Output} &= Wh \\ \text{and} \qquad \text{Input} &= \frac{\text{Output}}{\eta} = \frac{Wh}{\eta} \end{aligned}$$

$$\therefore \text{Work lost in overcoming friction,} = \text{Input} - \text{Output} = \frac{Wh}{\eta} - Wh = Wh\left(\frac{1}{\eta} - 1\right)$$

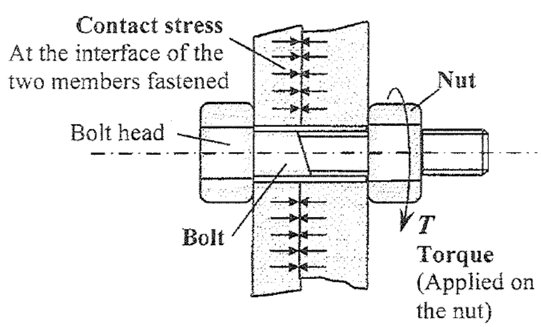
For self locking,

$$\begin{aligned} \text{Work lost in overcoming friction} &\geq \text{Output} \\ Wh\left(\frac{1}{\eta} - 1\right) &\geq Wh \\ \frac{1}{\eta} - 1 &\geq 1 \\ \eta &\leq \frac{1}{2} \text{ or } 50\% \end{aligned}$$

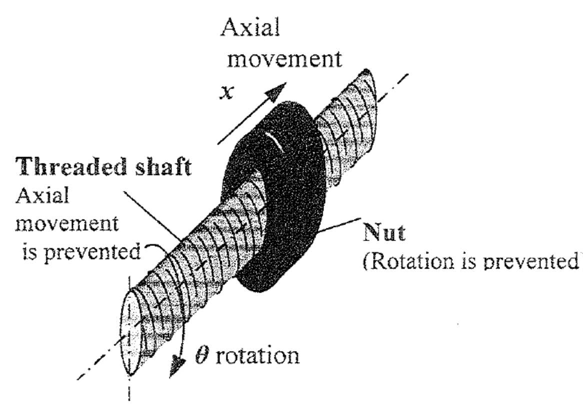
11 Friction in Helical Screw Threads

The two basic applications of screw threads are; as

1. **Fasteners** – (Figure 7a) Join two members together providing a specified contact stress at the interface of the members being fastened.
   - E.g. bolts in cylinder heads of IC Engines, pressure vessels, various structural assemblies
2. **Drivers** – (Figure 7b) Traverse some components of machine axially on helical screw.
   - E.g. Lead-screw in a lathe machine, thread shaft used in sluice gate operation, fly press etc.



(a) Fasteners



(b) Drivers

Figure 7: Two basic applications of Screw Threads

12 Basic Geometry of Helical Screw Threads

A helical screw can be formed (schematically) by wrapping a thread on a cylindrical surface as shown in Figure 8. **Pitch (P):** is the axial distance between two consecutive threads. **Lead (L):** is the axial movement of the thread per one revolution. i.e. for $\theta = 2\pi$ rad, $x = L$. In multi-start type treads $L = np$, where n is the number of starts. Then,

$$\frac{x}{\theta} = \frac{\dot{x}}{\dot{\theta}} = \frac{\ddot{x}}{\ddot{\theta}} = \frac{L}{2\pi} = \frac{np}{2\pi}$$

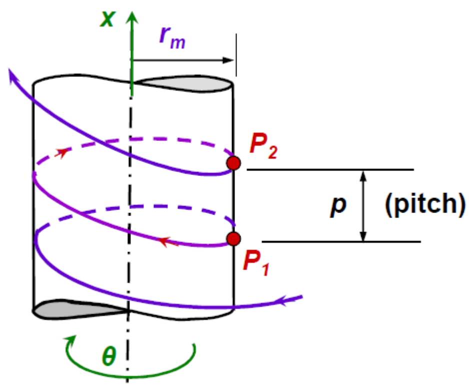


Figure 8: Single Start Helical Screw Threads

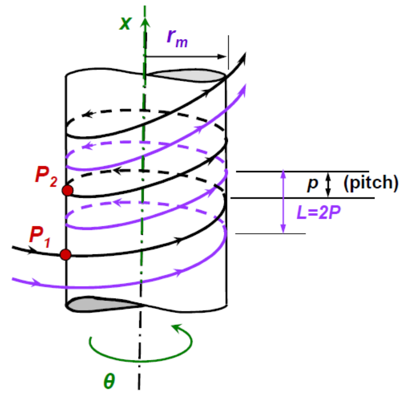


Figure 9: Multi Start Helical Screw Threads

Figure 9 shows a double start type thread (**n**=2). Multi-start type threads provide higher load carrying capacity and a fastener axial rotation. Figure 10 shows the development of line P_1P_2 (length of the thread for a complete revolution). The tangent drawn to the thread at P makes an angle α to horizontal as shown. This angle is known as the **Helix Angle** of the thread. d_m = mean thread diameter.

$$\alpha = \tan^{-1}\left(\frac{L}{\pi d_m}\right) = \tan^{-1}\left(\frac{np}{\pi d_m}\right)$$

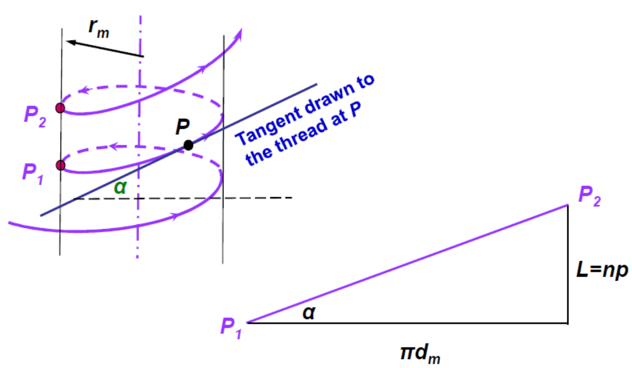


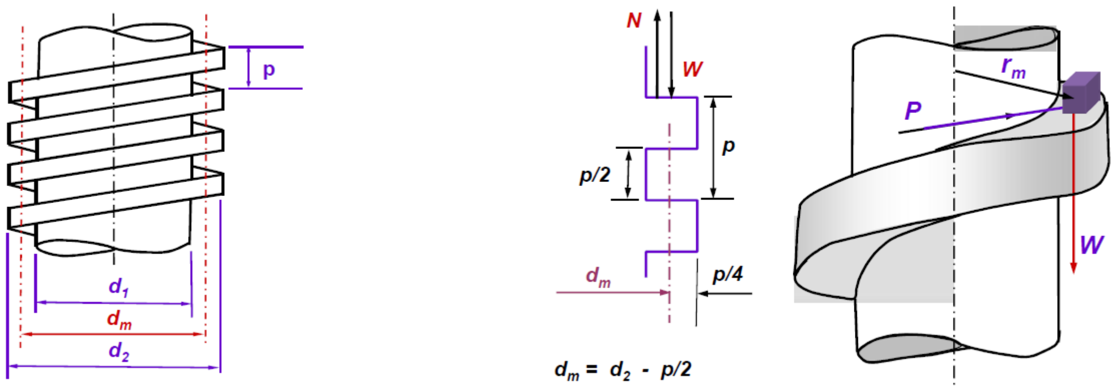
Figure 10: Development of line P_1P_2

13 Types of helical screw threads (based on thread profiles)

1. Square Threads

Can be formed by wrapping a square sectioned thread around a cylinder Figure 11.
d_1 = Minor diameter of the thread, d_2 = major diameter of the thread, d_m = Mean diameter,

$$d_m = \frac{(d_1 + d_2)}{2}$$



(a) (b)

Figure 11: Square Threads

2. V-Threads

Figure 12 shows a V-thread profile. β = the semi-apex angle or semi V-angle of the thread. In this case, the normal reaction N at the thread is not in the plane of P and W is $N\cos\alpha$. Nevertheless, the friction force is still μN.
ϕ' = apparent (or virtual) angle of friction.
Then, the equivalent coefficient of friction can be expressed as

$$\mu' = \tan\phi' = \frac{\mu}{\cos\beta}$$

The friction force is therefore increased in the ration $\sec\beta : 1$, so that V-thread is equivalent to a square thread having a apparent or virtual, coefficient of friction $\mu\sec\beta$

Other types of threads (Based on the direction of the movements)
(a)Right-hand threads, (b) Left-hand threads

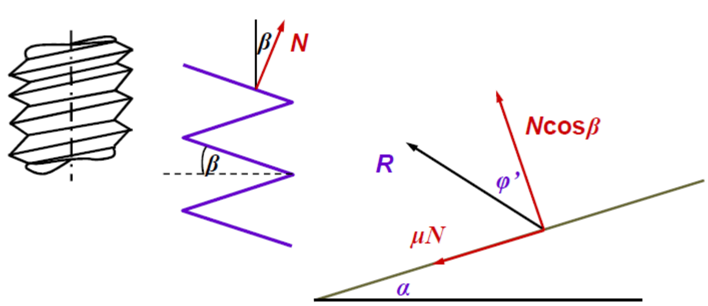


Figure 12: V-Thread

Applications: Turnbuckle

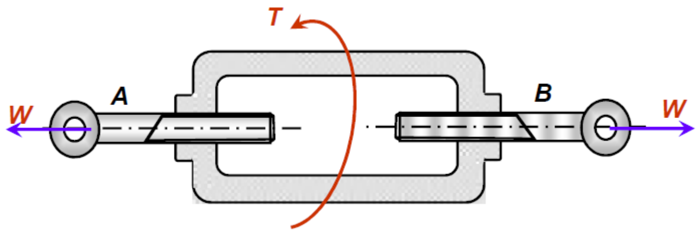


Figure 13: Turnbuckle Type 1

$$T = W\left(\tan(\phi_A \pm \alpha_A)\frac{d_{m_A}}{2} + \tan(\phi_B \pm \alpha_B)\frac{d_{m_B}}{2}\right)$$

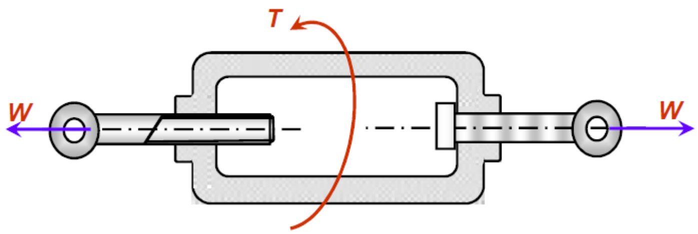


Figure 14: Turnbuckle Type 2

$$T = W\tan(\phi \pm \alpha)\frac{d_m}{2}$$

Applications: Differential screw jack

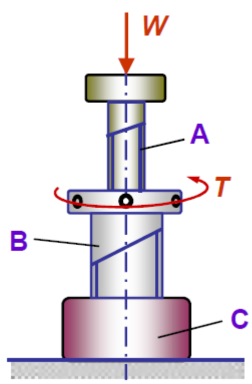


Figure 15: Differential screw jack

When raising the load required torque,

$$T = W\left(\tan(\phi_A - \alpha_A)\frac{d_{m_A}}{2} + \tan(\phi_B + \alpha_B)\frac{d_{m_B}}{2}\right)$$

When lowering the load required torque,

$$T = W\left(\tan(\phi_A + \alpha_A)\frac{d_{m_A}}{2} + \tan(\phi_B - \alpha_B)\frac{d_{m_B}}{2}\right)$$

Scissors jack

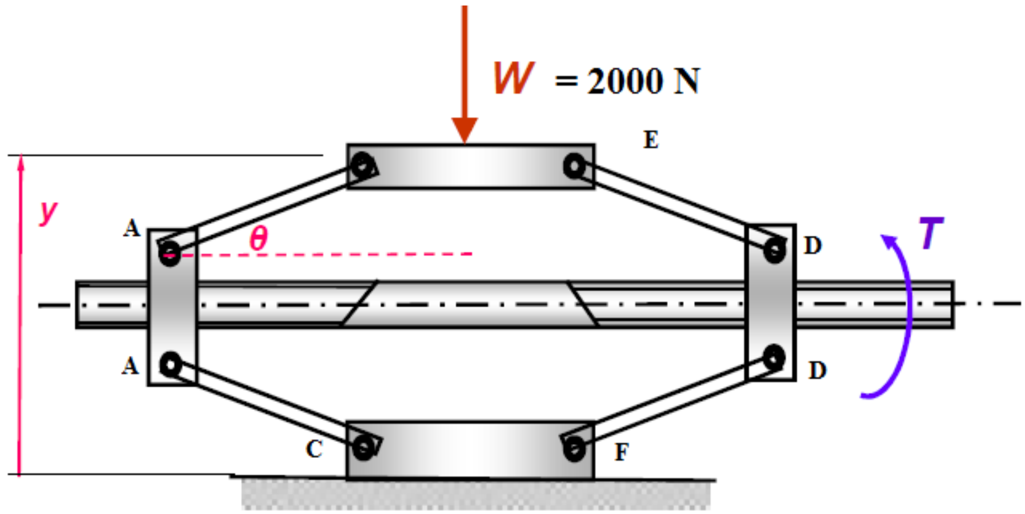


Figure 16: Scissors jack

The left half of the operating spindle of a lifting jack/scissors jack that is shown in the Figure 16 given below is threaded and is inserted through a nut to which the side links **AB** and **AC** are hinged ar point **A**. The screw has a single start square thread with an outer diameter of 14.0 mm and pitch of 4.0 mm. The right end of the operating spindle is inserted through a bush to which the side links **DE** and **DF** are hinged at point **D**. The bush rests against a collar attached to the shaft. The thrust bearing between the bush and the collar has a mean diameter of 22.0 mm. The lower plate of the jack rests on the ground and it is used to life a load of 2000 N, which acts on the upper plate. The coefficient of friction at the screw as well as at the thrust bearing is 0.13. All your four links are inclined to the horizontal by 35 degrees.
i.e. $\theta = 35°$. **AB=AC=DE=DF**

Determine the torque which should be exerted on the operating spindle in order to lift the load and the shear stress in the most heavily loaded thread in the screw.
You may assume that the most heavily loaded thread carries one third of the total axial load on the screw.
You may assume following results where standard notation has been used.
Friction torque at a thrust bearing = $\mu W r_m$ where μ = coefficient of friction at the bearing.
W = Thrust force carried by the bearing and r_m = mean radius of the thrust bearing.

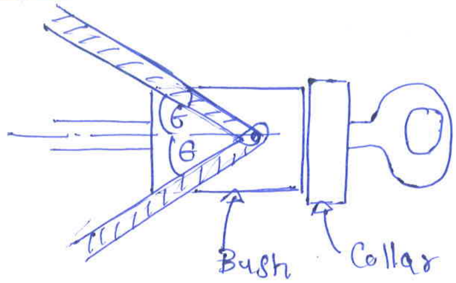
Torque which should be exerted on a screw in order to lift a load = $\frac{W D_m}{2} \times \tan[\lambda + \alpha]$

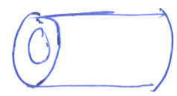
where **W** = load carried by the screw, D_m = mean diameter of the screw,
λ = friction angle at the thread and α = helix angle of the thread

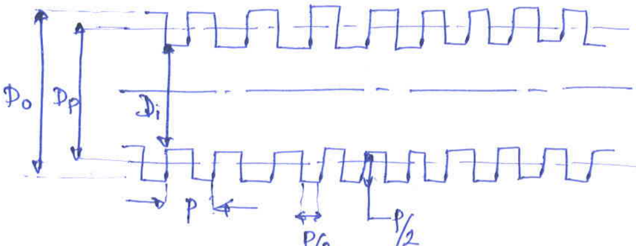
Reference

1. R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd.

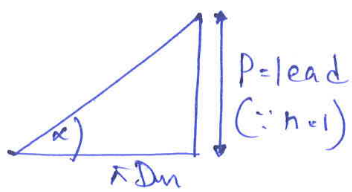
Answer for the scissor Jack Problem







n – No of Starts (thread types)



$D_o = 14\,mm$

$P = 4\,mm$

$D_m = \frac{D_o + D_i}{2} \Rightarrow D_o + D_i = 2D_m$

$D_o - D_i = P \Rightarrow D_o - D_i = P$

$\therefore D_m = D_o - P/2$ — ①

$D_m = D_i + P/2$ — ②

$\tan\alpha = \frac{P}{\pi D_m} = \frac{4}{\pi(12)}$

$\tan\alpha = 0.106$

$\alpha = 6.06^\circ$

① ⇒ $D_m = 14 - \frac{4}{2} = 12\,mm$

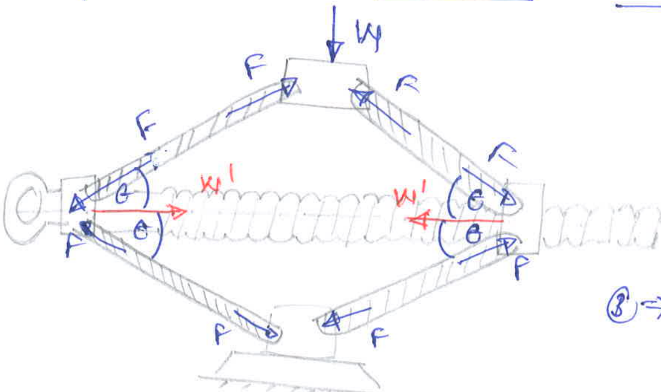
② ⇒ $D_i = 12 - \frac{4}{2} = 10\,mm$

$\mu = 0.13$

$\tan\phi = 0.13$

$\phi = 7.407^\circ$

$\boxed{\phi > \alpha}$ ∴ The thread is self locking.



$\uparrow\; 2F\sin\theta = W$

$F = \frac{W}{2\sin\theta}$ — ③

$\rightarrow$ Tension force on the Screw

$W' = 2F\cos\theta$

③ ⇒ $= 2\frac{W}{2\sin\theta}\cdot\cos\theta$

$\boxed{W' = W\cot\theta}$

b) Torque on the screw to lift the load. $= T_{screw} + T_{thrust\ bearing}$

$$= W' \times \left(\frac{D_m}{2}\right) \tan(\alpha + \phi) + \mu W' r'_m$$

$$= W' \left[\frac{D_m}{2} \tan(\alpha + \phi) + \mu r'_m\right]$$

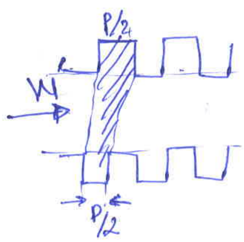
$$= W \cot\theta \left[\frac{D_m}{2} \tan(\alpha + \phi) + \mu \left(\frac{D'_m}{2}\right)\right]$$

$$= 2000 \cot(35^\circ) \left[\frac{12}{2} \tan(7.407^\circ + 6.06^\circ) + 0.13 \times \left(\frac{22}{2}\right)\right] \times 10^{-3}$$

$$= \underline{\underline{8.188\ Nm}}$$

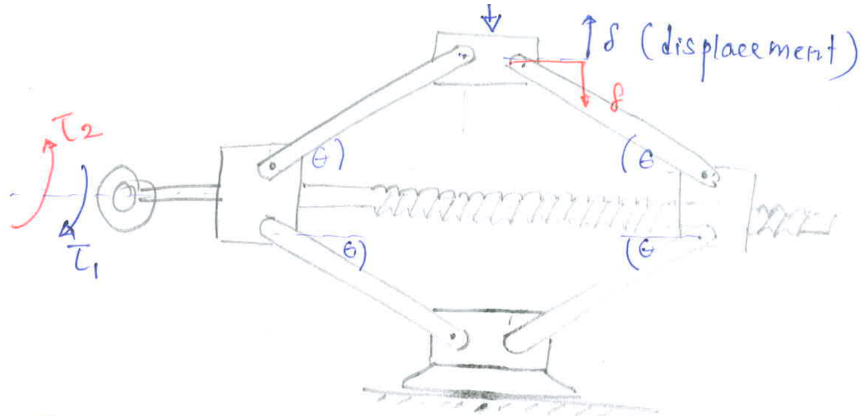
Data: - most heavily loaded thread carrying 1/3 of the total load.

$\therefore$ Shear stress in the 1^{st} thread $= \dfrac{W'/3}{(\pi D_i) P/2}$



$$= \frac{2000 \cot 35 / 3}{\pi \times 10 \times \left(\frac{4}{2}\right) \times 10^{-6}}$$

$$= 15.153 \times 10^{6}\ N m^{-2}$$



Relationship between τ_1 & δ is not linear. It depends on θ

Lifting

τ_1 = drive torque which is exerted in order to turn the screw and lift the load.

When the screw rotates by 1 rev (2π rad)
Energy supplied (output) to the screw jack $= \tau_1 \times 2\pi$ Nm rad
$= 2\pi\tau_1$ (J)

W load acting on the jack

δ(m) in the distance by which the load in lifted when screw rotates by 1 rev.

Work output of the jack $= W(N) \times \delta(m)$
$= W\delta$. (J)

When the work done against the load, Energy Input > Energy Output.
$2\pi\tau_1 > W\delta$

$2\pi\tau_1 = W\delta$ + Energy loss

(Energy loss in due to Work done against friction)

Mechanical efficiency of the Screw jack $\Big\} = \dfrac{W\delta}{2\pi\tau_1} \times 100\% < 100\%$

Lowering

T_2 = The torque which should be exerted on the screw in order to lower the load. $T_2 < T_1$

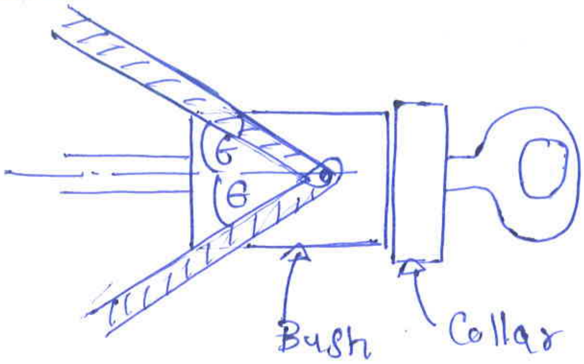
Energy supplied to the Jack $= 2\pi T_2$ (J)

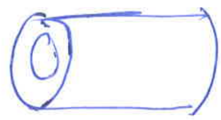
Work done by the load on the screw (Energy input $\Big\} = W\delta$ (J)

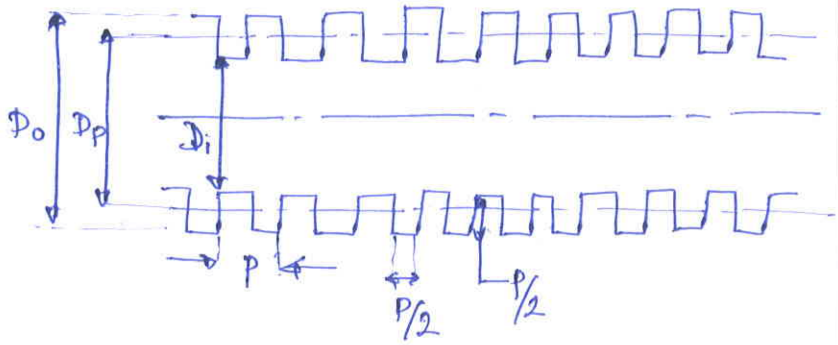
Energy Supplied to the jack + Work done by the load = Energy loss in order to overcome the friction.
(Work done against friction/ friction loss)

$2\pi T_2 + W\delta$ = Energy loss

Answer for the Scissor Jack Problem







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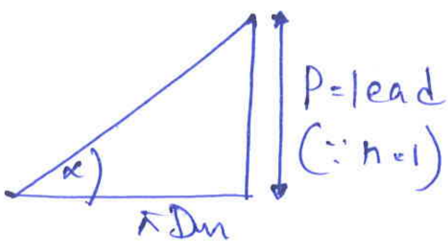

$$\tan\alpha = \frac{P}{\pi D_m} = \frac{4}{\pi(12)}$$

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$$D_m = D_i + P/2 \quad -(2)$$

$$(1) \Rightarrow D_m = 14 - \frac{4}{2} = 12mm$$

$$(2) \Rightarrow D_i = 12 - \frac{4}{2} = 10mm$$

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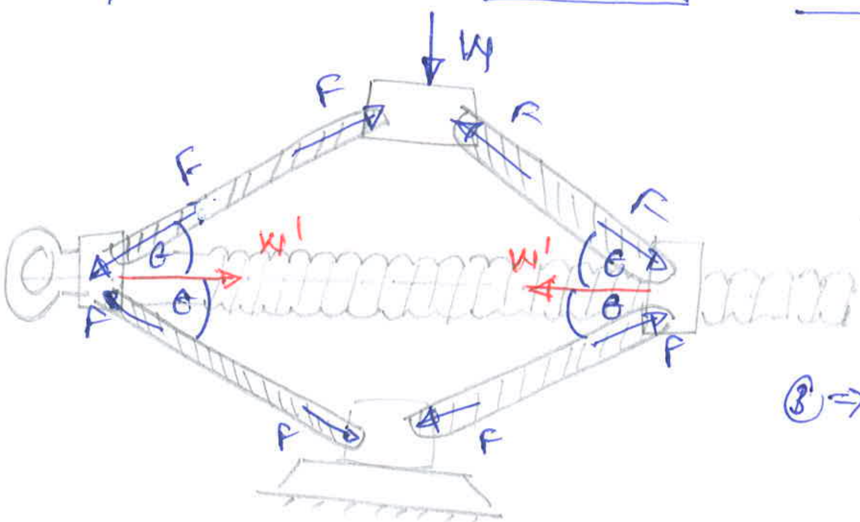

$$\uparrow\ 2F\sin\theta = W$$

$$F = \frac{W}{2\sin\theta} \quad -(3)$$

$\rightarrow$ Tension force on the screw

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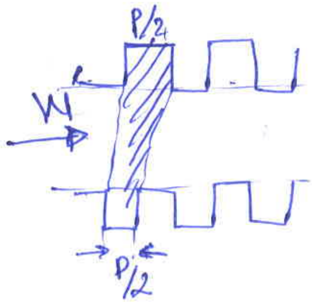

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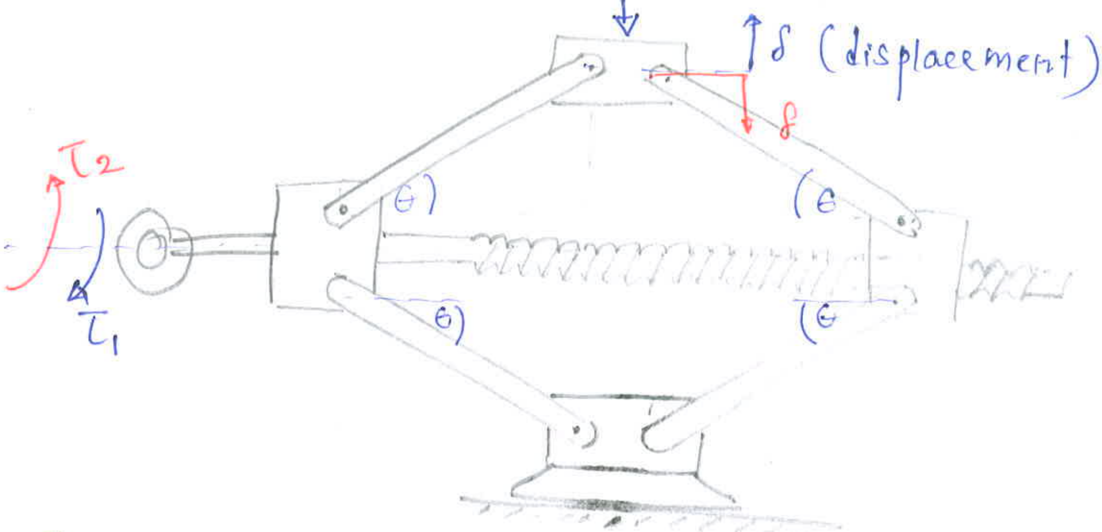

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When the work done against the load, Energy Input $>$ Energy Output.
$2\pi\tau_1 > W\delta$

$$2\pi\tau_1 = W\delta + \text{Energy loss}$$

(Energy loss is due to (Work done against friction)

Mechanical efficiency of the screw jack $\Big\} = \dfrac{W\delta}{2\pi\tau_1} \times 100\% < 100\%$

Lowering

T_2 = The torque which should be exerted on the screw in order to lower the load. $T_2 < T_1$

Energy supplied to the Jack = $2\pi T_2$ (J)

$\left.\begin{array}{l}\text{Work done by the load on the screw} \\ \text{(Energy input}\end{array}\right\} = W\delta$ (J)

Energy Supplied to the jack + Work done by the load = Energy loss in order to overcome the friction.
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$2\pi T_2 + W\delta$ = Energy loss

ME3531: Solid Mechanics and Mechanical Design – S1:2019
Lecture 10–15 May 2019 Thilina H. Weerakkody
Design of Welded Joints

Material Covered

1. Introduction
2. Advantages and Disadvantages of Welded Joints over Riveted Joints.
3. Welding Processes
   (a) Fusion Welding:– (I).Thermit Welding, (II).Gas Welding, (III).Electric Arc Welding
   (b) Forge Welding
4. Types of Welded Joints
   (a) Lap Joint/ Fillet joint
   (b) Butt Joint
   (c) Corner Joint
   (d) Tee Joint
   (e) Edge Joint
5. Weld Symbols
   (a) Basic Weld Symbols
   (b) Supplementary Weld Symbols
   (c) Elements of a Welding Symbol
   (d) Standard Location of Elements of a Welding Symbol
6. Strength of Lap/ Fillet Welded Joints
   - Strength of Transverse Fillet Welded Joints
   - Strength of Parallel Fillet Welded Joints
   - Special Cases of Fillet Welded Joints
7. Strength of Butt Joints
8. Stresses for Welded Joints
   - Stress Concentration Factor for Welded Joints
   - Axially Loaded Unsymmetrical Welded Sections
   - Eccentrically Loaded Welded Joints

Course Text:

R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd. – Chapter 10. Welded Joints (pp. 341-376).

ME3531: Solid Mechanics and Mechanical Design
Design of Welded Joints

Thilina H. Weerakkody
Department of Mechanical Engineering
SLIIT-Sri Lanka

Year 3 −Semester 1, 2019

Outline

- Introduction
- Advantages and Disadvantages of Welded Joints over Riveted Joints.
- Welding Processes
  1. Fusion Welding:− (I).Thermit Welding, (II).Gas Welding, (III).Electric Arc Welding
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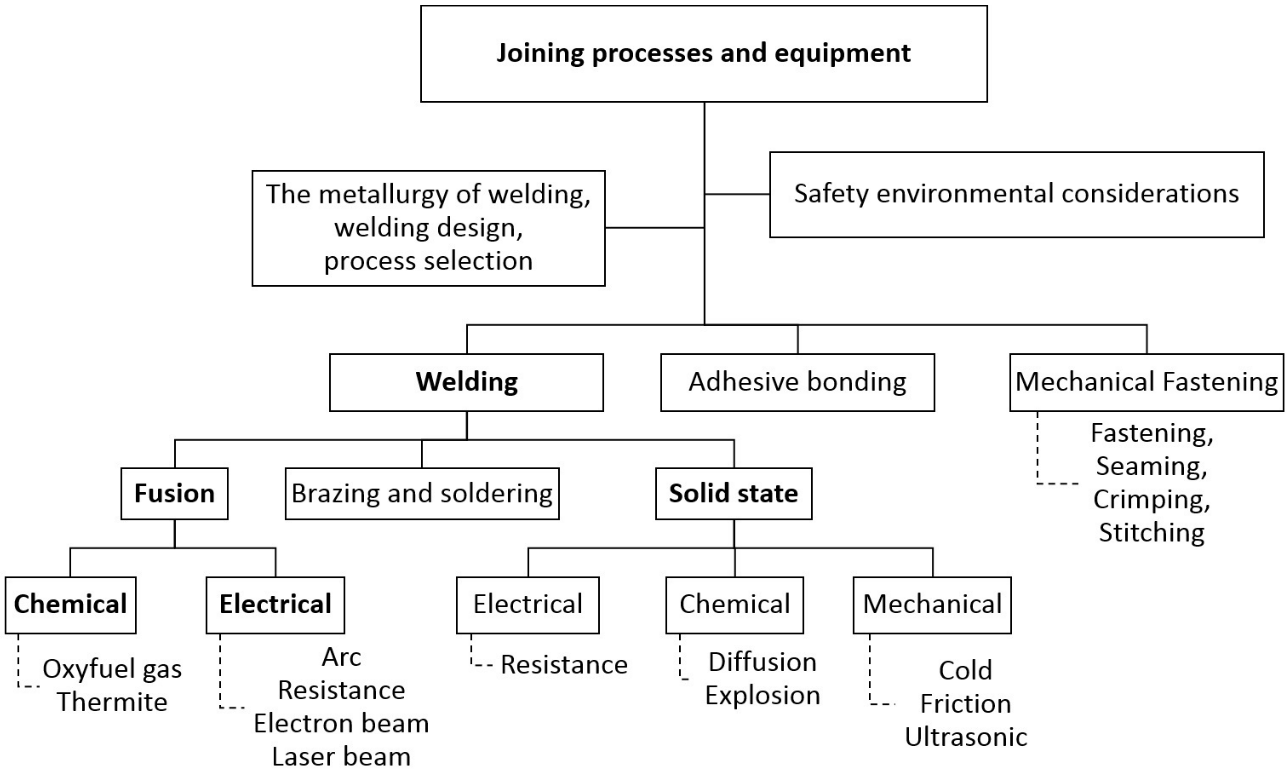


Figure 1: Joining processes classification

Introduction

A welded joint is **a permanent joint** which is obtained by the **fusion of the edges of the two parts to be joined together, <u>with or without</u> the application of pressure and a filler material.** The heat required for the fusion of the material may be obtained by burning of gas (in case of gas welding) or by an electric arc (in case of electric arc welding). The latter method is extensively used because of greater speed of welding. Welding is extensively used in fabrication as an alternative method for casting or forging and as a replacement for bolted and riveted joints. It is also used as a repair medium e.g. to reunite metal at a crack, to build up a small part that has broken off such as gear tooth or to repair a worn surface such as a bearing surface.

Advantages and Disadvantages of Welded Joints over Riveted Joints.

Advantages

1. The welded structures are usually lighter than riveted structures. This is due to the reason, that in welding, gussets or other connecting components are not used.
2. The welded joints provide maximum efficiency (may be 100%) which is not possible in case of riveted joints.
3. Alterations and additions can be easily made in the existing structures.
4. As the welded structure is smooth in appearance, therefore it looks pleasing.
5. In welded connections, the tension members are not weakened as in the case of riveted joints.

6. A welded joint has a great strength. Often a welded joint has the strength of the parent metal itself.
7. Sometimes, the members are of such a shape (*i.e.* circular steel pipes) that they afford difficulty for riveting. But they can be easily welded.
8. The welding provides very rigid joints. This is in line with the modern trend of providing rigid frames.
9. It is possible to weld any part of a structure at any point. But riveting requires enough clearance.
10. The process of welding takes less time than the riveting.

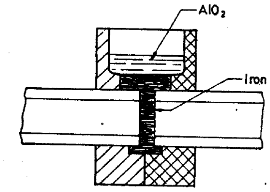
Disadvantages

1. Since there is an uneven heating and cooling during fabrication, therefore the members may get distorted or additional stresses may develop.
2. It requires a highly skilled labour and supervision.
3. Since no provision is kept for expansion and contraction in the frame, therefore there is a possibility of cracks developing in it.
4. The inspection of welding work is more difficult than riveting work.

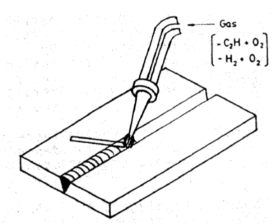
Welding Processes

The welding processes may be broadly classified into the following two groups:

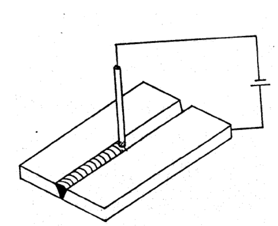
1. Welding processes that use heat alone *e.g.* **fusion welding.**
2. Welding processes that use a combination of heat and pressure *e.g.* **forge welding.**



(a) Thermit welding



(b) Gas welding



(c) Electric Arc welding

Figure 2: Fusion welding classification

1. Fusion Welding

In case of fusion welding, the parts to be jointed are held in position while the molten metal is supplied to the joint. The molten metal may come from the parts themselves (i.e. parent metal) or filler metal which normally have the composition of the parent metal. The joint surface become plastic or even molten because of the heat from the molten filler metal or other source. Thus, when the molten metal solidifies or fuses, the joint is formed.
The fusion welding, according to the method of heat generated, may be classified as:

1. Thermit welding,
2. Gas welding
3. Electric arc welding

(I) Thermit Welding

In thermit welding, **a mixture of iron oxide and aluminium** called ***thermit*** is ignited and the iron oxide is reduced to molten iron. The molten iron is poured into a mould made around the joint and fuses with the parts to be welded. A major advantage of the thermit welding is that all parts of weld section are molten at the same time and the weld cools almost uniformly. This results in a minimum problem with residual stresses. It is fundamentally a melting and casting process.

The thermit welding is often used in joining iron and steel parts that are too large to be manufactured in one piece, such as rails, truck frames, locomotive frames, other large sections used on steam and rail roads, for stern frames, rudder frames etc. In steel mills, thermit electric welding is employed to replace broken gear teeth, to weld new necks on rolls and pinions, and to repair broken shears.
Refer Figure 2a.

(a) Thermit Welding process

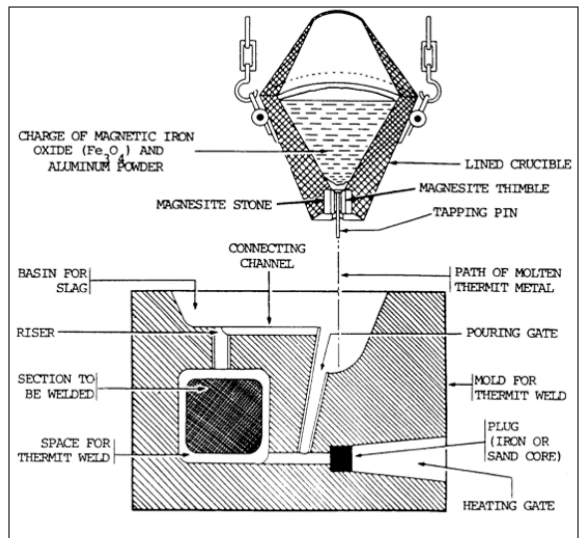


Figure 5-41. Thermit welding crucible and mold.

(b) Thermit Welding crucible and mold

Figure 3: Thermit Welding

(II) Gas Welding

A gas welding is made by applying the flame of an ***oxy-acetylene*** *or* ***hydrogen gas*** from a welding torch upon the surfaces of the prepared joint. The intense heat at the white cone of the flame heats up the local surfaces to fusion point while the operator manipulates a welding rod to supply the metal for the weld. A flux is being used to remove the slag. Since the heating rate in gas welding is slow, therefore it can be used on thinner materials.
Refer Figure 2b.

(III) Electric Arc Welding

In electric arc welding, the work is prepared in the same manner as for gas welding. In this case the filler metal is supplied by metal welding electrode. The operator, with his eyes and face protected, strikes an arc by touching the work of base metal with the electrode. The base metal in the path of the arc stream is melted, forming a pool of molten metal, which seems to be forced out of the pool by the blast from the arc, as shown in Figure 4. A small depression is formed in the base metal and

the molten metal is deposited around the edge of this depression, which is called the ***arc crater***. The slag is brushed off after the joint has cooled.
Refer Figure 2c.

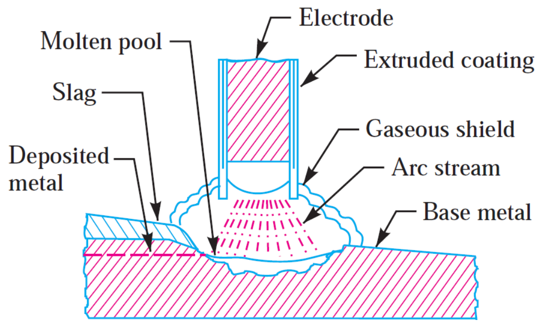


Figure 4: Shielded electric arc welding

The arc welding does not require the metal to be preheated and since the temperature of the arc is quite high, therefore the fusion of the metal is almost instantaneous. There are two kinds of arc welding methods depending upon the type of electrode.

1. Un−shielded arc welding
2. Shielded arc welding

When a large electrode or filler rod is used for welding, it is then said to be ***un-shielded arc welding***. In this case, the deposited weld metal while it is hot will absorb oxygen and nitrogen from the atmosphere. This decreases the strength of weld metal and lower its ductility and resistance to corrosion.

In ***shielded arc welding***, the welding rods coated with solid material are used, as shown in Figure4. The resulting projection of coating focuses a concentrated arc stream, which protects the globules of metal from the air and prevents the absorption of large amounts of harmful oxygen and nitrogen.

Figure 5: Blacksmith's Forge welding

2. Forge or Pressure Welding

In forge or Pressure welding, the parts to be jointed are first heated to a proper temperature in a furnace or forge and then hammered. This method of welding is rarely used now-a-days. An ***electric-resistance welding*** is an example of forge welding.
In this case, the parts to be joined are pressed together and an electric current is passed from one part to the other until the metal is heated to the fusion temperature of the joint. The principle of applying heat and pressure, either sequentially or simultaneously, is widely used in the processes known as *****spot, seam, projection, upset and flash welding.***

Types of Welded Joints

Following two types of welded joints are important from the subject point of view:

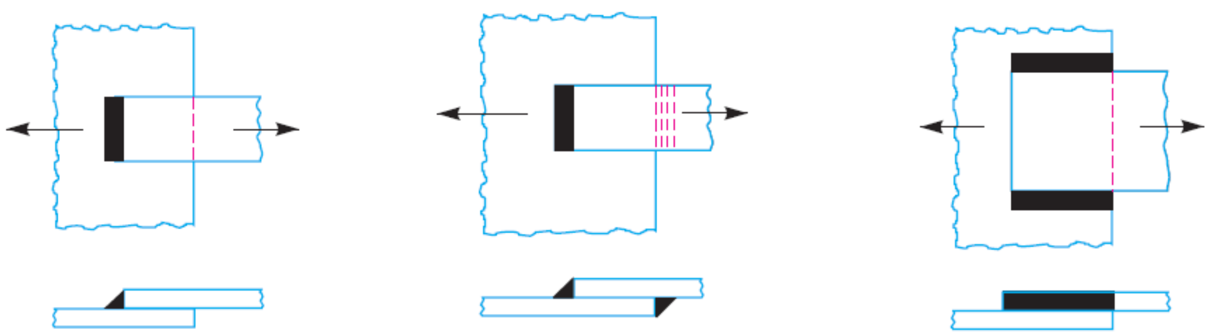
1. Lap joint or fillet joint
2. Butt joint

1. Lap Joint/ fillet joint

The lap joint or the fillet joint is obtained by overlapping the plates and then welding the edges of the plates. The cross-section of the fillet is approximately triangular. The fillet joints may be

1. Single transverse fillet.
2. Double transverse fillet.
3. Parallel fillet joints.

The fillet joints are shown in Figure 6. A single transverse fillet joint has the disadvantage that the edge of the plate which is not welded can buckle or warp out of shape.



(a) Single transverse (b) Double transverse (c) Parallel fillet

Figure 6: Types of lap or fillet joints

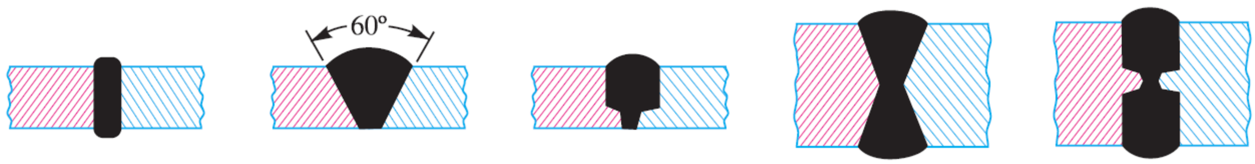
2. Butt Joint

The butt joint is obtained by placing the plates edge to edge as shown in Figure 7. In butt welds, the plate edges do not require bevelling if the thickness of plate is less than 5 mm. On the other hand, if the plate thickness is 5 mm to 12.5 mm, the edges should be bevelled to V or U−groove on both sides. The butt joints may be

1. Square butt joint − *(Figure7a)*
2. Single V-butt joint − *(Figure7b)*
3. Single U-butt joint − *(Figure7c)*
4. Double V-butt joint − *(Figure7d)*
5. Double U-butt joint − *(Figure7e)*

3. Corner Joint

Refer Figure 8



(a) Square butt joint (b) Single V-butt j. (c) Single U-butt j. (d) Double V-butt j. (e) Double U-butt j.

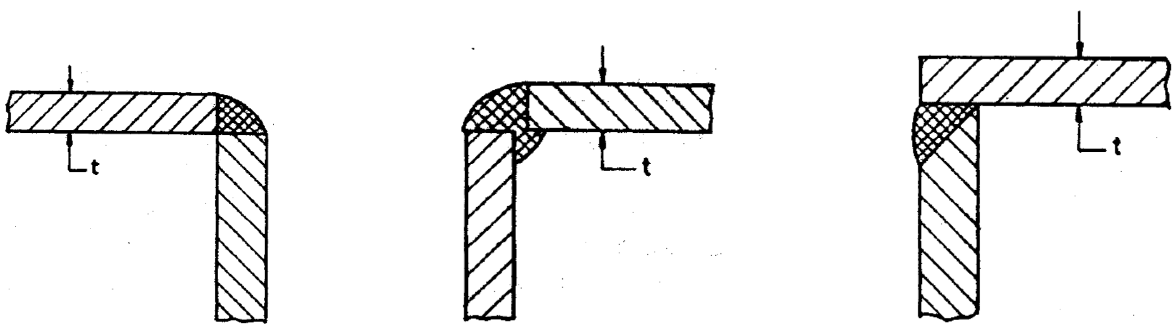
Figure 7: Types of butt joints

4. Edge Joint

Refer Figure 9

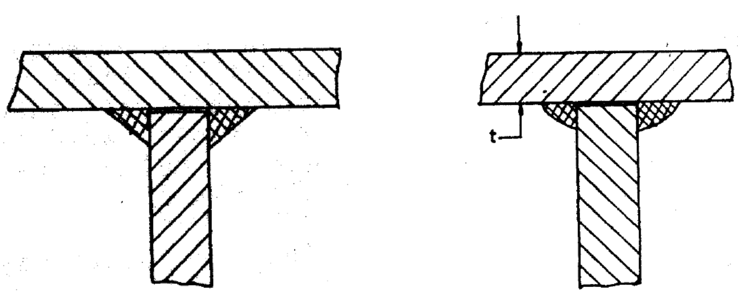
5. Tee – Joint

Refer Figure 10



(a) Single Fillet Corner Joint (b) Double Fillet Corner Joint (c) Flush Corner Joint

Figure 8: Corner Joint types



(a) Double fillet Tee joint (b) Double Bevel Tee joint

Figure 9: Tee joint types

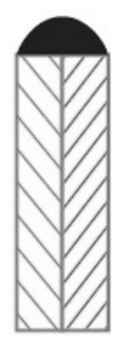


Figure 10: Edge Joint

The main considerations involved in the selection of weld type are:

1. The shape of the welded component required.
2. The thickness of the plates to be welded.
3. The direction of the forces applied.

Weld Symbols

1. Basic Weld Symbols

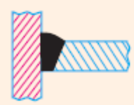
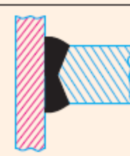
S. No.	*Form of weld*	*Sectional representation*	*Symbol*
1.	Fillet		
2.	Square butt		
3.	Single-*V* butt		
4.	Double-*V* butt		
5.	Single-*U* butt		
6.	Double-*U* butt		
7.	Single bevel butt		
8.	Double bevel butt		

Figure 11: Basic Weld Symbols

2. Elements of a Welding Symbol

A welding symbol consists of the following eight elements:

1. Reference line
2. Arrow
3. Basic weld symbols
4. Dimensions and other data
5. Supplementary symbols
6. Finish symbols
7. Tail
8. Specification, process or other references.

3. Standard Location of Elements of a Welding Symbol

According to Indian Standards, IS: 813 – 1961 (Reaffirmed 1991), the elements of a welding symbol shall have standard locations with respect to each other.
The arrow points to the location of weld, the basic symbols with dimensions are located on one or both sides of reference line. The specification if any is placed in the tail of arrow. Figure 12 shows the standard locations of welding symbols represented on drawing.

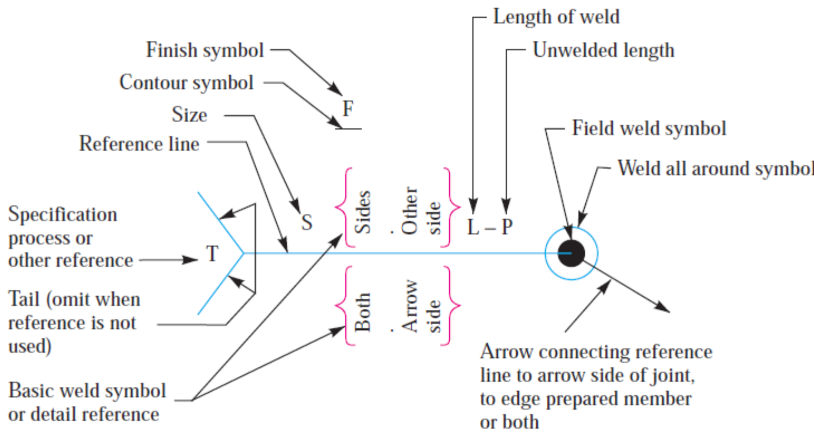


Figure 12: Standard location of welding symbols

Following figure 13 represented some of the examples of welding symbols.

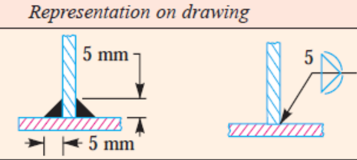
S. No.	*Desired weld*	*Representation on drawing*
1.	Fillet-weld each side of Tee- convex contour	
2.	Single V-butt weld -machining finish	
3.	Double V- butt weld	

Figure 13: Representation of welding symbols

Note: **Strength Calculation of Welded Joints under Static Loads**
Stresses in welded joints are difficult to determine due to

- Unpredictable parameters like homogeneity of the weld metal.
- Changes of material properties due to high rate of cooling.

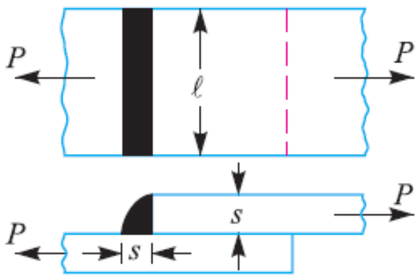
∴ The strength calculations are based on the following assumptions

- Load is distributed evenly along the entire weld length.
- Stress is distributed evenly over its effective section.

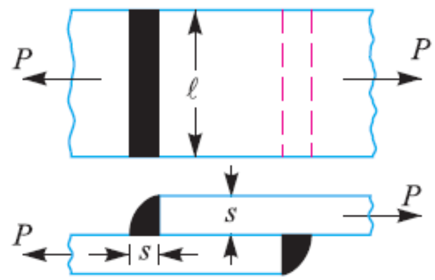
Strength of Lap/ Fillet Welded Joints

1. Strength of Transverse Fillet Welded Joints

the fillet or lap joint is obtained by overlapping the plates and then welding the edges of the plates. The transverse fillet welds are designed for tensile strength. Let us consider a single and double transverse fillet welds as shown in Fig. 14a and 14b respectively.



(a) Single Transverse Fillet



(b) Double Transverse Fillet

Figure 14: Transverse Fillet Welds

In order to determine the strength of the fillet joint, it is assumed that the section of fillet is a right angled triangle ABC with hypotenuse AC making equal angles with other two sides AB and BC. The enlarged view of the fillet is shown in Fig. 15. The length of each side is known as leg or size of the weld and the perpendicular distance of the hypotenuse from the intersection of legs (i.e. BD) is known as throat thickness. The minimum area of the weld is obtained at the throat BD, which is given by the product of the throat thickness and length of weld.

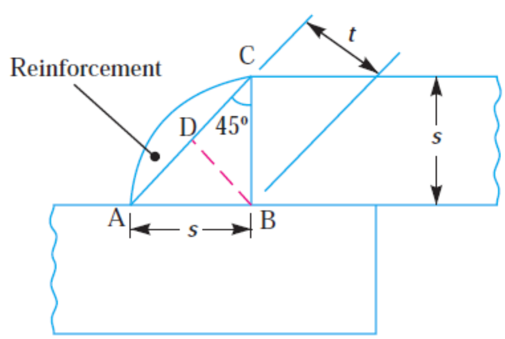


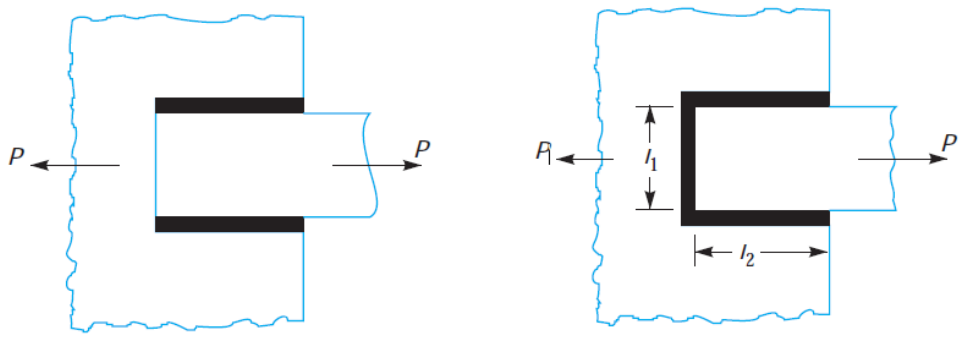
Figure 15: Enlarged view of a fillet weld

Let, t = Throat thickness (BD),
s = Leg or size of weld = Thickness of plate, and
l = Length of weld,
$t = s \times \sin 45° = 0.707s$
$\therefore$ Minimum area of the weld or throat area,
A = Throat thickness × Length of weld
A $= t \times l = 0707s \times l$

2. Strength of Parallel Fillet Welded Joints

The parallel fillet welded joints are designed for shear strength. Consider a double parallel fillet welded joint. We have already discussed in the previous article, that the minimum area of weld or the throat area.
A $= t \times l = 0707s \times l$

If τ is the allowable shear stress for the weld metal, then the shear strength of the joint for single parallel fillet weld
P = Throat area × Allowable shear stress = 0.707 $s \times l \times$ τ



(*a*) Double parallel fillet weld.

(*b*) Combination of transverse and parallel fillet weld.

Figure 16

SUMMARY - Refer Fig.14a, Fig.14b, Fig.15 and Fig.16

Load = Throat area × Allowable tensile stress

Strength of a **Single Transverse** Fillet Welded Joint – $\boxed{\mathrm{P} = \mathrm{t}\times\mathrm{l}\ \times\sigma_t = 0.707s \times l \times \sigma_t}$

Strength of a **Double Transverse** Fillet Welded Joint – $\boxed{\mathrm{P} = 2\mathrm{t}\times\mathrm{l}\times\sigma_t = 2 \times 0.707s \times l \times \sigma_t}$

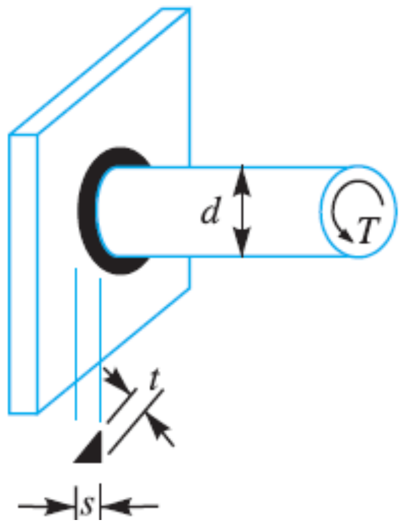
Strength of a **Double Parallel** Fillet Welded Joint – $\boxed{\mathrm{P} = 2\mathrm{t}\times\mathrm{l}\times\sigma_t = 2 \times 0.707s \times l \times \tau}$

Strength of a **Combination of Transverse and Parallel** Fillet Welded Joint (Fig 16 (b)) –
$\boxed{\mathrm{P} = (0.707s \times l_1 \times \sigma_t) + (2 \times 0.707s \times l_2 \times \tau)}$

Special Cases of Fillet Welded Joints

1. Circular fillet weld subjected to torsion

Consider a circular rod connected to a rigid plate by a fillet weld as shown in the Figure.



$d =$ Diameter of rod,
$r =$ Radius of rod,
$T =$ Torque acting on the rod,
$s =$ Size (or leg) of weld,
$t =$ Throat thickness,
$J =$ Polar moment of inertia of the weld section
$J = \frac{\pi t d^3}{4}$

We know that shear stress for the material,

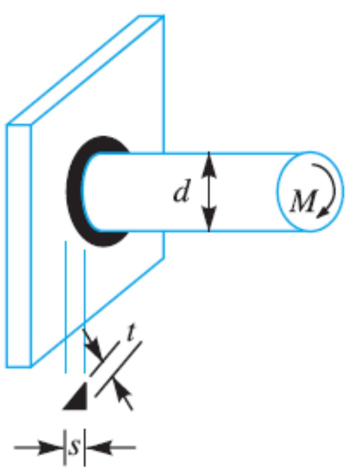
$$\tau = \frac{Tr}{J} = \frac{T \times \frac{d}{2}}{J} \qquad \left(\because \frac{T}{J} = \frac{\tau}{r}\right)$$
$$= \frac{T \times d/2}{\pi t d^3/4} = \frac{2T}{\pi t d^2}$$

This shear stress occurs in a horizontal plane along a leg of the fillet weld. The maximum shear occurs on the throat of weld which is inclined at 45° to the horizontal plane.

$\therefore$ Length of throat, $\quad t = s \sin 45° = 0.707s$
and Maximum shear stress

$$\tau_{max} = \frac{2T}{\pi \times 0.707s \times d^2} = \frac{2.83T}{\pi \ s \ d^2}$$

2. Circular fillet weld subjected to bending moment



$d =$ Diameter of rod,
$M =$ Bending moment acting on the rod,
$s =$ Size (or leg) of weld,
$t =$ Throat thickness,
$Z =$ Section modulus of the weld section$= \frac{\pi t d^2}{4}$

We know that the bending stress,

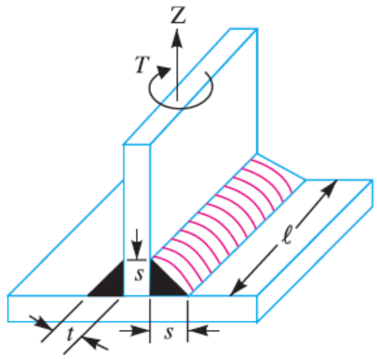
$$\sigma_b = \frac{M}{Z} = \frac{M}{\pi t d^2/4} = \frac{4T}{\pi t d^2}$$

This bending stress occurs in a horizontal plane along a leg of the fillet weld. The maximum bending stress occurs on the throat of the weld which is inclined at 45° to the horizontal plane.
$\therefore$ Length of throat, $\quad t = s \sin 45° = 0.707s$
and Maximum bending stress

$$\sigma_{b(max)} = \frac{4M}{\pi \times 0.707s \times d^2} = \frac{5.66M}{\pi \ s \ d^2}$$

3. Long fillet weld subjected to torsion

Consider a vertical plate attached to a horizontal plate by two identical fillet welds as shown in the Figure.



T = Torque acting on the vertical plate,
l = Length of weld,
s = Size (or leg) of weld,
t = Throat thickness,
J = Polar moment of inertia of the weld section
$J = 2 \times \frac{t \times l^3}{6}$ $(\because of both sides weld)$

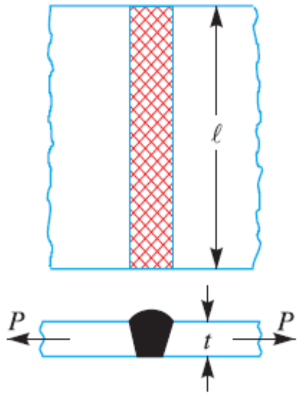
It may be noted that the effect of the applied torque is to rotate the vertical plate about the Z-axis through its mid point.
This rotation is resisted by shearing stresses developed between two fillet welds and the horizontal plate. It is assumed that these horizontal shearing stresses vary from zero at the Z-axis and maximum at the ends of the plate.
This variation of shearing stress is analogous to the variation of normal stress over the depth (l) of a beam subjected to pure bending.

$$\therefore \text{shear stress,} \qquad \tau = \frac{T \times l/2}{t \times l^3/6} = \frac{3T}{t \times l}$$

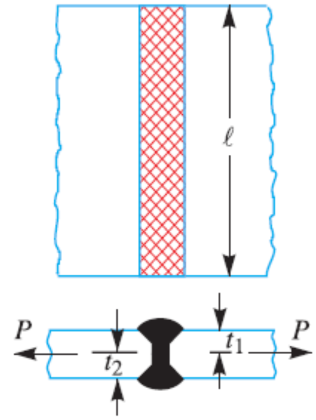
The maximum shear stress occurs at the throat and is given by

$$\tau_{max} = \frac{3T}{0.707s \times l^2} = \frac{4.242\ T}{s \times l^2}$$

Strength of Butt Joints



(a) Single-V or square butt joint



(b) Double-V butt joint

The butt joints are designed for tension or compression.In case of butt joint, the length of leg or size of weld is equal to the throat thickness which is equal to thickness of plates.
l = **Length of weld**. It is generally equal to the width of plate.

$\therefore$ Tensile strength of the single-V or square butt joint, $\boxed{P = t \times l \times \sigma_t}$

$\therefore$ Tensile strength of the double-V butt joint, $\boxed{P = (t_1 + t_2) \times l \times \sigma_t}$
t_1 = Throat thickness at the top, and
t_2 = Throat thickness at the bottom

Axially Loaded Unsymmetrical Welded

Sometimes unsymmetrical sections such as **angles, channels, T-sections** etc., welded on the flange edges are loaded axially as shown in Figure. In such cases, the lengths of weld should be proportioned in such a way that the sum of resisting moments of the welds about the gravity axis is zero. Consider an angle section as shown in Figure.

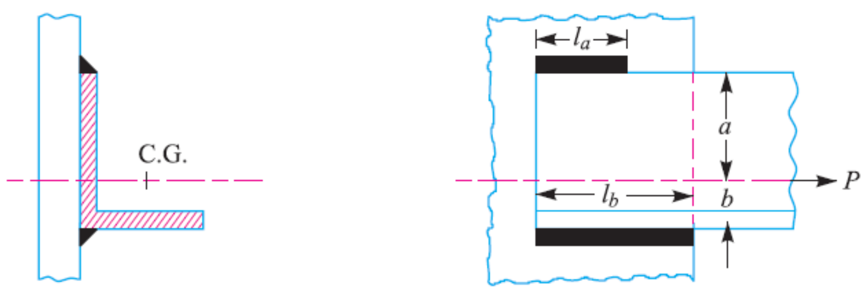


Figure 18: Axially loaded unsymmetrical welded section

Let, l_a =Length of weld at the top
l_b =Length of weld at the bottom
l = Total length of weld = $l_a + l_b$
P =Axial load

$a =$Distance of top weld from gravity axis
$b =$Distance of bottom weld from gravity axis
$f =$Resistance offered by the weld per unit length

$\therefore$Moment of the top weld about gravity axis $= l_a \times f \times a$
and moment of the bottom weld about gravity axis $= l_b \times f \times b$

Since the sum of the moments of the weld about the gravity axis must be zero,
$\therefore (l_a \times f \times a) - (l_b \times f \times b) = 0$

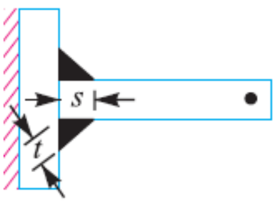
$$l_a \times a = l_b \times b$$

Also $\quad l = l_a + l_b$
Using both equations

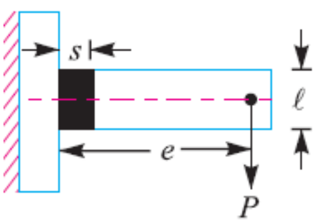
$$l_a = \frac{l \times b}{a+b} \qquad \text{and} \qquad l_b = \frac{l \times a}{a+b}$$

Eccentrically Loaded Welded Joints

An eccentric load may be imposed on welded joints in many ways. The stresses induced on the joint may be of different nature or of the same nature. The induced stresses are combined depending upon the nature of stresses.



- When the shear and bending stresses are simultaneously present in a joint (case 1).
- When the stresses are of the same nature, these may be combined vectorially (case 2)



Maximum normal stress, $\quad \sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b^2 + 4\tau^2)}$

Maximum shear stress, $\quad \tau_{max} = \frac{1}{2}\sqrt{(\sigma_b^2 + 4\tau^2)}$

where, $\quad \sigma_b =$ Bending stress
$\tau =$ Shear stress

<u>**Case 1:**</u> Consider a T-joint fixed at one end and subjected to an eccentric load P at a distance e as shown in the above Figure.
Let $s =$ Size of weld, $l =$ Length of weld, and $t =$ Throat thickness.

The joint will be subjected to the following two types of stresses:

(1) Direct shear stress due to the shear force P acting at the welds.
(2) Bending stress due to the bending moment $P \times e$

Area at the throat for double fillet weld,

$$\begin{aligned} A &= \text{Throat thickness} \times \text{Length of weld} \\ &= 2 \times t \times l \\ &= 2 \times 0.707\ s \times l = 1.414\ s \times l \qquad (\because t = s\ \sin 45^\circ) \end{aligned}$$

$\therefore$ Shear stress in the weld (assuming uniformly distributed),

$$\tau = \frac{P}{A} = \frac{P}{1.414\ s \times l}$$

Section modulus of the weld metal through the throat   For both sided weld Z will be given

$$Z = \frac{t \times l^2}{6} \times 2 = \frac{0.707\ s \times l^2}{6} \times 2 = \frac{s \times l^2}{4.242}$$

Bending moment, $M = P \times e$

$$\therefore \text{Bending stress,} \quad \sigma_b = \frac{M}{Z} = \frac{P \times e \times 4.242}{s \times l^2}$$

$$\text{Maximum normal stress,} \qquad \sigma_{t(max)} = \frac{\sigma_b}{2} + \frac{1}{2}\sqrt{(\sigma_b^2 + 4\tau^2)}$$

$$\text{Maximum shear stress,} \qquad \tau_{max} = \frac{1}{2}\sqrt{(\sigma_b^2 + 4\tau^2)}$$

where, σ_b = Bending stress
τ = Shear stress

Case 2: When a welded joint is loaded eccentrically, the following two types of the stresses are induced:

(1) Direct or primary shear stress, and
(2) Shear stress due to turning moment

$$\tau_1 = \frac{P}{A} = \frac{P}{2t \times l}$$
$$\tau_2 = \frac{T \times r_2}{J} = \frac{P \times e \times r_2}{J}$$

Reference

1. R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd.

ME3531: Solid Mechanics and Mechanical Design – S1:2019
Lecture 11–20 May 2019 Thilina H. Weerakkody
Reliability and Statistics

Material Covered

1. Definitions, Terminologies and Formulas.
2. Loading and Stresses
3. Stresses in Rotating Machinery
4. Mechanical Failures
5. Failure Prediction Methods
6. Reliability
7. Statistical Approach
8. Examples

ME3531: Solid Mechanics and Mechanical Design
Reliability & Statistics

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Department of Mechanical Engineering
SLIIT-Sri Lanka

Year 3 −Semester 1, 2019

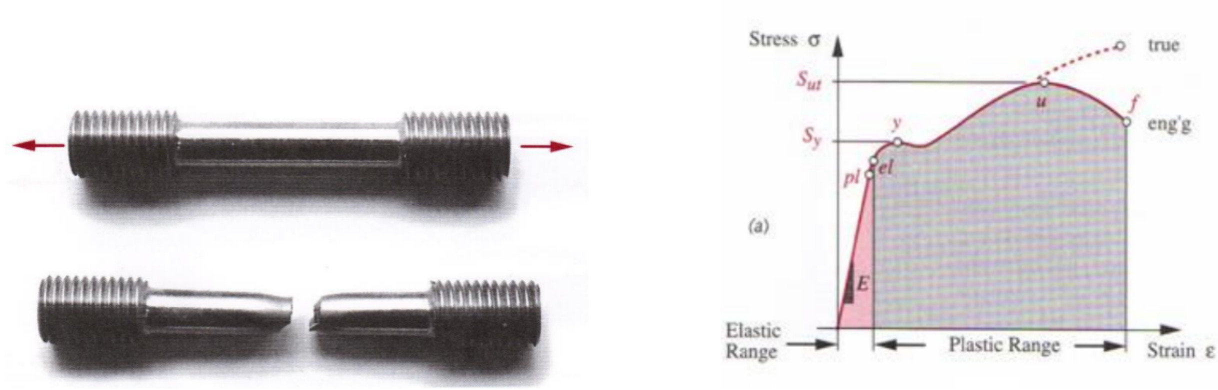
Outline

- Definitions, Terminologies and Formulas.
- Loading and Stresses
- Stresses in Rotating Machinery
- Mechanical Failures
- Failure Prediction Methods
- Reliability
- Statistical Approach
- Examples

1 Definitions, Terminologies and Formulas

Ductility

The tendency of a material to deform significantly before fracturing is a measure of its ductility. The absence of significant deformation before fracture is called brittleness.
The distortion (also known as necking-down) can clearly be seen at the break. The fracture surface appears torn and is laced with hills and valleys, also indicating a ductile failure. The ductility of a material is measured by ***its percent elongation to fracture***, or ***percent reduction in area at fracture***. Materials with more than 5% elongation at fracture are considered ductile.

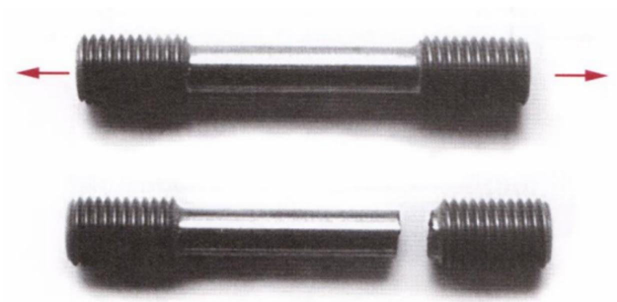


(a) Ductile Material Failure (b) Stress-strain curve for ductile material

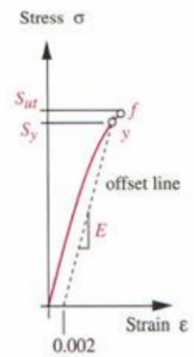
Figure 1: Ductile Materials

Brittleness

The break shows no evidence of necking and has the fine surface contours typical of a brittle material. It is noted that the yield point is not clearly defined in stress-strain curve for brittle materials. In such situations, yield strength S_Y is defined by an offset method as shown in Figure, where a line is drawn at slope E. The line offset distance is usually 0.2% of the original gauge length.



(a) Brittle material failure



(b) Stress-strain curve for brittle Material

Figure 2: Brittle Materials

Stress(σ) and Strain(ϵ)

Stress (σ) is defined as load per unit area of the specimen,

$$\sigma = \frac{P}{A}$$

Strain (ϵ) is the change in the length per unit length and is expressed as,

$$\epsilon = \frac{l - l_\circ}{l_\circ}$$

It's also known as elongation and it is a dimensionless number. However, it is customary to speak of it in terms of mm per mm or inch per inch.

Deformation (δ)

The total change in a uniform body caused by an axial load is called the deformation.

$$\epsilon = \frac{\delta}{l_\circ}$$

For most materials used in engineering, stress and strain are directly proportional. Under this condition, the material is said to follow Hook's law. The linear relationship between stress and strain can be expressed as,

$$\sigma = \epsilon E \quad \text{or} \quad \epsilon = \frac{\sigma}{E} \quad \text{or} \quad \delta = \frac{P \times l_\circ}{A_\circ \times E}$$

The constant **E** is called the Modulus of Elasticity or Young's Modulus, for the material. **E** is a measure of the stiffness of the material in its elastic range and has dimension of stress.

Material Stiffness

Stiffness (k) is the resistance of an elastic body to deflection or deformation by an applied force.

$$k = \frac{P}{\delta}$$

Yield Point

At yield point the strain begins to increase very rapidly without a corresponding increase in stress. This point is call the yield point.

Ultimate Strength

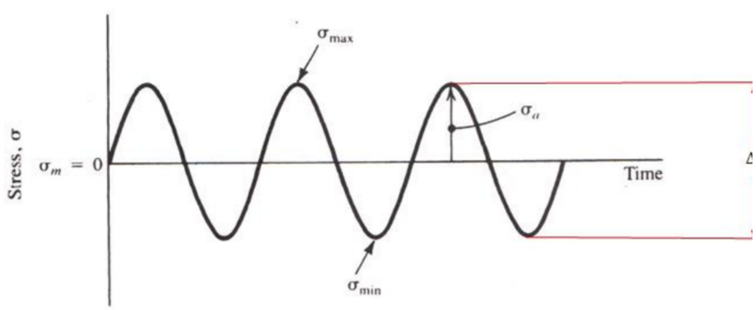
This strength corresponds to the maximum stress reached on the stress-strain curve.

2 Loading and Stresses

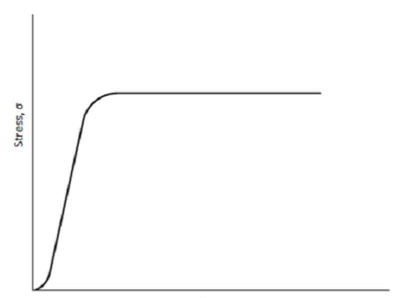
Stress categories

- Maximum stress, σ_{max}
- Minimum stress, σ_{min}
- Mean stress, σ_m
- Alternating stress, σ_a

Stress range ($\Delta\sigma$) is defined as the difference between the maximum and minimum stresses
Static stress – A gradually increasing load is applied to a part, without shock and is held at a constant value.



(a) Loading and Stresses – Stress vs Time diagram



(b) Static stress – Stress vs Time diagram

3 Stresses in Rotating Machinery

Fully Reversed Stresses

This type of stresses occurs when a given element of a load-carrying member is subjected to a certain level of stress followed by the same level of compressive stress.
This type of loading is also called fatigue loading. The material's ability to withstand fatigue loads is called its endurance strength.

$$\sigma_{min} = -\sigma_{max} \qquad \sigma_m = 0$$

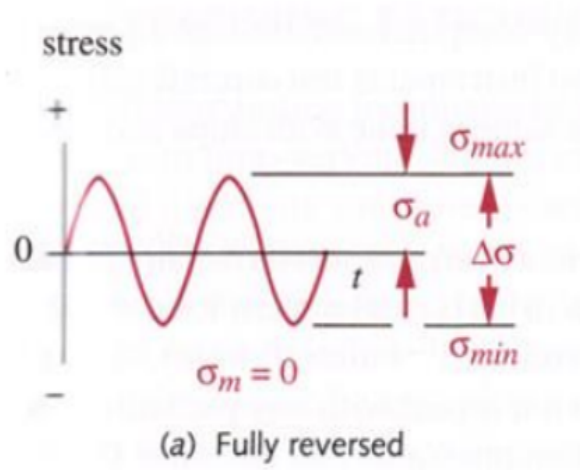


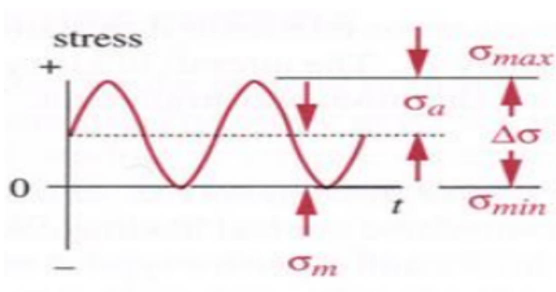
Figure 4: Fully Reversed Stresses

Repeated Stresses

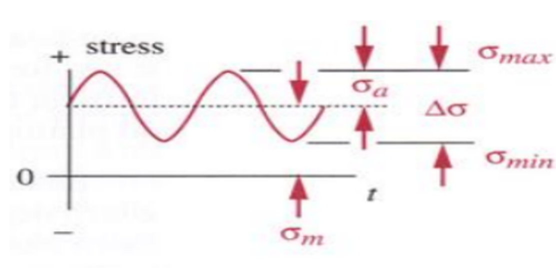
The stress value ranges from zero to a maximum with a mean value to the alternating component

Fluctuating stresses

There are applications in which a load-carrying member is subjected to an alternating stress with non zero value of mean stress.



(a) Repeated Stresses



(b) Fluctuating stresses

4 Mechanical Failures

Failure of a loaded member can be regarded as any behavior that renders it unsuitable for its intended function. Some of the most common failures are from corrosion and wear, but excluding these, components fail under load by yielding, fracture, fatigue, buckling and rapid fracture etc.

Yielding

If yield stress is exceeded due to bending, tension, compression, torsion or shear the material is permanently deformed – it will not return to its original length or shape if the load is removed. It usually still in one piece, but cannot normally perform its desired function, for example, a shaft is bent, a bolt or tension member is permanently stretched, a pressure vessel is bulged.

Fracture

If a material is strained beyond the point of yield, and sufficient deformation occurs that UTS (Ultimate Tensile Strength) is reached and passed, the components will eventually fracture (i.e. it comes apart). For example, pipes burst, pressure vessels explode, bolts break. A ductile material requires considerable deformation before fracture, a brittle material almost none.

Fatigue

Fatigue is caused by the application of cyclic loads in any form: tension, bending, torsion, shear etc. Multiple application of a cyclic stress can grow fatigue cracks, which start at places having some form of discontinuity.
A cyclic stress at a certain level, and after a large number of cycles, will cause a crack to start and grow. The final failure can occur by simple overload of the remaining cross section of the part accompanied by small deformation.

Rapid Fracture

This sort of fracture usually occurs when the length of crack reaches the 'critical length', and then propagates almost instantly through the remainder of the cross section.

Buckling

If a component is subjected to compressive stress it may fail by bucking well before it reaches its yield stress. A member which does not have sufficient lateral rigidity will fail if the load exceeds some critical value. This is a complete and instantaneous collapse.

5 Failure Prediction Methods

Following factors are considered when selecting a failure prediction method,

- The nature of the load (Static, repeated, reverse or fluctuating).
- The type of material involved (ductile or brittle).
- The amount of design effort and analysis that can be justifies by the nature of the component or product being designed.

Failure Prediction Method		Application
1	Maximum normal stress theory	Uni-axial static stress on brittle materials
2	Maximum shear stress theory	Biaxial static stress on ductile materials
3	Maximum distortion energy theory	Biaxial or tri-axial stress on ductile materials
4	Modified Mohr's circle	Biaxial static stress on brittle materials
5	Goodman	Fluctuating stress on ductile materials
6	Gerber	Fluctuating stress on ductile materials
7	Soderberg	Fluctuating stress on ductile materials

1. Maximum normal stress theory

This is the simplest of the failure theories. Material will fail when maximum normal stress exceeds the ultimate strength. Not correlated with ductile materials.

Design Stress:

$$K_t\sigma < \sigma_d = \frac{S_{UT}}{FOS} \qquad \text{For } \textbf{Tensile} \text{ Stress}$$

$$K_t\sigma < \sigma_d = \frac{S_{UC}}{FOS} \qquad \text{For } \textbf{Compressive} \text{ Stress}$$

Where, σ = Applied Stress
σ_d = Design Stress
K_t = Stress Concentration Factor
S_{UT} =Material's Ultimate Strength in tension
S_{UC} =Material's Ultimate Strength in compression
FOS = Factor of Safety

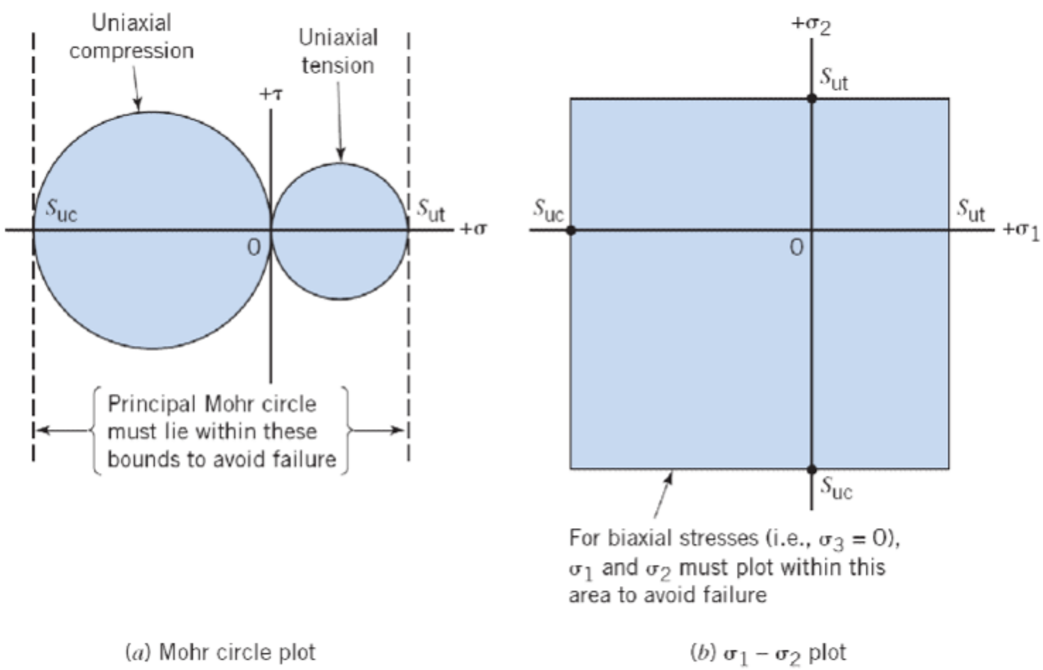


Figure 6

2. Maximum Shear Stress Theory

This theory states that a material subjected to any combination of loads will fail (by yielding or fracture) if the maximum shear stress exceeds the shear strength (yield or ultimate) of the material found from a standard uni-axial tensile test.

Correlates with ductile material

$$\tau_{max} < \tau_d = \frac{S_{sy}}{FOS} = \frac{0.5 S_{yt}}{FOS}$$

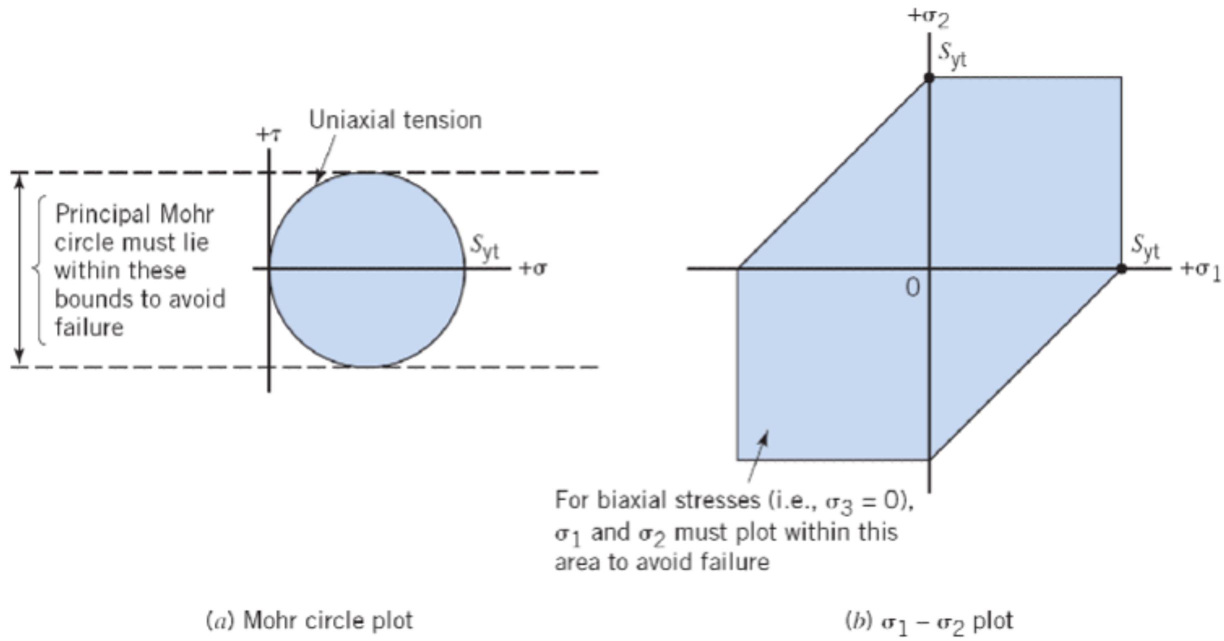


Figure 7

3. Maximum Distortion Energy Theory

- Theory is also known as Van Mises energy theory.
- This is the best predictor of failure for ductile materials.
- The failure is predicted based on energy absorbed by a body in distorting under applied load.
- The theory is shown graphically by plotting the envelope of an equivalent stress σ_e (Von Mises Equivalent) which if exceeded will cause failure.

$$\sigma_e = \frac{\sqrt{2}}{2}\left[(\sigma_2 - \sigma_1)^2 + (\sigma_3 - \sigma_1)^2 + (\sigma_3 - \sigma_2)^2\right]^{\frac{1}{2}}$$

For biaxial stress state ($\sigma_3 = 0$),

$$\sigma_e = [(\sigma_1)^2 + (\sigma_2)^2 - \sigma_1\sigma_2]^{\frac{1}{2}}$$

The direct stresses may also be used without first finding principal stresses

$$\sigma_e = [(\sigma_x)^2 + (\sigma_y)^2 - \sigma_x\sigma_y + 3(\tau_{xy})^2]^{\frac{1}{2}}$$

If only σ_x and τ_{xy} are present, the equation reduces to

$$\sigma_e = [(\sigma_x)^2 + 3(\tau_{xy})^2]^{\frac{1}{2}}$$

According to distortion-energy theory, the following equations are applied to design:

$$\sigma_e < \sigma_d = \frac{S_{yt}}{FOS}$$

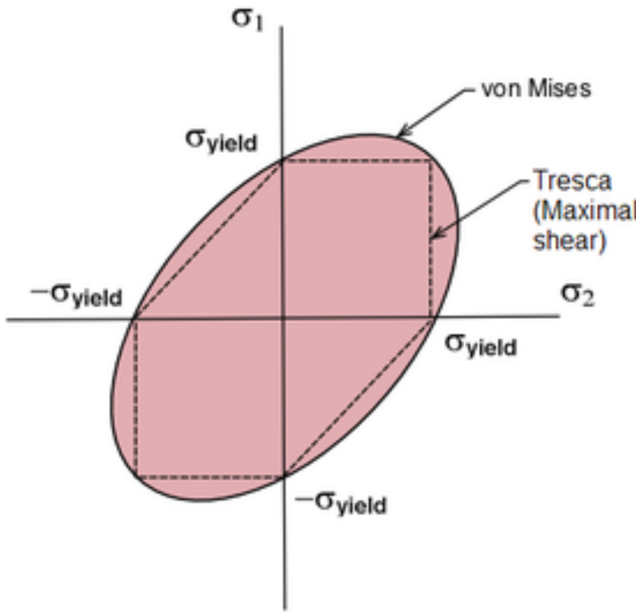


Figure 8: Von Mises yield criterion

4. Modified Mohr Theory,

- The modified Mohr theory is the recommended theory for use with brittle materials.
- The maximum normal stress theory is modified for mixture of compressive and tensile principal stresses.

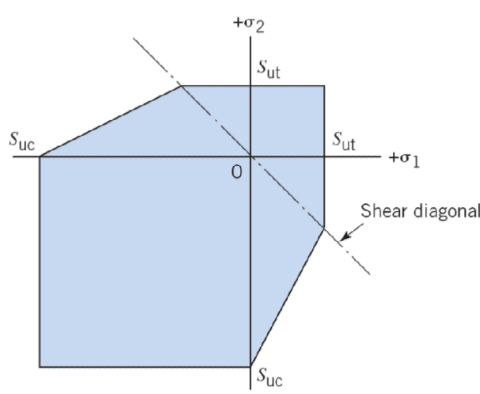


Figure 9: Modified Mohr Theory

6 Reliability

- The factor of safety > 1 does not guarantee that components don't fail.
- There exists an uncertainty with failure of any material.
- The reliability method of design is such that, the distribution of stresses and the distribution of strengths are obtained and then related in order to achieve an acceptable success rate.
- The reliability is denoted by R and can be expressed by a number having the range $(0 \leq R \leq 1)$.

7 Statistical Approach

Normal Distribution

A statistical quantity is described by its mean, standard deviation and distribution type. The most common distribution is the normal or Gaussian distribution.

The standard deviation,

$$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i - \mu)^2}$$

Where μ is the mean value,

$$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$$

Varying the value of the standard deviation changes the shape of the curve.

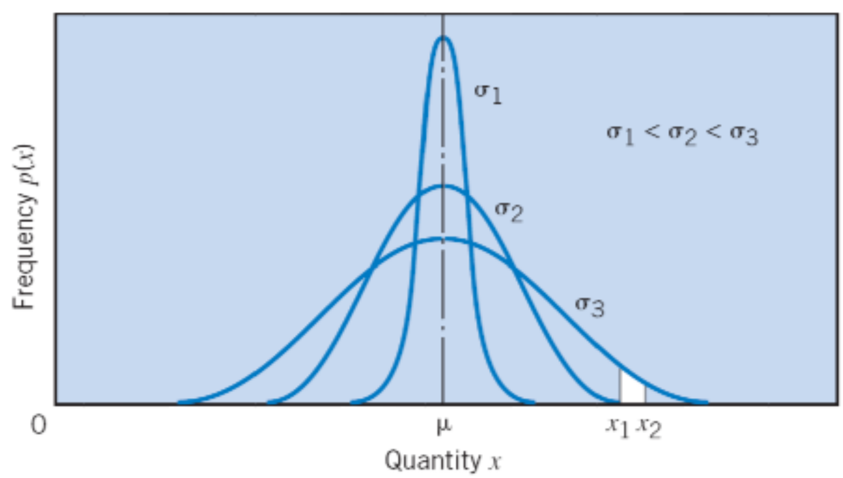


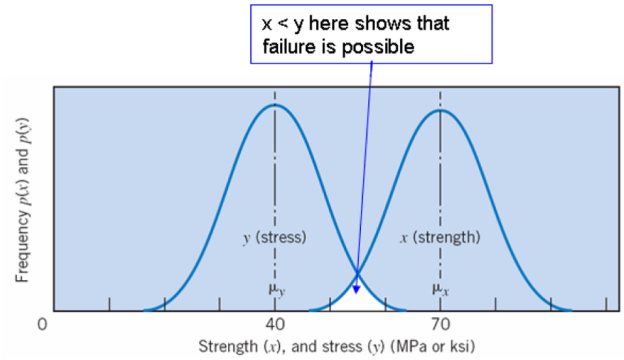
Figure 10: Normal curve by Standard deviation changes

The probability density is the number of occurrences divided by the total sample number.

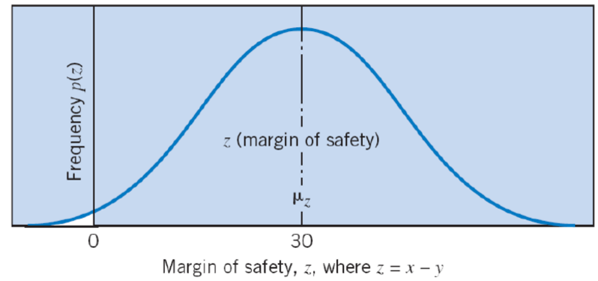
$$p(x) = \frac{1}{\sqrt{2\pi}\sigma}\exp\left[-\frac{(x-\mu)^2}{2\sigma^2}\right]$$

Interpretation of distribution curve

- $\mu \pm 1\sigma$ – 68% failure
- $\mu \pm 2\sigma$ – 95% failure



(a)



(b)

Figure 11

By definition the probability of failure is p(z¡0). For the mean

$$\mu_z = \mu_x - \mu_y$$

$$\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$$

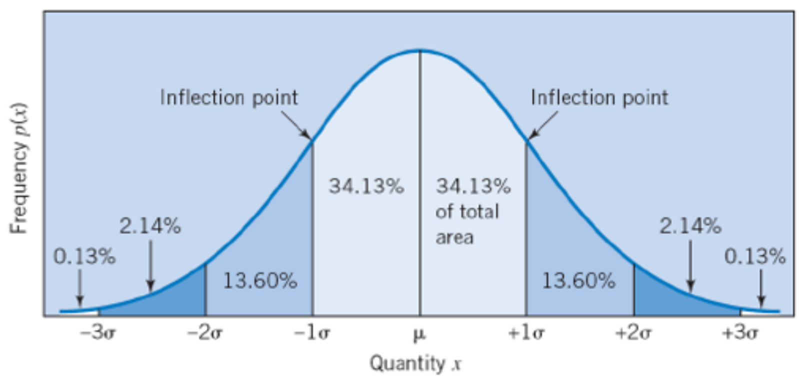


Figure 12

Example

A particular machine part is made of 1030 hot-rolled steel. Ten samples were randomly selected from a lot of 1000 components. The samples were tested for yield strength (the data is shown in Table 2.3).The in-service loading data was also recorded (the data is shown in Table below). What failure percentage would be expected?

Table 2.3: Material strength data

Specimen	Yield strength, S_y (MPa)
1	512.6
2	500.2
3	435.0
4	495.0
5	442.0
6	485.3
7	450.8
8	474.6
9	468.3
10	460.9

Table 2.4: In-service loading data

Stress measurements	In-service loading (MPa)
1	410.7
2	450.2
3	455.6
4	425.0
5	430.9
6	455.0
7	455.1
8	445.6
9	439.6
10	435.3

Figure 13: Example Data

Calculating the Mean (μ) and Standard deviation (σ)

Data	**Mean**	**Standard deviation**
	$\mu = \frac{1}{n}\sum_{i=1}^{n} x_i$	$\sigma = \sqrt{\frac{1}{n-1}\sum_{i=1}^{n}(x_i-\mu)^2}$
Material Strength	472.5 MPa	25.8 MPa
Applied Stress	440.3 MPa	15 MPa

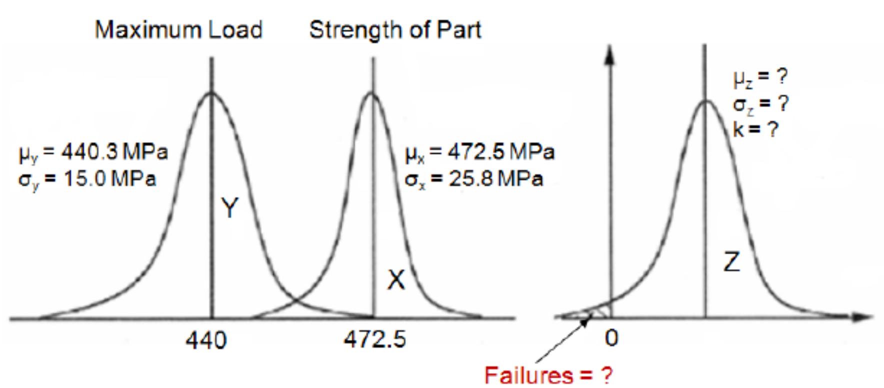


Figure 14

$$\mu_z = \mu_x - \mu_y$$
$$\mu_z = 472.5 - 440.3$$
$$\mu_z = 32.2\text{MPa}$$

$$\sigma_z = \sqrt{\sigma_x^2 + \sigma_y^2}$$
$$\sigma_z = \sqrt{25.8^2 + 15^2}$$
$$\sigma_z = 29.84\text{MPa}$$

Calculating the number of deviations from the mean value

$$k = \frac{\mu_z}{\sigma_z}$$
$$k = \frac{32.2}{29.84}$$
$$k = 1.08$$

From the table below, for k=1.08 the failures are 0.1401 (or 14.01%)

ME3531: Solid Mechanics and Mechanical Design – S1:2019
Lecture 12–24 May 2019 Thilina H. Weerakkody
Design of Shafts

Material Covered

1. Introduction.
2. Material Used for Shafts
3. Types of Shafts
4. Mounting of Shafts
5. Standard Sizes of Shafts
6. Stresses in Shafts
7. Design of Shafts
   (a) Strength
   (b) Rigidity and Stiffness
8. Design of Shafts on the Basis of Strength
   (a) Shafts Subjected to Twisting Moment Only
   (b) Shafts Subjected to Bending Moment Only
   (c) Shafts Subjected to Combined Twisting Moment and Bending Moment
9. Shafts Subjected to Fluctuating Loads (Fatigue)
10. Design of Shafts on the Basis of Rigidity
    (a) Torsional rigidity
    (b) Lateral rigidity

Course Text:

R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd. – Chapter 14. Shafts (pp. 509-557).

ME3531: Solid Mechanics and Mechanical Design
Design of Shafts

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Department of Mechanical Engineering
SLIIT-Sri Lanka

Year 3 –Semester 1, 2019

Outline

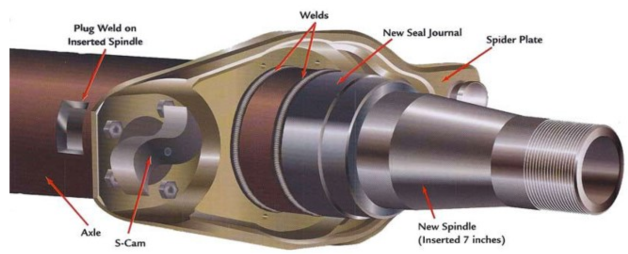
- Introduction.
- Material Used for Shafts
- Types of Shafts
- Mounting of Shafts
- Standard Sizes of Shafts
- Stresses in Shafts
- Design of Shafts
  1. Strength
  2. Rigidity and Stiffness
- Design of Shafts on the Basis of Strength
  1. Shafts Subjected to Twisting Moment Only
  2. Shafts Subjected to Bending Moment Only
  3. Shafts Subjected to Combined Twisting Moment and Bending Moment
- Shafts Subjected to Fluctuating Loads (Fatigue)
- Design of Shafts on the Basis of Rigidity
  1. Torsional rigidity
  2. Lateral rigidity

1 Introduction

- Power transmission is transmit power from one place to another.
- Shaft is a rotating machine element used for power transmission.
- The power is delivered to the shaft by some tangential force.
- Shafts transmit power by twisting
- The resultant torque (or twisting moment) set up within the shaft, permits the power to be transferred to various machines linked up to the shaft
- Other machine elements such as Pulleys and gears are mounted on shafts
- These members along with the forces exerted upon them causes the shaft to bending
- The various members are mounted on the shaft by means of keys or splines.

- The shafts are usually cylindrical, but may be square or cross-shaped in section.
- They are solid in cross-section but sometimes hollow shafts are also used.
- An **axle**, though similar in shape to the shaft, is a stationary machine element and is used for the transmission of bending moment only. It simply acts as a support for some rotating body such as hoisting drum, a car wheel or a rope sheave.
- A spindle is a short shaft that imparts motion either to a cutting tool (e.g. drill press spindles) or to a work piece (e.g. Drilling spindle, Lathe spindles)

(a) Bicycle front axle



(b) Spindle

Figure 1: Axles and Spindles

2 Material Used for Shafts

The material used for ordinary shafts is carbon steel.The material used for shafts should have the following properties. When **a shaft of high strength is required**, then an alloy steel such as nickel, nickel-chromium or chrome-vanadium steel is used.

1. It should have high strength
2. It should have good machinability
3. It should have low notch sensitivity factor
4. It should have good heat treatment properties
5. It should have high wear resistant properties

3 Types of Shafts

(a) Transmission shafts. −These shafts transmit power between the source and the machines absorbing power.
The counter shafts, line shafts, over head shafts and all factory shafts are transmission shafts.
Since these shafts carry machine parts such as pulleys, gears etc., therefore they are **subjected to bending in addition to twisting**.

(b) Machine shafts − These shafts form an integral part of the machine itself. The crank shaft is an example of machine shaft.

(a) Transmission Line Shaft

(b) Transmission Propeller Shaft

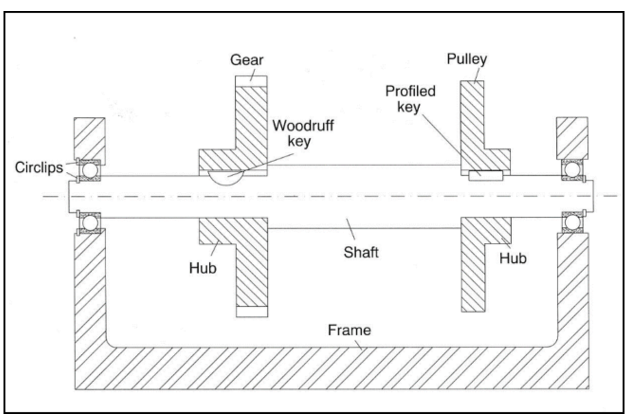
Figure 2: Transmission Shafts

(a)

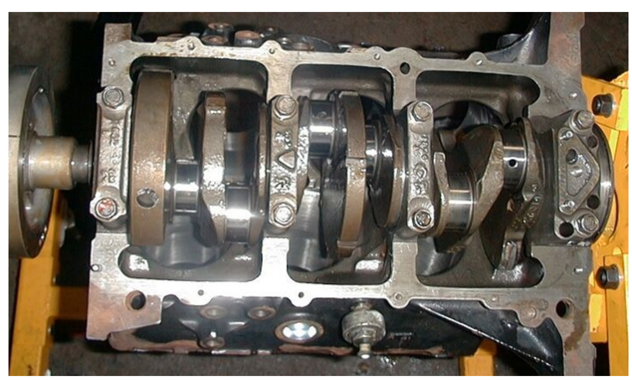
(b)

Figure 3: Transmission Shafts

4 Mounting of Shafts



(a)



(b)

Figure 4: Mounting of Shafts

5 Standard Sizes of Shafts

The standard sizes(diameter) of transmission shafts are :

- 25 mm to 60 mm with 5 mm steps
- 60 mm to 110 mm with 10 mm steps
- 110 mm to 140 mm with 15 mm steps
- 140 mm to 500 mm with 20 mm steps

The standard length of the shafts are 5 m, 6 m and 7 m.The maximum length of shaft usually does not exceed 7 m.
The standard sizes(diameter) of transmission shafts are :

- 10 mm to 75 mm with 5 mm steps
- 40 mm to 110 mm with 10 mm steps

6 Stresses in Shafts

The following stresses are induced in the shafts :

- **Shear stresses** due to the transmission of torque (i.e. due to *torsional load*)
- **Bending stresses** (*tensile or compressive*) due to the forces acting upon machine elements like gears, pulleys etc. as well as due to the weight of the shaft itself.
- **Stresses** due to combined torsional and bending loads

Design of Shafts

The shafts may be designed on the basis of

1. **Strength**
2. **Rigidity and stiffness**

7 Design of Shafts on the Basis of Strength

In designing shafts on the **basis of strength**, the following cases may be considered :

(a) Shafts subjected to twisting moment or torque only.
(b) Shafts subjected to bending moment only.
(c) Shafts subjected to combined twisting and bending moments.
(d) Shafts subjected to axial loads in addition to combined torsional and bending loads.

Shafts must be designed so that:

(a) **The deflections are within acceptable levels** – High deflections cause noise and vibration while in operation

(b) **Critical speed** – Start vibrating violently

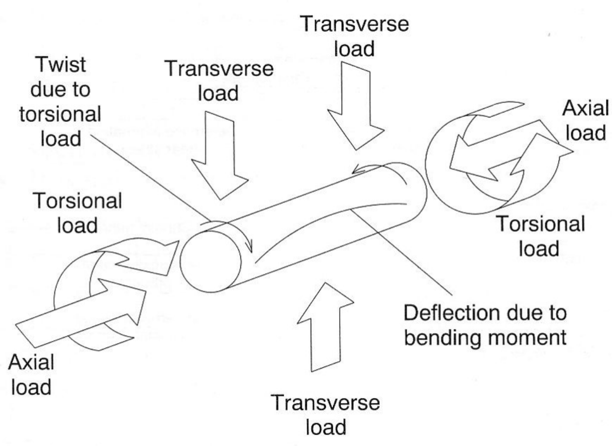


Figure 5: Typical Shaft Loading and Deflection

Design Considerations

- Size and spacing of components, tolerances
- Type of material to be used
- Deflection and rigidity
- Stress and strength (static strength, fatigue, reliability)
- Type of loading (static, shock or cyclic)
- Manufacturing constraints

7.1 Shafts Subjected to Twisting Moment Only

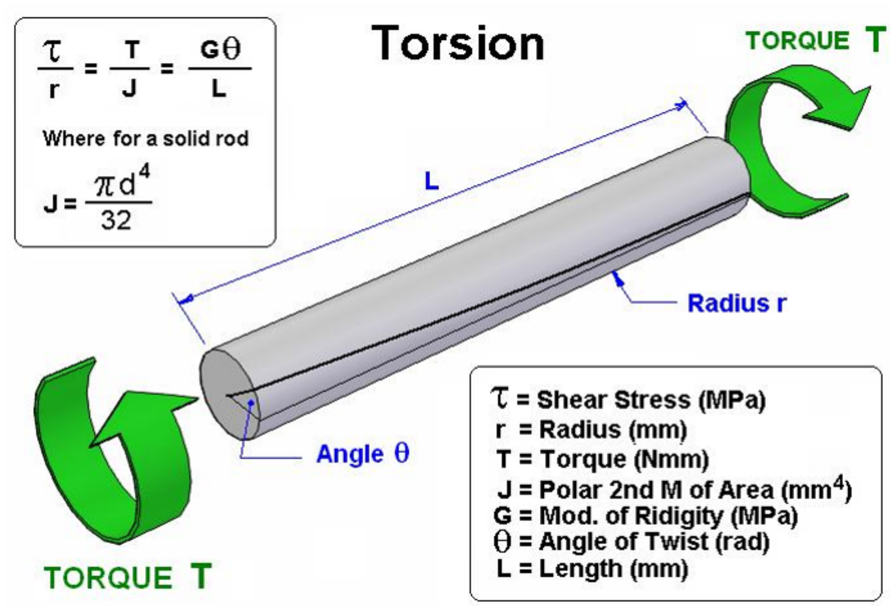


Figure 6: Shafts Subjected to Twisting Moment Only

$$\frac{\tau}{r} = \frac{T}{J} = \frac{G\theta}{L} \tag{1}$$

For a solid rod equation 1 can be written as

$$\frac{T}{\frac{\pi}{32} \times d^4} = \frac{\tau}{\frac{d}{2}} \qquad \text{or} \qquad T = \frac{\pi}{16} \times \tau \times d^3 \tag{2}$$

From this equation, we may determine the diameter of round solid shaft (d). We also know that for hollow shaft, polar moment of inertia.

$$J = \frac{\pi}{32}[(d_\circ)^4 - (d_i)^4]$$

$d_\circ$ and d_i = Outside and inside diameter of the shaft, and $r = \frac{d_\circ}{2}$
Substituting these values in equation 1,

$$\frac{T}{\frac{\pi}{32} \times [(d_\circ)^4 - (d_i)^4]} = \frac{\tau}{\frac{d_\circ}{2}} \qquad \text{or} \qquad T = \frac{\pi}{16} \times \tau \times \left[\frac{(d_\circ)^4 - (d_i)^4}{d_\circ}\right] \tag{3}$$

When We know that the power transmitted by the shaft (in Watts) the twisting moment (T) may be obtained by using the following relation

$$\begin{aligned} P &= T\omega = \frac{2\pi N \times T}{60} \\ T &= \frac{P \times 60}{2\pi N} \end{aligned} \tag{4}$$

T = Twisting moment in N-m
N = Speed of the shaft in rpm.

Example 1– A line shaft rotating at 200 r.p.m. is to transmit 20 kW. The shaft may be assumed to be made of mild steel with an allowable shear stress of 42 MPa. Determine the diameter of the shaft, neglecting the bending moment on the shaft.

Example 2– A solid shaft is transmitting 1 MW at 240 r.p.m. Determine the diameter of the shaft if the maximum torque transmitted exceeds the mean torque by 20%. Take the maximum allowable shear stress as 60 MPa.

7.2 Shafts Subjected to Bending Moment Only

When the shaft is subjected to a bending moment only, then the maximum stress (tensile or compressive) is given by the bending equation.

$$\frac{M}{I} = \frac{\sigma_b}{y} = \frac{E}{R}$$

M = Bending moment.
I = Moment of inertia of cross-sectional area of the shaft about the axis of rotation.
σ_b =Bending stress.
y = Distance from neutral axis to the outer-most fibre.
E = Young's Modulus of the Material of the beam.

R = Radius of curvature of the bent beam.

We know that for a round solid shaft, moment of inertia,

$$I = \frac{\pi}{64} \times d^4 \qquad \text{or} \qquad y = \frac{d}{2}$$
$$\frac{M}{\frac{\pi}{64} \times d^4} = \frac{\sigma_b}{\frac{d}{2}} \qquad \text{or} \qquad M = \frac{\pi}{32} \times \sigma_b \times d^3 \tag{5}$$

From this equation, diameter of the solid shaft (d) may be obtained. We also know that for a hollow shaft, moment of inertia.

Example 3– A pair of wheels of a railway wagon carries a load of 50 kN on each axle box, acting at a distance of 100 mm outside the wheel base. The gauge of the rails is 1.4 m. Find the diameter of the axle between the wheels, if the stress is not to exceed 100 MPa.

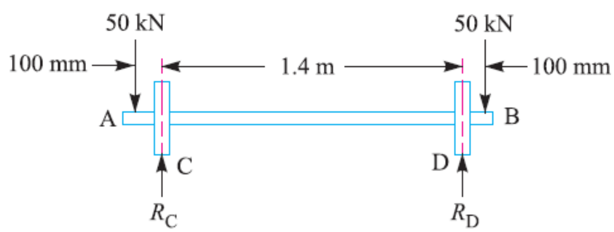


Figure 7: Example 3

7.3 Shafts Subjected to Combined Twisting Moment and Bending Moment

Shaft should be designed by taking two moments simultaneously

1. **Maximum shear stress theory** – for ductile materials such as mild steel
2. **Maximum normal stress theory** – for brittle materials such as cast iron

According to maximum shear stress theory, the maximum shear stress in the shaft

$$\tau_{max} = \frac{1}{2}\sqrt{(\sigma_b)^2 + 4\tau^2}$$

τ = Shear stress induced due to twisting moment
σ_b = Bending stress (tensile or compressive) induced due to bending moment.

Substituting $T = \frac{\pi}{16} \times \tau \times d^3$ and $M = \frac{\pi}{32} \times \sigma_b \times d^3$ to the above τ_{max} equation, you may obtain that

$$\tau_{max} = \frac{16}{\pi d^3}\sqrt{M^2 + T^2}$$

The expression $\sqrt{M^2 + T^2}$ is known as equivalent ***twisting moment*** and is denoted by T_e.

The ***equivalent twisting moment*** may be defined as that twisting moment, which when acting alone, produces the same shear stress (τ) as the actual twisting moment.

By limiting the maximum shear stress (τ_{max}) equal to the allowable shear stress (τ) for the material, the equation may be written as

$$T_e = \sqrt{M^2 + T^2} = \frac{\pi}{16} \times \tau \times d^3$$

Example 4– A solid circular shaft is subjected to a bending moment of 3000 N-m and a torque of 10 000 N-m. The shaft is made of 45 C 8 steel having ultimate tensile stress of 700 MPa and a ultimate shear stress of 500 MPa. Assuming a factor of safety as 6, determine the diameter of the shaft.

Example 5– A shaft supported at the ends in ball bearings carries a straight tooth spur gear at its mid span and is to transmit 7.5 kW at 300 r.p.m. The pitch circle diameter of the gear is 150 mm.The distances between the centre line of bearings and gear are 100 mm each. If the shaft is made of steel and the allowable shear stress is 45 MPa, determine the diameter of the shaft. Show in a sketch how the gear will be mounted on the shaft; also indicate the ends where the bearings will be mounted? The pressure angle of the gear may be taken as 20°.

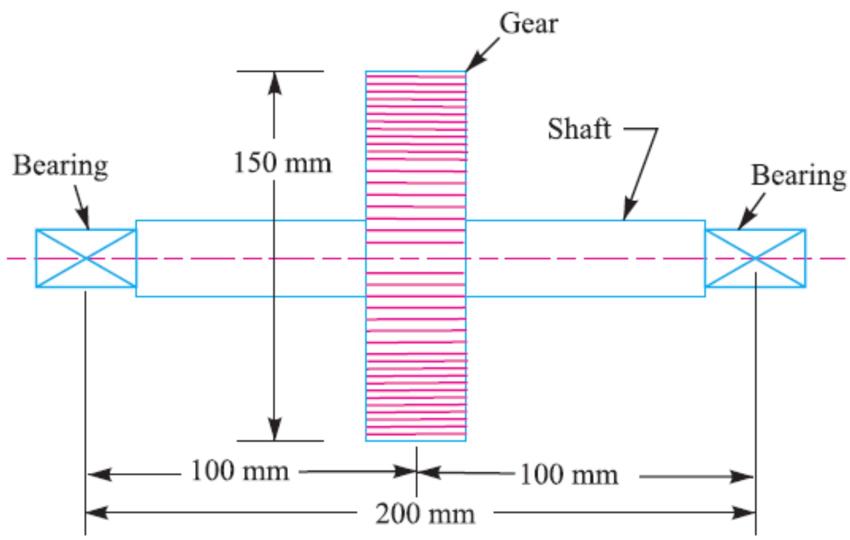


Figure 8: Example 5

8 Shafts Subjected to Fluctuating Loads (Fatigue)

- In the previous articles we have assumed that the shaft is subjected to constant torque and bending moment.
- But in actual practice, the shafts are subjected to fluctuating torque and bending moments.
- In order to design such shafts like line shafts and counter shafts, the combined shock and fatigue factors must be taken into account for the computed twisting moment(T) and bending moment(M).
- Thus for a shaft subjected to combined bending and torsion, the equivalent twisting moment,

$$T_e = \sqrt{(K_m \times M)^2 + (K_t \times T)^2}$$

and equivalent bending moment,

$$M_e = \frac{1}{2}\left[K_m \times M + \sqrt{(K_m \times M)^2 + (K_t \times T)^2}\right]$$

K_m = Combined shock and fatigue factor for bending
K_t = Combined shock and fatigue factor for torsion

Table 1: Recommended values for K_m and K_t

Nature of load	K_m	K_t
1. Stationary shafts		
(a) Gradually applied load	1.0	1.0
(b) Suddenly applied load	1.5 to 2.0	1.5 to 2.0
2. Rotating shafts		
(a) Gradually applied or steady load	1.5	1.0
(b) Suddenly applied load with minor shocks only	1.5 to 2.0	1.5 to 2.0
(c) Suddenly applied load with heavy shocks	2.0 to 3.0	1.5 to 3.0

Example 6– A mild steel shaft transmits 20 kW at 200 r.p.m. It carries a central load of 900 N and is simply supported between the bearings 2.5 metres apart. Determine the size of the shaft, if the allowable shear stress is 42 MPa and the maximum tensile or compressive stress is not to exceed 56 MPa. What size of the shaft will be required, if it is subjected to gradually applied loads?

9 Design of Shafts on the Basis of Rigidity

Shafts must be rigid enough to avoid excessive deflection. There are two types of rigidity.

1. Torsional rigidity
2. Lateral rigidity

9.1 Torsional rigidity

1. Important for camshafts where timing of the valves are crucial.
2. Estimate the total angle of twist in radians – Use torsion equation.

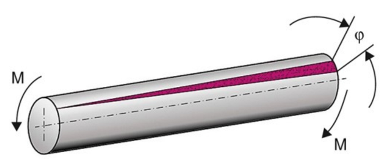


Figure 9: Torsional Rigidity

The torsional deflection may be obtained by using the torsion equation

$$\frac{T}{J} = \frac{G.\theta}{L} \qquad \text{or} \qquad \theta = \frac{T.L}{J.G}$$

θ = Torsional deflection or angle of twist in radians
T = Twisting moment or torque on the shaft
J = Polar moment of inertia of the cross-sectional area about the axis of rotation
L = Length of the shaft.
G = Modulus of rigidity for the shaft material

$$J = \frac{\pi}{32} \times d^4 \qquad \text{(For solid shaft)}$$
$$J = \frac{\pi}{32} \times [(d_\circ)^4 - (d_i)^4] \qquad \text{(For hollow shaft)} \tag{6}$$

9.2 Lateral rigidity

- Important for
  - Transmission shafting
  - Shafts running at high speed
- Lateral deflection must be minimized to avoid
  - Gear teeth alignment problems
  - Bearing related problems
- The lateral deflection (y) and the slope (θ) may be determined by equations from the strength of materials.

$$\frac{d^2y}{dx^2} = \frac{M}{EI}$$

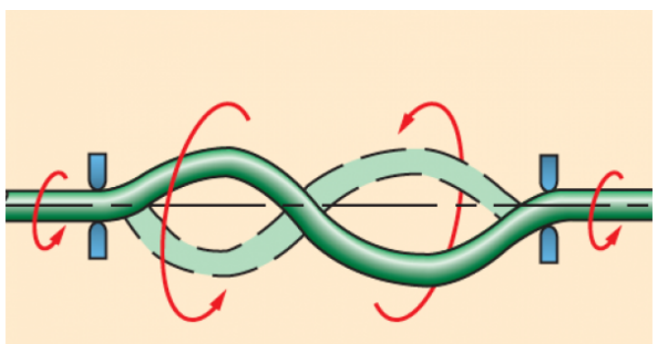


Figure 10: Lateral Rigidity

Example 7– A steel spindle transmits 4 kW at 800 r.p.m. The angular deflection should not exceed 0.25○ per metre of the spindle. If the modulus of rigidity for the material of the spindle is 84 GPa, find the diameter of the spindle and the shear stress induced in the spindle.

Reference

1. R.S.Khurmi and J.K.Gupta, *"A Textbook of Machine Design"*, 14th edition (2015), New Delhi, India: S. Chand & Company Ltd.– Chapter 14. Shafts (pp. 509-557)