Professor George Mann,
Department of Mechanical Engineering
Sri Lanka Institute of Information Technology (SLIIT)

# Assignment 1

Provide answers for Q1, Q5(a) and Q6.
DUE DATE: MARCH 7, 10:30 a.m
SUBMIT DURING THE CLASS TIME.

**Q1** Determine the mobility of the mechanisms shown in Figure Q1.

$n = 8$, $j_p = 10$, $j_h = 0$

$M = 3(8-1) - 2 \times 10 - 0$

$M = 1$

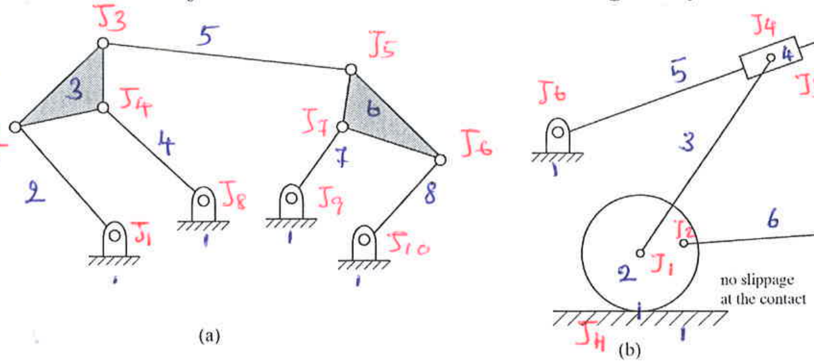



Figure Q1

$n = 6$

$J_p = 6$

$j_h = 1$

$M = 3(6-1) - 2 \times 6 - 1$

$M = 2$

**Q2** Consider Figure Q2 and assume all the position variables are available. Is it possible to estimate velocity of all other points given $\omega$?

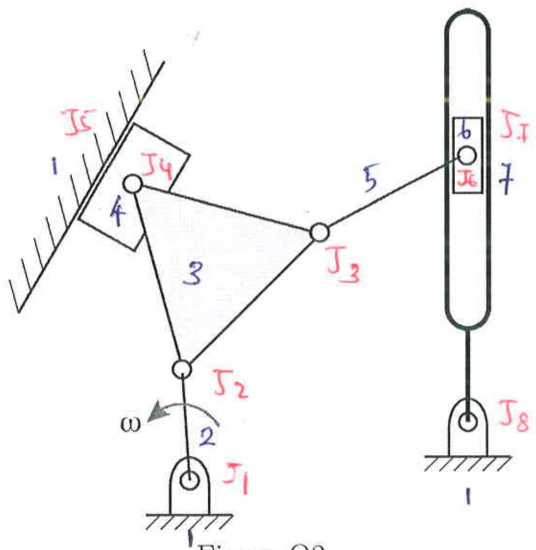



Figure Q2

$n = 7$

$J_p = 8$

$J_h = 0$

$M = 3(n-1) - 2J_p - J_h$

$= 3(7-1) - 2 \times 8 - 0$

$M = 2$

Need 2 inputs

**Q3** Determine the mobility of the planer mechanisms shown in Figure Q3.

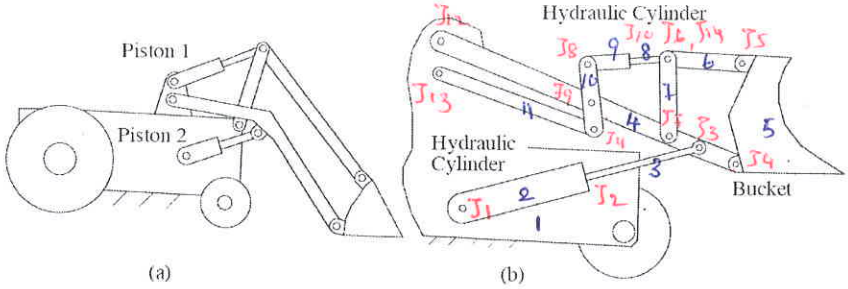



Figure Q3

$n = 11$

$j_p = 14$

$j_h = 0$

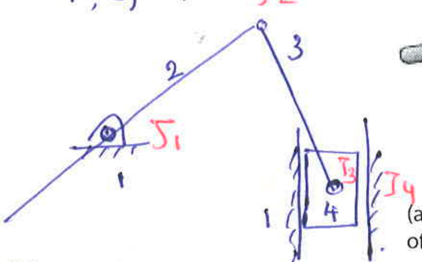
$M = 3(11-1) - 2 \times 14 - 0$

$= 30 - 28 - 0$

$\underline{M = 2}$

**Q4** Figure Q4 shows different mechanisms we generally found in industry. Draw kinematic diagram for each mechanism and specify the reference (or fixed links). Specify number of links and the number of joints and calculate the mobility of each of the mechanism.

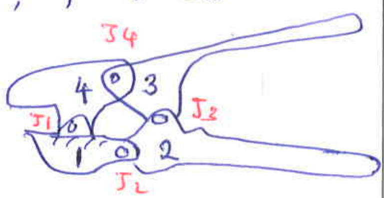
$n = 4,\ j_p = 4$



$M = 3(4-1) - 2 \times 4 - 0$

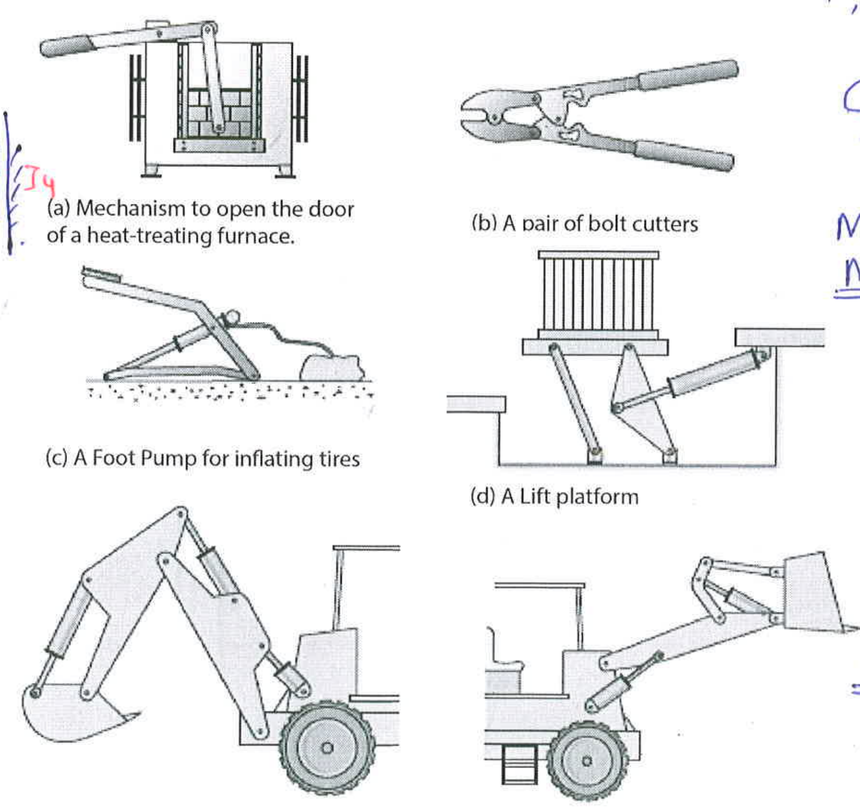
$\underline{M = 1}$

$n = 4,\ j_p = 4,\ j_h = 0$



$M = 3(4-1) - 2 \times 4 - 0$

$\underline{M = 1}$



(a) Mechanism to open the door of a heat-treating furnace.

(b) A pair of bolt cutters

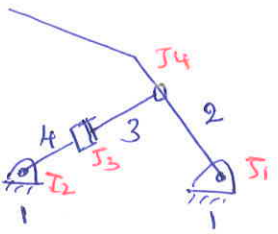
(c) A Foot Pump for inflating tires

(d) A Lift platform

(e) An excavator

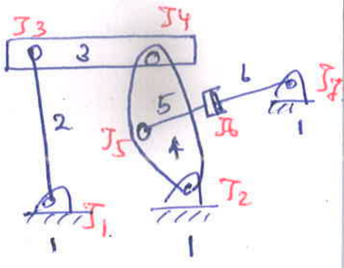
(f) A front loader

Figure Q4



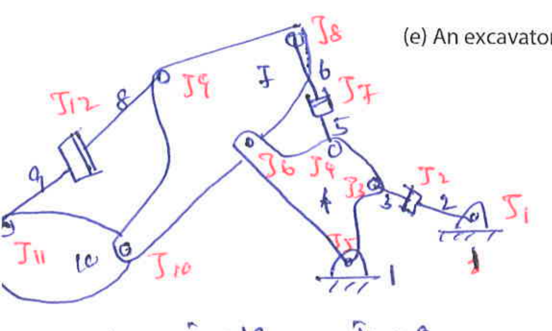
$M = 3(4-1) - 2 \times 4 - 0$

$\underline{M = 1}$



$M = 3(6-1) - 2 \times 7 - 0$

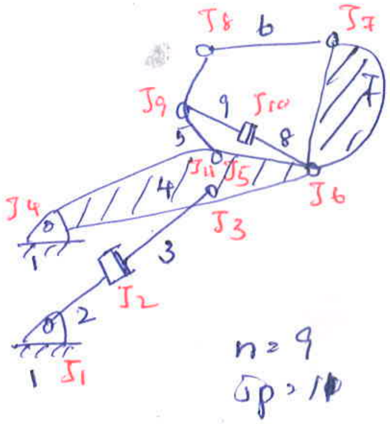
$\underline{M = 1}$



$n = 10 \quad j_p = 12 \quad j_h = 0$

$M = 3(10-1) - 2 \times 12 - 0$

$\underline{M = 3}$



$n = 9$

$j_p = 10$

$M = 3(9-1) - 2 \times 10 - 0$

$\underline{M = 2}$

**Q5** A four bar mechanism is shown in Figure Q5.
(a) Determine the range of values of the length of link 4 ($r_4$) so that the mechanism can be classified into different classes as given Grashof's criterion.
$r_1 = 1.0$ cm, $r_2 = 3.0$ cm, $r_3 = 2.5$ cm.
(b) Determine the range of values of the length of link 2 ($r_2$) so that the mechanism can be classified into different classes as given Grashof's criterion.
$r_1 = 1.0$ cm, $r_3 = 2.5$ cm, $r_4 = 2.0$ cm.

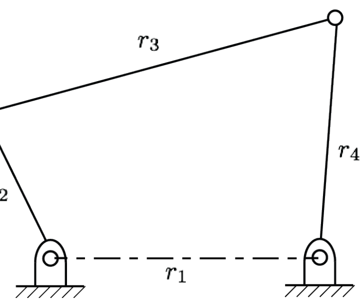



Figure Q5

**Q6** Draw kinematic diagrams to describe operating principles of the following mechanisms (You may use internet to find solutions). A clear diagram with some explanations is required. (a) Door damper mechanisms (b) Car window mechanism. (c) Dump truck lifting mechanism.

**Q7** An adjustable slider drive mechanism consists of a crank-slider with an adjustable pivot, which can be moved up and down and is shown in Figure Q7.
(a) How many bodies (links) can be identified in this mechanism?
(b) Identify the type (and corresponding number) of all kinematic joints.
(c) What is the function of this mechanism and how will it be affected by moving the pivot point up and down?

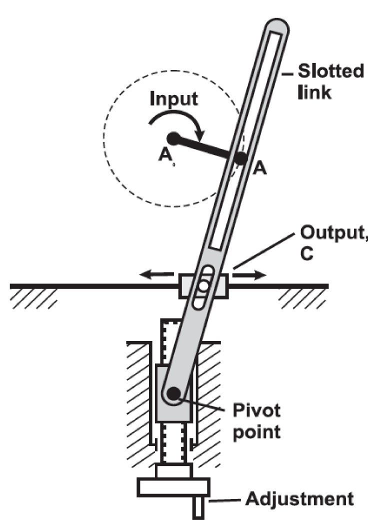
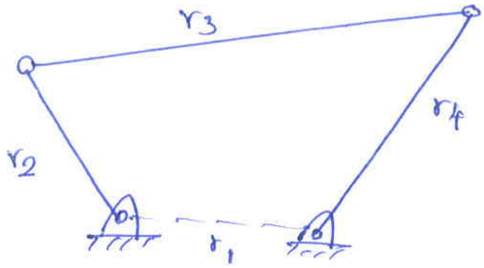



Figure Q7

Q5)
(a)




Apply Grashof's Criteria

$$\begin{cases} s+l < p+q & \text{- Type I} \\ s+l > p+q & \text{- Type II} \end{cases}$$

$r_1 = 1.0$ cm  $r_2 = 3.0$ cm  $r_3 = 2.5$ cm  $r_4$ - Vary

$0 < r_4 < 1.0$ cm

$s = r_4$; $l = r_2$ ; $p+q = r_1 + r_3 = 3.5$ cm ; s - attached to the base

$s + l < p + q$

$r_4 + 3.0 < 3.5$

$r_4 < 0.5$ cm

$0 < r_4 < 0.5$  Crank - Rocker

$r_4 = 0.5$  Change Point

$0.5 < r_4 < 1.0$  Rocker - Rocker/Double Rocker

$1 < r_4 < 3.0$

$s = r_1$ ; $l = r_2$ ; $p+q = r_3 + r_4 = r_4 + 2.5$ ; s - the Base

$s + l < p + q$

$1.0 + 3.0 < r_4 + 2.5$

$1.5 < r_4$

$1.0 < r_4 < 1.5$  Rocker - Rocker/Double Rocker

$r_4 = 1.5$  Change Point

$1.5 < r_4 < 3.0$  Double Crank / Crank - Crank / Drag Link.

$3.0 < r_4 < 6.5$

$s = r_1$ ; $l = r_4$ ; $p+q = r_2 + r_3 = 5.5$ ; s - Base

$s + l < p + q$

$1.0 + r_4 < 5.5$

$r_4 < 4.5$ cm

$3.0 < r_4 < 4.5$  Double Crank / Crank - Crank / Drag link

$r_4 = 4.5$  Change Point

$4.5 < r_4 < 6.5$  Rocker - Rocker / Double Rocker

SUMMARY

$0 < r_4 < 0.5$  Crank - Rocker

$r_4 = 0.5$  Change Point

$0.5 < r_4 < 1.5$  Rocker - Rocker / Double Rocker

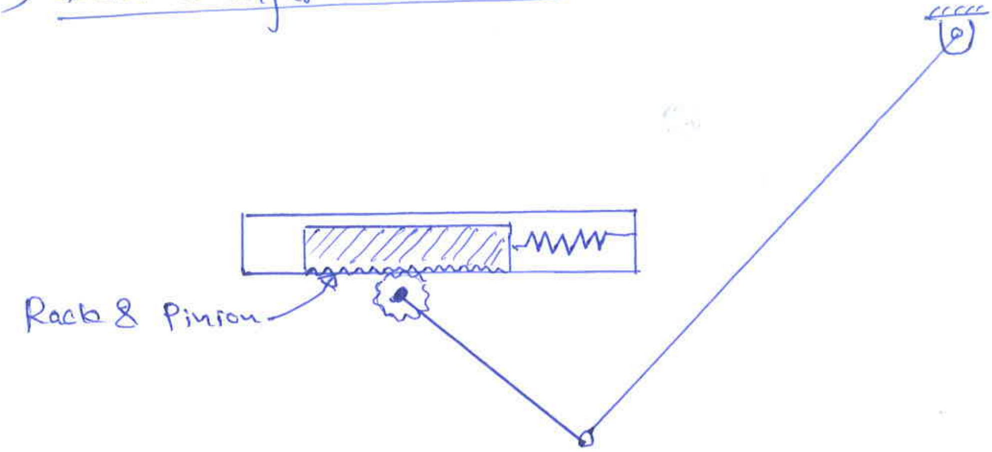
$r_4 = 1.5$  Change Point

$1.5 < r_4 < 4.5$  Double Crank / Drag Link / Crank - Crank

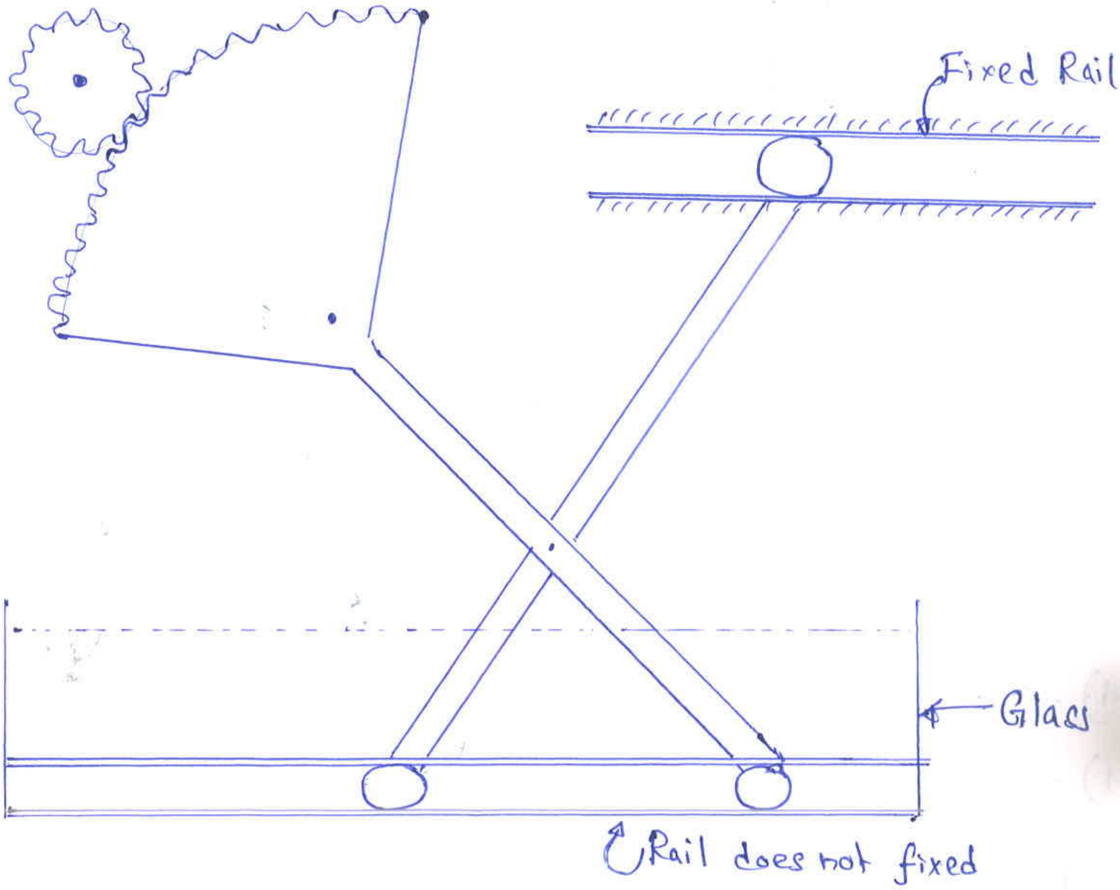
$r_4 = 4.5$  Change Point

$4.5 < r_4 < 6.5$  Rocker - Rocker / Double Rocker

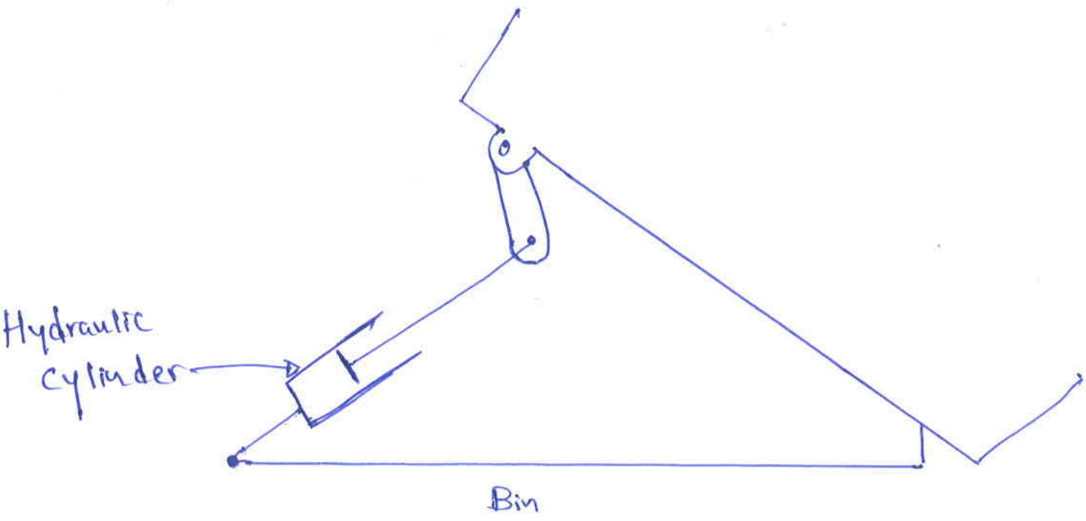
Q6) a) Door Damper Mechanism



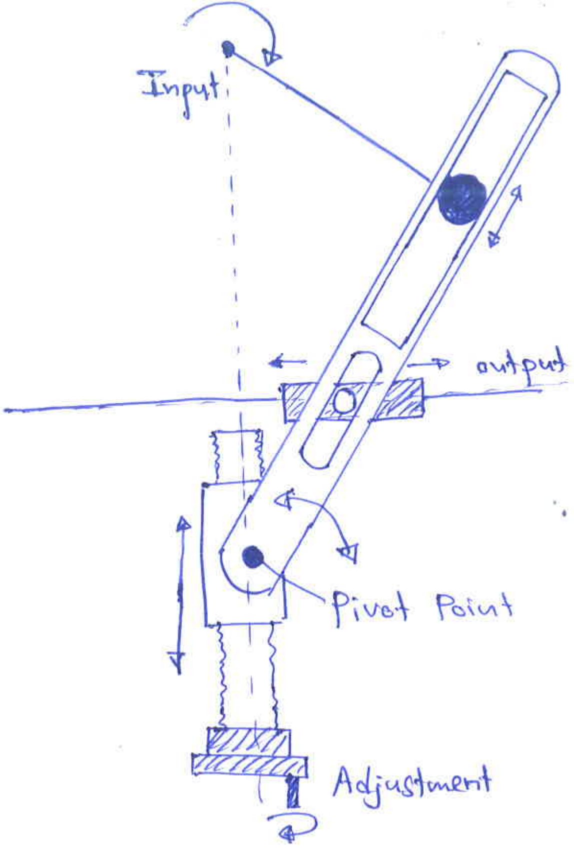

b) Car Window Mechanism




c) Dump Truck Mechanism



(Q7)




Kinematic Diagram

$n = 4$

$J_p = 2$

$J_h = 3$

Gruebler's eqⁿ

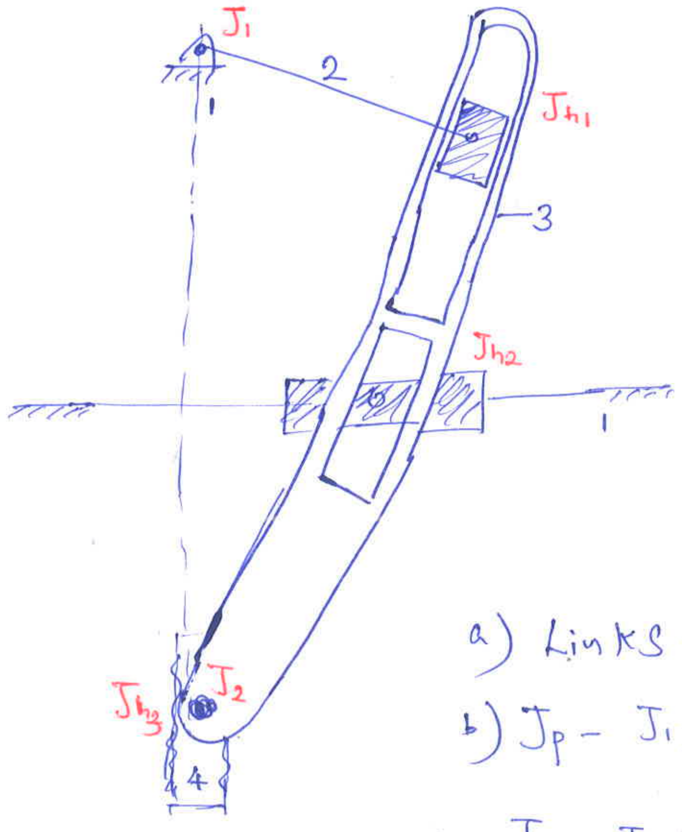
$M = 3(n-1) - 2j_p - j_h$

$= 3(4-1) - 2 \times 2 - 3$

$= 9 - 4 - 3$

$= 2$

Inputs - Input (Link 2)
- Adjuster




a) Links - 4

b) $J_p$ - $J_1$, $J_2$ - Revolute Joints

$J_h$ - $J_{h_1}$ - Pin in a slot

$J_{h_2}$ - Pin in a slot

$J_{h_3}$ - Rack & pinion (Gear)

$J_p$ - 2

$J_h$ - 3

c) Pivot point move down

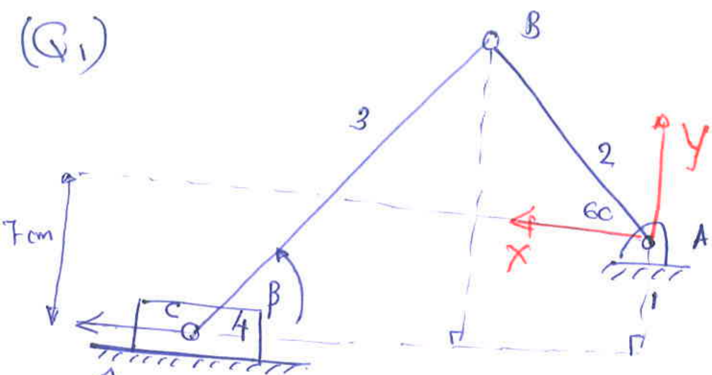
Reduce the angle slot link can move

Pivot point move up

Increase the angle (Range) slot link can move

Problem Set ②

(Q1)




$\hat{V}_c = 40\hat{i}$ cm/s

$\hat{a}_c = 0$

AB = 20 cm

BC = 40 cm

$$\sin\beta = \frac{17 + 20\sin 60}{40}$$

$$\beta = 59.1°$$

$$\hat{V}_B = \hat{V}_A + \hat{\omega}_2 \times \hat{r}_{B/A}$$
$$= \omega_2 \hat{k} \times (20\cos 60\hat{i} + 20\sin 60\hat{j})$$
$$= 20\omega_2(-\sin 60\hat{i} + \cos 60\hat{j}) \quad \text{— ①}$$

$$\hat{V}_B = \hat{V}_c + \hat{\omega}_3 \times \hat{r}_{B/c}$$
$$= 40\hat{i} + \omega_3\hat{k} \times (-40\cos\beta\hat{i} + 40\sin\beta\hat{j})$$
$$= (40 - 40\sin\beta\omega_3)\hat{i} - 40\cos\beta.\omega_3 \quad \text{— ②}$$

① = ②

$$40 - 40\sin\beta\,\omega_3 = -20\,\omega_2\sin 60$$
$$-40\cos\beta\,\omega_3 = 20\omega_2\cos 60$$

$$\left.\begin{array}{l}\omega_2 = -1.175 \text{ rad/s (CCW)} \\ \omega_3 = 0.572 \text{ rad/s (CW)}\end{array}\right\} //$$

$$\hat{a}_B = \hat{a}_A - \omega_2^2\hat{r}_{B/A} + \hat{\alpha}_2 \times \hat{r}_{B/A}$$
$$= 0 - (1.175)^2(20\cos 60\hat{i} + 20\sin 60\hat{j}) + \alpha_2\hat{k} \times (20\cos 60\hat{i} + 20\sin 60\hat{j})$$
$$= (-1.175^2 \times 20\cos 60 - 20\sin 60\,\alpha_2)\hat{i} + (-1.175^2 \times 20\sin 60 + 20\cos 60\,\alpha_2)\hat{j}$$
$$= (-17.32\alpha_2 - 13.806)\hat{i} + (10\alpha_2 - 23.91)\hat{j} \quad \text{— ③}$$

$$\hat{a}_B = \hat{a}_c - \omega_3^2\hat{r}_{B/c} + \hat{\alpha}_3 \times \hat{r}_{B/c}$$
$$= 0 - (0.572)^2(-40\cos\beta\hat{i} + 40\sin\beta\hat{j}) + \alpha_3\hat{k} \times (-40\cos\beta\hat{i} + 40\sin\beta\hat{j})$$
$$= (0.572^2.40\cos\beta - 40\sin\beta\,\alpha_3)\hat{i} + (-0.572^2.40\sin\beta - 40\cos\beta\,\alpha_3)\hat{j}$$
$$= (6.72 - 34.32\alpha_3)\hat{i} + (-11.23 - 20.54\alpha_3)\hat{j} \quad \text{— ④}$$

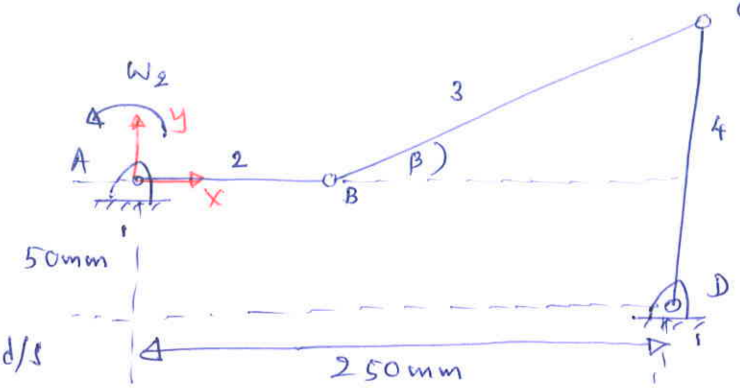
③ = ④

$$34.32\alpha_3 - 17.32\alpha_2 = 20.526$$
$$20.54\alpha_3 + 10\alpha_2 = 12.68$$

$$\left.\begin{array}{l}\alpha_2 = 0.02 \text{ rad/s}^2 \text{ (clockwise)} \\ \alpha_3 = 0.608 \text{ rad/s}^2 \text{ (CW)}\end{array}\right\} //$$

(Q2)

a)




AB = 75 mm

CD = 100 m

$BC = \sqrt{50^2 + 175^2}$

$= 182$ mm

$\beta$

$\hat{\omega}_2 = 2\hat{k}$ rad/s

$\hat{\alpha}_2 = 0$

$$\hat{V}_B = \hat{V}_A + \hat{\omega}_2 \times \hat{r}_{B/A}$$
$$= 0 + \omega_2 \hat{k} \times (75\hat{i})$$
$$= 150\hat{j} \text{ mm/s}$$

$$\hat{V}_c = \hat{V}_B + \hat{\omega}_3 \times \hat{r}_{c/B}$$
$$= 150\hat{j} + \omega_3 \hat{k} \times (175\hat{i} + 50\hat{j})$$
$$= (-50\omega_3 \hat{i}) + (150 + 175\omega_3)\hat{j} \quad \text{—— (1)}$$

$$\hat{V}_c = \hat{V}_D + \hat{\omega}_4 \times \hat{r}_{c/D}$$
$$= 0 + \omega_4 \hat{k} \times 100\hat{j}$$
$$= -100\omega_4 \hat{i} \quad \text{—— (2)}$$

(1) = (2) $\qquad \omega_3 - 2\omega_4 = 0$

$\omega_3 = -\frac{6}{7}$ rad/s

$\omega_4 = -\frac{3}{7}$ rad/s //

$\hat{V}_c = \frac{300}{7}\hat{i}$ mm/s $= 42.86$ mm/s

$$\hat{a}_B = \hat{a}_A - \omega_2^2 \hat{r}_{B/A} + \hat{\alpha}_2 \times \hat{r}_{B/A}$$
$$= 0 - 2^2(75\hat{i}) + 0$$
$$= -300\hat{i} \text{ mm/s}^2$$

$$\hat{a}_c = \hat{a}_B - \omega_3^2 \hat{r}_{c/B} + \hat{\alpha}_3 \times \hat{r}_{c/B}$$
$$= -300\hat{i} - (6/7)^2 (175\hat{i} + 50\hat{j}) + \alpha_3 \hat{k} \times (175\hat{i} + 50\hat{j}) \quad \text{—— (3)}$$
$$= (-300 - (6/7)^2 175 - 50\alpha_3)\hat{i} + (-(6/7)^2 50 + 175\alpha_3)\hat{j}$$

$$\hat{a}_c = \hat{a}_D - \omega_4^2 \hat{r}_{c/D} + \hat{\alpha}_4 \times \hat{r}_{c/D}$$
$$= 0 - \left(\frac{-3}{7}\right)^2 (100\hat{j}) + \alpha_4 \hat{k} \times (100\hat{j}) \quad \text{—— (4)}$$
$$= -100\alpha_4 \hat{i} - (3/7)^2 \cdot 100\hat{j}$$

(3) = (4)

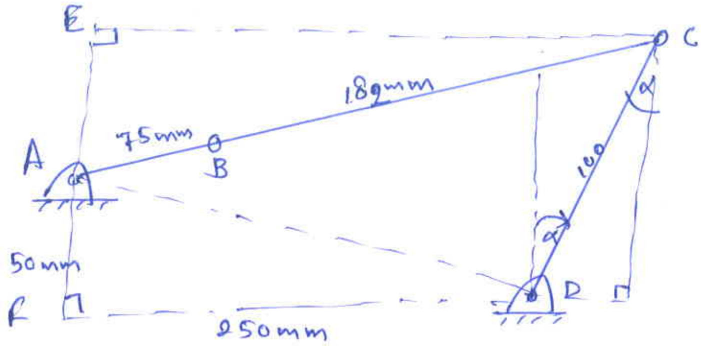
$$50\alpha_3 - 100\alpha_4 = -300 - (6/7)^2 \cdot 175$$
$$-(6/7)^2 \cdot 50 + 175\alpha_3 = -(3/7)^2 \cdot 100$$

$\alpha_3 = 0.105$ rad/s$^2$ (CCW)

$\alpha_4 = 4.34$ rad/s$^2$ (CCW)

(4) $\Rightarrow$ $\hat{a}_c = -434\hat{i} - 18.37\hat{j}$ mm/s$^2$ //

b)




Backward

Consider $ACE\,\Delta$;

$$AE^2 + EC^2 = AC^2$$

$$EC^2 = AC^2 - AE^2$$

$$EC^2 = (75+182)^2 - (100\cos\alpha - 50)^2 \quad \text{---} \quad (1)$$

$EC = 250 + 100\sin\alpha$ (by $ADF\,\Delta$)

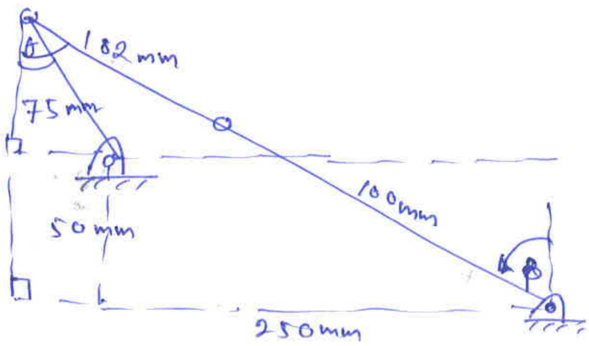
$$\therefore \quad 257^2 - (100\cos\alpha - 50)^2 = (100\sin\alpha + 250)^2$$

$$257^2 - 100^2\cos^2\alpha - 50^2 + 100^2\cos\alpha = 100^2\sin^2\alpha + 250^2 + 50{,}000\sin\alpha$$

$$100^2(\cos^2\alpha + \sin^2\alpha) + 50{,}000\sin\alpha - 10{,}000\cos\alpha = 1049$$

$$50{,}000\sin\alpha - 10{,}000\cos\alpha + 8951 = 0$$

$$\alpha = 1.2^\circ$$




Forward

$$(75\cos\theta + 50)^2 + (75\sin\theta + 250)^2 = (182+100)^2$$

$$75^2 + 50^2 + 7500\cos\theta + 250^2 + 75\times500\sin\theta = 282^2$$

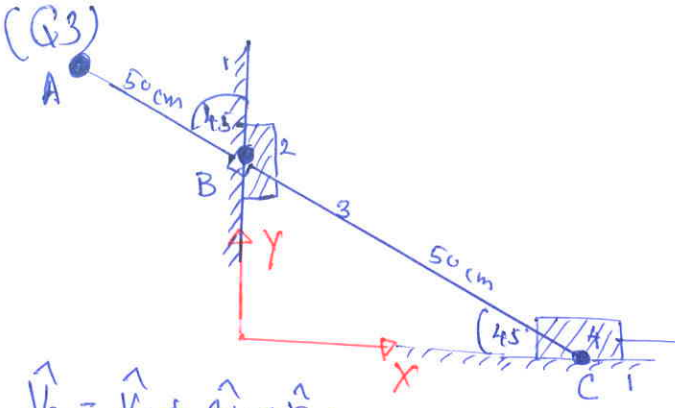
$$7500\cos\theta + 37500\sin\theta - 8899 = 0$$

$$\theta = 80.83^\circ$$

$$\cos\beta = \frac{75\cos\theta + 50}{182+100}$$

$$\beta = 77.3^\circ$$

(Q3)




$\hat{V}_{C_4} = 20\hat{i}$ cm/s ; $\hat{a}_c = 0$

$$\hat{V}_B = \hat{V}_C + \hat{\omega}_3 \times \hat{r}_{B/A}$$

$$= 20\hat{i} + \omega_3 \hat{k} \times (-50\cos 45\hat{i} + 50\sin 45\hat{j})$$

$$V_B\hat{j} = (20 - 50\sin 45\,\omega_3)\hat{i} - 50\cos 45\,\omega_3\hat{j}$$

$$20 - \frac{50}{\sqrt{2}}\omega_3 = 0$$

$$\omega_3 = \frac{2\sqrt{2}}{5} \text{ rad/s}$$

$$\omega_3 = 0.5657 \text{ rad/s}$$

$$V_B = -\frac{50}{\sqrt{2}} \times \frac{2\sqrt{2}}{5}$$

$$V_B = -20 \text{ cm/s}$$

$$\hat{V}_A = \hat{V}_C + \hat{\omega}_3 \times \hat{r}_{A/C}$$

$$= +20\hat{i} + \frac{2\sqrt{2}}{5}\hat{k} \times (-100\cos 45\hat{i} + 100\sin 45\hat{j})$$

$$= \left(20 - \frac{2\sqrt{2}}{5} \times \frac{100}{\sqrt{2}}\right)\hat{i} - \frac{2\sqrt{2}}{5}\frac{100}{\sqrt{2}}\hat{j}$$

$$\underline{\hat{V}_A = -20\hat{i} - 40\hat{j} \text{ cm/s}}$$

$$\hat{a}_B = \hat{a}_C - \omega_3^2 \hat{r}_{B/C} + \hat{\alpha}_3 \times \hat{r}_{B/C}$$

$$= 0 - \left(\frac{2\sqrt{2}}{5}\right)^2 (-50\cos 45\hat{i} + 50\sin 45\hat{j}) + \alpha_3\hat{k} \times (-50\cos 45\hat{i} + 50\sin 45\hat{j})$$

$$= \left\{+\frac{8}{25} \times \frac{50}{\sqrt{2}} - \frac{50}{\sqrt{2}}\alpha_3\right\}\hat{i} + \left\{-\frac{8}{25} \times \frac{50}{\sqrt{2}} - \frac{50}{\sqrt{2}}\alpha_3\right\}\hat{j}$$

$$a_B\hat{j} = \left(\frac{16}{\sqrt{2}} - \frac{50\alpha_3}{\sqrt{2}}\right)\hat{i} + \left(-\frac{16}{\sqrt{2}} - \frac{50\alpha_3}{\sqrt{2}}\right)\hat{j}$$

$\hat{i} = 0$ ; $\alpha_3 = \frac{8}{25} = \underline{0.32 \text{ rad/s}^2}$ ; $a_B = -\frac{16}{\sqrt{2}} - \frac{50}{\sqrt{2}} \times \frac{16}{50}$

$$\underline{a_B = (-16\sqrt{2}) \text{ cm/s}^2}$$

$$\hat{a}_A = \hat{a}_C - \omega_3^2 \hat{r}_{A/B} + \hat{\alpha}_3 \times \hat{r}_{A/B}$$

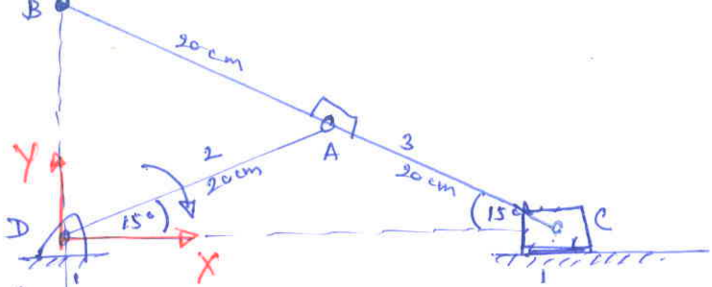
$$= 0 - \left(\frac{2\sqrt{2}}{5}\right)^2 (-100\cos 45\hat{i} + 100\sin 45\hat{j}) + \alpha_3\hat{k} \times (-100\cos 45\hat{i} + 100\sin 45\hat{j})$$

$$= \left(+\frac{8}{25} \times \frac{100}{\sqrt{2}} - \frac{8}{25} \times \frac{100}{\sqrt{2}}\right)\hat{i} + \left(\frac{-8}{25} \times \frac{100}{\sqrt{2}} - \frac{8}{25} \times \frac{100}{\sqrt{2}}\right)\hat{j}$$

$$= -32\sqrt{2}\hat{j} \text{ cm/s}^2$$

$$\underline{\hat{a}_A = -45.3\hat{j} \text{ cm/s}^2}$$

Q4)




$\hat{\omega}_2 = -20\hat{k}$ rad/s

$\hat{\alpha}_2 = -140\hat{k}$ rad/s$^2$

$\hat{V}_A = \hat{V}_D + \hat{\omega}_2 \times \hat{r}_{A/D} = 0 + -20\hat{k} \times (20\cos 15\hat{i} + 20\sin 15\hat{j}) = 20^2(\sin 15\hat{i} - \cos 15\hat{j})$ cm/s

$\hat{V}_C = \hat{V}_A + \hat{\omega}_3 \times \hat{r}_{C/A} = \hat{V}_A + \omega_3\hat{k} \times (20\cos 15\hat{i} - 20\sin 15\hat{j})$

$V_C\hat{i} = (20^2\sin 15 + 20\omega_3\sin 15)\hat{i} + (-20^2\cos 15 + 20\cos 15\,\omega_3)\hat{j}$

$V_C\hat{j} = 0 \quad \therefore \quad \underline{\omega_3 = 20 \text{ rad/s}}$

$V_C = 20^2 \times 2\sin 15$

$\underline{V_C = 207.1 \text{ cm/s}}$

$\hat{V}_B = \hat{V}_C + \hat{\omega}_3 \times \hat{r}_{B/C}$

$= 800\sin 15\hat{i} + 20\hat{k} \times (-40\cos 15\hat{i} + 40\sin 15\hat{j})$

$= (800\sin 15 - 800\sin 15)\hat{i} - 800\cos 15\hat{j}$

$= -800\cos 15\hat{j}$

$\underline{\hat{V}_B = -772.74\hat{j} \text{ cm/s}}$

$\hat{a}_A = \hat{a}_D - \omega_2^2 \cdot \hat{r}_{A/D} + \hat{\alpha}_2 \times \hat{r}_{A/D}$

$= 0 - 20^2(20\cos 15\hat{i} + 20\sin 15\hat{j}) + (-140)\hat{k} \times (20\cos 15\hat{i} + 20\sin 15\hat{j})$

$= (-8000\cos 15 + 2800\sin 15)\hat{i} + (-8000\sin 15 - 2800\cos 15)\hat{j}$

$= (-7002.71\hat{i} - 4775.14\hat{j})$ cm/s$^2$

$\hat{a}_C = \hat{a}_A - \omega_3^2\hat{r}_{C/A} + \hat{\alpha}_3 \times \hat{r}_{C/A}$

$= \hat{a}_A - 20^2(20\cos 15\hat{i} - 20\sin 15\hat{j}) + \alpha_3\hat{k} \times (20\cos 15\hat{i} - 20\sin 15\hat{j})$

$a_C\hat{i} = (-7002.71 - 20^3\cos 15 + 20\alpha_3\sin 15)\hat{i} + (-4775.14 + 20^3\sin 15 + 20\alpha_3\cos 15)\hat{j}$

$a_C\hat{j} = 0 \quad \therefore \quad \underline{\alpha_3 = 140 \text{ rad/s}^2}$

$\underline{\hat{a}_C = -14{,}005.43\hat{i} \text{ cm/s}^2}$

$\hat{a}_B = \hat{a}_C - \omega_3^2\hat{r}_{B/C} + \hat{\alpha}_3 \times \hat{r}_{B/C}$

$= \hat{a}_C - \omega_3^2(-40\cos 15\hat{i} + 40\sin 15\hat{j}) + \alpha_3\hat{k} \times (-40\cos 15\hat{i} + 40\sin 15\hat{j})$

$= (a_C + 40\omega_3^2\cos 15 - 40\alpha_3\sin 15)\hat{i} + (-40\omega_3^2\sin 15 - 40\alpha_3\cos 15)\hat{j}$

$= (-14{,}005 + 40 \times 20^2\cos 15 - 40 \times 140\sin 15)\hat{i} + (-40 \times 20^2\sin 15 - 40 \times 140\cos 15)\hat{j}$

$\underline{\hat{a}_B = -9550.28\hat{j} \text{ cm/s}^2}$

Point B moves directly towards point D ($\because$ AD = AB = AC)

(3) = (4)

$$-30\sqrt{2} - 7\omega_3 \sin\theta_1 = -4\omega_4 \sin\theta_3 \Rightarrow 1.778\,\omega_3 - 3.9\,\omega_4 = -30\sqrt{2} \quad \text{—— (5)}$$

$$30\sqrt{2} + 7\omega_3 \cos\theta_1 = 4\omega_4 \cos\theta_3 \Rightarrow 6.77\,\omega_3 - 0.8917\,\omega_4 = -30\sqrt{2} \quad \text{—— (6)}$$

by (5) & (6)

$$\left.\begin{aligned}\omega_3 &= (-5.1428)\ \text{rad/s}\\ \omega_4 &= 8.53\ \text{rad/s}\end{aligned}\right\} \parallel$$

$$\hat{V}_D = \hat{V}_Q + \hat{\omega}_4 \times \hat{r}_{D/Q} = 0 + 8.53\hat{k} \times (2\cos\theta_1 \hat{i} + 2\sin\theta_1 \hat{j})$$

$$= 17.06\,(\cos\theta_1 \hat{j} - \sin\theta_1 \hat{i})$$

$$\hat{V}_D = 17.06\,(-\sin\theta_1 \hat{i} + \cos\theta_1 \hat{j}) = \underline{\underline{-16.63\hat{i} + 3.8\hat{j}}}$$

$$\hat{V}_E = \hat{V}_D + \hat{\omega}_5 \times \hat{r}_{E/D} = \hat{V}_D + \omega_5 \hat{k} \times (8\cos\theta_4 \hat{i} + 8\sin\theta_4 \hat{j})$$

$$= \hat{V}_D + 8\omega_5(-\sin\theta_4 \hat{i} + \cos\theta_4 \hat{j})$$

$$\hat{V}_E = (-16.63 - 8\omega_5 \sin\theta_4)\hat{i} + (3.8 + 8\omega_5 \cos\theta_4)\hat{j}$$

E moves along only on X direction $\therefore\ 3.8 + 8\omega_5 \cos\theta_4 = 0$

$$\underline{\underline{\omega_5 = -0.546\ \text{rad/s}}}$$

$$\underline{\underline{\hat{V}_E = -14.43\hat{i}\ \text{cm/s}}}$$

b) Acceleration analysis

$$\hat{a}_B = \hat{a}_O - \omega_2^2 \hat{r}_{B/O} + \hat{\alpha}_2 \times \hat{r}_{B/O} = 0 - 20^2(3\cos 45^\circ \hat{i} + 3\sin 45 \hat{j}) + 0$$

$$\hat{a}_B = +1200(\sin 45 \hat{i} - \cos 45 \hat{j})$$

$$\hat{a}_B = 600\sqrt{2}\,(\hat{i} - \hat{j})$$

$$\hat{a}_C = \hat{a}_B - \omega_3^2 \hat{r}_{C/B} + \hat{\alpha}_3 \times \hat{r}_{C/B}$$

$$= \hat{a}_B - (-5.1428)^2 (7\cos\theta_1 \hat{i} + 7\sin\theta_1 \hat{j}) + \alpha_3 \hat{k} \times (7\cos\theta_1 \hat{i} + 7\sin\theta_1 \hat{j})$$

$$\hat{a}_C = (600\sqrt{2} - 179.07 - \alpha_3 \times 1.778)\hat{i} + (-600\sqrt{2} - 47.03 + 6.77\alpha_3)\hat{j} \longrightarrow (7)$$

$$\hat{a}_C = \hat{a}_Q - \omega_4^2 \hat{r}_{C/Q} + \hat{\alpha}_4 \times \hat{r}_{C/Q}$$

$$= 0 - 8.53^2 (4\cos\theta_3 \hat{i} + 4\sin\theta_3 \hat{j}) + \alpha_4 \hat{k}\,(4\cos\theta_3 \hat{i} + 4\sin\theta_3 \hat{j})$$

$$\hat{a}_C = (-253.11 - 1.97\alpha_4)\hat{i} + (-143.67 + 3.48\alpha_4) \longrightarrow (8)$$

(7) = (8)

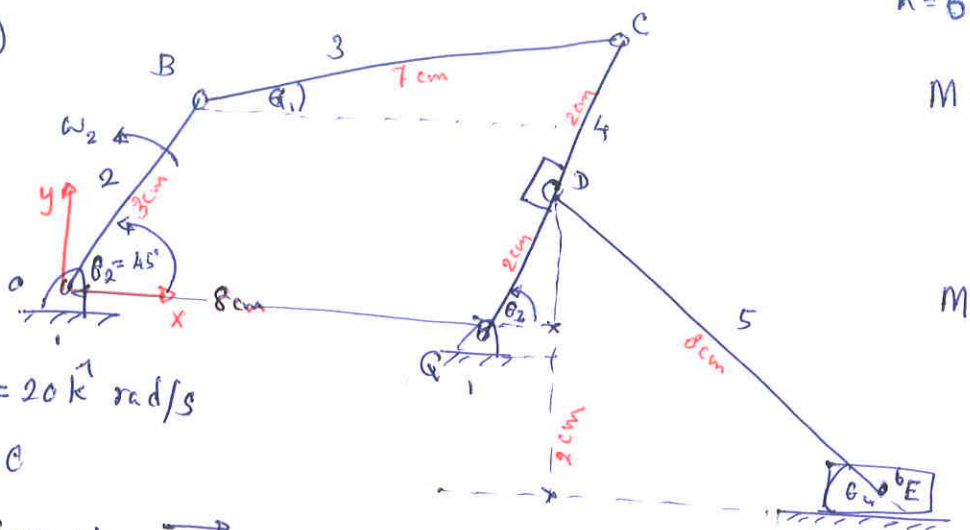
$$1.778\,\alpha_3 - 1.97\,\alpha_4 = 600\sqrt{2} - 179.07 + 253.11 = 600\sqrt{2} + 74.07$$

$$6.77\,\alpha_3 - 3.48\,\alpha_4 = 600\sqrt{2} + 47.03 - 143.67 = 600\sqrt{2} - 96.64$$

$$\left.\begin{aligned}\alpha_3 &= -241.89\ \text{rad/s}^2 \quad \text{(cw)}\\ \alpha_4 &= -686.64\ \text{rad/s}^2 \quad \text{(cw)}\end{aligned}\right\} \parallel$$

Problem set 2(a)

Q5)




$n = 6,\ j_p = 6,\ j_h = 1$

$M = 3(n-1) - 2 \times j_p - j_h$

$= 3(6-1) - 2 \times 6 - 1$

$= 15 - 12 - 1$

$M = 2$

$\omega_2 = 20\hat{k}$ rad/s

$\alpha_2 = 0$

by Geometry; $\rightarrow$ $3\cos 45 + 7\cos\theta_1 = 8 + 4\cos\theta_3$ —— ①

$3\cos 45 + 7\cos\theta_1 - 8 = 4\cos\theta_3$

$\uparrow$ $3\sin 45 + 7\sin\theta_1 = 4\sin\theta_3$ —— ②

$①^2 + ②^2$

$9(\sin^2 45 + \cos^2 45) + 49(\sin^2\theta_1 + \cos^2\theta_1) + 64 +$

$\frac{42}{\sqrt{2}}\cos\theta_1 - \frac{48}{\sqrt{2}} - 112\cos\theta_1 + \frac{42}{\sqrt{2}}\sin\theta_1 = 16(\sin^2\theta_3 + \cos^2\theta_3)$

$9 + 49 + 64 + 42\left(\frac{\cos\theta_1}{\sqrt{2}} \cancel{\cos 45} + \frac{\sin\theta_1}{\sqrt{2}} \cancel{\sin 45}\right) - 112\cos\theta_1 - \frac{48}{\sqrt{2}} - 16 = 0$

$21\sqrt{2}\sin\theta_1 + (21\sqrt{2} - 112)\cos\theta_1 = 24\sqrt{2} - 106$

$\left.\begin{aligned} \theta_1 &= 14.71° \\ \theta_3 &= 77.12° \\ \theta_4 &= 29.58° \end{aligned}\right\} //$

$\sin\theta_4 = \frac{2 + 2\sin\theta_3}{8}$

$\hat{V}_B = \hat{V}_o + \hat{\omega}_2 \times \hat{r}_{B/O} = 0 + 20\hat{k} \times (3\cos 45\,\hat{i} + 3\sin 45\,\hat{j})$

$\hat{V}_B = 60(-\sin 45\,\hat{i} + \cos 45\,\hat{j})$

$\hat{V}_B = 30\sqrt{2}(-\hat{i} + \hat{j})$

$\hat{V}_C = \hat{V}_B + \hat{\omega}_3 \times \hat{r}_{C/B} = \hat{V}_B + \hat{\omega}_3\hat{k} \times (7\cos\theta_1\,\hat{i} + 7\sin\theta_1\,\hat{j})$

$\hat{V}_C = (-30\sqrt{2} - 7\omega_3\sin\theta_1)\hat{i} + (30\sqrt{2} + 7\omega_3\cos\theta_1)\hat{j}$ —— ③

$\hat{V}_C = \hat{V}_Q + \hat{\omega}_4 \times \hat{r}_{C/Q} = 0 + \omega_4\hat{k} \times (4\cos\theta_3\,\hat{i} + 4\sin\theta_3\,\hat{j})$

$\hat{V}_C = 4\omega_4(-\sin\theta_3\,\hat{i} + \cos\theta_3\,\hat{j})$ —— ④

$$\hat{a}_D = \hat{a}_Q - \omega_4^2 \hat{r}_{D/Q} + \hat{\alpha}_4 \times \hat{r}_{D/Q}$$
$$= 0 - 8.53^2 (2\cos\theta_3 \hat{i} + 2\sin\theta_3 \hat{j}) + (-686.64)\hat{k} \times (2\cos\theta_3 \hat{i} + 2\sin\theta_3 \hat{j})$$
$$= (-2\times 8.53^2 \cos\theta_3 + 2\times 686.64 \sin\theta_3)\hat{i} + (2\times 8.53^2 \sin\theta_3 - 2\times 686.64 \cos\theta_3)\hat{j}$$

$$\hat{a}_D = 1306.29\,\hat{i} - 164.27\,\hat{j} \quad cm/s^2$$

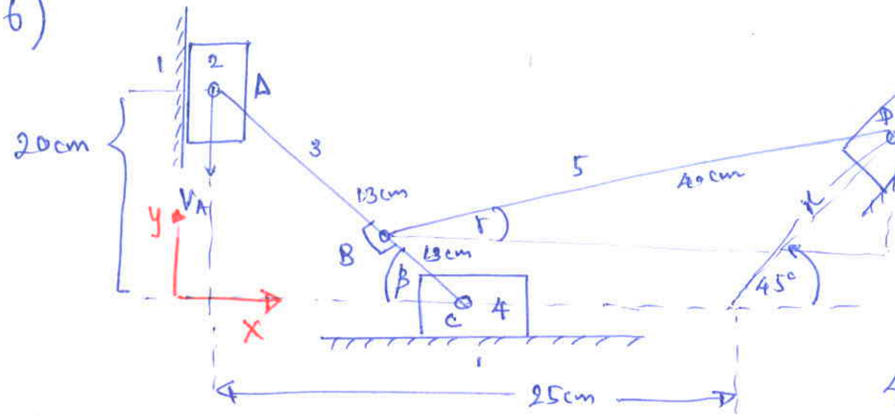
$$\hat{a}_E = \hat{a}_D - \omega_5^2 \hat{r}_{E/D} + \hat{\alpha}_5 \times \hat{r}_{E/D}$$
$$= \hat{a}_D - \omega_5^2 (8\cos\theta_4 \hat{i} - 8\sin\theta_4 \hat{j}) + \alpha_5 \hat{k} \times (8\cos\theta_4 \hat{i} - 8\sin\theta_4 \hat{j})$$
$$= (a_D - 8\omega_5^2 \cos(\theta_4) + 8\alpha_5 \sin\theta_4)\hat{i} + (a_D + 8\omega_5^2 \sin\theta_4 + 8\alpha_5 \cos\theta_4)\hat{j}$$
$$= (1306.29 - 8\times(-0.546)^2 \cos 29.58 + 8\alpha_5 \sin 29.58)\hat{i} + (a_D + 8(-0.546)^2 \sin 29.58 + 8\alpha_5 \cos 29.58)\hat{j}$$

E moves along the plane. $\therefore$ $a_E \hat{j} = 0$

$$-164.27 + 8(-0.546)^2 \sin 29.58 + 8\alpha_5 \cos 29.58 = 0$$
$$\underline{\alpha_5 = +23.44 \text{ rad/s}^2} \text{ (CCW)}$$

$$\underline{\hat{a}_E = 1396.78 \text{ cm/s}^2}$$

(Q6)




$\sin\beta = \dfrac{20}{26}$

$\beta = 50.28°$

$x\sin 45 = 40\sin\gamma + 13\sin\beta$

$x\cos 45 = 40\cos\gamma + 13\cos\beta - 25$

$40\sin\gamma - 4\cos\gamma + 26.69 = 0$

$\gamma = 16.9°$

$\hat{V}_A = -1.5\hat{j}\ m/s\ ,\quad \hat{a}_A = 0$

$\hat{V}_C = \hat{V}_A + \hat{\omega}_3 \times \hat{r}_{C/A} = -150\hat{j} + \omega_3\hat{k}\times(26\cos\beta\hat{i} - 26\sin\beta\hat{j})$

$V_C\hat{i} = 26\sin\beta\omega_3\hat{i} + (26\cos\beta\omega_3 - 150)\hat{j}\quad cm/s$

$\omega_3 = 9.03\ rad/s\ (CCW)\qquad \hat{V}_C = 180.59\hat{i}\ cm/s$

$\hat{V}_B = \hat{V}_A + \hat{\omega}_3 \times \hat{r}_{B/A} = -150\hat{j} + \omega_3\hat{k}\times(13\cos\beta\hat{i} - 13\sin\beta\hat{j})$

$= (13\omega_3\sin\beta)\hat{i} + (13\omega_3\cos\beta - 150)\hat{j}$

$\hat{V}_B = 90.3\hat{i} - 75\hat{j}\quad cm/s$

$\hat{V}_D = \hat{V}_B + \hat{\omega}_5\times\hat{r}_{D/B} = \hat{V}_B + \omega_5\hat{k}\times(40\cos\gamma\hat{i} + 40\sin\gamma\hat{j})$

$= (90.3 - 40\omega_5\sin\gamma)\hat{i} + (-75 + 40\omega_5\cos\gamma)\hat{j}$

$\hat{V}_D = V_D\cos 45\hat{i} + V_D\sin 45\hat{j}\quad \therefore\ V_D\hat{i} = V_D\hat{j}$

$\therefore\ 90.3 - 40\omega_5\sin\gamma = -75 + 40\omega_5\cos\gamma$

$\cancel{40\omega_5}$, $\omega_5 = 3.31\ rad/s\ (CCW)$, $\hat{V}_D = 51.8\hat{i} + 51.8\hat{j}\ cm/s$

$\hat{a}_C = \hat{a}_A - \omega_3^2\hat{r}_{C/A} + \hat{\alpha}_3\times\hat{r}_{C/A}$

$= 0 - 9.03^2(26\cos\beta\hat{i} - 26\sin\beta\hat{j}) + \alpha_3\hat{k}\times(26\cos\beta\hat{i} - 26\sin\beta\hat{j})$

$= (-9.03^2\times 26\cos\beta + \alpha_3\times 26\sin\beta)\hat{i} + (9.03^2\times 26\sin\beta + \alpha_3\times 26\cos\beta)\hat{j}$

$a_C\hat{j} = 0\quad \therefore\ \alpha_3 = -98.15\ rad/s^2,\quad \hat{a}_C = -3317.7\ cm/s^2$

$\hat{a}_B = \hat{a}_A - \omega_3^2\hat{r}_{B/A} + \hat{\alpha}_3\times\hat{r}_{B/A}$

$= 0 - 9.03^2(13\cos\beta\hat{i} - 13\sin\beta\hat{j}) + (-98.15)\hat{k}\times(13\cos\beta\hat{i} - 13\sin\beta\hat{j})$

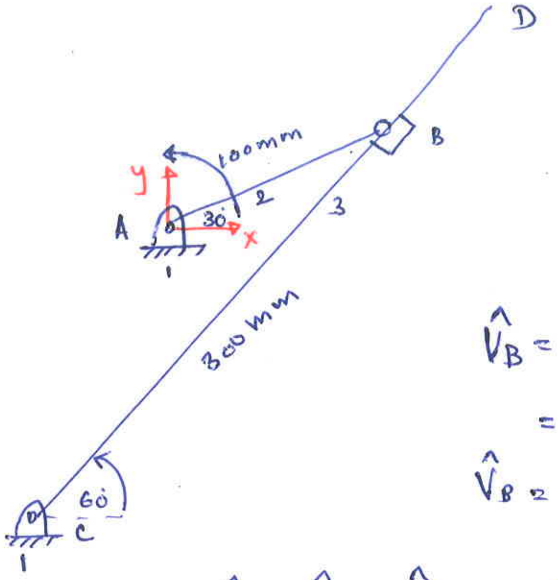
$\hat{a}_B = -1658.83\hat{i}\ cm/s^2$

$\hat{a}_D = \hat{a}_B - \omega_5^2\hat{r}_{D/B} + \hat{\alpha}_5\times\hat{r}_{D/B} = \hat{a}_B - 3.31^2(40\cos\gamma\hat{i} + 40\sin\gamma\hat{j}) + \alpha_5\hat{k}\times(40\cos\gamma\hat{i} + 40\sin\gamma\hat{j})$

$= (-2078.15 - 11.63\alpha_5)\hat{i} + (-127.4 + 38.27\alpha_5)\hat{j}$

$a_D\hat{i} = a_D\hat{j}$ (45° plane) $\quad\therefore\ \alpha_5 = -39.1\ rad/s^2\qquad \hat{a}_D = -1623.42(\hat{i}+\hat{j})\ cm/s^2$

(Q1)




$\hat{\omega}_{AB} = \hat{\omega}_2 = 3\hat{k}$ rad/s

$\hat{\alpha}_{AB} = \hat{\alpha}_2 = 9\hat{k}$ rad/s$^2$

$AB = 0.1$ m

$BC = 0.3$ m

$$\hat{V}_B = \hat{V}_A + \hat{\omega}_2 \times \hat{r}_{B/A}$$
$$= 0 + 3\hat{k} \times (0.1\cos 30\,\hat{i} + 0.1\sin 30\,\hat{j})$$
$$\hat{V}_B = 0.3(-\sin 30\,\hat{i} + \cos 30\,\hat{j}) \quad \text{—— (1)}$$

$$\hat{V}_B = \hat{V}_C + \hat{V}_{B/C} + \hat{\omega}_3 \times \hat{r}_{B/C}$$
$$= 0 + V_s(\cos 60\,\hat{i} + \sin 60\,\hat{j}) + \omega_3 \hat{k}(0.3\cos 60\,\hat{i} + 0.3\sin 60\,\hat{j})$$
$$\hat{V}_B = (V_s \cos 60 - 0.3\sin 60\,\omega_3)\hat{i} + (V_s \sin 60 + 0.3\cos 60\,\omega_3)\hat{j} \quad \text{—— (2)}$$

(1) = (2)

$$\cos 60\, V_s - 0.3\sin 60\,\omega_3 = -0.3\sin 30$$
$$\sin 60\, V_s + 0.3\cos 60\,\omega_3 = 0.3\cos 30$$

$$\left.\begin{aligned} V_s &= 0.15 \text{ m/s} \\ \omega_3 &= \frac{\sqrt{3}}{2} \text{ rad/s} = 0.866 \text{ rad/s} \end{aligned}\right\} //$$

$$\hat{a}_B = -\omega_2^2 \hat{r}_{B/A} + \hat{\alpha}_2 \times \hat{r}_{B/A}$$
$$= -(3\hat{k})^2(0.1\cos 30\,\hat{i} + 0.1\sin 30\,\hat{j}) + 9\hat{k} \times (0.1\cos 30\,\hat{i} + 0.1\sin 30\,\hat{j})$$
$$= +9 \times 0.1\,(-\cos 30\,\hat{i} - \sin 30\,\hat{j} - \sin 30\,\hat{i} + \cos 30\,\hat{j})$$
$$= 0.9(-\cos 30 - \sin 30)\hat{i} + 0.9(-\sin 30 + \cos 30)\hat{j}$$
$$\hat{a}_B = \frac{0.9}{2}\left\{-(\sqrt{3}+1)\hat{i} + (\sqrt{3}-1)\hat{j}\right\}$$
$$\hat{a}_B = -1.229\,\hat{i} + 0.329\,\hat{j} \quad \text{—— (3)}$$

$$\hat{a}_B = \hat{a}_C + \hat{a}_{B/C} - \omega_3^2 \hat{r}_{B/C} + 2\hat{\omega}_3 \times \hat{V}_{B/C} + \hat{\alpha}_3 \times \hat{r}_{B/C}$$
$$= 0 + a_s(\cos 60\,\hat{i} + \sin 60\,\hat{j}) - \left(\frac{\sqrt{3}}{2}\right)^2(0.3\cos 60\,\hat{i} + 0.3\sin 60\,\hat{j}) + \alpha_3\hat{k} \times (0.3\cos 60\,\hat{i} + 0.3\sin 60\,\hat{j})$$
$$+ 2\frac{\sqrt{3}}{2}\hat{k} \times V_s(\cos 60\,\hat{i} + \sin 60\,\hat{j})$$
$$\hat{a}_B = \left(\cos 60\, a_s - \frac{0.9}{4}\cos 60 - \sqrt{3}(0.15)\sin 60 - 0.3\sin 60\,\alpha_3\right)\hat{i}$$
$$+ \left(\sin 60\, a_s - \frac{0.9}{4}\sin 60 + \sqrt{3}(0.15)\cos 60 + 0.3\cos 60\,\alpha_3\right)\hat{j}$$
$$\hat{a}_B = \frac{1}{2}\left\{\left(a_s - 0.3\sqrt{3}\,\alpha_3 - \frac{27}{40}\right)\hat{i} + \left(\sqrt{3}\,a_s + 0.3\,\alpha_3 - \frac{3\sqrt{3}}{40}\right)\hat{j}\right\} \quad \text{—— (4)}$$

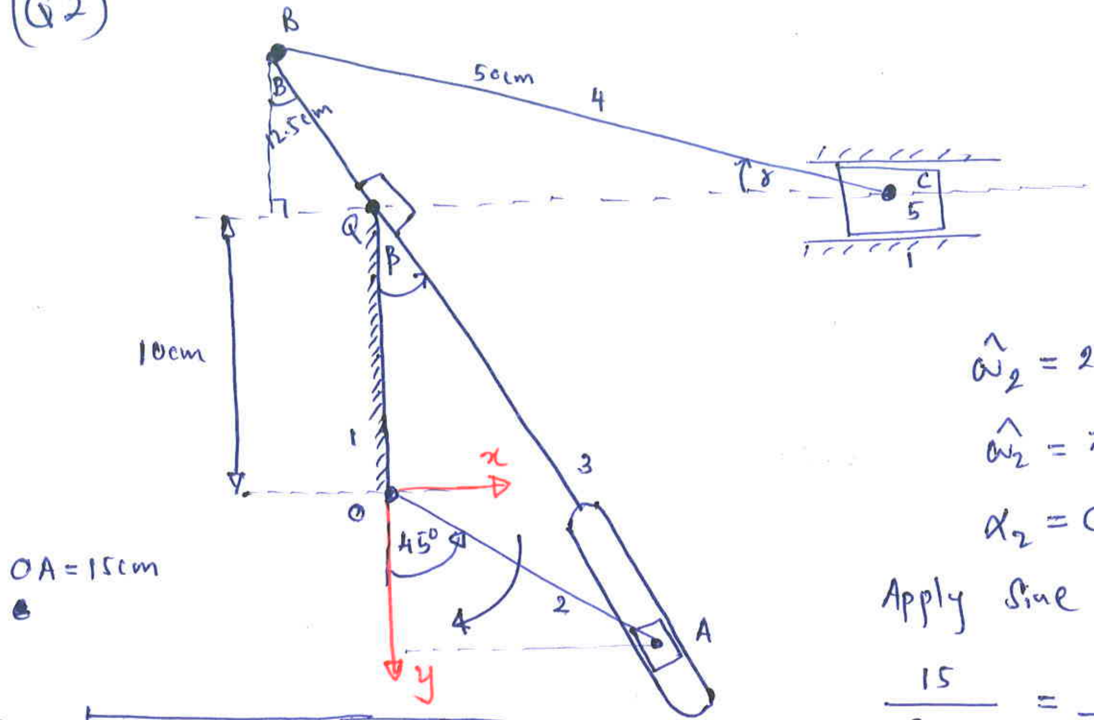
$(3) = (4)$

$$a_s - 0.3\sqrt{3}\,\alpha_3 = \frac{27}{40} - 1.229 \times 2$$

$$\sqrt{3}\,a_s + 0.3\,\alpha_3 = \frac{3\sqrt{3}}{40} + 0.329 \times 2$$

$$\left.\begin{aligned} a_s &= -0.105\ m/s^2 \\ \alpha_3 &= 3.23\ rad/s^2 \end{aligned}\right\} //$$

(Q2)




$$\hat{\omega}_2 = 2\pi f \hat{k} = 2\pi \frac{30}{60} \hat{k}$$

$$\hat{\omega}_2 = \pi \text{ rad/s}$$

$$\alpha_2 = 0$$

Apply Sine rule to OQA $\Delta$

$$\frac{15}{\sin\beta} = \frac{10}{\sin(45-\beta)} = \frac{QA}{\sin(180-45)}$$

$$QA = \sqrt{(15\sin 45)^2 + (10+15\cos 45)^2}$$

$QA = 23.176$ cm

$\sin\beta = 0.4576$

$\beta^\circ = 27.24^\circ$

$$\sin\gamma = \frac{12.5\cos\beta}{50} \quad ; \quad QC = 50\cos\gamma - 12.5\sin\beta$$

$\gamma^\circ = 12.843^\circ$ $\qquad$ $QC = 43$ cm

$$\hat{V}_A = \hat{\omega}_2 \times \hat{r}_{A/O}$$

$$= \pi\hat{k} \times (15\sin 45\hat{i} + 15\cos 45\hat{j}) = 15\pi(-\cos 45\hat{i} + \sin 45\hat{j})$$

$$\hat{V}_A = -10.61\pi\hat{i} + 10.61\hat{j} \text{ cm/s} \quad \text{—— (1)}$$

$$\hat{V}_A = \hat{V}_Q + \hat{V}_{A/Q} + \hat{\omega}_3 \times \hat{r}_{A/Q}$$

$$= 0 + V_s(\sin\beta\hat{i} + \cos\beta\hat{j}) + \omega_3\hat{k} \times (23.176\sin\beta\hat{i} + 23.176\cos\beta\hat{j})$$

$$\hat{V}_A = (-20.606\,\omega_3 + 0.4577\,V_s)\hat{i} + (10.608\,\omega_3 + 0.8891\,V_s)\hat{j} \text{ cm/s} \quad \text{—— (2)}$$

by (1) & (2)

$$-10.61\pi = -20.606\,\omega_3 + 0.4577\,V_s$$

$$10.61\pi = 10.608\,\omega_3 + 0.8891\,V_s$$

$$\left.\begin{aligned} \omega_3 &= 1.937 \text{ rad/s} = 0.617\pi \text{ rad/s} \\ V_s &= 14.38 \text{ cm/s} = 4.577\pi \text{ cm/s} \end{aligned}\right\}$$

$$\hat{V}_B = \hat{\omega}_3 \times \hat{r}_{B/Q}$$

$$= 0.617\pi\,\hat{k} \times (-12.5\sin\beta\hat{i} - 12.5\cos\beta\hat{j})$$

$$\hat{V}_B = 6.857\pi\hat{i} - 3.53\pi\hat{j} \text{ cm/s}$$

$$\hat{V}_B = \hat{V}_c + \hat{\omega}_4 \times \hat{r}_{B/c}$$
$$= V_c\hat{i} + \omega_4\hat{k}\times(-50\cos\gamma\,\hat{i} - 50\sin\gamma\,\hat{j})$$
$$= V_c\hat{i} + 11.114\,\omega_4\hat{i} - 48.749\,\omega_4\hat{j}$$

$$V_c + 11.114\,\omega_4 = 6.857\pi$$
$$-48.749\,\omega_4 = -3.53\pi$$

$$\left.\begin{aligned}\omega_4 &= 0.072\pi \text{ rad/s}\\ V_c &= 6.05\pi \text{ cm/s}\end{aligned}\right\}//$$

Accelerations

$$\hat{a}_A = -\omega_2^2\hat{r}_{A/0} = -(\pi)^2(15\sin 45\,\hat{i} + 15\cos 45\,\hat{j})$$
$$\hat{a}_A = -10.61\pi^2\hat{i} - 10.61\pi^2\hat{j}$$

$$\hat{a}_A = \hat{a}_Q + (\hat{a}_{A/Q})_{\text{sliding}} - \omega_3^2\hat{r}_{A/Q} + 2\hat{\omega}_3\times\hat{V}_{A/Q} + \hat{\alpha}_3\times\hat{r}_{A/Q}$$
$$= 0 + a_s(\sin\beta\,\hat{i} + \cos\beta\,\hat{j}) - (0.617\pi)^2(23.176\sin\beta\,\hat{i} + 23.176\cos\beta\,\hat{j})$$
$$+ 2\times(0.617\pi)\hat{k}\times V_s(\sin\beta\,\hat{i} + \cos\beta\,\hat{j}) + \alpha_3\hat{k}\times(23.176\sin\beta\,\hat{i} + 23.176\cos\beta\,\hat{j})$$

$$\hat{a}_A = (0.4577\,a_s - 20.606\,\alpha_3 - 9.06\pi^2)\hat{i}$$
$$+ (0.889\,a_s + 10.608\,\alpha_3 - 5.26\pi^2)\hat{j}$$

$$\left.\begin{aligned}a_s &= -5.467\pi^2 \text{ cm/s}^2\\ \alpha_3 &= -0.046\pi^2 \text{ rad/s}^2\end{aligned}\right\}//$$

$$\hat{a}_B = -\omega_3^2\hat{r}_{B/Q} + \hat{\alpha}_3\times\hat{r}_{B/Q} = -(0.617\pi)^2(-12.5\sin\beta\,\hat{i} - 12.5\cos\beta\,\hat{j})$$
$$+ (-0.046\pi^2)\hat{k}\times(-12.5\sin\beta\,\hat{i} - 12.5\cos\beta\,\hat{j})$$

$$\hat{a}_B = 1.665\pi^2\hat{i} + 4.495\pi^2\hat{j} \text{ cm/s}^2$$

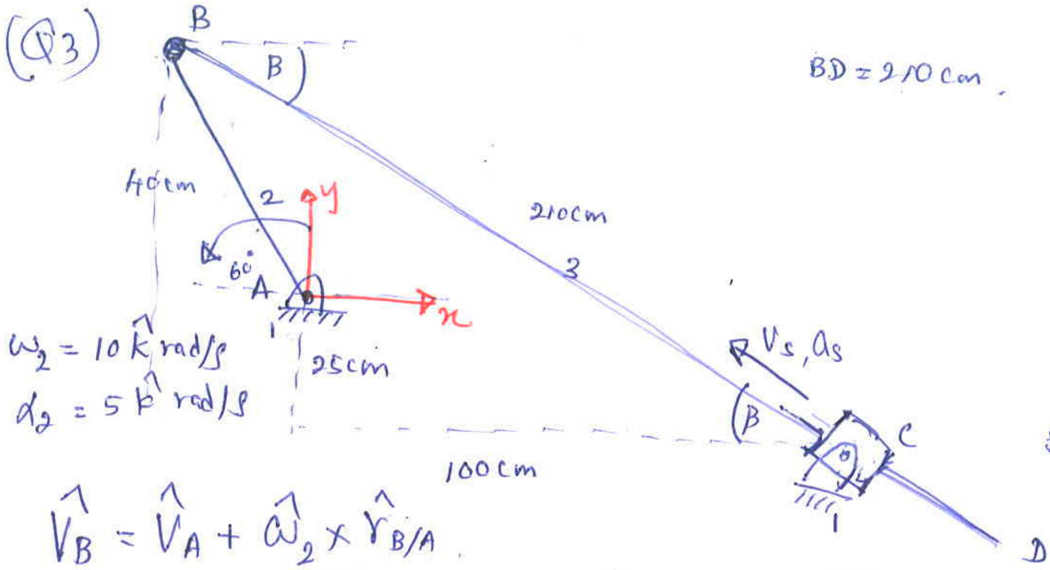
$$\hat{a}_B = \hat{a}_c + (\hat{a}_{B/c})^{=0} - \omega_4^2\hat{r}_{B/c} + (2\hat{\omega}_4\times V_{B/c})^{=0} + \hat{\alpha}_4\times\hat{r}_{B/c}$$
$$= a_c\hat{i} - (0.072\pi)^2(-50\cos\gamma\,\hat{i} - 50\sin\gamma\,\hat{j}) + \alpha_4\hat{k}\times(-50\cos\gamma\,\hat{i} - 50\sin\gamma\,\hat{j})$$
$$\hat{a}_B = (a_c + 11.114\,\alpha_4 + 0.2527\pi^2)\hat{i} + (-48.749\,\alpha_4 + 0.0576\pi^2)\hat{j}$$

$$-48.749\,\alpha_4 + 0.0576\pi^2 = 4.495\pi^2$$
$$a_c + 11.114\,\alpha_4 + 0.2527\pi^2 = 1.665\pi^2$$

$$\left\}\begin{aligned}\hat{\alpha}_4 &= -0.091\pi^2 \text{ rad/s}^2\\ a_c &= 2.4237\pi^2 \text{ cm/s}^2\end{aligned}\right.$$

$$\alpha_4 = -0.091\pi^2 \text{ rad/s}^2$$
$$a_c = 2.4237\pi^2 \text{ cm/s}^2 = 23.92 \text{ cm/s}^2$$

(Q3)




$$\tan\beta = \frac{25 + 40\sin 60}{100 + 40\cos 60}$$

$$\underline{\beta = 26.43^0}$$

$$BC = \frac{(40\sin 60 + 25)}{\sin\beta}$$

$$\underline{BC = 134\ \text{cm}}$$

$$\hat{V}_B = \hat{V}_A + \hat{\omega}_2 \times \hat{r}_{B/A}$$

$$= 0 + 10\hat{k} \times (-40\cos 60\,\hat{i} + 40\sin 60\,\hat{j})$$

$$\hat{V}_B = -346.41\,\hat{i} - 200\,\hat{j} \quad \text{cm/s} \quad \text{—— (1)}$$

$$\hat{V}_B = \hat{V}_C + \hat{\omega}_3 \times \hat{r}_{B/C} + (\hat{V}_{B/C})_{\text{sliding}}$$

$$= 0 + \omega_3\hat{k} \times (-134\cos\beta\,\hat{i} + 134\sin\beta\,\hat{j}) + V_s(-\cos\beta\,\hat{i} + \sin\beta\,\hat{j})$$

$$= (-134\sin\beta\,\omega_3 - \cos\beta\,V_s)\,\hat{i} + (-134\cos\beta\,\omega_3 + \sin\beta\,V_s)\,\hat{j} \quad \text{—— (2)}$$

(1) & (2)

$$-59.644\,\omega_3 - 0.8955\,V_s = -346.41$$

$$-119.994\,\omega_3 + 0.4451\,V_s = -200$$

$$\hat{\omega}_3 = 2.4871\,\hat{k} \quad \text{rad/s}$$

$$\underline{V_s = 221.17} \quad \text{cm/s}$$

$$\hat{V}_D = \hat{V}_B + \hat{\omega}_3 \times \hat{r}_{D/B}$$

$$= \hat{V}_B + 2.4871\,\hat{k} \times (210\cos\beta\,\hat{i} - 210\sin\beta\,\hat{j})$$

$$\underline{\hat{V}_D = -113.94\,\hat{i} + 267.7\,\hat{j}} \quad \text{cm/s}$$

$$\hat{a}_B = -\omega_2^2\,\hat{r}_{B/A} + \hat{\alpha}_2 \times \hat{r}_{B/A} = -10^2(-40\cos 60\,\hat{i} + 40\sin 60\,\hat{j}) + 5\hat{k} \times (-40\cos 60\,\hat{i} + 40\sin 60\,\hat{j})$$

$$= 1826.8\,\hat{i} - 3564.1\,\hat{j} \quad \text{—— (3)}$$

$$\hat{a}_B = \hat{a}_C + (\hat{a}_{B/C})_{\text{sliding}} - \omega_3^2\,\hat{r}_{B/C} + 2\hat{\omega}_3 \times (\hat{V}_{B/C}) + \hat{\alpha}_3 \times \hat{r}_{B/C}$$

$$= 0 + a_s(-\cos\beta\,\hat{i} + \sin\beta\,\hat{j}) - (2.4871)^2(-134\cos\beta\,\hat{i} + 134\sin\beta\,\hat{j}) + 2(2.4871\,\hat{k}) \times (221.17)(-\cos\beta\,\hat{i} + \sin\beta\,\hat{j}) + \alpha_3\hat{k} \times (-134\cos\beta\,\hat{i} + 134\sin\beta\,\hat{j})$$

$$= (-0.8955\,a_s - 59.64\,\alpha_3 + 252.57)\,\hat{i} + (0.445\,a_s - 120\,\alpha_3 - 1354.09)\,\hat{j} \quad \text{—— (4)}$$

by (3) & (4)

$$\underline{a_s = -2393.38\ \text{cm/s}^2}$$

$$\underline{\alpha_3 = 9.54\ \text{rad/s}^2}$$

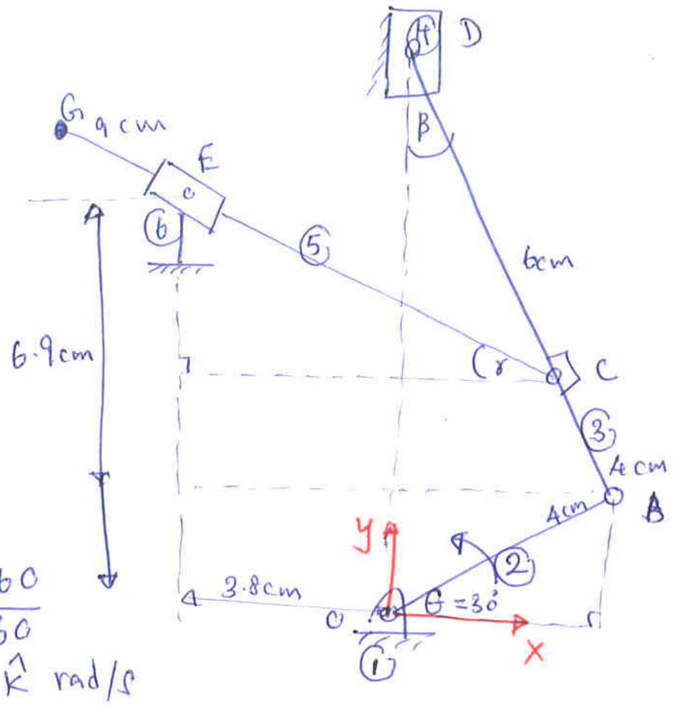
$$\hat{a}_D = \hat{a}_B - \omega_3^2 \hat{r}_{D/B} + \hat{\alpha}_3 \times \hat{r}_{B/B}$$

$$= (1826.8\hat{i} - 3564.1\hat{j}) - 2.4871^2(+210\cos\beta\hat{i} - 210\sin\beta\hat{j})$$

$$+ 9.54\hat{k} \times (210\cos\beta\hat{i} - 210\sin\beta\hat{j})$$

$$\underline{\hat{a}_D = 1818.2\hat{i} + 5805.22\hat{j} \ \ cm/s^2}$$

(Q4)




$$\sin\beta = \frac{4\cos\theta}{10}$$

$$\beta = 20.27^\circ$$

$$\tan\gamma = \frac{6.9 - 4\sin\theta - 4\cos\beta}{3.8 + 4\cos\theta - 4\sin\beta}$$

$$\gamma = 11.047^\circ$$

$$CE = x = \frac{6.9 - 4\sin\theta - 4\cos\beta}{\sin\gamma}$$

$$x = 5.989 \text{ cm} \simeq 6 \text{ cm}$$

$$\hat{\omega}_2 = 2\pi\,\frac{160}{60}$$

$$\hat{\omega}_2 = \frac{16\pi}{3}\hat{k} \text{ rad/s}$$

$$\hat{\alpha}_2 = (-50)\hat{k} \text{ rad/s}^2$$

$$\hat{V}_B = \hat{\omega}_2 \times \hat{r}_{B/O} = \frac{16\pi}{3}\hat{k} \times (4\cos 30\,\hat{i} + 4\sin 30\,\hat{j}) = \left(-\frac{32\pi}{3}\hat{i} + \frac{32\sqrt{3}\pi}{3}\hat{j}\right) \text{cm/s}$$

$$\hat{V}_D = \hat{V}_B + \hat{\omega}_3 \times \hat{r}_{D/B} = \hat{V}_B + \omega_3\hat{k} \times (-10\sin\beta\,\hat{i} + 10\cos\beta\,\hat{j})$$

$$= \left(\frac{-32\pi}{3} - 9.38\,\omega_3\right)\hat{i} + \left(\frac{32\sqrt{3}}{3}\pi - 3.464\,\omega_3\right)\hat{j}$$

D is moving along "y" axis only $\therefore$ $\hat{\omega}_3 = -3.57\hat{k}$ rad/s (CW)

$$\hat{V}_D = 70.42\hat{j} \text{ cm/s}$$

$$\hat{V}_C = \hat{V}_B + \hat{\omega}_3 \times \hat{r}_{C/B} = \hat{V}_B + \omega_3\hat{k} \times (-4\sin\beta\,\hat{i} + 4\cos\beta\,\hat{j})$$

$$= \left(\frac{-32\pi}{3} - 4\cos\beta\,\omega_3\right)\hat{i} + \left(\frac{32\sqrt{3}\pi}{3} - 4\sin\beta\,\omega_3\right)\hat{j}$$

$$\hat{V}_C = -20.11\hat{i} + 62.99\hat{j} \text{ cm/s}$$

$$\hat{V}_E = \hat{V}_C + (\hat{V}_{E/C})_{\text{sliding}} + \hat{\omega}_5 \times \hat{r}_{E/C} =$$

$$= \hat{V}_C + V_s(-\cos\gamma\,\hat{i} + \sin\gamma\,\hat{j}) + \omega_5\hat{k} \times (-x\cos\gamma\,\hat{i} + x\sin\gamma\,\hat{j})$$

$$= (-20.11 - \cos\gamma\,V_s - x\sin\gamma\,\omega_5)\hat{i} + (62.99 + \sin\gamma\,V_s + x\cos\gamma\,\omega_5)\hat{j}$$

$\hat{V}_E = 0$ (Fixed point)

$$\omega_5(5.989\sin\gamma) + \cos\gamma\,V_s = -20.11$$

$$\omega_5(5.989\cos\gamma) - \sin\gamma\,V_s = 62.99$$

$$\omega_5 = 9.68 \text{ rad/s (CCW)}$$

$$V_s = -31.8 \text{ cm/s}$$

$$\hat{V}_G = \hat{V}_C + \hat{\omega}_5 \times \hat{r}_{G/C} = \hat{V}_C + \omega_5 \hat{k} \times (-9\cos\gamma\,\hat{i} + 9\sin\gamma\,\hat{j})$$
$$= (-20.11 - 9\sin\gamma\,\omega_5)\hat{i} + (62.99 - 9\cos\gamma\,\omega_5)\hat{j}$$
$$\hat{V}_G = (-36.8\,\hat{i} - 22.5\,\hat{j}) \text{ cm/s}$$

---

$$\hat{a}_B = \hat{a}_A - \omega_2^2 \hat{r}_{B/A} + \hat{\alpha}_2 \times \hat{r}_{B/A} = -\left(\frac{16\pi}{3}\right)^2 (4\cos 30\,\hat{i} + 4\sin 30\,\hat{j}) + 50\hat{k} \times (4\cos 30\,\hat{i} + 4\sin 30\,\hat{j})$$
$$= \left(-\left(\frac{16\pi}{3}\right)^2 .4\cos 30 + 200\sin 30\right)\hat{i} + \left(-\left(\frac{16\pi}{3}\right)^2 .4\sin 30 - 200\cos 30\right)\hat{j}$$
$$= -872.5\,\hat{i} - 734.68\,\hat{j}$$
$$\hat{a}_D = \hat{a}_B + \hat{\alpha}_3 \times \hat{r}_{D/B} - \omega_3^2 \hat{r}_{D/B}$$
$$= (-872.5\,\hat{i} - 734.68\,\hat{j}) + \alpha_3 \hat{k} \times (-10\sin\beta\,\hat{i} + 10\cos\beta\,\hat{j}) - 3.57^2 (-10\sin\beta\,\hat{i} + 10\cos\beta\,\hat{j})$$
$$\hat{a}_D = (-872.5 - 9.38\alpha_3 + 44.15)\hat{i} + (-734.68 - 3.464\alpha_3 - 119.56)\hat{j}$$
$a_D \hat{i} = 0 \quad \therefore \quad \alpha_3 = -88.31 \text{ rad/s}^2 \text{ (cw)}$

$a_D = -548.33\,\hat{j} \text{ cm/s}^2$

$$\hat{a}_C = \hat{a}_B - \omega_3^2 \hat{r}_{C/B} + \hat{\alpha}_3 \times \hat{r}_{C/B} = \hat{a}_B - \omega_3^2 (-4\sin\beta\,\hat{i} + 4\cos\beta\,\hat{j}) + \alpha_3 \hat{k} \times (-4\sin\beta\,\hat{i} + 4\cos\beta\,\hat{j})$$
$$= (-872.5 + \omega_3^2 .4\sin\beta - 4\cos\beta\,\alpha_3)\hat{i} + (-734.68 - \omega_3^2 4\cos\beta - 4\cos\beta\,\alpha_3)\hat{j}$$
$$= -858.59\,\hat{i} - 783.89\,\hat{j} \text{ cm/s}^2$$

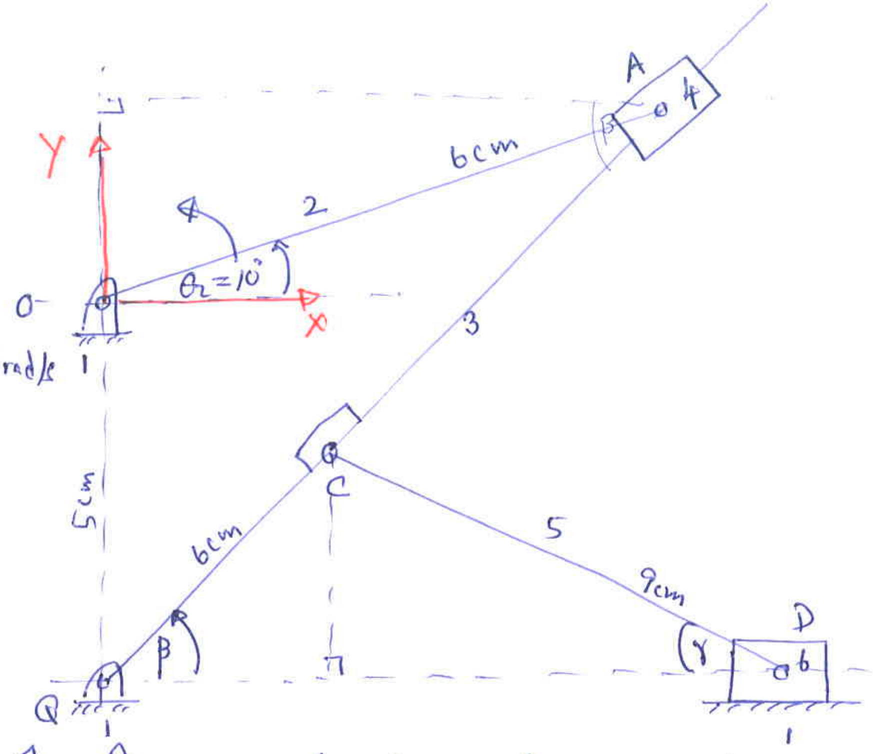
$$\hat{a}_E = \hat{a}_C + (\hat{a}_{E/C})_{\text{sliding}} - \omega_5^2 \hat{r}_{E/C} + 2\hat{\omega}_5 \times \hat{V}_{E/C} + \hat{\alpha}_5 \times \hat{r}_{E/C}$$
$$= \hat{a}_C + a_s(-\cos\gamma\,\hat{i} + \sin\gamma\,\hat{j}) - \omega_5^2(-x\cos\gamma\,\hat{i} + x\sin\gamma\,\hat{j}) + 2\omega_5 \hat{k} \times V_s(-\cos\gamma\,\hat{i} + \sin\gamma\,\hat{j})$$
$$+ \alpha_5 \hat{k} \times (-x\cos\gamma\,\hat{i} + x\sin\gamma\,\hat{j})$$
$$\hat{a}_E = (-858.59 - 0.98a_s + 5.62\omega_5^2 + 12.12\omega_5 - 1.15\alpha_5)\hat{i}$$
$$+ (-783.89 + 0.19a_s - 2.07\omega_5^2 + 62.42\omega_5 - 5.88\alpha_3)\hat{j}$$

$\omega_5 = 9.68$ rad/s $\quad$ & $\quad \hat{a}_E = 0$ $\qquad a_s = -282.88 \text{ cm/s}^2$

$\alpha_5 = 54.4 \text{ rad/s}^2$

$$\hat{A}_G = \hat{a}_C - \omega_5^2 \hat{r}_{G/C} + \hat{\alpha}_5 \times r_{G/C} = \hat{a}_C - \omega_5^2(-9\cos\gamma\,\hat{i} + 9\sin\gamma\,\hat{j}) + \alpha_5 \hat{k} \times (-9\cos\gamma\,\hat{i} + 9\sin\gamma\,\hat{j})$$
$$= (a_C + 9(9.68)^2\cos\gamma - 9\sin\gamma\,\alpha_5)\hat{i} + (a_C + 9(9.68)^2\sin\gamma - 9\cos\gamma.\alpha_5)\hat{j}$$
$$= -124.7\,\hat{i} - 1376.8\,j \quad \text{cm/s}^2$$

(Q5)




$\hat{\omega}_2 = 10\hat{k}$ rad/s

$\alpha_2 = 0$

$$\tan\beta = \frac{5 + 6\sin 10}{6\cos 10}$$

$$\beta = 45.64°$$

$$CA = \frac{(5 + 6\sin 10)}{\sin\beta} - 6$$

$$CA = 2.45\ \text{cm}$$

$$\sin\gamma = \frac{6\sin\beta}{9}$$

$$\gamma = 28.47°$$

$$\hat{V}_A = \hat{\omega}_2 \times \hat{r}_{A/O} = 10\hat{k} \times (6\cos 10\hat{i} + 6\sin 10\hat{j}) = -10.42\hat{i} + 59.09\hat{j} \quad \text{—— (1)}$$

$$\hat{V}_A = \hat{V}_Q + (\hat{V}_{A/Q})_{\text{Sliding}} + \hat{\omega}_3 \times \hat{r}_{A/Q}$$

$$= 0 + V_s(\cos\beta\hat{i} + \sin\beta\hat{j}) + \omega_3\hat{k} \times (8.45\cos\beta\hat{i} + 8.45\sin\beta\hat{j})$$

$$= (\cos\beta V_s - 8.45\sin\beta\,\omega_3)\hat{i} + (\sin\beta V_s + 8.45\cos\beta\,\omega_3)\hat{j} \quad \text{—— (2)}$$

by (1) & (2)

$$\left.\begin{aligned} V_s &= 34.96\ \text{cm/s} \\ \omega_3 &= 5.77\ \text{rad/s} \end{aligned}\right\} \parallel$$

$$\hat{V}_C = \hat{\omega}_3 \times \hat{r}_{C/Q} = 5.77\hat{k} \times (6\cos\beta\hat{i} + 6\sin\beta\hat{j}) = -24.75\hat{i} + 24.2\hat{j} \quad \text{—— (3)}$$

$$\hat{V}_C = \hat{V}_D + \hat{\omega}_5 \times \hat{r}_{C/D} = V_D\hat{i} + \omega_5\hat{k} \times (-9\cos\gamma\hat{i} + 9\sin\gamma\hat{j})$$

$$= (V_D - 9\sin\gamma.\omega_5)\hat{i} - 9\cos\gamma\,\omega_5\hat{j} \quad \text{—— (4)}$$

by (3) & (4)

$$\left.\begin{aligned} -9\cos\gamma\,\omega_5 &= 24.2 \\ V_D - 9\sin\gamma\,\omega_5 &= -24.75 \end{aligned}\right\} \qquad \begin{aligned} \omega_5 &= -3.06\ \text{rad/s} \\ V_D &= -37.87\ \text{cm/s} \end{aligned}$$

$$\therefore \hat{V}_D = -37.87\hat{i}\ \text{cm/s}$$

$$\hat{a}_A = -\omega_2^2\hat{r}_{A/O} + \hat{\alpha}_2 \times r_{A/O}$$

$$= -10^2 \times (6\cos 10\hat{i} + 6\sin 10\hat{j}) + \alpha_2\hat{k} \times (6\cos 10\hat{i} + 6\sin 10\hat{j})$$

$$= (-600\cos 10 - 6\sin 10\,\alpha_2)\hat{i} + (-600\sin 10 + 6\cos 10\,\alpha_2)\hat{j} \quad ; \alpha = 0$$

$$\hat{a}_A = -590.88\hat{i} - 104.19\hat{j}\ \text{cm/s}^2$$

$$\hat{a}_A = (\hat{a}_{A/Q})_{sliding} - \omega_3^2 \hat{r}_{A/Q} + 2\hat{\omega}_3 \times \hat{V}_{A/Q} + \hat{\alpha}_3 \times \hat{r}_{A/Q}$$

$$= a_s(\cos\beta\hat{i} + \sin\beta\hat{j}) - 5.77^2(8.45\cos\beta\hat{i} + 8.45\sin\beta\hat{j})$$
$$+ 2(5.77)\hat{k} \times 34.96(\cos\beta\hat{i} + \sin\beta\hat{j}) + \alpha_3\hat{k} \times (8.45\cos\beta\hat{i} + 8.45\sin\beta\hat{j})$$

$$= (0.699\,a_s - 6.04\,\alpha_3 - 485.135)\hat{i} + (0.715\,a_s + 5.91\,\alpha_3 + 80.93)$$

$$a_s = -206.29 \text{ cm/s}^2$$
$$\alpha_3 = -6.37 \text{ rad/s}^2$$

$$\hat{a}_C = -\omega_3^2 \hat{r}_{C/Q} + \hat{\alpha}_3 \times \hat{r}_{C/Q}$$

$$= -0.577^2(6\cos\beta\hat{i} + 6\sin\beta\hat{j}) + (-6.37)\hat{k} \times (6\cos\beta\hat{i} + 6\sin\beta\hat{j})$$

$$\hat{a}_C = -112.34\hat{i} - 169.54\hat{j} \text{ cm/s}^2$$

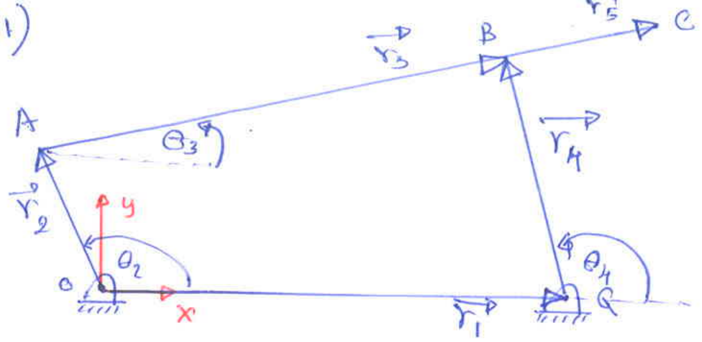
$$\hat{a}_C = \hat{a}_D - \omega_5^2 \hat{r}_{C/D} + \hat{\alpha}_5 \times \hat{r}_{C/D}$$

$$= a_D\hat{i} - (-3.06)^2(-9\cos\gamma\hat{i} + 9\sin\gamma\hat{j}) + \alpha_5\hat{k} \times (-9\cos\gamma\hat{i} + 9\sin\gamma\hat{j})$$

$$= (a_D - 4.29\,\alpha_5 + 74.08)\hat{i} + (-7.9116\,\alpha_5 - 40.17253)\hat{j}$$

$$\left.\begin{aligned} \alpha_5 &= 16.35 \text{ rad/s}^2 \\ \hat{a}_D &= -116.27\hat{i} \text{ cm/s}^2 \end{aligned}\right\} //$$

Q1)




$\vec{r_1} = 5\hat{i}$

$\vec{r_2} = 1.0(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$

$\vec{r_3} = 4.0(\cos\theta_3\hat{i} + \sin\theta_3\hat{j})$

$\vec{r_4} = 3.0(\cos\theta_4\hat{i} + \sin\theta_4\hat{j})$

$\vec{r_5} = 6.0(\cos\theta_3\hat{i} + \sin\theta_3\hat{j})$

Apply vector loop $\vec{r_1} + \vec{r_4} = \vec{r_2} + \vec{r_3}$

Collect $\hat{i}$; $\quad 5 + 3\cos\theta_4 = \cos\theta_2 + 4\cos\theta_3$

collect $\hat{j}$; $\quad 3\sin\theta_4 = \sin\theta_2 + 4\sin\theta_3$

INPUT $\theta_2 = (0^\circ \sim 360^\circ)$ Solve for $\theta_3$ and $\theta_4$

$$4\cos\theta_3 = (5 - \cos\theta_2) + 3\cos\theta_4 \quad \text{——— } (1)$$

$$4\sin\theta_3 = -\sin\theta_2 + 3\sin\theta_4 \quad \text{——— } (2)$$

$(1)^2 + (2)^2$

$$4^2 = (5-\cos\theta_2)^2 + \sin^2\theta_2 + 3^2 + 6\cos\theta_4(5-\cos\theta_2) - 6\sin\theta_4 \cdot \sin\theta_2$$

$$A\cos\theta_4 + B\sin\theta_4 + C = 0$$

$A = 6(5-\cos\theta_2)$,

$B = -6\sin\theta_2$

$C = (5-\cos\theta_2)^2 + \sin^2\theta_2 - 7$

Let $t = \tan\left(\frac{\theta_4}{2}\right)$ ; $\quad t = \dfrac{-B \pm \sqrt{B^2 - C^2 + A^2}}{(C-A)}$

$$\theta_4 = 2.\tan^{-1}(t)$$

Substitute $\theta_2 = 120^\circ$ ; $A = 33$, $B = -5.1962$, $C = 24$.

$t = -3.1593$ or $2.0046$

$\theta_4 = -144.87^\circ$ or $\theta_4 = 126.97^\circ$

for the given configuration

$$\cos\theta_3 = \frac{(5-\cos\theta_2) + 3\cos\theta_4}{4}$$

$$\sin\theta_3 = \frac{-\sin\theta_2 + 3\sin\theta_4}{4}$$

$$\theta_3 = \text{Atan2}(\sin\theta_3, \cos\theta_3)$$

$\vec{P_C}$ = Position of C

$$\vec{P_C} = \vec{r_2} + \vec{r_5} = 1.0(\cos\theta_2\hat{i} + \sin\theta_2\hat{j}) + 6.0(\cos\theta\hat{i} + \sin\theta\hat{j})$$

$$\vec{P_C} = (\cos\theta_2 + 6\cos\theta_3)\hat{i} + (\sin\theta_2 + 6\sin\theta_3)\hat{j} \longrightarrow *$$

$$\vec{P_B} = \vec{r_1} + \vec{r_4} = 5\hat{i} + 3.0(\cos\theta_4\hat{i} + \sin\theta_4\hat{j})$$

$$\vec{P_B} = (5 + 3\cos\theta_4)\hat{i} + 3\sin\theta_4\hat{j} \longrightarrow *$$

INPUT VELOCITY $\dot{\theta}_2 = 10\,rad/s$ , $\ddot{\theta}_2 = 0$

$$\vec{\dot{r}_1} = 0$$
$$\vec{\dot{r}_2} = 1.0(-\sin\theta_2\hat{i} + \cos\theta_2\hat{j})\dot{\theta}_2$$
$$\vec{\dot{r}_3} = 4(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})\dot{\theta}_3$$
$$\vec{\dot{r}_4} = 3(-\sin\theta_4\hat{i} + \cos\theta_4\hat{j})\dot{\theta}_4$$
$$\vec{\dot{r}_5} = 6(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})\dot{\theta}_3$$

Vector loop

$$\vec{r_1} + \vec{r_4} = \vec{r_2} + \vec{r_3}$$
$$\vec{\dot{r}_1} + \vec{\dot{r}_4} = \vec{\dot{r}_2} + \vec{\dot{r}_3}$$

collect $\hat{i}$; $-3\sin\theta_4.\dot{\theta}_4 = -\sin\theta_2.\dot{\theta}_2 - 4\sin\theta_3.\dot{\theta}_3$

collect $\hat{j}$; $+3\cos\theta_4.\dot{\theta}_4 = +\cos\theta_2.\dot{\theta}_2 + 4\cos\theta_3.\dot{\theta}_3$

$$-4\sin\theta_3.\dot{\theta}_3 + 3\sin\theta_4.\dot{\theta}_4 = \sin\theta_2.\dot{\theta}_2$$
$$-4\cos\theta_3.\dot{\theta}_3 + 3\cos\theta_4.\dot{\theta}_4 = \cos\theta_2.\dot{\theta}_2$$

$$\begin{bmatrix} -4\sin\theta_3 & 3\sin\theta_4 \\ -4\cos\theta_3 & 3\cos\theta_4 \end{bmatrix} \begin{bmatrix} \dot{\theta}_3 \\ \dot{\theta}_4 \end{bmatrix} = \begin{bmatrix} \sin\theta_2.\dot{\theta}_2 \\ \cos\theta_2.\dot{\theta}_2 \end{bmatrix}$$

Velocity for C.

$$\vec{\dot{P}_C} = \vec{\dot{r}_2} + \vec{\dot{r}_5} = (-\sin\theta_2.\dot{\theta}_2 - 6\sin\theta_3.\dot{\theta}_3)\hat{i} + (\cos\theta_2.\dot{\theta}_2 + 6\cos\theta_3.\dot{\theta}_3)\hat{j}$$

ACCELERATIONS $\ddot{\theta}_2 = 0$

$$\vec{\ddot{r}_1} = 0$$
$$\vec{\ddot{r}_2} = (-\sin\theta_2\hat{i} + \cos\theta_2\hat{j})\ddot{\theta}_2 - (\cos\theta_2\hat{i} + \sin\theta_2\hat{j})\dot{\theta}_2^2$$
$$\vec{\ddot{r}_3} = 4(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})\ddot{\theta}_3 - 4(\cos\theta_3\hat{i} + \sin\theta_3\hat{j})\dot{\theta}_3^2$$
$$\vec{\ddot{r}_4} = 3(-\sin\theta_4\hat{i} + \cos\theta_4\hat{j})\ddot{\theta}_4 - 3(\cos\theta_4\hat{i} + \sin\theta_4\hat{j})\dot{\theta}_4^2$$
$$\vec{\ddot{r}_5} = 6(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})\ddot{\theta}_3 - 6(\cos\theta_3\hat{i} + \sin\theta_3\hat{j})\dot{\theta}_3^2$$

$$\vec{\ddot{r}}_1 + \vec{\ddot{r}}_4 = \vec{\ddot{r}}_2 + \vec{\ddot{r}}_3$$

$\hat{i}$; $-3\sin\theta_4 . \ddot{\theta}_4 - 3\cos\theta_4 . \dot{\theta}_4^2 = -\sin\theta_2 . \ddot{\theta}_2 - \cos\theta_2 . \dot{\theta}_2^2 - 4\sin\theta_3 . \ddot{\theta}_3 - 4\cos\theta_3 . \dot{\theta}_3^2$

$\hat{j}$; $+3\cos\theta_4 . \ddot{\theta}_4 - 3\sin\theta_4 . \dot{\theta}_4^2 = \cos\theta_2 . \ddot{\theta}_2 - \sin\theta_2 . \dot{\theta}_2^2 + 4\cos\theta_3 . \ddot{\theta}_3 - 4\sin\theta_3 . \dot{\theta}_3^2$

$\ddot{\theta}_2 = 0$ ; unknown $\ddot{\theta}_3$ & $\ddot{\theta}_4$

$$4\sin\theta_3 . \ddot{\theta}_3 - 3\sin\theta_4 . \ddot{\theta}_4 = -\cos\theta_2 . \dot{\theta}_2^2 - 4\cos\theta_3 . \dot{\theta}_3^2 + 3\cos\theta_4 . \dot{\theta}_4^2$$
$$4\cos\theta_3 \ddot{\theta}_3 - 3\cos\theta_4 . \ddot{\theta}_4 = \sin\theta_2 . \dot{\theta}_2^2 + 4\sin\theta_3 . \dot{\theta}_3^2 - 3\sin\theta_4 . \dot{\theta}_4^2$$

$$\begin{bmatrix} 4\sin\theta_3 & -3\sin\theta_4 \\ 4\cos\theta_3 & -3\cos\theta_4 \end{bmatrix} \begin{bmatrix} \ddot{\theta}_3 \\ \ddot{\theta}_4 \end{bmatrix} = \begin{bmatrix} -\cos\theta_2 . \dot{\theta}_2^2 - 4\cos\theta_3 . \dot{\theta}_3^2 + 3\cos\theta_4 . \dot{\theta}_4^2 \\ \sin\theta_2 . \dot{\theta}_2^2 + 4\sin\theta_3 . \dot{\theta}_3^2 - 3\sin\theta_4 . \dot{\theta}_4^2 \end{bmatrix}$$

$$\vec{\ddot{P}}_c = \vec{\ddot{r}}_2 + \vec{\ddot{r}}_5$$

$$\vec{\ddot{P}}_c = -(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j})\dot{\theta}_2^2 + 6(-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j})\ddot{\theta}_3 - 6(\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j})\dot{\theta}_3^2$$
$$= (-\cos\theta_2 . \dot{\theta}_2^2 - 6\sin\theta_3 . \ddot{\theta}_3 - 6\cos\theta_3 . \dot{\theta}_3^2)\hat{i} + (-\sin\theta_2 . \dot{\theta}_2^2 + 6\cos\theta_3 . \ddot{\theta}_3 - 6\sin\theta_3 . \dot{\theta}_3^2)\hat{j}$$

SUMMARY

$\theta_2 = 0 \sim 360°$ $\quad \dot{\theta}_2 = 10\ rad/s$ , $\ddot{\theta}_2 = 0$

$A = 6(5 - \cos\theta_2)$ , $B = -6\sin\theta_2$ , $C = (5 - \cos\theta_2)^2 + \sin^2\theta_2 - 7$

$$t = \frac{-B \pm \sqrt{B^2 - C^2 + A^2}}{C - A}$$

$\theta_4 = 2\tan^{-1}(t)$

$$\cos\theta_3 = \frac{(5 - \cos\theta_2) + 3\cos\theta_4}{4} \qquad \sin\theta_3 = \frac{-\sin\theta_2 + 3\sin\theta_4}{4}$$

$\theta_3 = A\tan 2(\sin\theta_3, \cos\theta_3)$

$P_{Bx} = 5 + 3\cos\theta_4$

$P_{By} = 3\sin\theta_4$

$P_{cx} = \cos\theta_2 + 6\cos\theta_3$

$P_{cy} = \sin\theta_2 + 6\sin\theta_3$

$$\begin{bmatrix} \dot\theta_3 \\ \dot\theta_4 \end{bmatrix} = \begin{bmatrix} -4\sin\theta_3 & 3\sin\theta_4 \\ -4\cos\theta_3 & 3\cos\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} \sin\theta_2 \cdot \dot\theta_2 \\ \cos\theta_2 \cdot \dot\theta_2 \end{bmatrix}$$

$$V_{cx} = (-\sin\theta_2 . \dot\theta_2 - 6\sin\theta_3 . \dot\theta_3)$$

$$V_{cy} = (\cos\theta_2 . \dot\theta_2 + 6\cos\theta_3 . \dot\theta_3)$$

$$V_c = \sqrt{V_{cx}^2 + V_{cy}^2} \qquad \measuredangle V_c = \mathrm{Atan2}\left(\frac{V_{cy}}{V_c}, \frac{V_{cx}}{V_c}\right)$$

$$\begin{bmatrix} \ddot\theta_3 \\ \ddot\theta_4 \end{bmatrix} = \begin{bmatrix} 4\sin\theta_3 & -3\sin\theta_4 \\ 4\cos\theta_3 & -3\cos\theta_4 \end{bmatrix}^{-1} \begin{bmatrix} -\cos\theta_2 . \dot\theta_2^2 - 4\cos\theta_3 . \dot\theta_3^2 + 3\cos\theta_4 . \dot\theta_4^2 \\ \sin\theta_2 . \dot\theta_2^2 + 4\sin\theta_3 . \dot\theta_3^2 - 3\sin\theta_4 . \dot\theta_4^2 \end{bmatrix}$$

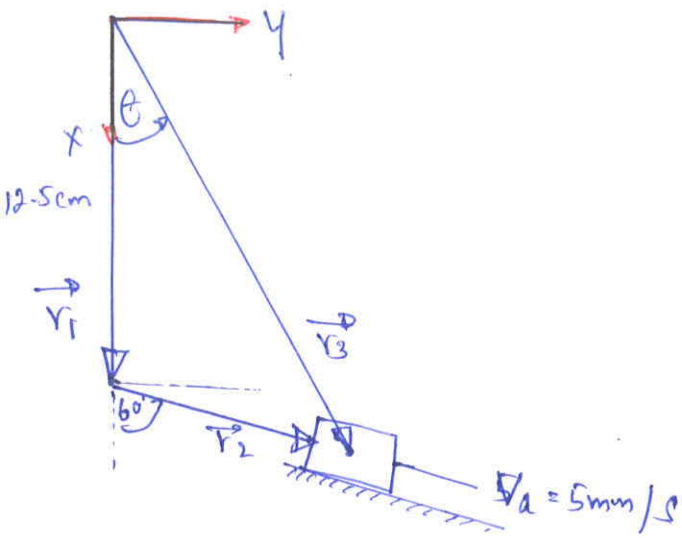
$$a_{cx} = -\cos\theta_2 . \dot\theta_2^2 - 6\sin\theta_3 . \ddot\theta_3 - 6\cos\theta_3 . \dot\theta_3^2$$

$$a_{cy} = -\sin\theta_2 . \dot\theta_2^2 + 6\cos\theta_3 . \ddot\theta_3 - 6\sin\theta_3 . \dot\theta_3^2$$

$$a_c = \sqrt{a_{cx}^2 + a_{cy}^2}$$

$$\measuredangle a_c = \mathrm{Atan2}\left(\frac{a_{cy}}{a_c}, \frac{a_{cx}}{a_c}\right)$$

Q2)




$\vec{r_1} = 12.5\hat{i}$ cm

$\vec{r_2} = r_2(\cos 60\hat{i} + \sin 60\hat{j})$

$= r_2[0.5\hat{i} + 0.866\hat{j}]$

$\vec{r_3} = r_3[\cos\theta\hat{i} + \sin\theta_3\hat{j}]$

Apply vector loop;

$\vec{r_1} + \vec{r_2} = \vec{r_3}$

INPUT ANGLE; $\theta$

$\vec{r_1} + \vec{r_2} = \vec{r_3}$

$12.5i + r_2[0.5i + 0.866\hat{j}] = r_3[\cos\theta\hat{i} + \sin\theta_3\hat{j}]$

$\hat{i}$; $12.5 + r_2(0.5) = r_3\cos\theta$

$\hat{j}$; $r_2(0.866) = r_3\sin\theta$

$$r_3 = \frac{0.866}{\sin\theta} r_2$$

$$12.5 + r_2(0.5) = \frac{0.866 r_2}{\sin\theta}\cdot\cos\theta$$

$$r_2 = \frac{12.5}{\frac{0.866\cos\theta}{\sin\theta} - 0.5} \quad \text{— (1)}$$

$$r_3 = \frac{0.866 r_2}{\sin\theta} \quad \text{— (2)}$$

When $\theta = 20^\circ$ $\quad r_2 = \dfrac{12.5}{\frac{0.866\cos 20}{\sin 20} - 0.5} = 6.651$ cm

$$r_3 = \frac{0.866 \times 6.651}{\sin 20} = 16.84 \text{ cm}$$

VELOCITIES; $\boxed{\text{INPUT } \dot{r}_2}$

$\vec{\dot{r}_1} = 0$

$\vec{\dot{r}_2} = \dot{r}_2(0.5\hat{i} + 0.866\hat{j})$

$\vec{\dot{r}_3} = \dot{r}_3(\cos\theta\hat{i} + \sin\theta\hat{j}) + r_3\dot{\theta}(-\sin\theta\hat{i} + \cos\theta\hat{j})$

$$\vec{\dot r_1} + \vec{\dot r_2} = \vec{\dot r_3}$$

$$0.5\dot r_2 = \dot r_3 \cos\theta - r_3\dot\theta \sin\theta$$

$$0.866\dot r_2 = \dot r_3 \sin\theta + r_3\dot\theta \cos\theta$$

$$\begin{bmatrix} \cos\theta & -r_3\sin\theta \\ \sin\theta & r_3\cos\theta \end{bmatrix} \begin{bmatrix} \dot r_3 \\ \dot\theta \end{bmatrix} = \begin{bmatrix} 0.5\dot r_2 \\ 0.866\dot r_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot r_3 \\ \dot\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -r_3\sin\theta \\ \sin\theta & r_3\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} 0.5\dot r_2 \\ 0.866\dot r_2 \end{bmatrix} \longrightarrow (3)$$

ACCELERATION ;

$$\vec{\ddot r_1} = 0$$

$$\vec{\ddot r_2} = \ddot r_2 (0.5\hat i + 0.866\hat j)$$

$$\vec{\ddot r_3} = \ddot r_3(\cos\theta\hat i + \sin\theta\hat j) + \dot r_3(-\sin\theta\hat i + \cos\theta\hat j)\dot\theta + r_3\dot\theta^2(-\cos\theta\hat i - \sin\theta\hat j)$$
$$+ r_3\ddot\theta(-\sin\theta\hat i + \cos\theta\hat j) + \dot r_3\dot\theta(-\sin\theta_3\hat i + \cos\theta_3\hat j)$$

$$\vec{\ddot r_3} = (\ddot r_3 - r_3\dot\theta^2)(\cos\theta\hat i + \sin\theta\hat j) + (2\dot r_3\dot\theta + r_3\ddot\theta)(-\sin\theta\hat i + \cos\theta\hat j)$$

$$\vec{\ddot r_1} + \vec{\ddot r_2} = \vec{\ddot r_3}$$

$\hat i$; $\quad 0.5\ddot r_2 = (\ddot r_3 - r_3\dot\theta^2)\cos\theta - (2\dot r_3\dot\theta + r_3\ddot\theta)\sin\theta$

$\hat j$; $\quad 0.866\ddot r_2 = (\ddot r_3 - r_3\dot\theta^2)\sin\theta + (2\dot r_3\dot\theta + r_3\ddot\theta)\cos\theta$

$$\ddot r_3\cos\theta - r_3\ddot\theta\sin\theta = 0.5\ddot r_2 + r_3\dot\theta^2\cos\theta + 2\dot r_3\dot\theta\sin\theta$$

$$\ddot r_3\sin\theta + r_3\ddot\theta\cos\theta = 0.866\ddot r_2 + r_3\dot\theta^2\sin\theta - 2\dot r_3\dot\theta\cos\theta$$

$$\begin{bmatrix} \cos\theta & -r_3\sin\theta \\ \sin\theta & r_3\cos\theta \end{bmatrix} \begin{bmatrix} \ddot r_3 \\ \ddot\theta \end{bmatrix} = \begin{bmatrix} 0.5\ddot r_2 + r_3\dot\theta^2\cos\theta + 2\dot r_3\dot\theta\sin\theta \\ 0.866\ddot r_2 + r_3\dot\theta^2\sin\theta - 2\dot r_3\dot\theta\cos\theta \end{bmatrix}$$

$$\begin{bmatrix} \ddot r_3 \\ \ddot\theta \end{bmatrix} = \begin{bmatrix} \cos\theta & -r_3\sin\theta \\ \sin\theta & r_3\cos\theta \end{bmatrix}^{-1} \begin{bmatrix} 0.5\ddot r_2 + r_3\dot\theta^2\cos\theta + 2\dot r_3\dot\theta\sin\theta \\ 0.866\ddot r_2 + r_3\dot\theta^2\sin\theta - 2\dot r_3\dot\theta\cos\theta \end{bmatrix} \longrightarrow (4)$$

GIVEN $\quad \theta = 90^\circ$, $\dot r_2 = V_A = 0.5\ cm/s \qquad \ddot r_2 = 0$

$r_2 = 6.651$ cm

$r_3 = 16.84$ cm

by ③ ⇒

$$\begin{bmatrix}\dot{r}_3\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}\cos 20 & -16.841\sin 20\\ \sin 20 & 16.841\cos 20\end{bmatrix}^{-1}\begin{bmatrix}0.5\times 0.5\\ 0.866\times 0.5\end{bmatrix}$$

$$= \begin{bmatrix}0.9397 & -5.760\\ 0.3420 & 15.8254\end{bmatrix}^{-1}\begin{bmatrix}0.25\\ 0.433\end{bmatrix}$$

RECALL

$$\begin{bmatrix}a & b\\ c & d\end{bmatrix}^{-1} = \frac{1}{ad-bc}\begin{bmatrix}+d & -b\\ -c & a\end{bmatrix}$$

$\therefore a = 0.9397$, $b = -5.760$, $c = 0.3420$, $d = 15.8254$, $ad - bc = 16.841$

$$\begin{bmatrix}0.9397 & -5.760\\ 0.3420 & 15.8254\end{bmatrix}^{-1} = \frac{1}{16.841}\begin{bmatrix}15.8254 & 5.760\\ -0.3420 & 0.9397\end{bmatrix} = \begin{bmatrix}0.9397 & 0.3420\\ -0.0203 & 0.0558\end{bmatrix}$$

$$\begin{bmatrix}\dot{r}_3\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}0.9397 & 0.3420\\ -0.0203 & 0.0558\end{bmatrix}\begin{bmatrix}0.25\\ 0.433\end{bmatrix}$$

$$\begin{bmatrix}\dot{r}_3\\ \dot{\theta}\end{bmatrix} = \begin{bmatrix}0.3816\\ 0.0191\end{bmatrix}$$

$$\left.\begin{array}{l}\text{Sliding velocity } \dot{r}_3 = 0.3816 \text{ cm/s}\\ \text{Rotational Speed } \dot{\theta} = 0.0191 \text{ rad/s}\end{array}\right\}$$

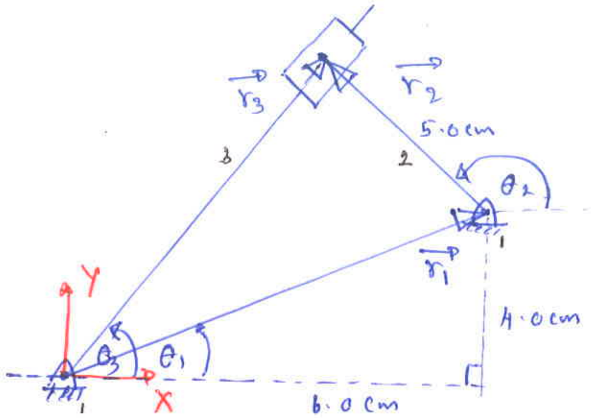
ACCELERATIONS $\qquad \ddot{r}_2 = 0 \qquad \theta_2 = 20°$

④ ⇒ $$\begin{bmatrix}\ddot{r}_3\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}\cos 20 & -16.841\sin 20\\ \sin 20 & 16.841\cos 20\end{bmatrix}^{-1}\begin{bmatrix}0 + 16.841(0.0191)^2\cos 20 + 2(.3816)(0.0191)\sin 20\\ 0 + 16.841(0.0191)^2\sin 20 - 2(.3816)(0.0191)\cos 20\end{bmatrix}$$

$$= \begin{bmatrix}0.9397 & 0.3420\\ -0.0203 & 0.0558\end{bmatrix}\begin{bmatrix}0.0108\\ -0.0116\end{bmatrix}$$

$$\begin{bmatrix}\ddot{r}_3\\ \ddot{\theta}\end{bmatrix} = \begin{bmatrix}0.0062 \text{ cm/s}^2\\ -0.0009 \text{ rad/s}^2\end{bmatrix} \quad //$$

Q3)




$\vec{r_1} = 6\hat{i} + 4\hat{j}$ cm

$\vec{r_2} = 5(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$ cm

$\vec{r_3} = r_3(\cos\theta_3\hat{i} + \sin\theta_3\hat{j})$ cm

$\vec{r_1} + \vec{r_2} = \vec{r_3}$

$\hat{i}$; $\quad 6 + 5\cos\theta_2 = r_3\cos\theta_3$ —— (1)

$\hat{j}$; $\quad 4 + 5\sin\theta_2 = r_3\sin\theta_3$ —— (2)

$(1)^2 + (2)^2 \quad r_3 = \sqrt{(6+5\cos\theta_2)^2 + (4+5\sin\theta_2)^2}$

$\theta_2 = 135^\circ \quad r_3 = \sqrt{(6+5\cos 135)^2 + (4+5\sin 135)^2}$

$r_3 = 7.928$ cm

$$\cos\theta_3 = \frac{6+5\cos 135}{7.928} = 0.3108$$

$$\sin\theta_3 = \frac{4+5\sin 135}{7.928} = 0.950$$

$$\theta_3 = \text{Atan2}(\sin\theta_3, \cos\theta_3)$$
$$= \text{Atan2}(0.950, 0.3108)$$
$$= 71.89^\circ$$

GIVEN ; $\quad \dot{\theta}_2 = -80 \times \frac{2\pi}{60}$

$\dot{\theta}_2 = -8.38$ rad/s , $\quad \ddot{\theta}_2 = 0$

$\dot{\vec{r_1}} = 0$

$\dot{\vec{r_2}} = 5(-\sin\theta_2\hat{i} + \cos\theta_2\hat{j})\dot{\theta}_2$

$\dot{\vec{r_3}} = \dot{r}_3(\cos\theta_3\hat{i} + \sin\theta_3\hat{j}) + r_3\dot{\theta}_3(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})$

$\dot{\vec{r_1}} + \dot{\vec{r_2}} = \dot{\vec{r_3}}$

$\hat{i}$; $\quad -5\sin\theta_2 \cdot \dot{\theta}_2 = \dot{r}_3\cos\theta_3 + r_3\dot{\theta}_3(-\sin\theta_3)$

$\hat{j}$; $\quad 5\cos\theta_2 \cdot \dot{\theta}_2 = \dot{r}_3\sin\theta_3 + r_3\dot{\theta}_3\cos\theta_3$

$$\begin{bmatrix} \cos\theta_3 & -r_3\sin\theta_3 \\ \sin\theta_3 & r_3\cos\theta_3 \end{bmatrix} \begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -5\sin\theta_2 \cdot \dot{\theta}_2 \\ 5\cos\theta_2 \cdot \dot{\theta}_2 \end{bmatrix}$$

$$\begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & -r_3\sin\theta_3 \\ \sin\theta_3 & r_3\cos\theta_3 \end{bmatrix}^{-1} \begin{bmatrix} 5\sin\theta_2 \cdot \dot{\theta}_2 \\ 5\cos\theta_2 \cdot \dot{\theta}_2 \end{bmatrix}$$

$$\begin{bmatrix} \cos\theta_3 & -r_3\sin\theta_3 \\ \sin\theta_3 & r_3\cos\theta_3 \end{bmatrix}^{-1} = \frac{1}{(r_3\cos^2\theta_3 + r_3\sin^2\theta_3)} \begin{bmatrix} r_3\cos\theta_3 & r_3\sin\theta_3 \\ -\sin\theta_3 & \cos\theta_3 \end{bmatrix}$$

$$= \begin{bmatrix} \cos\theta_3 & \sin\theta_3 \\ -\frac{\sin\theta_3}{r_3} & \frac{\cos\theta_3}{r_3} \end{bmatrix}$$

$$\begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 \\ -\frac{\sin\theta_3}{r_3} & \frac{\cos\theta_3}{r_3} \end{bmatrix} \begin{bmatrix} 5\sin\theta_2 \cdot \dot{\theta}_2 \\ 5\cos\theta_2 \cdot \dot{\theta}_2 \end{bmatrix}$$

$\theta_2 = 135^\circ$, $\theta_3 = 71.89^\circ$, $r_3 = 7.928\,cm$ $\quad \dot{\theta}_2 = -8.38\ rad/s$

$$\begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 0.3108 & 0.9505 \\ -0.1199 & 0.0392 \end{bmatrix} \begin{bmatrix} 29.63 \\ 29.63 \end{bmatrix}$$

$$\begin{bmatrix} \dot{r}_3 \\ \dot{\theta}_3 \end{bmatrix} = \begin{bmatrix} 37.37\ cm/s \\ -2.39\ rad/s \end{bmatrix}$$

$\underline{\dot{r}_3 = 37.37\ cm/s}$ $\qquad \underline{\dot{\theta}_3 = -2.39\ rad/s}$

**ACCELERATION** $\qquad \ddot{r}_1 = 0 \qquad \ddot{\theta}_2 = 0$

$$\vec{\ddot{r}_2} = -5\,\dot{\theta}_2^2(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$$

$$\vec{\ddot{r}_3} = \ddot{r}_3(\cos\theta_3\hat{i} + \sin\theta_3\hat{j}) + \dot{r}_3\dot{\theta}_3(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j}) + r_3\dot{\theta}_3^2(-\cos\theta_3\hat{i} - \sin\theta_3\hat{j})$$
$$+ r_3\ddot{\theta}_3(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j}) + \dot{r}_3\dot{\theta}_3(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})$$

$$\vec{\ddot{r}_3} = (\ddot{r}_3 - r_3\dot{\theta}_3^2)(\cos\theta_3\hat{i} + \sin\theta_3\hat{j}) + (2\dot{r}_3\dot{\theta}_3 + r_3\ddot{\theta}_3)(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j})$$

$$\vec{\ddot{r}_1} + \vec{\ddot{r}_2} = \vec{\ddot{r}_3}$$

$$-5\dot{\theta}_2^2\cos\theta_2 = (\ddot{r}_3 - r_3\dot{\theta}_3^2)\cos\theta_3 + (2\dot{r}_3\dot{\theta}_3 + r_3\ddot{\theta}_3)(-\sin\theta_3)$$

$$-5\dot{\theta}_2^2\sin\theta_2 = (\ddot{r}_3 - r_3\dot{\theta}_3^2)\sin\theta_3 + (2\dot{r}_3\dot{\theta}_3 + r_3\ddot{\theta}_3)\cos\theta_3$$

$$\begin{bmatrix} \cos\theta_3 & -r_3\sin\theta_3 \\ \sin\theta_3 & r_3\cos\theta_3 \end{bmatrix} \begin{bmatrix} \ddot{r}_3 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -5\dot{\theta}_2^2\cos\theta_2 + r_3\dot{\theta}_3^2\cos\theta_3 + 2\dot{r}_3\dot{\theta}_3\sin\theta_3 \\ -5\dot{\theta}_2^2\sin\theta_2 + r_3\dot{\theta}_3^2\sin\theta_3 - 2\dot{r}_3\dot{\theta}_3\cos\theta_3 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{r}_3 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & -r_3\sin\theta_3 \\ \sin\theta_3 & r_3\cos\theta_3 \end{bmatrix}^{-1} \begin{bmatrix} 92.578 \\ -149.71 \end{bmatrix}$$

$$\begin{bmatrix} \ddot{r}_3 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} \cos\theta_3 & \sin\theta_3 \\ -\frac{\sin\theta_3}{r_3} & \frac{\cos\theta_3}{r_3} \end{bmatrix} \begin{bmatrix} 92.578 \\ -149.71 \end{bmatrix}$$

$$= \begin{bmatrix} 0.3108 & 0.9505 \\ -0.1199 & 0.0392 \end{bmatrix} \begin{bmatrix} 92.578 \\ -149.71 \end{bmatrix}$$

$\theta_3 = 71.89^\circ$
$r_3 = 7.928$

$$\begin{bmatrix} \ddot{r}_3 \\ \ddot{\theta}_3 \end{bmatrix} = \begin{bmatrix} -113.52 \\ -16.97 \end{bmatrix}$$

$\ddot{r}_3 = -113.52\ cm/s^2$ $\qquad$ $\ddot{\theta}_3 = -16.97\ rad/s^2$

a) Angular velocity of link 3 ; $\dot{\theta}_3 = -2.39\ rad/s$

b) Sliding velocity at the Slider; $\dot{r}_3 = 37.37\ cm/s$

c) Coriolis acceleration at the Slider;

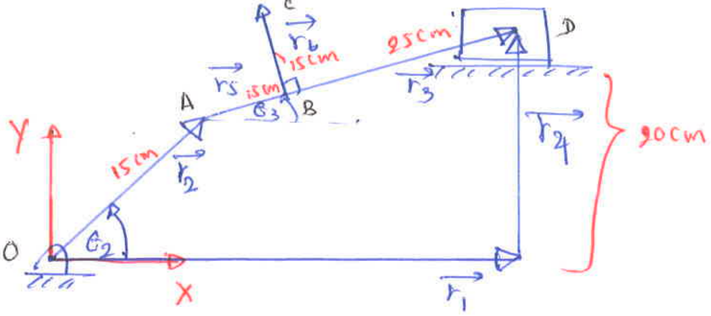
$$= 2\dot{r}_3\dot{\theta}_3\left(-\sin\theta_3\hat{i} + \cos\theta_3\hat{j}\right)$$

$$= 2(37.37)(-2.39)\left(-\sin 71.89\hat{i} + \cos 71.89\hat{j}\right)$$

$$= 169.78\hat{i} - 55.53\hat{j}$$

d) Angular acceleration of link 3 ; $\ddot{\theta}_3 = -16.97\ rad/s^2$

Q4)




$\ddot{\theta}_2 = 0$

$\vec{r_1} = r_1 \hat{i}$

$\vec{r_2} = 15\left(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}\right)$

$\vec{r_3} = 40\left(\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}\right)$

$\vec{r_4} = 20\hat{j}$

$\vec{r_5} = 15\left(\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}\right)$

$\vec{r_6} = 15\left(\cos(\theta_3+90)\hat{i} + \sin(\theta_3+90)\hat{j}\right)$

$$\vec{r_2} + \vec{r_3} = \vec{r_1} + \vec{r_4}$$

$\hat{i}$; $\quad 15\cos\theta_2 + 40\cos\theta_3 = r_1$

$\hat{j}$; $\quad 15\sin\theta_2 + 40\sin\theta_3 = r_4 = 20$

$\theta_2 = 45^\circ$ ; $\quad \sin\theta_3 = \dfrac{20 - 15\sin 45}{40} = 0.2348$

$$\underline{\underline{\theta_3 = 13.58^\circ}}$$

$$r_1 = 15\sin 45 + 40\sin(13.58)$$

$$\underline{\underline{r_1 = 49.49\,\text{cm}}}$$

VELOCITIES

$\left\{\begin{array}{l} \vec{r_1} = r_1(\cos 0\,\hat{i} + \sin 0\,\hat{j}) \\ \vec{\dot{r}_1} = r_1 \dot{0}(-\sin 0\,\hat{i} + \cos 0\,\hat{j}) + \dot{r}_1(\cos 0\,\hat{i} + \sin 0\,\hat{j}) \end{array}\right\}$ explanation only

$\dot{\theta}_2 = 2\pi \times \dfrac{150}{60} = 50\pi$ rad/s
$= 157.1$ rad/s

$\vec{\dot{r}_1} = \dot{r}_1 \hat{i}$

$\vec{\dot{r}_2} = 15\dot{\theta}_2\left(-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}\right)$

$\vec{\dot{r}_3} = 40\dot{\theta}_3\left(-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}\right)$

$\vec{\dot{r}_4} = 0$

$\vec{\dot{r}_5} = 15\dot{\theta}_3\left(-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}\right)$

$\vec{\dot{r}_6} = 15\dot{\theta}_3\left(-\cos\theta_3 \hat{i} - \sin\theta_3 \hat{j}\right)$

$$\vec{\dot{r}_2} + \vec{\dot{r}_3} = \vec{\dot{r}_1} + \vec{\dot{r}_4}$$

$$-15\dot{\theta}_2 \sin\theta_2 - 40\dot{\theta}_3 \sin\theta_3 = \dot{r}_1$$

$$15\dot{\theta}_2 \cos\theta_2 + 40\dot{\theta}_3 \cos\theta_3 = 0$$

$$\dot{\theta}_3 = \frac{-15\cos\theta_2\,\dot{\theta}_2}{40\cos\theta_3} = \frac{-15\cos 45}{40\cos 13.58}(50\pi)$$

$$\underline{\underline{\dot{\theta}_3 = -42.85\ \text{rad/s}}}$$

$$\dot{r}_1 = -15(50\pi)\sin 45 - 40(-42.85)\sin(13.58)$$

$$\underline{\underline{\dot{r}_1 = -1263.63\ \text{cm/s}}}$$

## ACCELERATION

$$\vec{\ddot{r}_1} = \ddot{r}_1 \hat{i}$$

$$\vec{\ddot{r}_2} = 15\left(-\cos\theta_2 \hat{i} - \sin\theta_2 \hat{j}\right)\dot{\theta}_2^2 + 15\left(-\sin\theta_2 \hat{i} + \cos\theta_2 \hat{j}\right)\cancel{\ddot{\theta}_2} \rightarrow 0$$

$$= -15\left(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}\right)\dot{\theta}_2^2$$

$$\vec{\ddot{r}_3} = 40\left(-\cos\theta_3 \hat{i} - \sin\theta_3 \hat{j}\right)\dot{\theta}_3^2 + 40\left(-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}\right)\ddot{\theta}_3$$

$$\vec{\ddot{r}_4} = 0$$

$$\vec{\ddot{r}_5} = 15\left(-\cos\theta_3 \hat{i} - \sin\theta_3 \hat{j}\right)\dot{\theta}_3^2 + 15\left(-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}\right)\ddot{\theta}_3$$

$$\vec{\ddot{r}_6} = -15\left(-\sin\theta_3 \hat{i} + \cos\theta_3 \hat{j}\right)\dot{\theta}_3^2 - 15\left(\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}\right)\ddot{\theta}_3$$

$$\vec{\ddot{r}_2} + \vec{\ddot{r}_3} = \vec{\ddot{r}_1} + \vec{\ddot{r}_4}$$

$$-15\cos\theta_2 . \dot{\theta}_2^2 + 40\cos\theta_3 . \dot{\theta}_3^2 - 40\sin\theta_3 . \ddot{\theta}_3 = \ddot{r}_1$$

$$-15\sin\theta_2 . \dot{\theta}_2^2 - 40\sin\theta_3 . \dot{\theta}_3^2 + 40\cos\theta_3 . \ddot{\theta}_3 = 0$$

$$\ddot{\theta}_3 = \frac{15\sin\theta_2 . \dot{\theta}_2^2 + 40\sin\theta_3 . \dot{\theta}_3^2}{40\cos\theta_3} = \frac{15\sin 45(50\pi) + 40\sin\theta_3 . \dot{\theta}_3^2}{40\cos(13.58)}$$

$$\underline{\underline{\ddot{\theta}_3 = 7174.39 \text{ rad/s}^2}}$$

$$\ddot{r}_1 = -15\cos 45 . (50\pi)^2 - 40\cos(13.58)(-42.85)^2 - 40\sin(13.58)(7174.39)$$

$$\underline{\underline{\ddot{r}_1 = -4004.82 \text{ m/s}}}$$

$$\vec{P_c} = \vec{r_2} + \vec{r_5} + \vec{r_6}$$

$$\vec{P_c} = 15\left(\cos\theta_2 \hat{i} + \sin\theta_2 \hat{j}\right) + 15\left(\cos\theta_3 \hat{i} + \sin\theta_3 \hat{j}\right) + 15\left[\cos(\theta_3 + 90)\hat{i} + \sin(\theta_3 + 90)\hat{j}\right]$$

$$\vec{P_c} = \left(15\cos\theta_2 + 15\cos\theta_3 + 15\cos(\theta_3 + 90)\right)\hat{i} + \left(15\sin\theta_2 + 15\sin\theta_3 + 15\sin(\theta_3 + 90)\right]\hat{j}$$

$$\vec{V_c} = \vec{\dot{P}_c} = 15\left[\left(-\sin\theta_2 . \dot{\theta}_2 - \sin\theta_3 . \dot{\theta}_3 - \sin(\theta_3 + 90)\dot{\theta}_3\right)\hat{i} + \left(\cos\theta_2 \dot{\theta}_2 + \cos\theta_3 . \dot{\theta}_3 + \cos(\theta_3 + 90) . \dot{\theta}_3\right)\hat{j}\right]$$

$$\underline{\underline{\vec{V_c} = -890.38\hat{i} + 1192.22\hat{j} \text{ cm/s}}}$$

$\theta_2 = 45°$, $\dot{\theta}_2 = 50\pi$ rad/s $\ddot{\theta}_2 = 0$

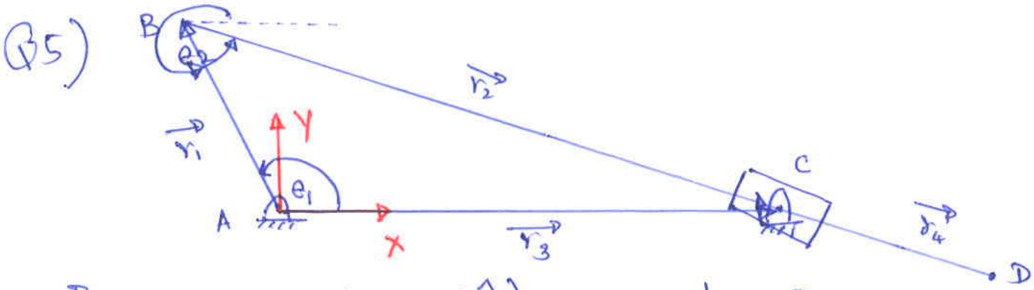
$\theta_3 = 13.58°$, $\dot{\theta}_3 = -42.85$ rad/s, $\ddot{\theta}_3 = 7174.39$ rad/$s^2$

$$\vec{a}_c = \ddot{\vec{P}}_c = 15\Big\{\big[-\sin\theta_2.\ddot{\theta}_2 - \cos\theta_2.\dot{\theta}_2^2 - \sin\theta_3.\ddot{\theta}_3 - \cos\theta_3.\dot{\theta}_3^2 - \sin(\theta_3+90).\ddot{\theta}_3 - \cos(\theta_3+90)\dot{\theta}_3^2\big]\hat{i}$$
$$+\big[-\sin\theta_2.\dot{\theta}_2^2 + \cos\theta_2\ddot{\theta}_2 - \sin\theta_2.\dot{\theta}_3^2 + \cos\theta_3\ddot{\theta}_3 - \sin(\theta_3+90)\dot{\theta}_3^2 + \cos(\theta_3+90)\ddot{\theta}_3\big]\hat{j}\Big\}$$

$$\vec{a}_c = \ddot{\vec{P}}_c = 15\Big\{\big[-\cos\theta_2.\dot{\theta}_2^2 - (\sin\theta_3+\cos\theta_3)\ddot{\theta}_3 + (\sin\theta_3 - \cos\theta_3)\dot{\theta}_3^2\big]\hat{i} +$$
$$\big[-\sin\theta_2.\dot{\theta}_2^2 - (\sin\theta_3 - \cos\theta_3)\ddot{\theta}_3 - (\sin\theta_3+\cos\theta_3)\dot{\theta}_3^2\big]\hat{j}\Big\}$$

$$\underline{\vec{a}_c = \ddot{\vec{P}}_c = (-4118.88\,\hat{i} - 2156.07\,\hat{j})\ m/s^2}$$

Q5)




$AB = 175\,cm$
$AC = 250\,cm$
$BD = 500\,cm$
$\dot{\theta}_1 = 100\,rad/s$

$$\vec{r}_1 = 175(\cos\theta_1\hat{i} + \sin\theta_1\hat{j})$$
$$\vec{r}_2 = r_2(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$$
$$\vec{r}_3 = 250\hat{i}$$
$$\vec{r}_4 = 500(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$$

APPLY VECTOR LOOP

$$\vec{r}_1 + \vec{r}_2 = \vec{r}_3$$

$\hat{i}$; $175\cos\theta_1 + r_2\cos\theta_2 = 250$

$\hat{j}$; $175\sin\theta_1 + r_2\sin\theta_2 = 0$

$\theta_1 = 90^\circ$

$$\sin\theta_2 = \frac{-175\sin 90}{r_2}$$
$$\cos\theta_2 = \frac{250 - 175\cos 90}{r_2}$$
$$\theta_2 = \mathrm{Atan}(\sin\theta_2, \cos\theta_2)$$
$$= -34.98^\circ$$
$$\underline{\theta_2 = (-35^\circ)}$$
$$r_2 = \frac{-175}{\sin(-35)} = \underline{305.1\,cm}$$

$$\dot{\vec{r}}_1 = 175(-\sin\theta_1\hat{i} + \cos\theta_1\hat{j})\dot{\theta}_1$$
$$\dot{\vec{r}}_2 = \dot{r}_2(\cos\theta_2\hat{i} + \sin\theta_2\hat{j}) + r_2(-\sin\theta_2\hat{i} + \cos\theta_2\hat{j})\dot{\theta}_2$$
$$\dot{\vec{r}}_3 = 0$$
$$\dot{\vec{r}}_4 = 500(-\sin\theta_2\hat{i} + \cos\theta_2\hat{j})\dot{\theta}_2$$

APPLY VECTOR LOOP;

$$\dot{\vec{r}}_1 + \dot{\vec{r}}_2 = \dot{\vec{r}}_3$$

$\hat{i}$; $175(-\sin\theta_1)\dot{\theta}_1 + \dot{r}_2\cos\theta_2 - r_2\dot{\theta}_2\sin\theta_2 = 0$

$\hat{j}$; $175\cos\theta_1.\dot{\theta}_1 + \dot{r}_2\sin\theta_2 + r_2\dot{\theta}_2\cos\theta_2 = 0$

$$\begin{bmatrix}\cos\theta_2 & -r_2\sin\theta_2\\ \sin\theta_2 & r_2\cos\theta_2\end{bmatrix}\begin{bmatrix}\dot{r}_2\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}175\dot{\theta}_1\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\dot{r}_2\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}\cos\theta_2 & -r_2\sin\theta_2\\ \sin\theta_2 & r_2\cos\theta_2\end{bmatrix}^{-1}\begin{bmatrix}175\dot{\theta}_1\\ 0\end{bmatrix}$$

$$= \begin{bmatrix}\cos\theta_2 & \sin\theta_2\\ \frac{-\sin\theta_2}{r_2} & \frac{\cos\theta_2}{r_2}\end{bmatrix}\begin{bmatrix}17{,}500\\ 0\end{bmatrix}$$

$$\begin{bmatrix}\dot{r}_2\\ \dot{\theta}_2\end{bmatrix} = \begin{bmatrix}14{,}335.16\ cm/s^2\\ 82.9\ rad/s^2\end{bmatrix}$$

$$\vec{P}_D = \vec{r}_1 + \vec{r}_4 = 175(\cos\theta_1\hat{i} + \sin\theta_1\hat{j}) + 500(\cos\theta_2\hat{i} + \sin\theta_2\hat{j})$$
$$= (175\cos\theta_1 + 500\cos\theta_2)\hat{i} + (175\sin\theta_1 + 500\sin\theta_2)\hat{j}$$
$$\underline{\vec{P}_D = 409.58\,\hat{i} - 111.79\,\hat{j}\quad (cm)}$$

$$\dot{\vec{P}}_D = \dot{\vec{r}}_1 + \dot{\vec{r}}_4 = 175(-\sin\theta_1\hat{i} + \cos\theta_1\hat{j})\dot{\theta}_1 + 500(-\sin\theta_2\hat{i} + \cos\theta_2\hat{j})\dot{\theta}_2$$
$$\underline{\dot{\vec{P}}_D = -8664.7\,\hat{i} + 13{,}475.05\,\hat{j}\quad (cm/s^2)}$$

Dr. George Mann
Department of Mechanical Engineering
SLIIT, Sri Lanka

**Assignment 3**

A test is conducted for this Tute on May 3, 2018 from 4:30 pm till 5:30 pm
Question 6 is optional, and students can submit to receive bonus marks

**Q1** The mechanism shown in Figure Q1 is drawn to a scale. Determine the magnitude and sense of the torque applied to link 2 to keep the mechanism in static equilibrium while being subjected to a force of magnitude of 50 N. User graphical approach to solve the problem and then verify your results using vector approach.

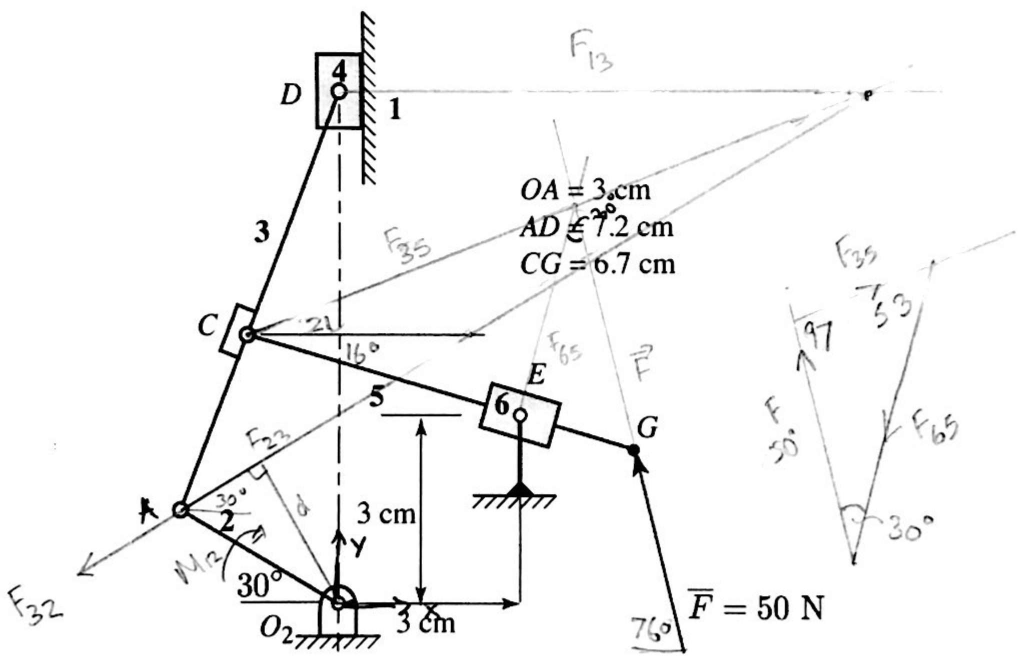
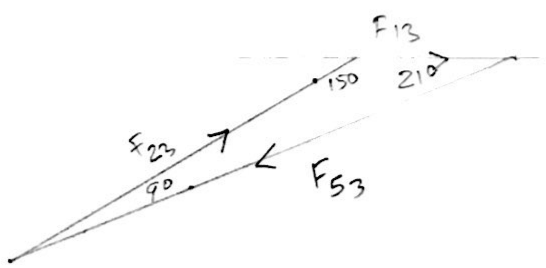



Figure Q1




$$\frac{F_{35}}{\sin 30^\circ} = \frac{50}{\sin 53^\circ}$$

$F_{35} = 31.3\,N$

$F_{53} = 31.3\,N$

$$\frac{F_{23}}{\sin(21^\circ)} = \frac{F_{53}}{\sin(150^\circ)}$$

$F_{23} = 22.43$

$M_{12} = F_{32} \times d = 22.43 \times OA \sin 60^\circ$

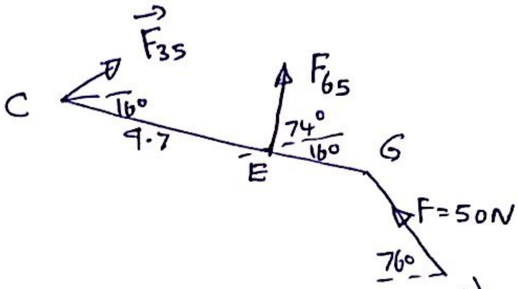
$= 22.43 \times \frac{3}{100} \sin 60^\circ\ Nm$

$= \underline{0.582\,Nm}$

Q1 $\quad OA = 26\text{mm}$ represents $30\text{mm}$

$CE = 41 \times \frac{30}{26} = 47.3\text{m} = 4.7\text{cm}$

$GE = 6.7 - 4.7 = 2\text{cm}$




$\circlearrowleft C$

$$\hat{r}_{E/C} \times \vec{F}_{65} + \hat{r}_{G/C} \times \vec{F} = 0$$

$$4.7 \times F_{65} + (6.7\cos 16°\hat{i} - 6.7\sin 16°\hat{j}) \times (-50\cos 76°\hat{i} + 50\sin 76°\hat{j}) = 0$$

$$4.7 \times F_{65}\hat{k} + 6.7 \times 50(\cos 16°\sin 76° - \sin 16°\cos 76°)\hat{k} = 0$$

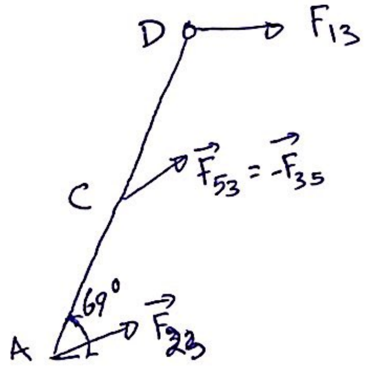
$$F_{65} = -\frac{6.7 \times 50}{4.7}\sin(60°) = -61.72$$

$$\vec{F}_{35} + \vec{F}_{65} + \vec{F} = 0$$

$$\vec{F}_{35} - 61.72(\cos 74°\hat{i} + \sin 74°\hat{j}) + 50(-\cos 76°\hat{i} + \sin 76°\hat{j}) = 0$$

$$\vec{F}_{35} + (-61.72\cos 74° - 50\cos 76°)\hat{i} + (-61.72\sin 74° + 50\sin 76°)\hat{j} = 0$$

$$\vec{F}_{35} = 29.1\hat{i} + 10.8\hat{j} \qquad |\vec{F}_{35}| = 31N$$




$AC = 26\text{mm} \times S.F = 3.0\text{cm}$

$\circlearrowleft A$

$$\hat{r}_{D/A} \times \vec{F}_{13} + \hat{r}_{C/A} \times \vec{F}_{53} = 0$$

$$(7.2\cos 69°\hat{i} + 7.2\sin 69°\hat{j}) \times F_{13}\hat{i}$$
$$+ (3.0\cos 69°\hat{i} + 3.0\sin 69°\hat{j}) \times (-29.1\hat{i} - 10.8\hat{j}) = 0$$

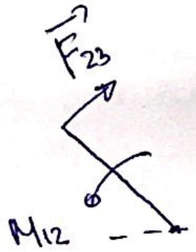
$$-7.2\sin 69° \cdot F_{13}\hat{k} + (-3.0\cos 69° \times 10.8 + 3.0\sin 69° \times 29.1)\hat{k} = 0$$

$$F_{13} = 10.4N$$

$$\vec{F}_{23} + \vec{F}_{53} + \vec{F}_{13} = 0$$

$$\vec{F}_{23} + (-29.1\hat{i} - 10.8\hat{j}) + 10.4\hat{i} = 0$$

$$\vec{F}_{23} = 18.7\hat{i} + 10.8\hat{j} = 21.6N$$




$$\hat{M}_{12} + \hat{r}_{A/O_2} \times F_{32} = 0$$

$$\hat{M}_{12}\hat{k} + (3.0\cos 30°\hat{i} + 3.0\sin 30°\hat{j}) \times (-18.7\hat{i} - 10.8\hat{j}) = 0$$

$$M_{12} + (+3.0\cos 30° \times 10.8 + 3.0\sin 30° \cdot 18.7) = 0$$

$$M_{12} = 56.1\text{ Ncm} = 0.56\text{ Nm}$$

**Problem 1[20 points]**

In the mechanism shown in Figure Q1, a moment is applied at the crank $M_{12} = 50$ N.m. The diagram is drawn to a scale. Using static force analysis determine (a) the slider force $\overline{Q}$ and (b) the ground reactions at $O_1$ and $O_4$.

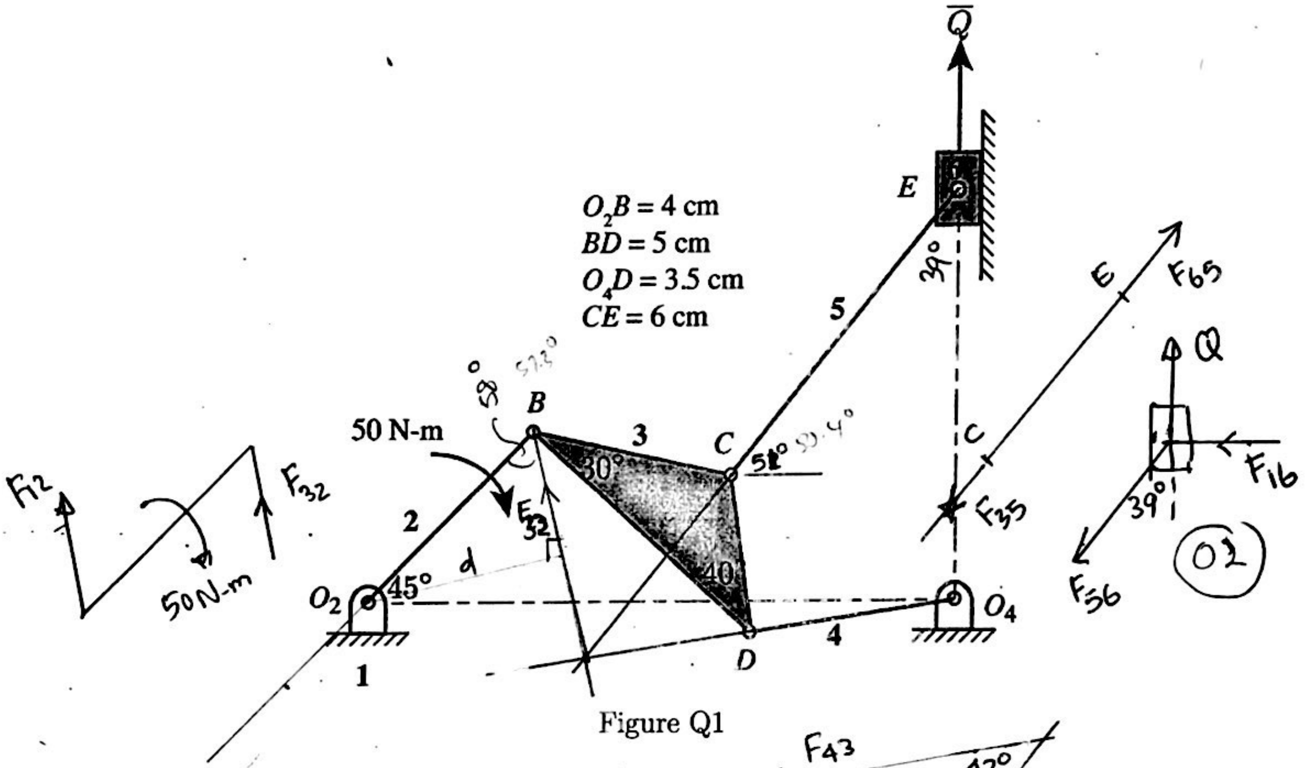



Figure Q1

$F_{32} \times d = 50\text{Nm}$

$F_{32} \times \frac{4}{100} \sin 58^\circ = 50$

$F_{32} = 1473.9\text{N}$

$F_{23} = 1473.9\text{N}$ (04)

$\frac{F_{53}}{\sin 86^\circ} = \frac{F_{43}}{\sin 52^\circ} = \frac{F_{23}}{\sin 42^\circ} = \frac{1473.9}{\sin 42^\circ}$

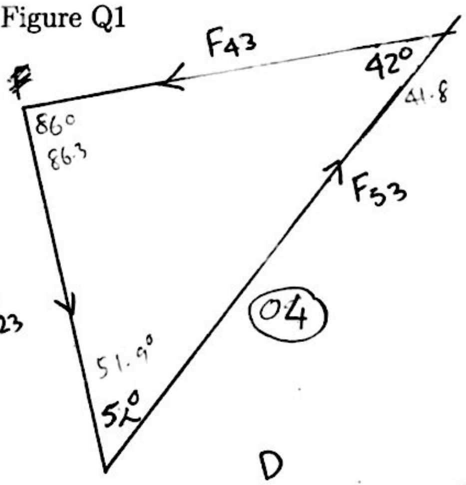
$F_{53} = 2197.3\text{N}$ (02)

$F_{43} = 1735.8\text{N}$ (02)

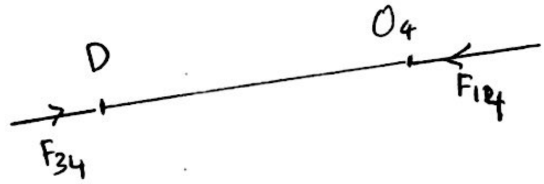
$F_{53} = F_{35}$

$F_{35} = F_{65}$

$F_{56} = F_{65} = 2197.3$




$F_{34} = F_{43}$




$Q = F_{56} \cos 39^\circ$

$= 2197.3 \cos 39^\circ$

$= 1707.6\text{N}$ (02)

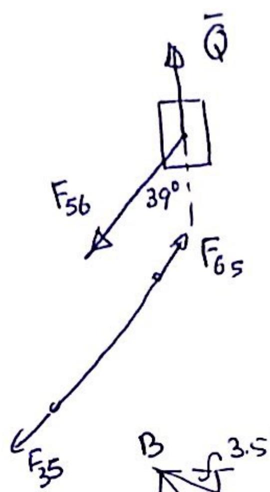
Ground Reaction (02)

at $O_2 = F_{12} = F_{32} = 1473.9\text{N}$

at $O_4 = F_{14} = F_{34} = 1735.8\text{N}$

(02)

$\underline{j}\ \ \to i$




$F_{56}\cos 39^\circ = Q$

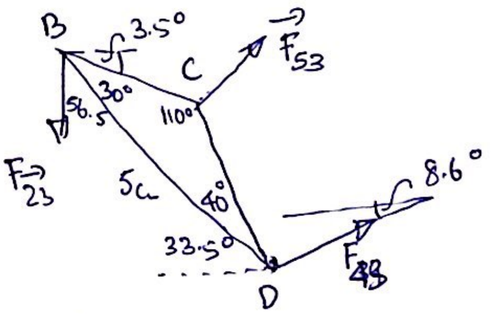
$F_{56} = 1.2868Q$

$\vec{F}_{56} = F_{56}\left(-\sin 39^\circ\hat{i} - \cos 39^\circ\hat{j}\right)$

$\vec{F}_{56} = -0.81Q\hat{i} - Q\hat{j}$

$\vec{F}_{35} = \vec{F}_{56}$

$\vec{F}_{53} = -\vec{F}_{56} = 0.81Q\hat{i} + Q\hat{j}$




$\frac{5}{\sin(110^\circ)} = \frac{BC}{\sin 40^\circ}$

$BC = 3.42\ cm$

B)

$$\hat{r}_{D/B} \times \vec{F}_{34} + \hat{r}_{C/B} \times \vec{F}_{53} = 0$$

$$5\left(\cos(33.5^\circ)\hat{i} - \sin(33.5^\circ)\hat{j}\right) \times \left(\cos(8.6^\circ)\hat{i} + \sin(8.6^\circ)\hat{j}\right)F_{43}$$

$$+ 3.42\left(\cos(3.5^\circ)\hat{i} - \sin(3.5^\circ)\hat{j}\right) \times \left(0.81Q\hat{i} + Q\hat{j}\right) = 0$$

$$5\left(\cos(33.5^\circ)\sin(8.6^\circ) + \sin(33.5^\circ)\cos(8.6^\circ)\right)F_{43}$$

$$+ 3.42Q\left(\cos(3.5^\circ) + \sin(3.5^\circ) \times 0.81\right) = 0$$

$$3.3521F_{43} + 3.5827Q = 0$$

$$F_{43} = -1.0688Q$$

$$\vec{F}_{23} + \vec{F}_{53} + \vec{F}_{43} = 0$$

$$\vec{F}_{23} + \left(0.81Q\hat{i} + Q\hat{j}\right) - 1.0688Q\left(\cos(8.6)\hat{i} + \sin(8.6)\hat{j}\right) = 0$$

$$\vec{F}_{23} + \left(-0.2468Q\hat{i} + 0.8402Q\hat{j}\right) = 0$$

$$\vec{F}_{23} = 0.2468Q\hat{i} - 0.8402Q\hat{j} \ : \ \vec{F}_{32} = -\vec{F}_{23}$$

$M_{12} = -50K \qquad -50 + \hat{r}_{B/O_2} \times \vec{F}_{32} = 0$

$$-50 + \frac{4}{100}\left(\cos 45^\circ\hat{i} + \sin 45^\circ\hat{j}\right) \times \left(-0.2468Q\hat{i} + 0.8402Q\hat{j}\right)$$

$$-\frac{50 \times 25}{\cos 45^\circ} + \left(0.8402Q + 0.2468Q\right) = Q =$$

$$\underline{\underline{Q = 1626\ N}}$$

**Q2** For the mechanism shown in Figure Q2 (drawn to a scale), determine the slider force $\overline{Q}$ and ground reactions. First, solve using graphical approach and then verify the results using vector approach.

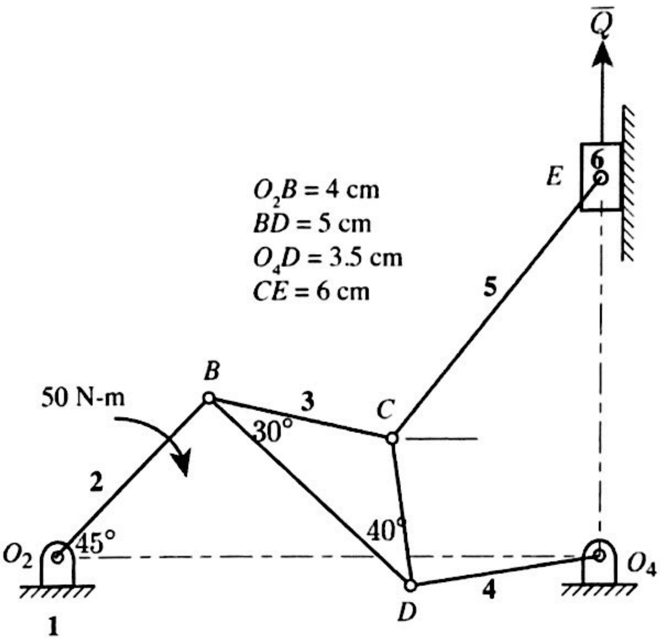



Figure Q2

**Q3** Figure Q3 shows a vise grip mechanism drawn to a scale. Using graphical approach (a) determine the applied force $\overline{P}$ necessary to produce a force $\overline{Q}$ of magnitude 600 N and (b) determine the force $\overline{Q}$ that can be produced by an applied force $\overline{P}$ of magnitude 15 N. Also, determine the reaction forces applied on link 3.

$\frac{Q}{\sin 28^\circ} = \frac{F_{23}}{\sin(36^\circ)} = \frac{F_{31}}{\sin 116^\circ}$

$F_{23} = 1.252\,Q$

$F_{31} = 1.914\,Q$

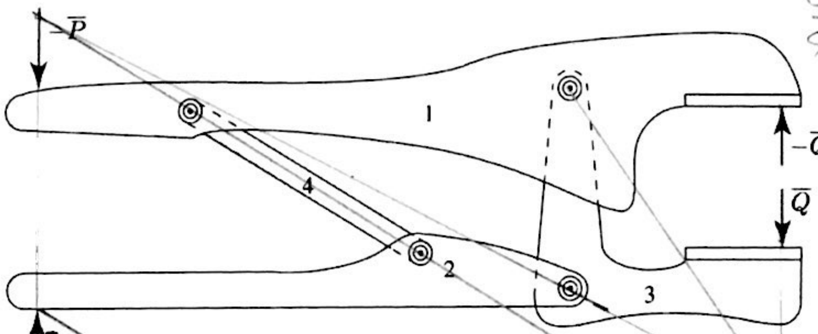
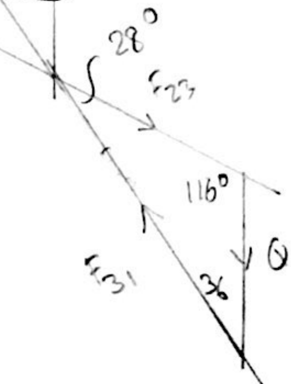



Figure Q3




(a) $Q = 600$ N

$P = 68.8$ N

$F_{23} = 751.2$ N

$F_{31} = 1148.4$ N

(b) $P = 15$ N

$Q = 131$ N

$F_{23} = 163.9$ N

$F_{31} = 250$ N

$F_{32} = 1.252Q$

4.5°

$\frac{F_{32}}{\sin 59^\circ} = \frac{P}{\sin(4.5^\circ)} = \frac{F_{42}}{\sin(116.5^\circ)}$

$P = 0.1146Q;$

$F_{42} = 1.307Q$

**Q4** Figure Q4 shows a quick return mechanism. An input torque of 50 N.m is applied at link 2. Using graphical approach determine the the resistive force $\overline{F}$ at the slider $D$.

$F_{32} \times 0.8 \cos(25^\circ) = 50$

$F_{32} = 69\ N$

$F_{24} = 69 N$

$\frac{F_{54}}{\sin 65^\circ} = \frac{69}{\sin(6^\circ)}$

$F_{54} = 598.3 N$

$F_{56} = 598.3$

$\vec{F} = F_{56} \cos 36^\circ$

$= \underline{484 N}$

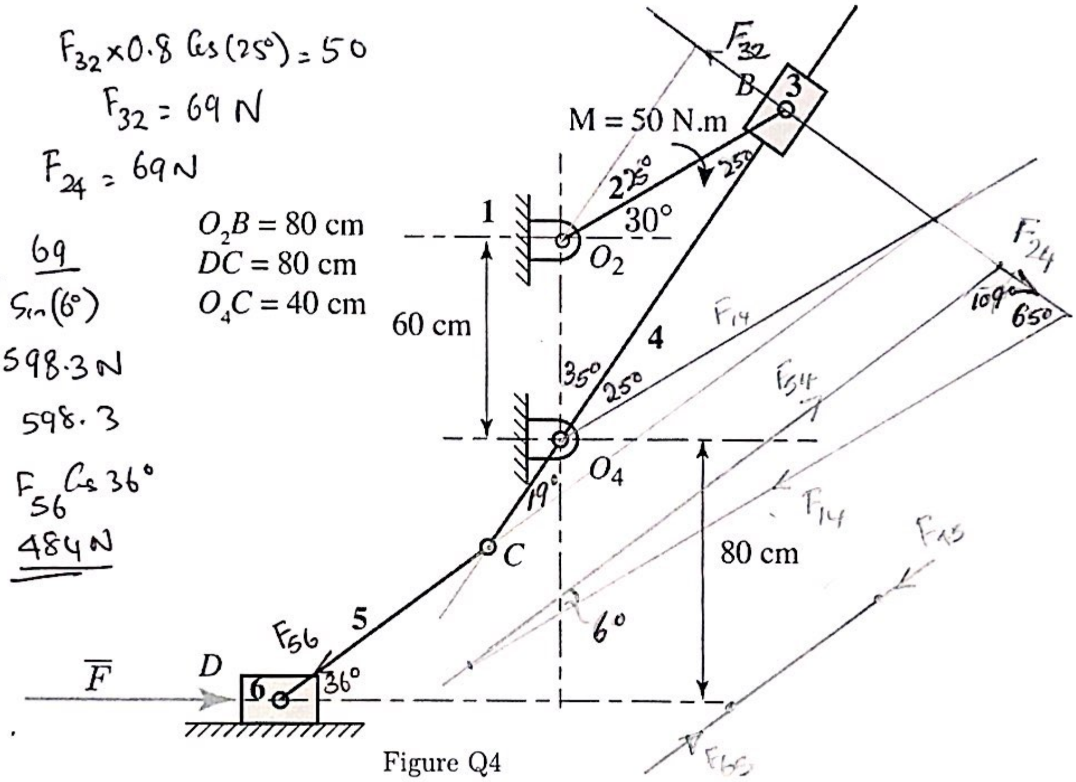



Figure Q4

**Q5** The mechanism shown in Figure Q5 has a linear hydraulic actuator joined between $B$ and $O_2$. The cylinder diameter is 2.0 cm. Determine the pressure required at the cylinder to maintain a horizontal force $\overline{F} = 100$ N.

$F_{45} \cos 30^\circ = 200$

$F_{45} = 231 N$

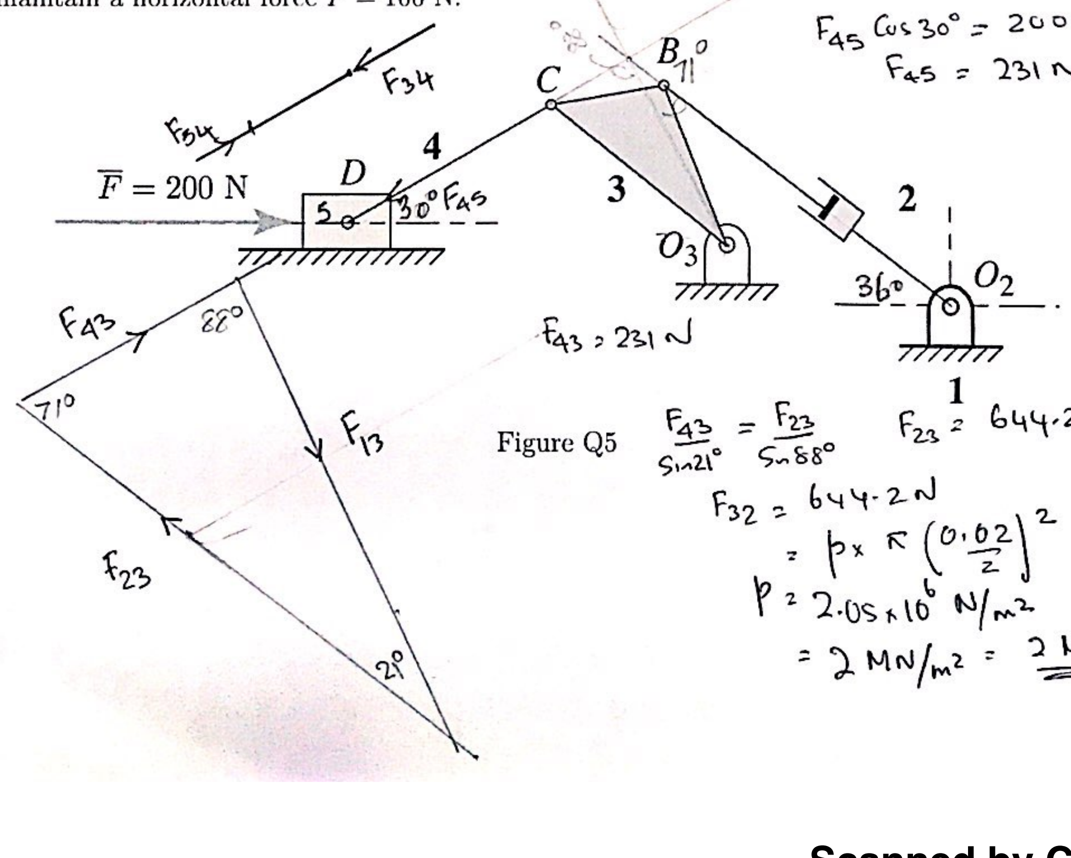



Figure Q5

$\frac{F_{43}}{\sin 21^\circ} = \frac{F_{23}}{\sin 88^\circ}$ $F_{23} = 644.2 N$

$F_{32} = 644.2 N$

$= p \times \pi \left(\frac{0.02}{2}\right)^2$

$p = 2.05 \times 10^6\ N/m^2$

$= 2\ MN/m^2 = \underline{2\ MPa}$

Dr. George Mann
Department of Mechanical Engineering
SLIIT, Sri Lanka

**Problem Set 6**

All the questions have been taken from the textbook Cleghorn.

### Q1

In the gear train of Figure Q1, shaft $A$ rotates at 200 rpm and shaft $B$ rotates at 300 rpm in the direction indicated. Determine the speed of shaft $C$ and its direction of rotation.

$$N_2 = 35; \quad N_3 = 25; \quad N_4 = 14; \quad N_5 = 46; \quad N_6 = 20; \quad N_7 = 16$$

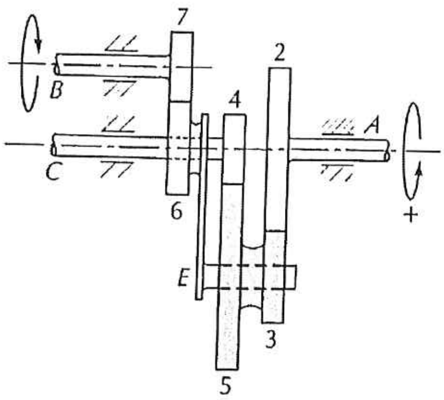



Figure Q1

### Q2

In Figure Q2, $C$ and $D$ represent brakes that can be used to stop the rotation of either arm $E$ or gear 4, one at a time. Determine the speed and direction of shaft $B$ when shaft $A$ rotating at 1000 rpm CW, while

(a) brake $C$ holds arm $E$ fixed.

(b) brake $D$ holds gear 4 fixed.

$N_2 = 90; \quad N_3 = 32; \quad N_4 = \cancel{14}; \quad N_5 = 94; \quad N_6 = 28.$

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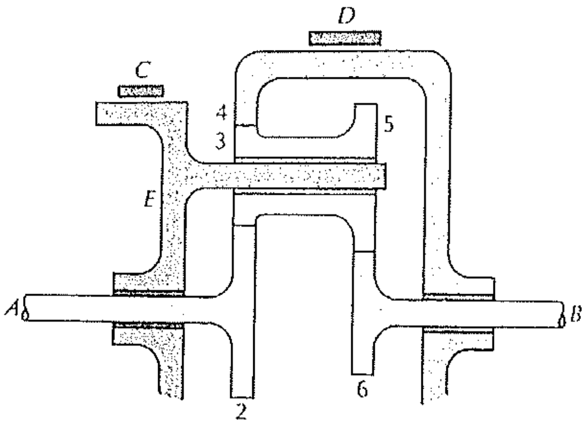



Figure Q2

**Q3**

For the gear train shown in Figure Q3, gear 2 has a module of 2.0 mm with 75 teeth; gear 5 has a module of 4.0 mm with 50 teeth; and gear 4 has 40 teeth. Determine the number of teeth on gear 3 and speed ratio of the gear train.

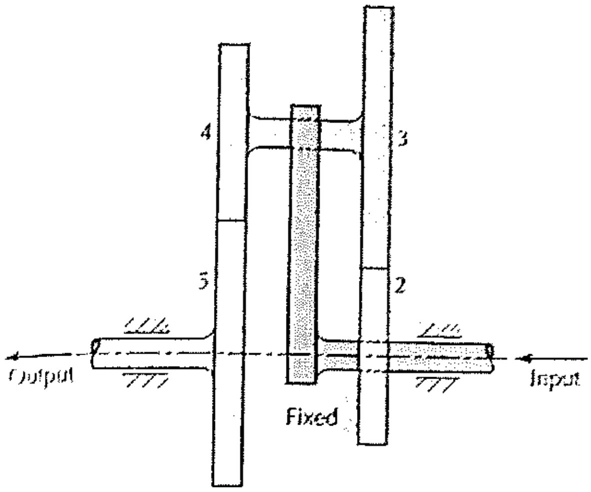



Figure Q3

**Q4**

Determine the speed ratios, $e_{2/1}$, $e_{5/1}$ and $e_{8/1}$ of the planetary gear train shown in Figure Q4. Use either tabular method or combination of standard gear trains given in the appendix. $N_1 = 15$; $N_3 = 45$, $N_4 = 105$, $N_5 = 13$, $N_6 = 10$, $N_7 = 27$, $N_8 = 87$.

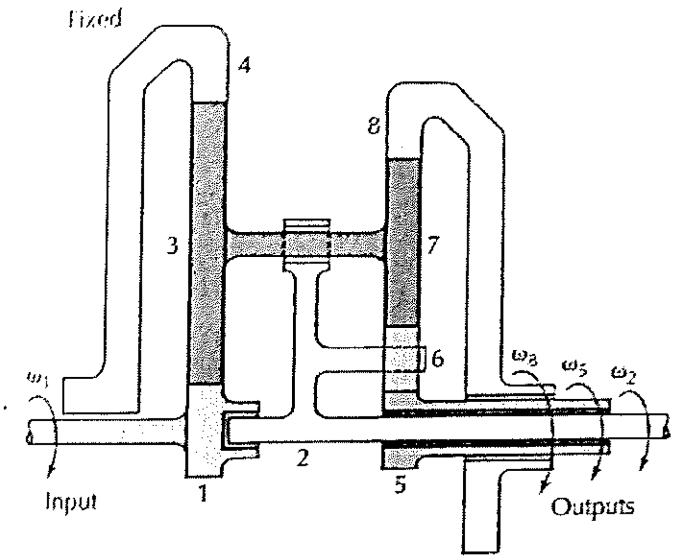



Figure Q4

## Q5

Determine the output rotational speed, $\omega_2$ for the gear train shown in Figure Q5 if $\omega_1 = 200$ rpm CW. $N_1 = 20$; $N_3 = 25$, $N_4 = 70$, $N_5 = 15$, $N_6 = 15$, $N_7 = 35$, $N_8 = 80$.

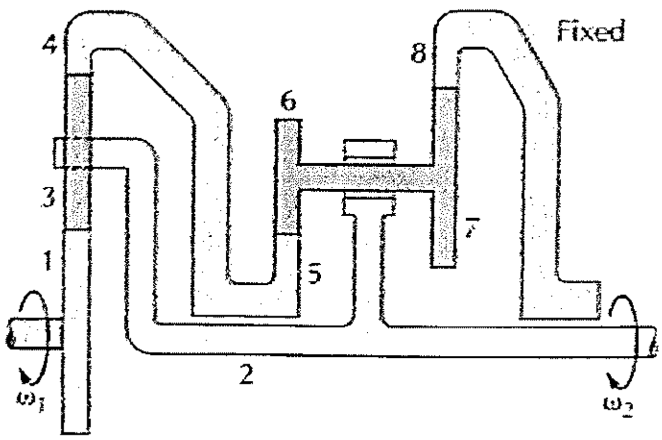



Figure Q5

## Q6

Determine the expression for the speed ratio of each of the following gear trains (refer to the basic planetary gear types given in the Appendix of Gear Notes);

(a) type D, $\omega_1 = 0$, $\omega_2 =$ input, $\omega_7 =$ output.

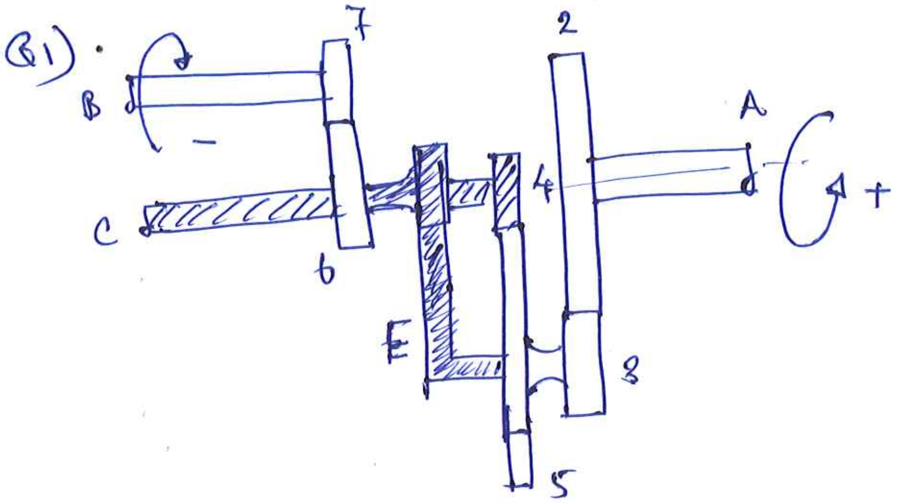
(b) type J, $\omega_7 = 0$, $\omega_1 =$ input, $\omega_2 =$ output.

(c) type C, $\omega_7 = 0$, $\omega_1 =$ input, $\omega_2 =$ output.

(d) type L, $\omega_1 = 0$, $\omega_2 =$ input, $\omega_6 =$ output.

# PROBLEM SET 6

(Q1)




$\omega_A = 200 \text{ rpm} = \omega_2$

$\omega_B = -300 \text{ rpm} = \omega_7$

$\omega_6 = -\dfrac{N_7}{N_6}\omega_7$

$\omega_3 = -\dfrac{N_2}{N_3}\omega_2$

$\omega_5 = \omega_3$

$\omega_E = \omega_6$

For the planet gear box of Gears 4, E, 5

| | Gear 4 ($\omega_4$) | Gear 5 ($\omega_5$) | ~~Crank~~ E ($\omega_E$) |
|---|---|---|---|
| All components rotates at $x$ rpm. | $x$ | $x$ | $x$ |
| Crank E fixed. Gear 4 at $y$ rpm | $y$ | $-\dfrac{N_4}{N_5}y$ | 0 |
| | $x+y$ | $x-\dfrac{N_4 y}{N_5}$ | $x$ |

from the table.

$\omega_5 = x - \dfrac{N_4}{N_5}y = -\dfrac{N_2}{N_3}\omega_2$

$\omega_E = \omega_6 = x = -\dfrac{N_7}{N_6}\omega_7$

$\omega_6 = x = -\dfrac{16}{20}(-300)$

$x = 240 \text{ rpm}$

$\omega_5 = 240 - \dfrac{14}{46}y = -\dfrac{35}{25}\times(200)$

$y = 1708.57$

$\therefore \omega_4 = x+y$

$= \underline{\underline{1948.57 \text{ rpm}}}$

Q2) $\omega_A = 1000\ rpm = \omega_2$

| | Gear 2 ($\omega_2$) | Crank E | Gear 3 ($\omega_3$) | Gear 4 ($\omega_4$) |
|---|---|---|---|---|
| All components rotate at $x$ rpm. | $x$ | $x$ | $x$ | $x$ |
| Crank E fixed. Gear 2 at $y$ rpm | $y$ | $0$ | $-\frac{N_2}{N_3}y$ | $-\frac{N_2}{N_4}y$ |
| Absolute Speed | $x+y$ | $x$ | $x-\frac{N_2 y}{N_3}$ | $x-\frac{N_2 y}{N_4}$ |

$\omega_3 = \omega_5$

$\omega_A = 1000\ rpm = \omega_2$

$\omega_B = \omega_6 = -\frac{N_5}{N_6}\omega_5$

a) Brake C holds. Arm E fixed

$\therefore \omega_E = x = 0$

$\omega_2 = x + y = 1000$

$y = 1000$

$\therefore \omega_3 = -\frac{N_2}{N_3}y$

$\omega_B = \omega_6 = -\frac{N_5}{N_6} \times \frac{-N_2}{N_3}y$

$\omega_B = \frac{-94}{28} \times \frac{-90}{32}(1000)$

$\omega_B = 9441.96\ rpm$

b) Brake D holds. Gear 4 fixed

$\therefore \omega_4 = x - \frac{N_2}{N_4}y = 0$

$x = \frac{N_2 y}{N_4}$

$\omega_2 = x + y = 1000$

$y = \frac{1000 N_4}{(N_2 + N_4)}$

$x = \frac{1000 N_2}{(N_2+N_4)}$

$\omega_3 = x - \frac{N_2}{N_3}y$

$= \left(\frac{N_2}{N_4} - \frac{N_2}{N_3}\right)y$

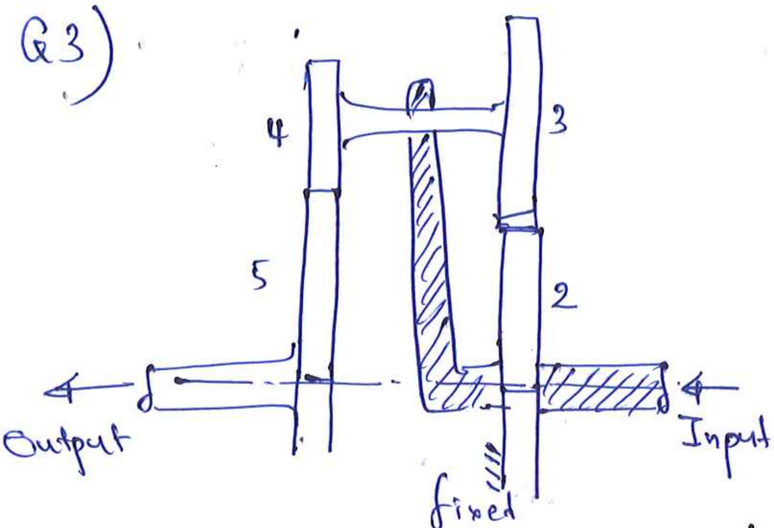
$\omega_B = \omega_6 = -\frac{N_5}{N_6}\left(\frac{N_2}{N_4} - \frac{N_2}{N_3}\right)\frac{1000 N_4}{(N_2+N_4)}$

$= -\frac{N_2 \cancel{N_4} N_5 (N_3 - N_4)1000}{(N_2+N_4)N_3 \cancel{N_4} N_6}$

$= -\frac{N_2(N_3-N_4)N_5\, 1000}{(N_2+N_4)N_3 N_6}$

$= 4720.98\ rpm.$

Q3)




$$r_2 + r_3 = r_4 + r_5$$

$$d_2 + d_3 = d_4 + d_5$$

$$m_2 N_2 + m_2 N_3 = m_4 N_4 + m_4 N_5$$

$$2(75 + N_3) = 4(40 + 50)$$

$$N_3 = 105$$

| | Gear 2 | Gear 3 | Crank |
|---|---|---|---|
| All components Rotate at $x$ rpm | $x$ | $x$ | $x$ |
| Gear 2 rotates at $y$ rpm. Crank fixed | $y$ | $-\frac{N_2}{N_3}y$ | 0 |
| Absolute speed | $x+y$ | $x - \frac{N_2}{N_3}y$ | $x$ |

~~Crank is f~~ Gear 2 is fixed $\therefore$ $x + y = 0$

$$x = -y$$

$$\therefore \omega_3 = x\left(1 + \frac{N_2}{N_3}\right)$$

$$\omega_4 = \omega_3$$

$$\omega_5 = -\frac{N_4}{N_5}\omega_4$$

$$\omega_5 = -\frac{N_4}{N_5} \times \left(1 + \frac{N_2}{N_3}\right)x$$

$$e_{out/in} = \frac{\omega_5}{x} = -\frac{40}{50}\left(1 + \frac{75}{105}\right) = -1.37$$

(Q4) $\boxed{\text{STAGE I}}$

| | Gear 1 | Crank 2 | Gear 3 | Gear 4 |
|---|---|---|---|---|
| All components rotate at $x_1$ rpm | $x_1$ | $x_1$ | $x_1$ | $x_1$ |
| Gear 1 rotates at $y_1$ rpm. Crank 2 fixed | $y_1$ | $0$ | $-\frac{N_1}{N_3}y_1$ | $-\frac{N_1}{N_4}y_1$ |
| | $x_1+y_1$ | $x_1$ | $x_1-\frac{N_1}{N_3}y_1$ | $x_1-\frac{N_1}{N_4}y_1$ |

$\boxed{\text{STAGE 2}}$

| | Gear 5 | Gear 6 | Gear 7 | Gear 8 | Crank 2 |
|---|---|---|---|---|---|
| All components rotate at $x_2$ rpm | $x_2$ | $x_2$ | $x_2$ | $x_2$ | $x_2$ |
| Crank fixed. Gear 5 rotates at $y_2$ rpm. | $y_2$ | $-\frac{N_5}{N_6}y_2$ | $\frac{N_5}{N_7}y_2$ | $\frac{N_5}{N_8}y_2$ | $0$ |
| | $x_2+y_2$ | $x_2-\frac{N_5}{N_6}y_2$ | $x_2+\frac{N_5}{N_7}y_2$ | $x_2+\frac{N_5}{N_8}y_2$ | $x_2$ |

Since crank is same $x_1 = x_2$

$\omega_3 = \omega_7$ (common shaft) $\therefore \quad x_1 - \frac{N_1}{N_3}y_1 = x_2 + \frac{N_5}{N_7}y_2$ (1)

Gear 4 is fixed $\therefore \omega_4 = 0$

$$x_2 = x_1 = \frac{N_1}{N_4}y_1 = \frac{15y_1}{105} = \frac{y_1}{7}$$

(1) $\Rightarrow$ $$y_2 = -\frac{N_1N_7}{N_3N_5}y_1 = \frac{-9y_1}{13}$$

$$e_{2/1} = \frac{\omega_2}{\omega_1} = \frac{x_1}{x_1+y_1} = \frac{x_1}{x_1+7x_1} = \underline{\underline{0.125}}$$

$$e_{5/1} = \frac{x_2+y_2}{x_1+y_1} = \frac{\frac{y_1}{7}+\left(-\frac{9y_1}{13}\right)}{\frac{y_1}{7}+y_1} = \underline{\underline{-0.48}}$$

$$e_{8/1} = \frac{x_2+\frac{N_5}{N_8}y_2}{x_1+y_1} = \frac{\frac{y_1}{7}+\frac{N_5}{N_8}\times\left(-\frac{9y_1}{13}\right)}{y_1/7+y_1} = \frac{1}{29} = \underline{\underline{0.035}}$$

Q5)

STAGE I

| | Gear 1 | Crank 2 | Gear 3 | Gear 4 |
|---|---|---|---|---|
| All components rotates at $x_1$ rpm. | $x_1$ | $x_1$ | $x_1$ | $x_1$ |
| Crank 2 fixed; Gear 1 at $y_1$ rpm | $y_1$ | 0 | $-\frac{N_1 y_1}{N_3}$ | $-\frac{N_1}{N_4} y_1$ |
| Absolute Speeds | $x_1 + y_1$ | $x_1$ | $x_1 - \frac{N_1 y_1}{N_3}$ | $x_1 - \frac{N_1}{N_4} y_1$ |

STAGE II

| | Gear 5 | Gear 6 | Crank 2 | Gear 8 |
|---|---|---|---|---|
| All components rotates at $x_2$ rpm | $x_2$ | $x_2$ | $x_2$ | $x_2$ |
| Crank 2 fixed. Gear 5 at $y_2$ rpm. | $y_2$ | $-\frac{N_5}{N_6} y_2$ | 0 | $-\frac{N_7}{N_8} \times \frac{N_5}{N_6} y_2$ |
| | $x_2 + y_2$ | $x_2 - \frac{N_5 y_2}{N_6}$ | $x_2$ | $x_2 - \frac{N_5 N_7 y_2}{N_6 N_8}$ |

Crank is same $\therefore x_1 = x_2$

$\omega_8 = 0 \quad \therefore x_2 = \frac{N_5 N_7 y_2}{N_6 N_8} = 0.4375 y_2$ (Gear 8 fixed)

Gear 4 & 5 are same gear. $\therefore \omega_4 = \omega_5$

$$x_1 - \frac{N_1}{N_4} y_1 = x_2 + y_2$$

$$y_1 = -\frac{7}{2} y_2$$

$$e_{2/1} = \frac{\omega_2}{\omega_1} = \frac{x_1}{x_1 + y_1} = \frac{0.4375 y_2}{0.4375 y_2 + (-3.5 y_2)} = -\frac{1}{7}$$

$\therefore \omega_2 = -\frac{200}{7} = \cancel{-28.57 \text{ rad/}}$

$= -28.75$ rpm

Q5) $\boxed{\text{STAGE I}}$

| | Gear 1 | Crank 2 | Gear 3 | Gear 4 |
|---|---|---|---|---|
| All components rotates at $x_1$ rpm. | $x_1$ | $x_1$ | $x_1$ | $x_1$ |
| Crank 2 fixed; Gear 1 at $y_1$ rpm | $y_1$ | 0 | $-\frac{N_1 y_1}{N_3}$ | $-\frac{N_1}{N_4} y_1$ |
| Absolute speeds | $x_1+y_1$ | $x_1$ | $x_1-\frac{N_1 y_1}{N_3}$ | $x_1-\frac{N_1}{N_4} y_1$ |

$\boxed{\text{STAGE II}}$

| | Gear 5 | Gear 6 | Crank 2 | Gear 8 |
|---|---|---|---|---|
| All components rotates at $x_2$ rpm | $x_2$ | $x_2$ | $x_2$ | $x_2$ |
| Crank 2 fixed. Gear 5 at $y_2$ rpm. | $y_2$ | $-\frac{N_5}{N_6} y_2$ | 0 | $-\frac{N_7}{N_8} \times \frac{N_5}{N_6} y_2$ |
| | $x_2+y_2$ | $x_2-\frac{N_5 y_2}{N_6}$ | $x_2$ | $x_2-\frac{N_5 N_7 y_2}{N_6 N_8}$ |

Crank is same $\therefore x_1 = x_2$

$\omega_8 = 0 \quad \therefore x_2 = \frac{N_5 N_7}{N_6 N_8} y_2 = 0.4375 y_2$ (Gear 8 fixed)

Gear 4 & 5 are same gear. $\therefore \omega_4 = \omega_5$

$$x_1 - \frac{N_1}{N_4} y_1 = x_2 + y_2$$

$$y_1 = -\frac{7}{2} y_2$$

$$e_{2/1} = \frac{\omega_2}{\omega_1} = \frac{x_1}{x_1+y_1} = \frac{0.4375 y_2}{0.4375 y_2 + (-3.5 y_2)} = -\frac{1}{7}$$

$\therefore \omega_2 = -\frac{200}{7} =$ ~~$-28.57$ rad/~~

$= -28.75$ rpm

Q6)

a) Type D

All components rotate at $x$ rpm

Crank 2 fixed.

Gear 1 at $y$ rpm

| | Gear 1 | Gear 3 | Crank | Gear 4 | Gear 5 | Gear 6 | 7 |
|---|---|---|---|---|---|---|---|
| All components rotate at $x$ rpm | $x$ | $x$ | $x$ | $x$ | $x$ | $x$ | $x$ |
| Crank 2 fixed. Gear 1 at $y$ rpm | $y$ | $+\frac{N_1}{N_3}y$ | 0 | $+\frac{N_1}{N_3}y$ | $-\frac{N_4}{N_5}\frac{N_1 y}{N_3}$ | $-\frac{N_1N_4 y}{N_3N_5}$ | |
| | $x+y$ | $x+\frac{N_1 y}{N_3}$ | $x$ | $x+\frac{N_1 y}{N_3}$ | $x-\frac{N_1N_4 y}{N_3N_5}$ | $x-\frac{N_1N_4 y}{N_3N_5}$ | |

$\omega_1 = 0 \therefore x = (-y)$   Gear 7 $= x - \frac{N_1N_4N_6}{N_3N_5N_7}y$

$$e_{7/2} = \frac{\omega_7}{\omega_2} = \frac{1+\frac{N_1N_4N_6}{N_3N_5N_7}}{1}$$

$$e_{7/2} = \left(1+\frac{N_1N_4N_6}{N_3N_5N_7}\right)$$

b) $\omega_7 = 0 \therefore$   $x = -\frac{N_1N_4N_6}{N_3N_5N_7}y$

$$e_{2/1} = \frac{x}{x+y} = \frac{1}{1+y/x} = \left(\frac{N_1N_4N_6}{N_1N_4N_6 - N_3N_5N_7}\right)$$

c) $\omega_7 = 0 \therefore$   $x = \frac{N_1N_4N_6}{N_3N_5N_7}y$

$$e_{2/1} = \frac{x}{x+y} = \left(\frac{N_1N_4N_6}{N_1N_4N_6 + N_3N_5N_7}\right)$$

d) $\omega_1 = 0 \therefore x = -y$

$$e_{6/2} = \frac{x+\frac{N_1N_5}{N_4N_6}y}{x} = \left(1 - \frac{N_1N_5}{N_4N_6}\right)$$