BTC 1102, ETC 1102, ITC 1102 MATHEMATICS II

FIRST YEAR SECOND SEMESTER 2022 MAY

ASSIGNMENT 02

Assignment posted date: 25th May, 2022.

Submit this assignment to LMS on or before 8th June, 2022.

1. a) Find the derivative of the following functions.

   i) $y = \left(\sqrt{x} - \frac{3}{x}\right)(2x^3 - 1)$

   ii) $f(w) = w^2 - w^{-\frac{3}{2}}$

   iii) $g(x) = ax^2 + bx + c$, where a, b, c are constants.

   iv) $y = \sin(3x^2) - 4\cos\left(\frac{x}{2}\right)$

   b) The position of an object at any time t is given by $s(t) = 1 - 150t^3 + 45t^2 - 2t^5$.

   i) Determine the velocity of the object at any time t.

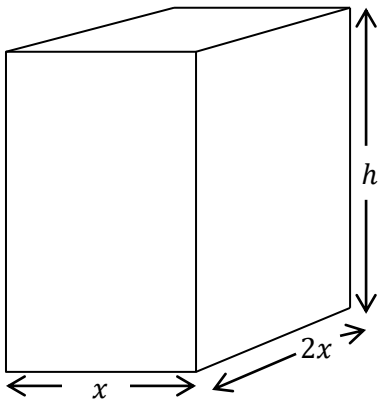
   ii) Does the position of the object ever stop changing?

2. a) For each of the following curves, find an equation of the tangent to the curve at the point whose x-coordinate is given.

   i) $y = x^2 - \frac{3}{x} - \frac{1}{2}$, where $x = -2$

   ii) $y = 4x^2 + \frac{5}{x} - 1$, where $x = 1$

   b)



The above figure shows the design of a milk carton with a capacity of 1000 cm^3. The design of the carton is that of a closed cuboid whose base measure is x cm by $2x$ cm and the height is h cm.

i) Show that the surface area of the carton is given by,

$$A = 4x^2 + \frac{3000}{x}$$

ii) Calculate the minimum value for A. Justify your answer.

3. Evaluate the following integrals.

i) $\int (2\cos x - \sin 3x \cos 3x)\, dx$

ii) $\int (t^3 - \frac{e^{-t}-4}{e^{-t}})\, dt$

iii) $\int (\sqrt[3]{w} + 10\sqrt[5]{w^3})\, dw$

iv) $\int_2^1 \left(\frac{2y^3-6y^2}{y^2}\right) dy$

v) $\int_{\pi/6}^{\pi/3} (2\sec^2 x - 8\sec^2 5x)\, dx$

vi) $\int_{-5}^{-2} \left(7e^x + \frac{2}{x}\right) dx$

4. a) Find the equation of the curve that passes through the point $(4,5)$ if its slope is given by, $\frac{dy}{dx} = \left(\frac{1}{x^{\frac{3}{2}}} + 1\right)$.

b) Find the area of the segment cut off from the curve $y = \sin x$, $0 \le x \le \pi$ by the line $y = \frac{1}{\sqrt{2}}$ as shown by the following figure.