Index no - BST 20110
Name - H.M Senavirathna.
Assingnment NO -

① a. $y = (\sqrt{x} - \frac{3}{x})(2x^3 - 1)$

$\frac{dy}{dx} = \left(\sqrt{x} - \frac{3}{x}\right) 6x^{2} + (x^{\frac{1}{2}} - 3x^{-1})(2x^3 - 1)$

$\frac{dy}{dx} = \left(\sqrt{x} - \frac{3}{x}\right) 6x^2 + \frac{1}{2} x^{-\frac{1}{2}} + 3x^{-2} \times 2x^3 - 1$

$\frac{dy}{dx} = \left(x - \frac{3}{x}\right) 6x^2 + \left(\frac{1}{2\sqrt{x}} + \frac{3}{x^2} \times 2x^3 - 1\right)$

II). $f(w) = w^2 - w^{-3/2}$

$f'(w) = 2w - (-\frac{3}{2}) w^{-\frac{3}{2} - 1}$

$f'(w) = 2w + \frac{3}{2} w^{-5/2}$

$= 2w + \frac{3}{2w^{5/2}}$

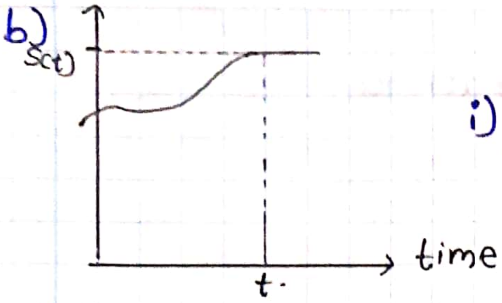
III) $g(x) = ax^2 + bx + c$
$g'(x) = 2ax + b$ (where a, b are constants)

iv) $y = \sin(3x^2) - 4 \cos\left(\frac{x}{2}\right)$

$\frac{dy}{dx} = [\cos 3x^2 \times 6x) - \{\cancel{4}^{2} (- \sin\left(\frac{x}{2}\right) \times \frac{1}{\cancel{2}})$

$6x \cos(3x^2) + 2 \sin\left(\frac{x}{2}\right)$

b)



$$s(t) = 1 - 150t^3 + 45t^2 - 2t^5$$

i) $$\frac{ds(t)}{dt} = 0 - 450t^2 + 90t - 10t^4$$

$$= \underline{\underline{-450t^2 + 90t - 10t^4}}$$

II) At the stationary point, the object should be stop.

At the stationary point

$$\frac{ds(t)}{dt} = 0$$

$$-450t^2 + 90t - 10t^4 = 0$$

$$t(-450t + 90 - 10t^3) = 0$$

$$t = 0 \quad \text{or} \quad -450t + 90 - 10t^3 = 0$$

$$0 = \cancel{450}t - \cancel{90} + \cancel{10}t^3$$

$$0 = 45t - 9 + t^3$$

$$t^3 + 45t - 9 = 0$$

$$t = 0.19982$$

$$t = 0 \quad \text{or} \quad t = 0.2$$

$\therefore$ The position of the object is stop at $\underline{\underline{t = 0.2\,s}}$

2) i) a. $$y = x^2 - \frac{3}{x} - \frac{1}{2}$$

$$y = x^2 - 3x^{-1} - \frac{1}{2}$$

$$\frac{dy}{dx} = 2x + 3x^{-2} - 0$$

$$= 2x + \frac{3}{x^2}$$

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$\frac{dy}{dx}\Big|_{x=-2}$

$2x + \frac{3}{x^2}$

$2(-2) + \frac{3}{(-2)^2}$

$m = -4 + \frac{3}{4}$

$m = \frac{-16+3}{4}$

$m = \frac{-13}{4}$

$y = x^2 - \frac{3}{x} - \frac{1}{2}$

when $x = -2$

$y = (-2)^2 - \frac{3}{-2} - \frac{1}{2}$

$y = 4 + \left(\frac{3-1}{2}\right)$

$4 + \left(\frac{2}{2}\right)$

$y = 5$

using $y = mx + c$

$5 = \frac{-13}{4} \times -2 + c$

$c = 5 - \frac{13}{2}$

$c = -\frac{3}{2}$

Then tangent equation of is $y = \frac{-13x}{4} - \frac{3}{2}$

II) $y = 4x^2 + \frac{5}{x} - 1$

$\frac{dy}{dx} = 8x - 5x^{-2} - 0$

$= 8x - \frac{5}{x^2}$

$\frac{dy}{dx}\Big|_{x=1}$ $8 \times 1 - \frac{5}{1^2}$

$m = 3$

when $x = 1$

$y = 4 \times 1^2 + \frac{5}{1} - 1$

$y = 4 + 5 - 1$

$y = 8$

using $y = mx + c$

$8 = 3 \times 1 + c$

$c = 5$

The equation of tangent = $y = 3x + 5$

b i) Surface area,

$A = (x \times 2x \times 2) + (2x \times h \times 2) + (x \times h \times 2)$

$= 4x^2 + 4xh + 2xh$

$= 4x^2 + \underbrace{2x^2 h} \times \frac{2+1}{x}$

$= 4x^2 + 1000 \times \frac{3}{x}$

$A = 4x^2 + \frac{3000}{x}$

volume

$x \times 2x \times h$

$2x^2 h$

1000

II) $A = 4x^2 + 3000x^{-1}$

$\frac{dA}{dx} = 8x - 3000x^{-2}$

$= 8x - \frac{3000}{x^2}$

At stationary point,

$\frac{dA}{dx} = 0$

$8x - \frac{3000}{x^2} = 0$

$8x - \frac{3000}{x^2} = 0$

$\frac{8x^3 - 3000}{x^2} = 0$

$8x^3 - 3000 = 0$

$8x^3 = 3000$

$x = \sqrt[3]{\frac{3000}{8}}$

$x = 7.21 cm$

$A = (4 \times 7.21^2) - \left(\frac{3000}{7.21}\right)$

$A = 624.025 cm^2$

3 i) $\int (2 \cos x - \sin 3x \cos 3x) dx$

$= \int 2 \cos dx - \int \sin 3x \cdot \cos 3x \, dx$

$= 2 \cos x \, dx - \int \frac{\sin 6x}{2} dx$

$2 \sin x - \frac{1}{2} \int \sin (6x) dx$

$2 \sin x - \frac{1}{2} \left[-\cos (6x) \times \frac{1}{6}\right] + c$

$2 \sin x + \frac{1}{12} \cos (6x) + c$ c is an aribitary constant.

$\sin(6x) = 2\sin(3x) \cdot \cos 3x$

$\frac{\sin (6x)}{2} = \sin 3x \cdot \cos 3x$

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II) $\int (t^3 - \frac{e^{-t} - 4}{e^{-t}}) \, dt$

$= t^3 dt - \int \frac{e^{-t} - 4}{e^{-t}} dt$

$= \frac{t^{3+1}}{3+1} - \int e^t (e^{-t} - 4) dt$

$= \frac{t^4}{4} - \int e^t . e^{-t} - 4) dt$

$\frac{t^4}{4} - \int e^t . e^{-t} - 4e^t dt$

$\frac{t^4}{4} - \int 1 dt - 4 \int e^t dt$

$= \frac{t^4}{4} - t - 4e^t + c$; c is an arbitary constant.

III. $\int (3\sqrt{w} + 10 \sqrt[5]{w^3}) dw$

$= \int (w^{\frac{1}{3}} + 10 \; w^{\frac{3}{5}}) dw$

$= \int w^{1/3} dw + \int 10 w^{\frac{3}{5}} dw$

$= \frac{w^{\frac{1}{3}+1}}{\frac{1}{3}+1} + 10 \int w^{\frac{3}{5}} dw$

$\frac{w^{\frac{4}{3}}}{\frac{4}{3}} + 10 \frac{w^{\frac{3}{5}+1}}{\frac{3}{5}+1} + C$

$\frac{3}{4} \, 3\sqrt{w^4} + 10 \times \frac{w^{\frac{8}{5}}}{\frac{8}{5}} + C$

$\frac{3}{4} \, 3\sqrt{w^4} + 10 \times \frac{5}{8} \times \sqrt[5]{w^8} + C$

$\frac{3}{4} \sqrt[3]{w^4} + \frac{25}{4} \sqrt[5]{w^8} + C$

c is an arbitary constant.

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iv) $\int_2^1 \left(\frac{2y^3-6y^2}{y^2}\right)dy$

$= \int_2^1 \left(\frac{2y^3}{y^2} - \frac{6y^2}{y^2}\right)dy$

$= \int_2^1 (2y-6)\,dy$

$= \left[\frac{2y^2}{2} - 6y\right]_2^1$

$= \left[\frac{2\times 1^2}{2} - 6\times 1\right] - \left[\frac{2\times 2^2}{2} - 6\times 2\right]$

$(1-6)-(4-12)$

$-5 + 8$

$\underline{\underline{3}}$

v) $\int_{\pi/6}^{\pi/3} (2\sec^2 x - 8\sec^2 5x)\,dx$

$\int_{\pi/6}^{\pi/3} [2\sec^2 x - 8\sec^2(5x)]\,dx$

$\left[2\tan x - 8\times \tan(5x)\frac{1}{5}\right]_{\pi/6}^{\pi/3}$

$\left[2\tan x - \frac{8}{5}\tan\left(\frac{5\pi}{3}\right)\right] - \left[2\tan\left(\frac{\pi}{6}\right) - \frac{8}{5}\tan\left(\frac{5\pi}{6}\right)\right]$

$\left(2\sqrt{3} + \frac{8\sqrt{3}}{5}\right) - \frac{2}{\sqrt{3}} + \frac{8}{5\sqrt{3}}$

$\frac{18\sqrt{3}}{5} - \frac{6\sqrt{3}}{5}$

$\frac{12\sqrt{3}}{5} \approx \underline{\underline{4.157}}$

4) a) $\frac{dy}{dx} = \frac{1}{x^{3/2}} + 1$

integrate,

$\int\left(\frac{dy}{dx}\right) dx = \int\left(\frac{1}{x^{3/2}} + 1\right) dx$

$y = \int x^{-3/2}\, dx + \int 1\, dx$

$y = \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + x + C$

$y = \frac{x^{-1/2}}{-\frac{1}{2}} + x + C$

$y = \frac{-2}{\sqrt{x}} + x + C$

(4, 5) point in the curve,

$5 = \frac{-2}{\sqrt{4}} + 4 + C$

$5 = \frac{-2}{2} + 4 + C$

$C = 2$

The equation of curve,

$y = \frac{-2}{\sqrt{x}} + x + 2$

b) Shaded area,

$= \int_{\pi/4}^{3\pi/4} \left(\sin x - \frac{1}{2}x\right)$

$= \left[-\cos x - \frac{1}{\sqrt{2}} \times \frac{x^2}{2}\right]_{\pi/4}^{3\pi/4}$

$= \left[-\cos x - \frac{x^2}{2\sqrt{2}}\right]_{\pi/4}^{3\pi/4}$

$= \left[\cos\left(\frac{3\pi}{4}\right) - \frac{\left(\frac{3\pi}{4}\right)^2}{2\sqrt{2}}\right] - \left[-\cos\left(\frac{\pi}{4}\right) - \frac{\left(\frac{\pi}{4}\right)^2}{2\sqrt{2}}\right]$

$= -1.256 - (-0.925)$

$= -1.256 + 0.925$

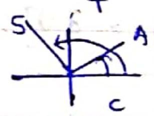
$= -0.331$

Area $= 0.331$ squre unit

$\sin x = \frac{1}{\sqrt{2}}$

$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$x = \frac{\pi}{4}$



$x = \frac{3\pi}{4}$