① a) i. $y = \left(\sqrt{x} - \frac{3}{x}\right)(2x^3 - 1)$

$y = \left(x^{\frac{1}{2}} - 3x^{-1}\right)(2x^3 - 1)$

$\left[6x^2 \times x^{\frac{1}{2}} - 6x^2 \times 3x^{-1}\right] + \left[2x^3 \times \frac{1}{2} x^{-\frac{1}{2}} + 2x^3 \times 3x^{-2}\right]$

$\frac{1}{2}x^{-\frac{1}{2}} + 3x^{-2}$

$6x^{5/2} - 18x + x^{5/2} + 6x + \frac{1}{2}x^{-\frac{1}{2}} + 3x^{-2}$

$\frac{dy}{dx} = \left(x^{\frac{1}{2}} - 3x^{-1}\right)6x^2 + (2x^3 - 1)\left(\frac{1}{2}x^{-\frac{1}{2}} + 3x^{-2}\right)$

~~$y = (\sqrt{x} - \frac{3}{x}$~~ $\frac{dy}{dx} = \left[6x^2\left(\sqrt{x} - \frac{3}{x}\right)\right] + \left[(2x^3 - 1)\left(\frac{1}{2\sqrt{x}} + \frac{3}{x^2}\right)\right]$

$= 7x^{5/2} - 12x + \frac{1}{2\sqrt{x}} + \frac{3}{x^2}$ //

ii. $f(w) = w^2 - w^{-\frac{3}{2}}$

$f'(w) = 2w + \frac{3}{2} \times w^{-\frac{3}{2} - 1}$

$f'(w) = 2w + \frac{3}{2} w^{-\frac{5}{2}}$

$f'(w) = 2w + \frac{3}{2w^{5/2}}$ //

iii. $g(x) = ax^2 + bx + c$

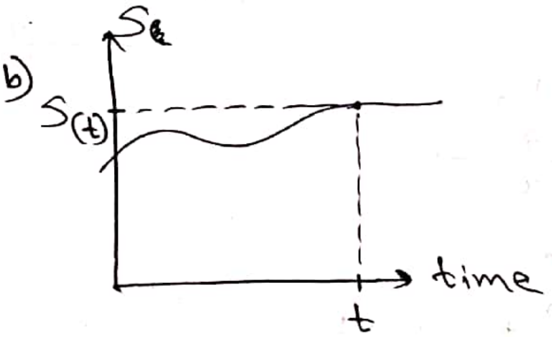
$g'(x) = 2ax + b$

iv. $y = \sin(3x^2) - 4\cos\left(\frac{x}{2}\right)$

$\frac{dy}{dx} = \left[\cos(3x^2) \times 6x\right] - \left\{\cancel{4}^{2}\left[-\sin\left(\frac{x}{2}\right)\right] \times \frac{1}{\cancel{2}}\right\}$

$\frac{dy}{dx} = 6x\cos(3x^2) + 2\sin\left(\frac{x}{2}\right)$

b)



$S(t) = 1 - 150t^3 + 45t^2 - 2t^5$

i. $S(t) = 1 - 150t^3 + 45t^2 - 2t^5$

$\frac{dS(t)}{dt} = 0 - 450t^2 + 90t - 10t^4$

$= -450t^2 + 90t - 10t^4$

ii. At the stationary points, the object should be stop.

At the stationary point,

$\frac{dS(t)}{dt} = 0$

$-450t^2 + 90t - 10t^4 = 0$

$t(-450t + 90 - 10t^3) = 0$

$t = 0$ or $-450t + 90 - 10t^3 = 0$

$10t^3 + 450t - 90 = 0$

$t^3 + 45t - 9 = 0$

$t = 0.19982$

$t = 0$ or $t = 0.2$

∴ The ~~object~~ position of the object is stop at $t = 0.25$

② a) i. $y = x^2 - \frac{3}{x} - \frac{1}{2}$

$y = x^2 - 3x^{-1} - \frac{1}{2}$

$\frac{dy}{dx} = 2x + 3x^{-2} - 0$

$\frac{dy}{dx} = 2x + \frac{3}{x^2}$

when $x = -2$,

$y = (-2)^2 - \frac{3}{-2} - \frac{1}{2}$

$y = 4 + \left(\frac{3 - 1}{2}\right)$

$y = 4 + \left(\frac{2}{2}\right)$

$y = 4 + 1$

$y = 5$

$\left.\frac{dy}{dx}\right|_{x=-2} = 2\times(-2) + \frac{3}{(-2)^2}$

$m = -4 + \frac{3}{4}$

$m = \frac{-16+3}{4}$

$m = \frac{-13}{4}$

Using $y = mx + c$

$5 = \frac{-13}{4} \times (-2) + c$

$c = 5 - \frac{13}{2}$

$c = -\frac{3}{2}$

∴ The equation of the tangent $\Rightarrow$ $y = \frac{-13}{4}x - \frac{3}{2}$

---

ii. $y = 4x^2 + \frac{5}{x} - 1$

$y = 4x^2 + 5x^{-1} - 1$

$\frac{dy}{dx} = 8x - 5x^{-2} - 0$

$= 8x - \frac{5}{x^2}$

When $x = 1$,

$y = 4x^2 + \frac{5}{x} - 1$

~~$= 4\times(1)^2 +$~~

$y = 4 \times (1)^2 + \frac{5}{1} - 1$

$y = 4 + 5 - 1$

$y = 8$

$\left.\frac{dy}{dx}\right|_{x=1} = 8\times1 - \frac{5}{1^2}$

$m = 8 - 5$

$m = 3$

Using $y = mx + c$,

$8 = 3\times1 + c$

$c = 8 - 3$

$c = 5$

∴ The equation of the tangent. $\rightarrow$ $y = 3x + 5$

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b) i. surface area $= (x \times 2x \times 2) + (2x \times h \times 2) + (x \times h \times 2)$

Volume,

$x \times 2x \times h$

$2x^2 h$

$\Downarrow$

1000

$$A = 4x^2 + 4xh + 2xh$$

$$A = 4x^2 + 2xh(2+1)$$

$$A = 4x^2 + \underbrace{2x^2 h} \times \frac{2+1}{x}$$

$$A = 4x^2 + 1000 \times \frac{3}{x}$$

$$A = 4x^2 + \frac{3000}{x}$$

ii. $A = 4x^2 + 3000x^{-1}$

$$\frac{dA}{dx} = 8x - 3000x^{-2}$$

$$\frac{dA}{dx} = 8x - \frac{3000}{x^2}$$

At stationary point;

$\cancel{8x - 300}$

$$\frac{dA}{dx} = 0$$

$$8x - \frac{3000}{x^2} = 0$$

$$x\left(8 - \frac{3000}{x}\right) = 0$$

$x = 0$ or $8 - \frac{3000}{x} = 0$

This value is impossible.

$$\frac{3000}{x} = 8$$

$$8x = 3000$$

$$x = 375$$

minimum value of A $= 8 \times 375 - \frac{3000}{(375)^2}$

$A = 2999.98787 \text{ cm}^2$

$A = 3000 \text{ cm}^2$

$$8x - \frac{3000}{x^2} = 0$$

$$\frac{8x^3 - 3000}{x^2} = 0$$

$$8x^3 - 3000 = 0$$

$$8x^3 = 3000$$

$$x = \sqrt[3]{\frac{3000}{8}}$$

$$x = 7.21 \text{ cm}$$

$$A = (4 \times 7.21^2) - \left(\frac{3000}{7.21}\right)$$

$$A = 624.025 \text{ cm}^2$$

③ i. $\int (2\cos x - \sin 3x \cos 3x)\, dx$

$= \int 2\cos x \, dx - \int \sin 3x \cdot \cos 3x \, dx$

$= 2\int \cos x \, dx - \int \frac{\sin(6x)}{2} \, dx$

$= 2\sin x - \frac{1}{2}\int \sin(6x)\, dx$

$= 2\sin x - \frac{1}{2}\left[-\cos(6x) \times \frac{1}{6}\right] + c$

$= 2\sin x + \frac{1}{12}\cos(6x) + c$; c is an arbitary constant.

$\sin(6x) = 2\sin(3x)\cdot\cos(3x)$

$\frac{\sin(6x)}{2} = \sin(3x)\cdot\cos(3x)$

ii. $\int \left(t^3 - \frac{e^{-t} - 4}{e^{-t}}\right) dt$

$= \int t^3 \, dt - \int \frac{e^{-t} - 4}{e^{-t}} \, dt$

$= \frac{t^{3+1}}{3+1} - \int e^t(e^{-t} - 4)\, dt$

$= \frac{t^4}{4} - \int e^t \cdot e^{-t} - 4e^t \, dt$

$= \frac{t^4}{4} - \int 1 \, dt - 4\int e^t \, dt$

$= \frac{t^4}{4} - t - 4e^t + c$; c is an arbitarey constant.

iii. $\int \left(\sqrt[3]{w} + 10\sqrt[5]{w^3}\right) dw$

$= \int \left(w^{\frac{1}{3}} + 10\, w^{\frac{3}{5}}\right) dw$

$= \int w^{\frac{1}{3}} \, dw + \int 10\, w^{\frac{3}{5}} \, dw$

$= \frac{w^{\frac{1}{3}+1}}{\frac{1}{3}+1} + 10\int w^{\frac{3}{5}} \, dw$

$= \frac{w^{\frac{4}{3}}}{\frac{4}{3}} + 10\, \frac{w^{\frac{3}{5}+1}}{\frac{3}{5}+1} + c$

$= \frac{3}{4}\sqrt[3]{w^4} + 10 \times \frac{w^{\frac{8}{5}}}{\frac{8}{5}} + c$

$= \frac{3}{4}\sqrt[3]{w^4} + 10 \times \frac{5}{8} \times \sqrt[5]{w^8} + c$

$= \frac{3}{4}\sqrt[3]{w^4} + \frac{25}{4}\sqrt[5]{w^8} + c$;

c is an arbitary constant.

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iv. $\int_2^1 \left(\frac{2y^3 - 6y^2}{y^2}\right) dy$

$= \int_2^1 \left(\frac{2y^3}{y^2} - \frac{6y^2}{y^2}\right) dy$

$= \int_2^1 (2y - 6)\, dy$

$= \left[2y - 6\right]_2^1$

$= \left[(2\times1) - 6\right] - \left[(2\times2) - 6\right]$

$= -4 - (-2)$

$= -4 + 2$

$= -2$

$= \int \left[\frac{2y^2}{\cancel{3}2} - 6y\right]_2^1$

$= \left[\frac{2\times1^2}{2} - 6\times1\right] - \left[\frac{2\times2^2}{2} - 6\times2\right]$

$= (1-6) - (4-12)$

$= -5 + 8$

$= \underline{\underline{3}}$

vi. $\int_{-5}^{-2} \left(7e^x + \frac{2}{x}\right) dx$

$= \int_{-5}^{-2} \left(7e^x + 2x^{-1}\right) dx$

$= \left[7e^x + \frac{2x^{-1+1}}{-1}\right]_{-5}^{-2}$

v. $\int_{\pi/6}^{\pi/3} \left(2\sec^2 x - 8\sec^2 5x\right) dx$

$= \int_{\pi/6}^{\pi/3} \left[2\sec^2 x - 8.\sec^2(5x)\right] dx$

$= \left[2\tan x - 8 \times \tan(5x) \times \frac{1}{5}\right]_{\pi/6}^{\pi/3}$

$= \left[2\tan x - \frac{8}{5}\tan(5x)\right]_{\pi/6}^{\pi/3}$

$= \left[2\tan\left(\frac{\pi}{3}\right) - \frac{8}{5}\tan\left(\frac{5\pi}{3}\right)\right] - \left[2\tan\left(\frac{\pi}{6}\right) - \frac{8}{5}\tan\left(\frac{5\pi}{6}\right)\right]$

$= \left[2\sqrt{3} - \frac{8}{5}\times(-\sqrt{3})\right] - \left[2\frac{1}{\sqrt{3}} - \frac{8}{5}\times\left(-\frac{1}{\sqrt{3}}\right)\right]$

$= \left(2\sqrt{3} + \frac{8\sqrt{3}}{5}\right) - \left(\frac{2}{\sqrt{3}} + \frac{8}{5\sqrt{3}}\right)$

$= \frac{18\sqrt{3}}{5} - \frac{6\sqrt{3}}{5}$

$= \frac{12\sqrt{3}}{5} \approx \underline{\underline{4.157}}$

vi. $\int_{-5}^{-2} \left(7e^x + \frac{2}{x}\right) dx$

$= \int_{-5}^{-2} \left(7e^x + 2\frac{1}{x}\right) dx$

$= \left[7e^x + 2\ln|x|\right]_{-5}^{-2}$

$= \left[7e^{-2} + 2\ln|-2|\right] - \left[7e^{-5} + 2\ln|5|\right]$

$= 2.33 - 3.27 = \underline{\underline{-0.94}}$

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④ a) $\frac{dy}{dx} = \frac{1}{x^{\frac{3}{2}}} + 1$

Integrate,

$$\int\left(\frac{dy}{dx}\right)dx = \int\left(\frac{1}{x^{3/2}} + 1\right)dx$$

$$y = \int\left(x^{-\frac{3}{2}} + 1\right)dx$$

$$y = \int x^{-\frac{3}{2}}dx + \int 1\,dx$$

$$y = \frac{x^{-\frac{3}{2}+1}}{-\frac{3}{2}+1} + x + c$$

$$y = \frac{x^{-\frac{1}{2}}}{-\frac{1}{2}} + x + c$$

$$y = \frac{-2}{\sqrt{x}} + x + c$$

$(4, 5)$ point is on the curve,

$$5 = \frac{-2}{\sqrt{4}} + 4 + c$$

$$5 = \frac{-2}{2} + 4 + c$$

$$c = 5 + 1 - 4$$

$$c = 2$$

∴ The equation of the curve $\Rightarrow$ $y = \frac{-2}{\sqrt{x}} + x + 2$

b) Shaded area $= \int_{\frac{\pi}{4}}^{\frac{3\pi}{4}} \left(\sin x - \frac{1}{\sqrt{2}}\right)$

$$= \left[-\cos x - \frac{1}{\sqrt{2}}x\right]_{\frac{\pi}{4}}^{\frac{3\pi}{4}}$$

$$= \left[-\cos\left(\frac{3\pi}{4}\right) - \frac{1}{\sqrt{2}} \times \frac{3\pi}{4}\right] - \left[-\cos\left(\frac{\pi}{4}\right) - \frac{1}{\sqrt{2}} \times \frac{\pi}{4}\right]$$

$$= \cancel{\left(\frac{\sqrt{2}}{2} - \frac{3\pi}{\sqrt{2}}\right) - \left(\frac{\sqrt{2}}{2} - \frac{\pi}{4\sqrt{2}}\right)}$$

$$= -0.959 - (-1.262)$$

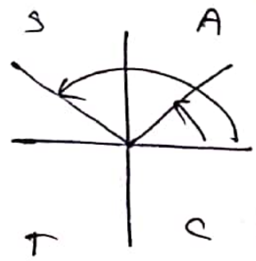
$$= -0.959 + 1.262$$

$= 0.303$ square unit.

$$\sin x = \frac{1}{\sqrt{2}}$$

$$x = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$$

$$x = \frac{\pi}{4}$$



$$x = \frac{3\pi}{4}$$

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