Lecture Module - Numerical Integration and Curve Fitting

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Module 3 - Numerical Integration and Curve Fitting

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- Topic 1 -
- Topic 2 -
- Topic 3 -

Topic 1 -

What is a Linear Equation

- "A linear equation is an algebraic equation in which each term is either a constant or the product of a constant and a single variable" - Wikipedia

- slope intercept form

- does not contain

What is a System of Linear Equations?

- multiple linear equations with...

- also known as...

The general system of linear equations is shown with variables $x_{1,2,..,n}$, coefficients $a_{11,12,..,nm}$, and knowns $b_{1,2,...,m}$

$$a_{11}x_1 + a_{12}x_2 + ... + a_{1n}x_n = b_1$$

$$a_{21}x_1 + a_{22}x_2 + ... + a_{2n}x_n = b_2$$

$$.$$

$$a_{m1}x_1 + a_{m2}x_2 + ... + a_{mn}x_n = b_m$$

The equations are cast into matrix form of the system.

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & . & & \\ & . & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_m \end{bmatrix}$$

To verify the matrix form $[A]\{x\} = \{b\}$ is correct, use matric multiplication and the result will match the individual equations.

$$\begin{pmatrix} a_{11} & a_{12} & \dots & a_{1n} \\ a_{21} & a_{22} & \dots & a_{2n} \\ & . & & \\ & . & & \\ a_{m1} & a_{m2} & \dots & a_{mn} \end{pmatrix} \times \begin{bmatrix} x_1 \\ x_2 \\ . \\ . \\ x_n \end{bmatrix} = \begin{bmatrix} b_1 \\ b_2 \\ . \\ . \\ b_m \end{bmatrix}$$

Consider the intersection of two Lines on the XY plane (2D).

- Write an equation for each line. $ax + by = c$
- Organize the equations.

Consider the intersection of two Lines on the XY plane (2D).

- Cast the system into matrix form.

- Solve the system. What exactly does this mean?
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Repeat the exercise, and now consider the intersection of three planes in space (3D). What does the solution represent?

- Write an equation for each plane. $ax + by + cz = d$
- Organize the equations.

- Cast the system into matrix form.
- Solve the system. What exactly does this mean?
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Topic 2 -

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- Why do we need to multiply matrices?

- Why do we need to use a computer?

Applications of Matrix Multiplication:

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Consider 2 conformable matrices F and G with elements f_{ij} and g_{ij}. Matrix Multiplication gives the product matrix E with elements e_{ij}.

$$E = F \times G \qquad\qquad e_{ij} = \Sigma_{k=1}^{n} f_{ik} \times g_{kj}$$

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix} \times \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix}$$

$e_{ij} = \Sigma_{k=1}^{n} f_{ik} \times g_{kj}$

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \left[\begin{array}{c|c} f_{11}g_{11} + f_{12}g_{21} + f_{13}g_{31} & f_{11}g_{12} + f_{12}g_{22} + f_{13}g_{32} \\ f_{21}g_{11} + f_{22}g_{21} + f_{23}g_{31} & f_{21}g_{12} + f_{22}g_{22} + f_{23}g_{32} \end{array}\right]$$

- What does that equation above mean?
- How can we write a *General Solution Technique* using the equation?

A Programming Exercise - Matrix Multiplication

Topic 3 -

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There are many different techniques for solving linear systems. This is not an exhaustive list.

- Kramer's Method
- Gaussian Elimination
- Gauss-Seidel Method
- Jacobi Method

Not all problems can be solved with this type of technique!

- non-homogeneous system is one in which ...

- most of the time the system will be non-homogeneous

- a non-homogeneous system has a proper solution if and only if

$$rank(A) = rank([A|b]) = n$$