Ordinary Differential Equations - Lecture 2

ME3001 - Mechanical Engineering Analysis

April 03, 2020

Analytical Solutions to Differential Equations

Lecture 2 - Analytical Solutions to Differential Equations:

- Review
- Analytical vs. Numerical Methods
- Example - Separation of Variables

What is a Differential Equation? Solution?

A **differential equation** is an equation which describes a function

and one or more of its __________________________ of the

__________________________ __________________________

with respect to the _______________________________.

The **solution** to a differential equation describes the

_____________________ _____________________ as a function

of the _____________________ _____________________.

Analytical Methods

A solution to a problem that can be written in "closed form" in terms of known functions, constants, etc., is often called an **analytic solution**. Note that this use of the word is completely different from its use in the terms analytic continuation, analytic function, etc.

Analytical solutions, also called closed-form solutions, are mathematical solutions in the form of math expressions. If you are developing algorithms or modeling engineering systems, analytical solutions offer the advantages of transparency and efficiency.

Numerical Solutions

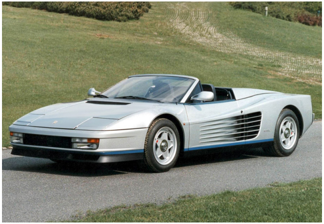
A **numerical solution** is an approximation to the solution of a mathematical equation, often used where analytical solutions are hard or impossible to find. All numerical solutions are approximations, some better than others, depending on the context of the problem and the numerical method used.

Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations (ODEs). Their use is also known as "numerical integration", although this term is sometimes taken to mean the computation of integrals.

Problem Statement

Remember our example from the previous lecture?

$$\dot{v} + \frac{c}{m}v = f(t)$$



We are going to find an **analytical solution** to this problem.

Separation of Variables

This is an **analytical** method that you learned in calculus.

Assume the external force $f(t)$ is zero. Re-write then separate.

$$\dot{v} + \frac{c}{m}v = 0$$

Solution

The solution $v(t)$ has been found. What does it mean? What do we do next?

$$v(t) =$$

Graph of Solution

What does the solution look like?

$$v(t) = v_0 e^{-\frac{c}{m}t}$$