

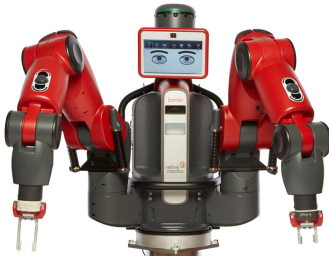
## Numerical Integration - Lecture 2

ME3001 - Mechanical Engineering Analysis

April 16, 2020

**Euler's Method for Higher-Order Equations**

## Lecture 2 - Euler's Method for Higher-Order Equations:



- Review Euler's Method
- A More Exciting Model
- Equation Decomposition
- MATLAB Solution

## Forward Integration

Last time we solved a **first order** ODE with Euler's method.

ODE and IC:      $m\dot{v} + cv = 0$       $v(t=0) = v_0$



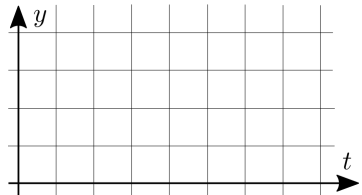
Slope Function:

$$v(t_{i+1}) = v(t_i) + f(t_i, v_i)\Delta t$$

$$v(\quad) = v(\quad) + f(\quad, \quad)\Delta t$$

$$v(\quad) = v(\quad) + f(\quad, \quad)\Delta t$$

$$v(\quad) = v(\quad) + f(\quad, \quad)\Delta t$$



## Euler's Method in MATLAB

```
% approximate with Euler's forward integration
v_eu(1)=v0;
for j=1:length(time)-1
    v_eu(j+1)=v_eu(j)+f(time(j),v_eu(j),m,c)*dt;
end

% If this is an 'Inline Definition' of the function
% it MUST go at the bottom of the script
function [dvdt]=f(t,v,M,C)
    dvdt=-C/M*v;
end
```

## A More Exciting Model

- First and second order linear models are frequently used in science and engineering
- However the world is \_\_\_\_\_ - \_\_\_\_\_
- Many exciting and important engineering problems involve **more complex models** involving rotational motion.



## Non-Linear Swinging Pendulum

An example of a non-linear system is an *inverted pendulum metronome*.



How will this system behave?

$$I_o \ddot{\theta} + k_T - (m \cdot g \cdot l) \sin(\theta) = 0$$

Finding an analytical solution is **very involved** and only mathematicians have time for all that...but you can look [here](#)

As we have seen using Euler's method is not hard, but we have to setup the problem correctly. This is a re-occurring theme!

$$I_o \ddot{\theta} + k_T - (m \cdot g \cdot l) \sin(\theta) = 0$$

To solve a second order system with an integration method like Euler's you must write the **slope function(s)**.

There are two derivatives so there are two \_\_\_\_\_ .

## x2 First Order from x1 Second Order

One second order ODE can be **decomposed** into *two* first order ODEs through a simple change of variables. This step can be confusing, but remember it is just an algebraic substitution!

$$I_o \ddot{\theta} + k_T - (m \cdot g \cdot l) \sin(\theta) = 0$$



## Execute Euler's Method

$$f_1 =$$

$$f_2 =$$

Use Euler's method just as before with both slope functions.  
Compute the values of the solution one-by-one **forward in time**  
for each variable side-by-side.

i	$z_1(t_{i+1}) = z_1(t_i) + f_1(t_i, z_{1i}, z_{2i})\Delta t$	$z_2(t_{i+1}) = z_2(t_i) + f_2(t_i, z_{1i}, z_{2i})\Delta t$
1	$z_1(\quad) = z_1(\quad) + f_1(\quad, \quad, \quad)\Delta t$	$z_2(\quad) = z_2(\quad) + f_2(\quad, \quad, \quad)\Delta t$
2	$z_1(\quad) = z_1(\quad) + f_1(\quad, \quad, \quad)\Delta t$	$z_2(\quad) = z_2(\quad) + f_2(\quad, \quad, \quad)\Delta t$
3	$z_1(\quad) = z_1(\quad) + f_1(\quad, \quad, \quad)\Delta t$	$z_2(\quad) = z_2(\quad) + f_3(\quad, \quad, \quad)\Delta t$

As you can see this can get messy quickly.

## Part 1 - Program Setup

```
clear variables;close all;clc

% define the constant parameters
global m g l kt; % global variables, gross...
m=2;g=9.8;
l=42*(1/100);kt=6;

% initial conditions
theta0=15;
omega0=0;

% create an array of time values
dt=.001;tstop=10;
time=0:dt:tstop;
```

## Part 2 - Euler's Method

```
z1(1)=theta0*pi/180; z2(1)=0;    % initial conditions

for j=1:length(time)-1    % approximate with Euler's
    z1(j+1)=z1(j)+f1(time(j),z1(j),z2(j))*dt;
    z2(j+1)=z2(j)+f2(time(j),z1(j),z2(j))*dt;
end

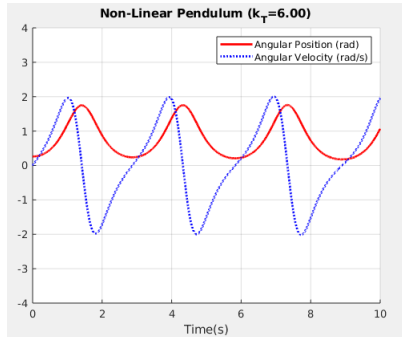
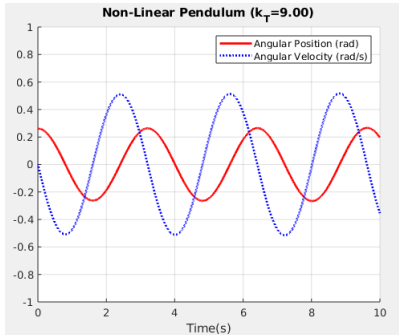
function [z1dot]=f1(t,Z1,Z2)    % fns. go at bottom
    global m g l kt    % even grosser
    z1dot=Z2;
end

function [z2dot]=f2(t,Z1,Z2)
    global m g l kt
    z2dot=(m*g*l*sin(Z1)-kt*Z1)/(m*l^2);
end
```

## Part 3 - Graph the Solutions

```
% plot the results of the method  
figure(1);hold on  
plot(time,z1,'r')  
plot(time,z2,'b')  
grid on  
title('Non-Linear Pendulum')  
legend('Angular Pos. (rad)','Angular Vel. (rad/s)')  
xlabel('Time(s)')  
axis([0 tstop -3 3])
```

## Do you believe the results?



This graph on the left makes sense, but what about this on the right?