

ME 3001 Lecture, Systems of Linear Equations

The Gaussian Elimination Algorithm

What is Matrix Multiplication

- Consider 2 conformable matrices F and G with elements f_{ij} and g_{ij} .
- Matrix Multiplication gives the product matrix E with elements e_{ij} .

$$E = F \times G \qquad e_{ij} = \sum_{k=1}^n f_{ik} \times g_{kj}$$

- Lets do an example with a few small matrices.

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \end{bmatrix} \times \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \\ g_{31} & g_{32} \end{bmatrix}$$

- A closer look at E

$$\begin{bmatrix} e_{11} & e_{12} \\ e_{21} & e_{22} \end{bmatrix} = \begin{bmatrix} f_{11}g_{11} + f_{12}g_{21} + f_{13}g_{31} & f_{11}g_{12} + f_{12}g_{22} + f_{13}g_{32} \\ f_{21}g_{11} + f_{22}g_{21} + f_{23}g_{31} & f_{21}g_{12} + f_{22}g_{22} + f_{23}g_{32} \end{bmatrix}$$

- So what is n ?

A Programming Exercise - Matrix Multiplication

- $e_{ij} = \sum_{k=1}^n f_{ik} \times g_{kj}$
- What does that equation mean?
- How can we write a *General Solution Technique*?

Simple Example (3x3)

Gaussian Elimination

This is a 2 part process

- **Step 1:** Forward Elimination of Unknowns

- Eliminate x_1 from equations 2 to n
 - * Eliminate x_1 from equation 2
 - define the eliminating factor f_{21} as a_{21}/a_{11}
 - redefine a_{21} as $a_{21} - a_{11} * f_{21}$
 - redefine a_{22} as $a_{22} - a_{12} * f_{21}$
 - . . .
 - redefine a_{2n} as $a_{2n} - a_{1n} * factor$
 - * Eliminate x_1 from equation 3
 - define the eliminating factor f_{31} as a_{31}/a_{11}
 - redefine a_{31} as $a_{31} - a_{11} * f_{31}$
 - redefine a_{32} as $a_{32} - a_{12} * f_{31}$
 - . . .
 - redefine a_{3n} as $a_{3n} - a_{1n} * f_{31}$
- Eliminate x_2 from equations 3 to n
 - * Eliminate x_2 from equation 3
 - define the eliminating factor f_{32} as a_{32}/a_{22}
 - redefine a_{32} as $a_{32} - a_{22} * f_{32}$
 - redefine a_{33} as $a_{33} - a_{23} * f_{32}$
 - . . .
 - redefine a_{3n} as $a_{3n} - a_{2n} * f_{32}$
 - . . .
- Eliminate x_{n-1} from equation n
 - define the eliminating factor $f_{n,n-1}$ as $a_{n,n-1}/a_{n-1,n-1}$
 - redefine $a_{n,n-1}$ as $a_{n,n-1} - a_{n-1,n-1} * f_{n,n-1}$

- **Step 2:** Backwards Substitution

- Solve Equations n through 1

- * Solve for x_n as $\frac{b_n}{a_{n,n}}$

- * Solve for x_{n-1} as $\frac{b_{n-1} - (a_{n-1,n}x_n)}{a_{n-1,n-1}}$

- * Solve for x_{n-2} as $\frac{b_{n-2} - (a_{n-2,n-1}x_{n-1}) - (a_{n-2,n}x_n)}{a_{n-2,n-2}}$

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- * Solve for x_1 as $\frac{b_1 - (a_{12}x_2) - \dots - (a_{1,n-1}x_{n-1}) - (a_{1,n}x_n)}{a_{1,1}}$

- Summary

- The Forward Elimination Algorithm:

for k *from* 1 *to* $n-1$

for i *from* $k+1$ *to* n

$\text{fact} = a_{i,k} / a_{k,k}$

for j *from* k *to* n

$a_{i,j} = a_{i,j} - \text{fact} \times a_{k,j}$

end

$b_i = b_i - \text{fact} \times b_k$

end

end

- The Backwards Substitution Algorithm:

$x_n = b_n / a_{n,n}$

for i *from* $n-1$ *to* 1

$x_i = (b_i - \sum_{j=i+1}^n (a_{i,j} x_j)) / a_{i,i}$

end

REMINDER - Homework 2 is due Wed. Sep. 26

**REMINDER - MATLAB script from today's lecture
will be posted on ilearn.**