Ordinary Differential Equations - Lecture 4

ME3001 - Mechanical Engineering Analysis

April 02, 2020

Validation of Analytical Solutions with ODE45

Lecture 4 - Validation of Analytical Solutions with ODE45:

- Review
- Solution Validation
- MATLAB ode45() function
- Example - 1986 (or 1968?) Ferrari Testarosa Spider

Analytical vs. Numerical Methods

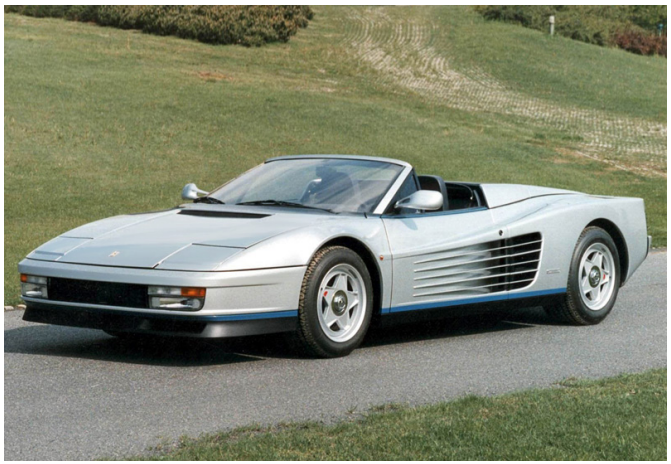
Analytical solutions (methods), also called closed-form solutions, are mathematical solutions in the form of math expressions. If you are developing algorithms or modeling engineering systems, analytical solutions offer the advantages of transparency and efficiency.

Numerical methods for ordinary differential equations are methods used to find numerical approximations to the solutions of ordinary differential equations (ODEs). Their use is also known as "numerical integration", although this term is sometimes taken to mean the computation of integrals.

Engineering Example - Validation of Analytical Solution

Remember our example from the previous lecture?

$$m\dot{v} + cv = f(t) \quad \rightarrow \quad v(t) = (v_0 - \frac{F}{c})e^{-\frac{c}{m}t} + \frac{F}{c}$$



How do we know that the analytical solution we derived is correct?

Solution Validation

Q: How do we know that the solution we derived is correct?

A:

Use the Help References

We are going to solve the differential same equation in *MATLAB* using the **ode45** function.

This *built-it* function uses **numerical integration** to approximate a solution an ODE as an **initial value problem**.

```
>> help ode45
```

Use the help in the MATLAB command window or look it up elsewhere. It can be hard to understand.

Using the ODE45 function

The **ode45** function is a powerful tool and it is easy to use.

```
[t_45,y_45]=ode45(@ODEFUN,TSPAN,Y0,OPTIONS,P...);
```

Here is a description of the arguments.

ODEFUN - name of the function containing the model

TSPAN - time range for the initial value problem

Y0 - initial value of the dependent variable

OPTIONS - options defined by OPTIMSET function

P... - additional parameters passed to ODEFUN

Graph of Solution

Do the results from the two methods agree?

```
figure(1);hold on
plot(t_45,y_45)
```