Lecture Module - Non-Linear Equations

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Module 2 - Non-Linear Equations

Module 2 - Non-Linear Equations

- Topic 1 - Solving Non-Linear Equations
- Topic 2 - The Newton-Raphson Method, Secant Method
- Topic 3 - The Bisection Method

Topic 1 - Solving Non-Linear Equations

- What is a Non-Linear Equation ?
- Solving Non-linear Equations
- Analytical vs. Numerical Methods
- Example

What is a Non-Linear Equation ?

Different Types of Non-Linear Equations

- **Polynomials (excluding first order)**
- **Transcendentals**

  " a transcendental function "transcends" algebra in that it cannot be expressed in terms of a finite sequence of the algebraic operations of addition, multiplication, and root extraction. Examples of transcendental functions include the exponential function, the logarithm, and the trigonometric functions. "

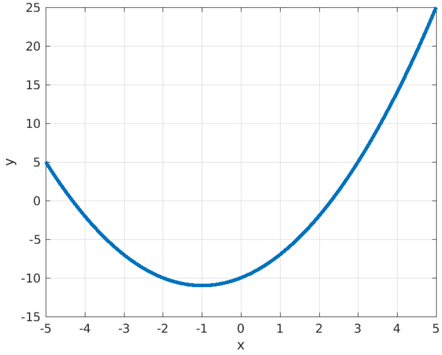
  - Exponentials
  - Logarithms
  - Trigonometrics

What is a Non-Linear Equation ?

Solving Non-linear Equations

Example: Solve the following equation.

$$y = x^2 + 2x - 10$$



Solving Non-linear Equations

Defintion of **Solution**

- -
- -
- -

Analytical vs. Numerical Methods

Analytical

- solution to a problem that can be written in **closed form**
- solution in terms of known functions, constants, etc.
- gives an **exact answer**

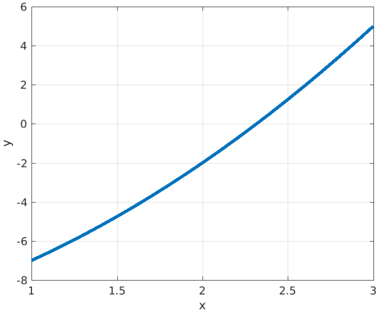
Numerical

- an **approximation** to the solution of a mathematical equation
- iterative procedure or algorithm
-

Example

We are looking for where the line crosses the x-axis, so how can we tell where this happens?

$y = x^2 + 2x - 10$



Topic 2 - The Newton-Raphson Method, Secant Method

- Classification of Methods
- Taylor Series Derivation
- The Newton Raphson Method
- The Finite Difference
- Modified Newton-Raphson, Secant Method
- Algorithm Comparison

Classification of Methods

Theoretical/Analytical Solution Techniques

- solving the equation using exact mathematics
- leads to an exact or *analytical* solution

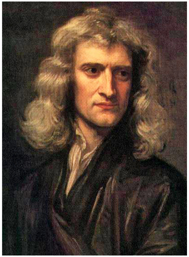
Numerical Solution Techniques

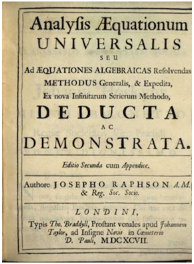
- approximating the solution to the equation using varying methods, or *algorithms*
- leads to a approximate solution
- a.k.a. *Numerical Method*

Classification of Methods

These scientists changed the world forever.

- Isaac Newton, mathematician and physicist, 1642-1727
- Joseph Raphson, English Mathematician, 1648-1715
- add Taylor to list





Analyſis Æquationum
UNIVERSALIS
SEU
Ad ÆQUATIONES ALGEBRAICAS Reſolvendas
METHODUS Generalis, & Expedita,
Ex nova Infinitarum Serierum Methodo,
DEDUCTA
AC
DEMONSTRATA.

Editio Secunda cum *Appendice*.

Authore JOSEPHO RAPHSON *A.M.*
& *Reg. Soc. Socio.*

LONDINI,
Typis *Tho. Braddyll*, Proſtant venales apud *Johannem Taylor*, ad Inſigne *Navis* in *Cœmeterio D. Pauli*, MDCXCVII.

Classification of Methods

The Newton-Raphson method is a *shooting method*.

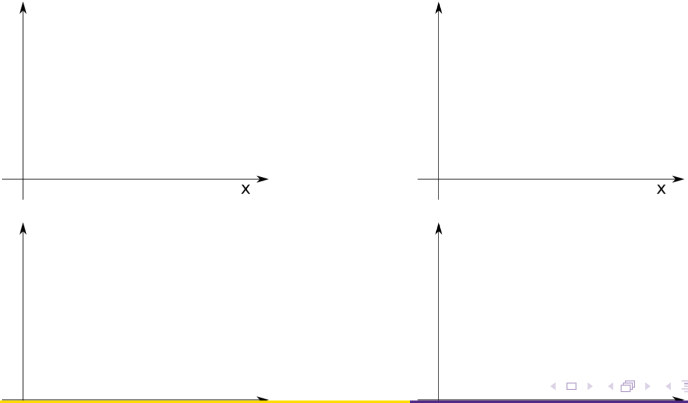
Taylor Series Derivation

The method can be derived from the Taylor series.

$$f(x) \approx f(a) + f'(a)(x-a) + \frac{f''(a)}{2!}(x-a)^2 + \ldots + \frac{f^{(n)}(a)}{n!}(x-a)^{(n)}$$

Taylor Series Derivation

The 4 possible sign (+/-) cases are handled by the algorithm.
Now you have much better hammer. However, must be used properly...



The Newton Raphson Method

Algorithm Summary:

- - Step 1: start doing stuff
- - Step 2: do more stuff
- - Step 3: keep doing stuff until you have the solution

The Newton Raphson Method

General Use Algorithm: a much better hammer ...

Pros:

- can be used for many different equations
- problem specific algebra not required to obtain value of solution
- executiion of numerical method is routine and can automated

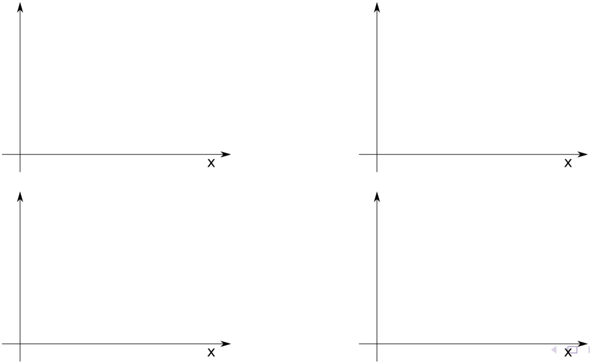
Cons:

- problem definition can be the difficult
- solution results and computation time dependent on initial estimation
- numerical solution must be computed for with defined equation parameters

The Newton Raphson Method

4 General Use Cases

- The general problem of solving for the root of a non linear equation can be extended to 4 useful variants.
- With careful equation setup, all cases can be solved with the same systematic routine.



The Newton Raphson Method

The Finite Difference

Our goal is to write a computer program to automate the Newton-Raphson method. We want our program to be (1) robust to different inputs and (2) user friendly.

The Finite Difference

The Newton-Raphson method is not purely numerical, why?

- The Equation
- The Derivation

How can we avoid this issue?

Hint: **Think about the title secant ...**

The Finite Difference

This idea or technique is the foundation of a family of methods known as the *Finite Difference Methods*.

Forward Difference

Backwards Difference

Central Difference

Modified Newton-Raphson, Secant Method

The secant method is a modified version of the Newton-Raphson method which uses the Finite difference method to compute the slope values of the function to be solved.

Newton-Raphson

Secant

Modified Newton-Raphson, Secant Method

Algorithm Summary:

- - Step 1: start doing stuff
- - Step 2: do more stuff
- - Step 3: keep doing stuff until you have the solution

Modified Newton-Raphson, Secant Method

General Use Algorithm: the secant method is a generalized, numerical tool for solving non linear equations

Pros:

- -
- -
- -

Cons:

- -
- -
- -

Topic 3 - The Bisection Method

- Analytical vs. Numerical
- A Bracketing Method: Graphical Explanation
- Algorithm Description

Analytical vs. Numerical

Theoretical/Analytical Solution Techniques

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Analytical vs. Numerical

A Bracketing Method: Graphical Explanation

The Bisection method is a *bracketing method*.

A Bracketing Method: Graphical Explanation

A Bracketing Method: Graphical Explanation

Algorithm Description

1.
2.
3.
4.

Algorithm Description

See MATLAB example.

Algorithm Description

General Use Algorithm: the bisection method can also be used to solve the general root finding problem

Pros:

- -
- -
- -

Cons:

- -
- -
- -

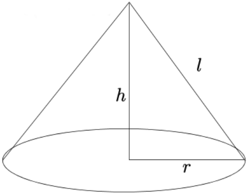
Topic 3 - Mechanical Design Problem

- Problem Statement
- Mathematical Model
- Solution Approach
- Design

Problem Statement

A Mechanical Design Problem

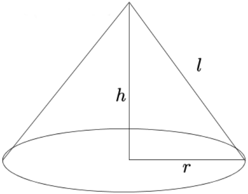
As an engineer you are asked to design a structure. The geometry of this structures is simple but certain properties are critical. Also you want to spend as little as possible on materials.



You are required to design is a cone with a surface area of exactly $25m^2$ to a tolerance of 0.1 m^2 and a height of exactly $1m$. Your goal is to find the radius in meters.

Problem Statement

What is the *mathematical model* of the cone?



surface area, $s = \pi r l = \pi r \sqrt{h^2 + r^2}$
volume, $v = \pi r^2 \frac{h}{3}$

Problem Statement

How are you going to solve this problem?

Problem Statement

How are you going to *design* the cone?