

Module 3 - Systems of Linear Equations

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Topic 4 - Gaussian Elimination

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- Various Row-Reduction Methods
- Gaussian Elimination Technique
- A Generalized Algorithm
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Various Row-Reduction Methods

The Gaussian Elimination method has many variations. You may have used a different version in linear algebra, but that is fine. This method is generalized so that it can be automated easily with a computer program.

Gaussian Elimination Technique

The Gaussian Elimination consists of two main steps. Some variations of the method combine the two steps into a single procedure.

- ➊ Forward Elimination of Unknowns
- ➋ Backwards Substitution

Gaussian Elimination Technique

Step 1: Forward Elimination of Unknowns

- Eliminate x_1 from equations 2 to n
 - Eliminate x_1 from equation 2
 - define the eliminating factor f_{21} as a_{21}/a_{11}
 - redefine a_{21} as $a_{21} - a_{11} * f_{21}$
 - redefine a_{22} as $a_{22} - a_{12} * f_{21}$
 - . . .
 - redefine a_{2n} as $a_{2n} - a_{1n} * factor$
 - Eliminate x_1 from equation 3
 - define the eliminating factor f_{31} as a_{31}/a_{11}
 - redefine a_{31} as $a_{31} - a_{11} * f_{31}$
 - redefine a_{32} as $a_{32} - a_{12} * f_{31}$
 - . . .
 - redefine a_{3n} as $a_{3n} - a_{1n} * f_{31}$

Gaussian Elimination Technique

- Eliminate x_2 from equations 3 to n
 - Eliminate x_2 from equation 3
 - define the eliminating factor f_{32} as a_{32}/a_{22}
 - redefine a_{32} as $a_{32} - a_{22} * f_{32}$
 - redefine a_{33} as $a_{33} - a_{23} * f_{32}$
 - . . .
 - redefine a_{3n} as $a_{3n} - a_{2n} * f_{32}$
 - . . .
- Eliminate x_{n-1} from equation n
 - define the eliminating factor $f_{n,n-1}$ as $a_{n,n-1}/a_{n-1,n-1}$
 - redefine $a_{n,n-1}$ as $a_{n,n-1} - a_{n-1,n-1} * f_{n,n-1}$

Gaussian Elimination Technique

Step 2: Backwards Substitution

- Solve Equations n through 1
 - Solve for x_n as $\frac{b_n}{a_{n,n}}$
 - Solve for x_{n-1} as $\frac{b_{n-1} - (a_{n-1,n}x_n)}{a_{n-1,n-1}}$
 - Solve for x_{n-2} as $\frac{b_{n-2} - (a_{n-2,n-1}x_{n-1}) - (a_{n-2,n}x_n)}{a_{n-2,n-2}}$
 - .
 - .
 - .
 - Solve for x_1 as $\frac{b_1 - (a_{12}x_2) - \dots - (a_{1,n-1}x_{n-1}) - (a_{1,n}x_n)}{a_{1,1}}$

A Generalized Algorithm

Step 1: Forward Elimination

```
for k from 1 to n-1
  for i from k+1 to n
    fact =  $a_{i,k} / a_{k,k}$ 
    for j from k to n
       $a_{i,j} = a_{i,j} - \text{fact} \times a_{k,j}$ 
    end
     $b_i = b_i - \text{fact} \times b_k$ 
  end
end
```

Step 2: Backwards Substitution

```
 $x_n = b_n / a_{n,n}$ 
for i from n-1 to 1
   $x_i = (b_i - \sum_{j=i+1}^n (a_{i,j} x_j)) / a_{i,i}$ 
end
```