ME 3023 - The Fourier Series

- **The Fourier Series - Full Expansion**

$$f(x) = a_0 + \Sigma_{n=1}^{\infty}\{a_n cos(nx) + b_n sin(nx)\}$$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} f(x)dx$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)cos(nx)dx$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)sin(nx)dx$$

- **Alternate Form (more useful)**

$$f(x) = a_0 + \Sigma_{n=1}^{\infty}\{a_n cos(\frac{n\pi x}{P}) + b_n sin(\frac{n\pi x}{P})\}$$

$$a_0 = \frac{1}{2P}\int_{-P}^{P} f(x)dx$$

$$a_n = \frac{1}{P}\int_{-P}^{P} f(x)cos(\frac{n\pi x}{P})dx$$

$$b_n = \frac{1}{P}\int_{-P}^{P} f(x)sin(\frac{n\pi x}{P})dx$$

- **The Fourier Series - Half Range Expansions**

  - **Cosine Series**

$$f(x) = a_0 + \Sigma_{n=1}^{\infty}\{a_n cos(\frac{n\pi x}{P})\}$$

$$a_0 = \frac{1}{P}\int_0^P f(x)dx$$

$$a_n = \frac{2}{P}\int_0^P f(x)cos(\frac{n\pi x}{P})dx$$

  - **Sine Series**

$$f(x) = \Sigma_{n=1}^{\infty}\{b_n sin(\frac{n\pi x}{P})\}$$

$$b_n = \frac{2}{P}\int_0^P f(x)sin(\frac{n\pi x}{P})dx$$

- **Some Useful Stuff...**

$sin(n\pi) = 0$ if n is an integer

$cos(n\pi) = (-1)^n$ if n is an integer