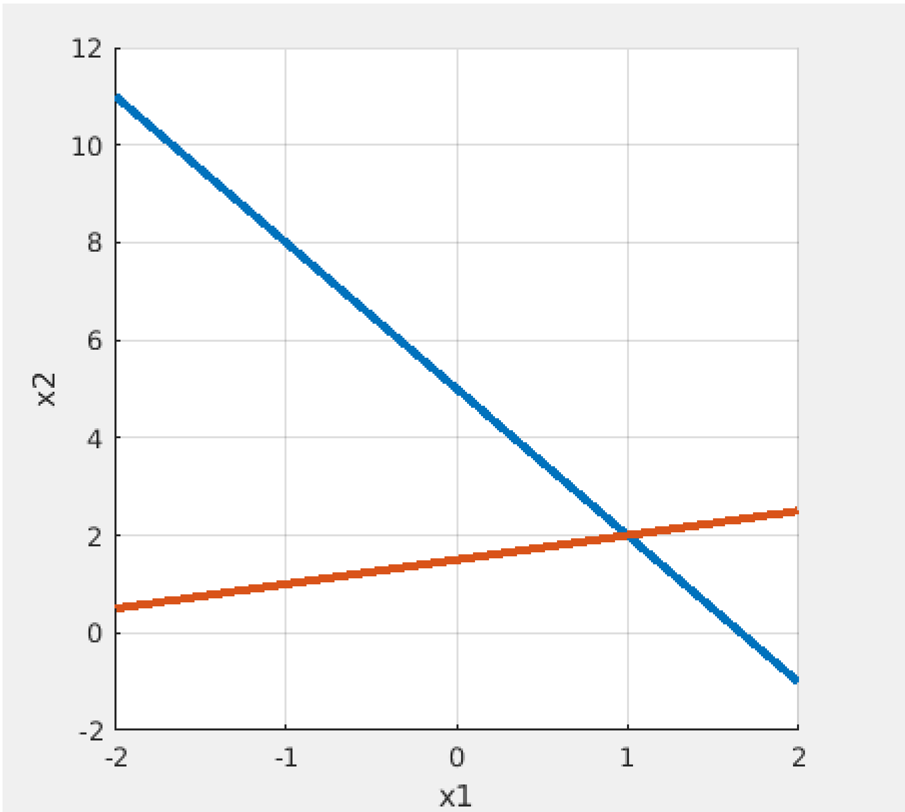
ME 3001 Lecture - Systems of Linear Equations
Solution Existence and Potential Problems

- **Some problems cannot be solved with this type of technique!**
  - non-homogeneous system is one in which ...
  - most of the time the system will be non-homogeneous
  - a non-homogeneous system has a proper solution if and only if ...

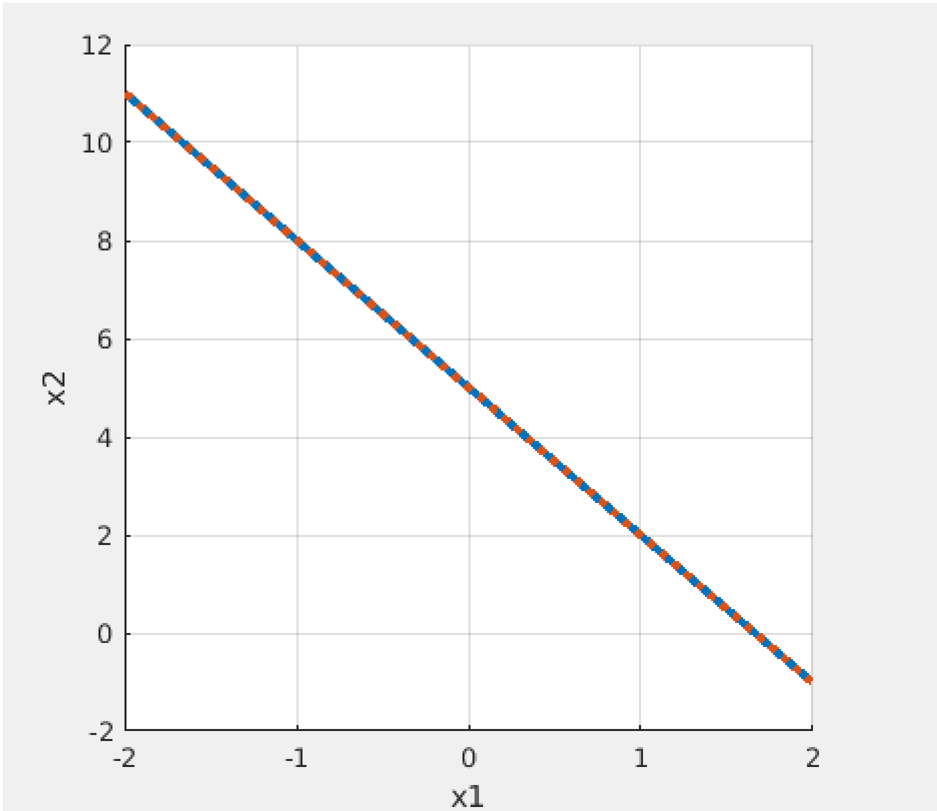
$$rank(A) = rank([A|b]) = n$$

– **Normal Case - 2 Equations - 2 Unknowns - 1 Solution**



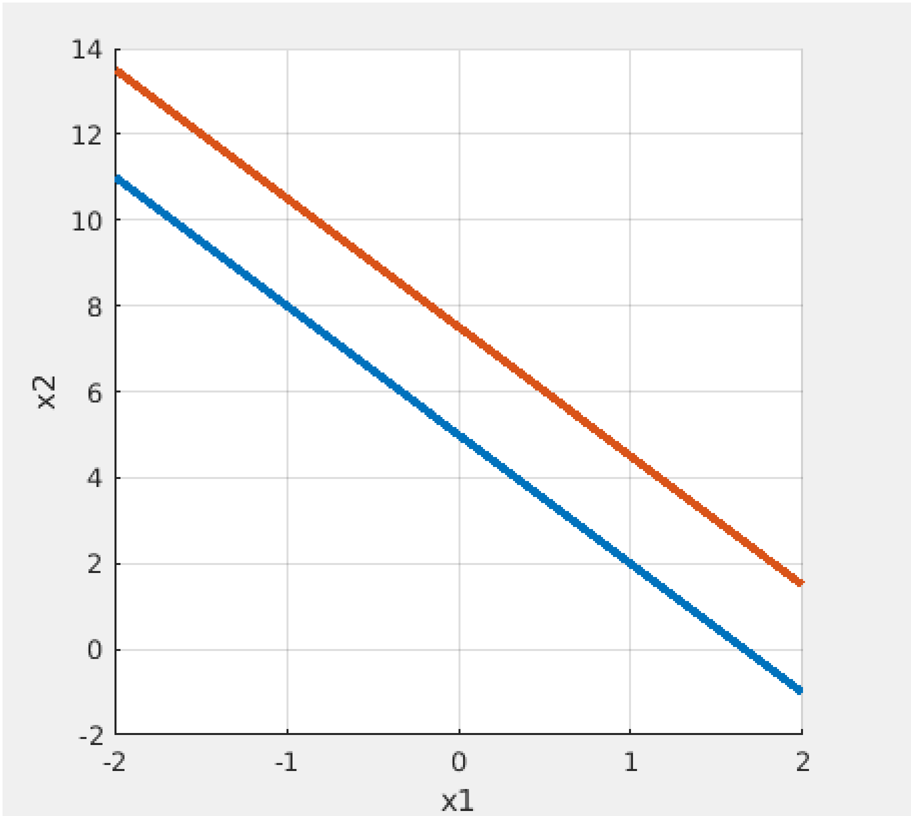
$$3x_1 + x_2 = 5$$
$$x_1 - 2x_2 = -3$$

– **Abnormal Case - 2 Equations - 2 Unknowns - ∞ Solutions**



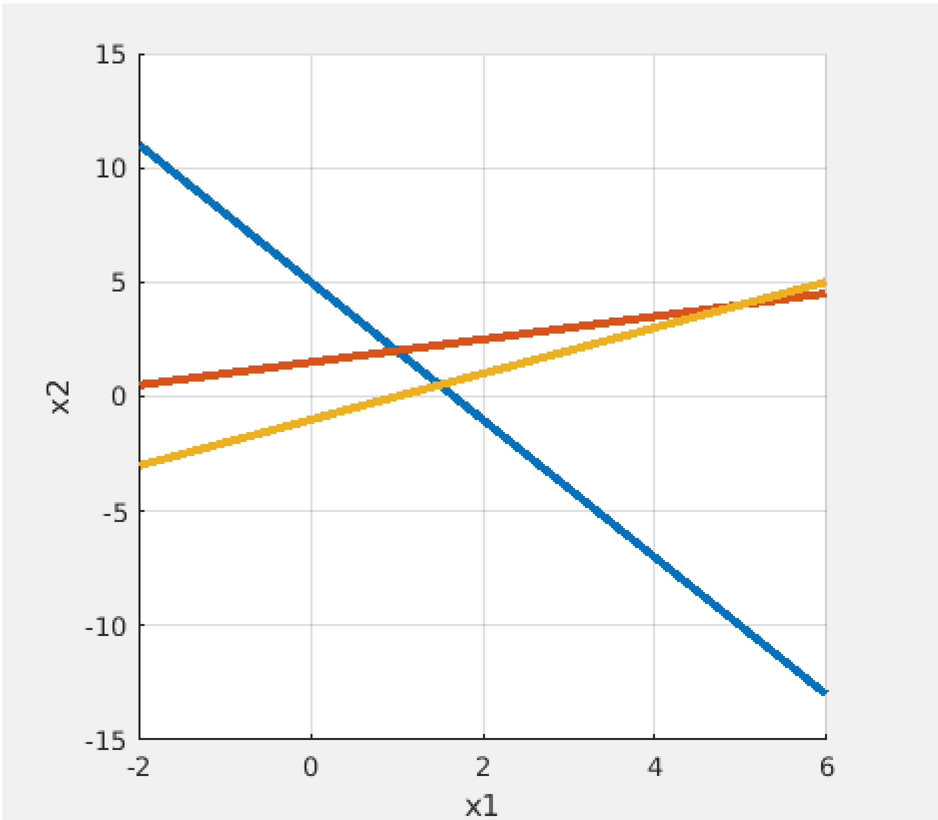
$$3x_1 + x_2 = 5$$
$$6x_1 + 2x_2 = 10$$

– **Abnormal Case - 2 Equations - 2 Unknowns - 0 Solutions**



$$3x_1 + x_2 = 5$$
$$6x_1 + 2x_2 = 15$$

– **Abnormal Case - 3 Equations - 2 Unknowns - 0 Solutions**

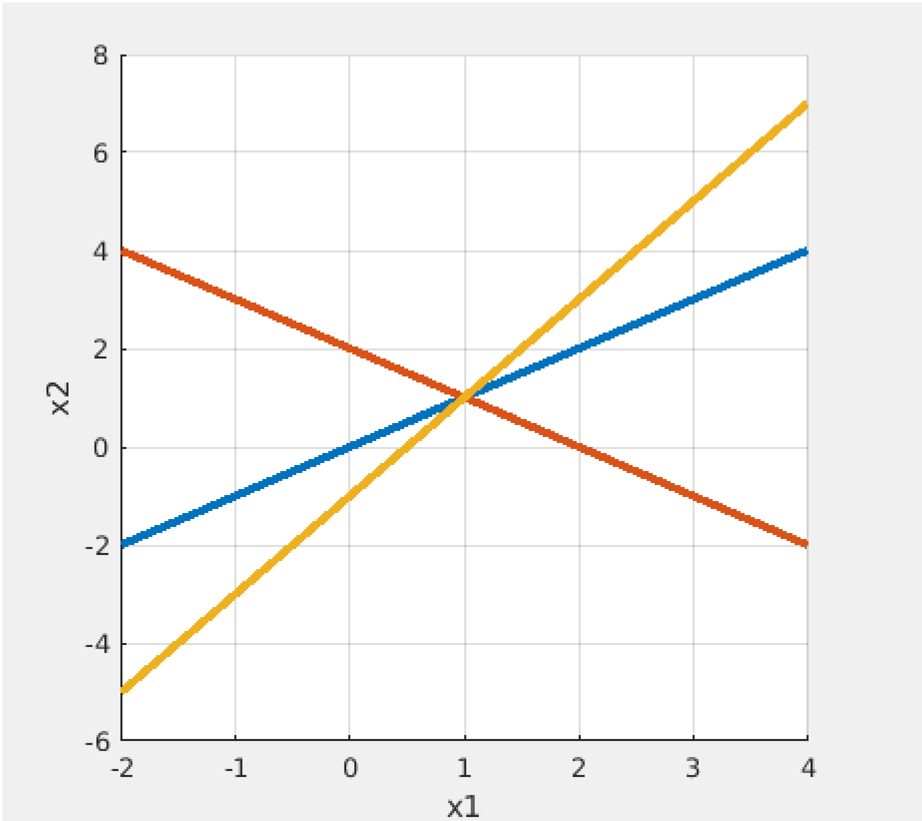


$$3x_1 + x_2 = 5$$

$$x_1 - 2x_2 = -3$$

$$x_1 - x_2 = 1$$

– **Abnormal Case - 3 Equations - 2 Unknowns - 1 Solution**



$$-x_1 + x_2 = 0$$
$$x_1 + x_2 = 2$$
$$-2x_1 + x_2 = -1$$

- **We also want our answer to have as little error as possible**

  - **What causes error in the numerical methods?**

    "In software engineering and mathematics, numerical error is the combined effect of two kinds of error in a calculation. The first is caused by the finite precision of computations involving floating-point or integer values. The second usually called truncation error is the difference between the exact mathematical solution and the approximate solution obtained when simplifications are made to the mathematical equations to make them more amenable to calculation."-wikipedia

  - **2 (or 3) Major Causes**

    * **floating point computations**

    * **truncation and solution simplification**

    * **system condition**

- **The System Condition can cause problems!**
  - An ill-conditioned system can cause error.
  - A system is ill-conditioned if small changes in the coefficients on the either side of the equation create large variations in the solution.
  - Let us look at a simple 2x2 example.

$$x_1 - x_2 = 5$$
$$kx_1 - x_2 = 4$$

  - The system shown will have huge variations in the solution if $k \approx 1$

$$x_1 - x_2 = 5$$
$$(0.99)x_1 - x_2 = 4$$

  - When $k = 0.99$, this gives a solution $(x_1, x_2) = (100, 95)$

$$x_1 - x_2 = 5$$
$$(1.01)x_1 - x_2 = 4$$

  - When $k = 1.01$, this gives a solution $(x_1, x_2) = (-100, 105)$