Lecture Module - Numerical Integration and Curve Fitting

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Module 5 - Numerical Integration and Curve Fitting

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- Topic 1 - Overview and Motivation
- Topic 2 - Linear Regression
- Topic 3 - Polynomial Splines
- Topic 4 - Polynomial Splines

Topic 1 - Overview and Motivation

- Problem Definition
- Engineering Applications
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Problem Definition

What is curve fitting?
- various techniques to fit a curve or function to discrete data

- "Data is often given for discrete values along a continuum. However, you may require estimates at points between the discrete values" - Numerical Methods for Engineers, Chapra and Canale

- additional problem is to find a simpler form of a complicated function by fitting function to data sampled from original function

Problem Definition

Two General Approaches

- 1) Given data with random error, find a single curve that represents the overall trend of the data.

  - "Because any individual data point may be incorrect, we make no effort to intersect every point" - Numerical Methods for Engineers, Chapra and Canale

- 2) Given data assumed to be precise or specified, find a curve that directly passes through each data point

Engineering Applications

Example Applications in Engineering

- Calibration Curves, Sensors and Instrumentation
- Table Interpolation, Mechanics, Thermo, Statistics
- Velocity Profile Generation, Dynamics of Machinery, Robotics

Two General Problems

- Trend Analysis - predictions from dataset using interpolation polynomial or lsr
- Hypothesis Testing - compare predicted to measured data for model performance or selection

Topic 2 - Linear Regression

- Overview
- Fit Criteria
- Linear Least Squares
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Overview

Consider fitting a straight line to a dataset

$$(x_1, y_1), (x_2, y_2), ..., (x_n, y_n)$$

with a function

$$y = a_o + a_1 + e$$

This can be rearranged to show the **error** as

$$e = y - a_0 - a_1 x$$

The general problem is to find a function that minimizes the error

Overview

To find the coefficents of the fit line, the minimization objective must be considered carefully. You might consider fitting a model that mimizes the error directly, but this will not work. The absolute value approach is also problematic.

- $\Sigma_{i=1}^{n} e_i = (y_i - a_0 - a_1 x_i)$
- $\Sigma_{i=1}^{n} |e_i| = |y_i - a_0 - a_1 x_i|$

To solve these issues, the common technique is to _____ the error.

- $\Sigma_{i=1}^{n} e_i^2 = (y_i - a_0 - a_1 x_i)^2$

Overview

Fit Criteria

To fit a straight line to the data, we must find the values a_o and a_1 that minimize the square of the error. First find the partial derivatives of the sqaured error and set these equal to zero

$$S_r = \Sigma_{i=1}^{n} e_i^2 = (y_i - a_0 - a_1 x_i)^2$$

$$\frac{\delta S_r}{\delta a_0}$$

Fit Criteria

Linear Least Squares

Linear Least Squares

Topic 3 - Polynomial Splines

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Topic 3 - Lagrange Polynomials

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