ME 3001 Lecture, Systems of Linear Equations
The Gaussian Elimination Algorithm

Simple Example (3x3)

This is a 2 part process

- **Step 1:** Forward Elimination of Unknowns
  - Eliminate x_1 from equations 2 to n
    * Eliminate x_1 from equation 2
      · define the eliminating factor f_{21} as a_{21}/a_{11}
      · redefine a_{21} as $a_{21} - a_{11} * f_{21}$
      · redefine a_{22} as $a_{22} - a_{12} * f_{21}$
      · · ·
      · redifine a_{2n} as $a_{2n} - a_{1n} * factor$

    * Eliminate x_1 from equation 3
      · define the eliminating factor f_{31} as a_{31}/a_{11}
      · redefine a_{31} as $a_{31} - a_{11} * f_{31}$
      · redefine a_{32} as $a_{32} - a_{12} * f_{31}$
      · · ·
      · redefine a_{3n} as $a_{3n} - a_{1n} * f_{31}$

  - Eliminate x_2 from equations 3 to n

    * Eliminate x_2 from equation 3
      · define the eliminating factor f_{32} as a_{32}/a_{22}
      · redefine a_{32} as $a_{32} - a_{22} * f_{32}$
      · redefine a_{33} as $a_{33} - a_{23} * f_{32}$
      · · ·
      · redefine a_{3n} as $a_{3n} - a_{2n} * f_{32}$

    · · ·
  - Eliminate x_{n-1} from equation n
    · define the eliminating factor $f_{n,n-1}$ as $a_{n,n-1}/a_{n-1,n-1}$
    · redefine $a_{n,n-1}$ as $a_{n,n-1} - a_{n-1,n-1} * f_{n,n-1}$

- **Step 2:** Backwards Subsitution

  – Solve Equations n through 1

    * Solve for x_n as $\dfrac{b_n}{a_{n,n}}$

    * Solve for x_{n-1} as $\dfrac{b_{n-1}-(a_{n-1,n}x_n)}{a_{n-1,n-1}}$

    * Solve for x_{n-2} as $\dfrac{b_{n-2}-(a_{n-2,n-1}x_{n-1})-(a_{n-2,n}x_n)}{a_{n-2,n-2}}$
    .
    .
    .

    * Solve for x_1 as $\dfrac{b_1-(a_{12}x_2)-...-(a_{1,n-1}x_{n-1})-(a_{1,n}x_n)}{a_{1,1}}$

- Summary

- **The Forward Eimination Algorithm:**

  for k *from* 1 *to* n-1

      for i *from* k+1 *to* n

          fact$= a_{i,k}/a_{k,k}$

          for j *from* k *to* n

  $$a_{i,j} = a_{i,j} - \text{fact} \times a_{k,j}$$

        end

  $$b_i = b_i - \text{fact} \times b_k$$

      end

  end

- **The Backwards Substitution Algorithm:**

  $$x_n = b_n/a_{n,n}$$

  for i *from* n-1 *to* 1

  $$x_i = (b_i - \sum_{j=i+1}^{n} (a_{i,j} x_j))/a_{i,i}$$

  end

- A close look at backwards subsitution

$$x_i = (b_i - \sum_{j=i+1}^{n} (a_{i,j} x_j))/a_{i,i}$$
$$end$$

REMINDER - Homework 2 is due Wed. Sep. 26

REMINDER - MATLAB script from today's lecture will be posted on ilearn.