Numerical Integration - Lecture 2

ME3001 - Mechanical Engineering Analysis

April 17, 2020

Solving Higher-Order Equations with ODE45

Lecture 2 - Solving Higher-Order Equations with ODE45:

- Review ODE45 Function
- A More Exciting Model
- Equation Decomposition
- MATLAB Solution

Using the ODE45 function

The **ode45** function is a powerful tool and it is easy to use.

```
[t_45,y_45]=ode45(@ODEFUN,TSPAN,Y0,OPTIONS,P...);
```

Here is a description of the arguments.

ODEFUN - name of the function containing the model

TSPAN - time range for the initial value problem

Y0 - initial value of the dependent variable

OPTIONS - options defined by OPTIMSET function

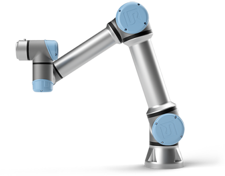
P... - additional parameters passed to ODEFUN

Euler's Method in MATLAB

```
% approximate with Euler's forward integration
v_eu(1)=v0;
for j=1:length(time)-1
    v_eu(j+1)=v_eu(j)+f(time(j),v_eu(j),m,c)*dt;
end

% If this is an 'Inline Definition' of the function
% it MUST go at the bottom of the script
function [dvdt]=f(t,v,M,C)
    dvdt=-C/M*v;
end
```

A More Exciting Model

- First and second order linear models are frequently used in science and engineering
- However the world is ________-____________________
- Many exciting and important engineering problems invlove **more complex models** involving rotational motion.



Non-Linear Swinging Pendulum

An example of a non-linear system is an *inverted pendulum metronome*.

How will this system behave?

$$I_o\ddot{\theta} + k_T - (m \cdot g \cdot l)\, sin(\theta) = 0$$

Finding an analytical solution is **very involved** and only mathematicians have time for all that...but you can look here

As we have seen using Euler's method is not hard, but we have to setup the problem correctly. This is a re-occuring theme!

$$I_o\ddot{\theta} + k_T - (m \cdot g \cdot l)\, sin(\theta) = 0$$

To solve a second order system with an integration method like Euler's you must write the **slope function(s)**.

There are two derivatives so there are two ________ __________.

x2 First Order from x1 Second Order

One second order ODE can be **decomposed** into *two* first order ODEs through a simple change of variables. This step can be confusing, but remember it is just an algebraic substitution!

$$I_o\ddot{\theta} + k_T - (m \cdot g \cdot l)\, sin(\theta) = 0$$

Execute Euler's Method

$f_1 =$

$f_2 =$

Use Euler's method just as before with both slope functions. Compute the values of the solution one-by-one **forward in time** for each variable side-by-side.

i	$z_1(t_{i+1}) = z_1(t_i) + f_1(t_i, z_{1i}, z_{2i})\Delta t$	$z_2(t_{i+1}) = z_2(t_i) + f_2(t_i, z_{1i}, z_{2i})\Delta t$
1	$z_1(\quad) = z_1(\quad) + f_1(\quad,\quad,\quad)\Delta t$	$z_2(\quad) = z_2(\quad) + f_2(\quad,\quad,\quad)\Delta t$
2	$z_1(\quad) = z_1(\quad) + f_1(\quad,\quad,\quad)\Delta t$	$z_2(\quad) = z_2(\quad) + f_2(\quad,\quad,\quad)\Delta t$
3	$z_1(\quad) = z_1(\quad) + f_1(\quad,\quad,\quad)\Delta t$	$z_2(\quad) = z_2(\quad) + f_3(\quad,\quad,\quad)\Delta t$

As you can see this can get messy quickly.

Part 1 - Program Setup

```
clear variables;close all;clc

% define the constant parameters
global m g l kt; % global variables, gross...
m=2;g=9.8;
l=42*(1/100);kt=6;

% initial conditions
theta0=15;
omega0=0;

% create an array of time values
dt=.001;tstop=10;
time=0:dt:tstop;
```

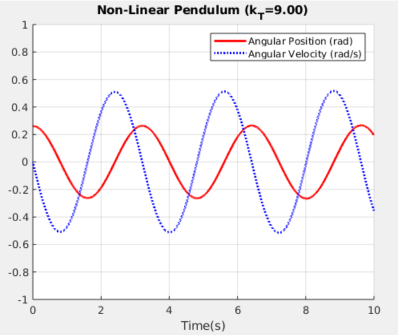
Part 2 - Euler's Method

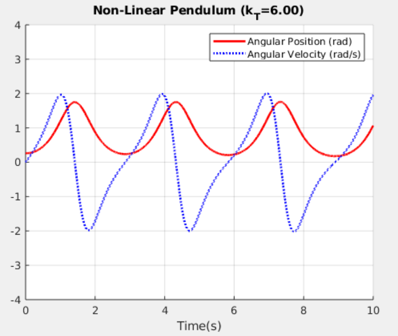
```
z1(1)=theta0*pi/180; z2(1)=0;  % initial conditions

for j=1:length(time)-1  % approximate with Euler's
    z1(j+1)=z1(j)+f1(time(j),z1(j),z2(j))*dt;
    z2(j+1)=z2(j)+f2(time(j),z1(j),z2(j))*dt;
end

function [z1dot]=f1(t,Z1,Z2)  % fns. go at bottom
    global m g l kt % even grosser
    z1dot=Z2;
end

function [z2dot]=f2(t,Z1,Z2)
    global m g l kt
    z2dot=(m*g*l*sin(Z1)-kt*Z1)/(m*l^2);
end
```

Part 3 - Graph the Solutions

```
% plot the results of the method
figure(1);hold on
plot(time,z1,'r')
plot(time,z2,'b')
grid on
title('Non-Linear Pendulum')
legend('Angular Pos. (rad)','Angular Vel. (rad/s)')
xlabel('Time(s)')
axis([0 tstop -3 3])
```

Do you believe the results?





This graph on the left makes sense, but what about this on the right?