Lecture Module - Systems of Linear Equations

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Topic 3 - Existence of Solutions

Topic 3 - Existence of Solutions

- Techniques for Solving Linear Systems
- Homogeneous and Inhomogeneous Systems
- Solution Existence Cases in 2D
- Numerical Error and System Condition

Techniques for Solving Linear Systems

There are many different techniques for solving linear systems. This is not an exhaustive list.

- Kramer's Method
- Gaussian Elimination
- Gauss-Seidel Method
- Jacobi Method

Homogeneous and Inhomogeneous Systems

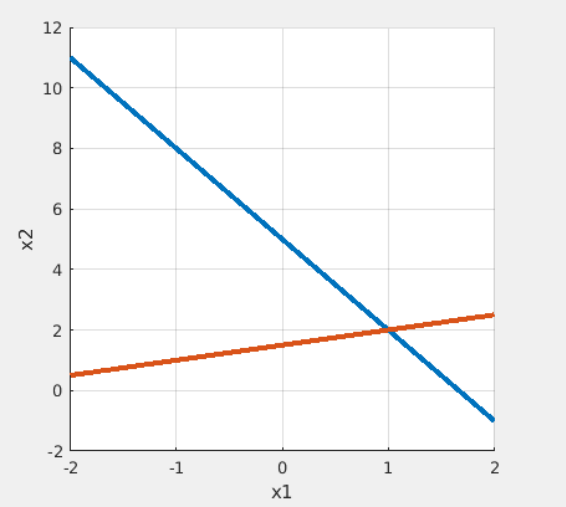
Not all problems can be solved with this type of technique!

- non-homogeneous system is one in which ...
- most of the time the system will be non-homogeneous
- a non-homogeneous system has a proper solution if and only if

$$rank(A) = rank([A|b]) = n$$

Solution Existence Cases in 2D

Normal Case - 2 Equations - 2 Unknowns - 1 Solution

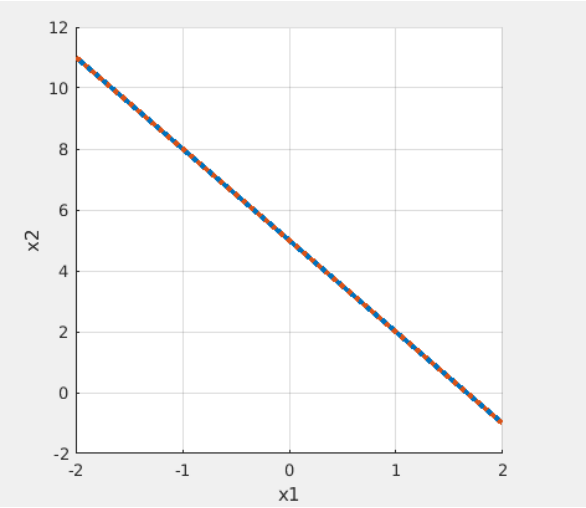


$$3x_1 + x_2 = 5$$

$$x_1 - 2x_2 = -3$$

Solution Existence Cases in 2D

Abnormal Case - 2 Equations - 2 Unknowns - ∞ Solutions

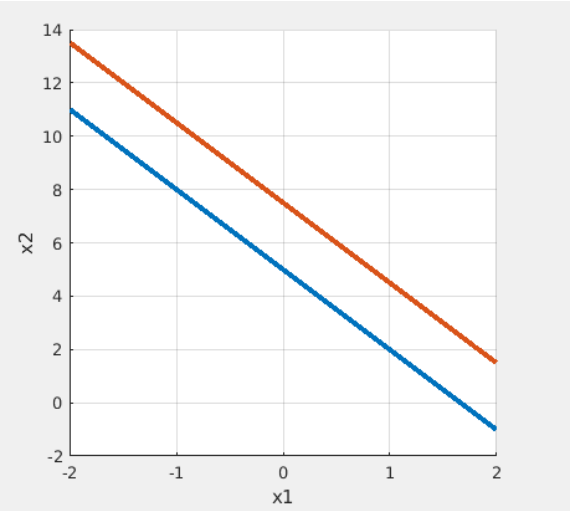


$$3x_1 + x_2 = 5$$

$$6x_1 + 2x_2 = 10$$

Solution Existence Cases in 2D

Abnormal Case - 2 Equations - 2 Unknowns - 0 Solutions

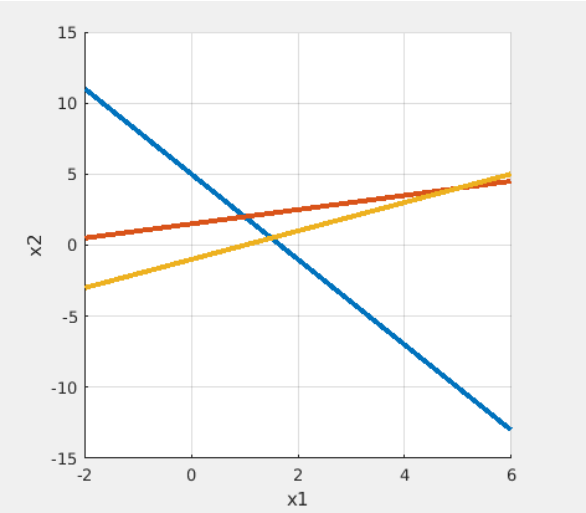


$$3x_1 + x_2 = 5$$

$$6x_1 + 2x_2 = 15$$

Solution Existence Cases in 2D

Abnormal Case - 3 Equations - 2 Unknowns - 0 Solutions



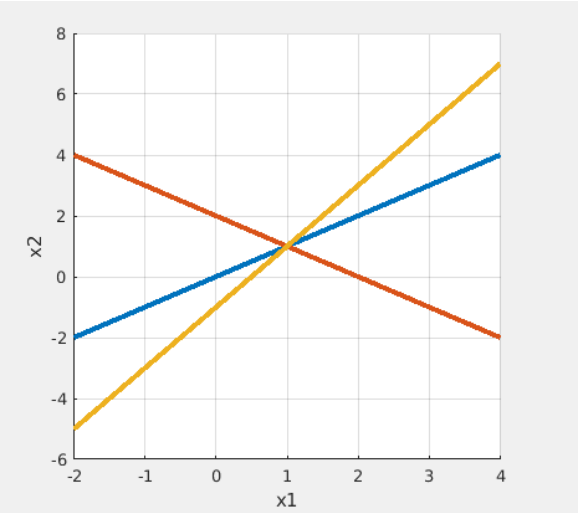
$$3x_1 + x_2 = 5$$

$$x_1 - 2x_2 = -3$$

$$x_1 - x_2 = 1$$

Solution Existence Cases in 2D

Abnormal Case - 3 Equations - 2 Unknowns - 1 Solution



$$-x_1 + x_2 = 0$$

$$x_1 + x_2 = 2$$

$$-2x_1 + x_2 = -1$$

Numerical Error and System Condition

We want our answer to have as little error as possible.

What causes error in the numerical methods?
"In software engineering and mathematics, numerical error is the combined effect of two kinds of error in a calculation. The first is caused by the finite precision of computations involving floating-point or integer values. The second usually called truncation error is the difference between the exact mathematical solution and the approximate solution obtained when simplifications are made to the mathematical equations to make them more amenable to calculation."-wikipedia

Numerical Error and System Condition

Major Causes of Error

- **floating point computations**
- **truncation and solution simplification**
- **system condition**

The System Condition can cause problems!

- An ill-conditioned system can cause error.
- A system is ill-conditioned if small changes in the coefficients on the either side of the equation create large variations in the solution.

Numerical Error and System Condition

Look at this simple 2x2 example. The solution will have huge variations if $k \approx 1$.

$$x_1 - x_2 = 5$$

$$kx_1 - x_2 = 4$$

When $k = 0.99$, this gives a solution $(x_1, x_2) = (100, 95)$

$$x_1 - x_2 = 5$$

$$(0.99)x_1 - x_2 = 4$$

When $k = 1.01$, this gives a solution $(x_1, x_2) = (-100, 105)$

$$x_1 - x_2 = 5$$

$$(1.01)x_1 - x_2 = 4$$