Lecture Module - Systems of Linear Equations

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Topic 4 - Gaussian Elimination

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- Various Row-Reduction Methods
- Gaussian Elimination Technique
- A Generalized Algorithm
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Various Row-Reduction Methods

The Gaussian Elimination method has many variations. You may have used a different version in linear algebra, but that is fine. This method in generalized so that is can be automated easily with a computer program.

Gaussian Elimination Technique

The Gaussian Elimination consists of two main steps. Some variations of the method combine the two steps into a single procedure.

1. Forward Elimination of Unknowns
2. Backwards Substitution

Gaussian Elimination Technique

Step 1: Forward Elimination of Unknowns

- Eliminate x_1 from equations 2 to n
  - Eliminate x_1 from equation 2
    - define the eliminating factor f_{21} as a_{21}/a_{11}
    - redefine a_{21} as $a_{21} - a_{11} * f_{21}$
    - redefine a_{22} as $a_{22} - a_{12} * f_{21}$

      . . .
    - redifine a_{2n} as $a_{2n} - a_{1n} * factor$
  - Eliminate x_1 from equation 3
    - define the eliminating factor f_{31} as a_{31}/a_{11}
    - redefine a_{31} as $a_{31} - a_{11} * f_{31}$
    - redefine a_{32} as $a_{32} - a_{12} * f_{31}$

      . . .
    - redefine a_{3n} as $a_{3n} - a_{1n} * f_{31}$

Gaussian Elimination Technique

- Eliminate x_2 from equations 3 to n
  - Eliminate x_2 from equation 3
    - define the eliminating factor f_{32} as a_{32}/a_{22}
    - redefine a_{32} as $a_{32} - a_{22} * f_{32}$
    - redefine a_{33} as $a_{33} - a_{23} * f_{32}$

      . . .
    - redefine a_{3n} as $a_{3n} - a_{2n} * f_{32}$

  . . .
- Eliminate x_{n-1} from equation n
  - define the eliminating factor $f_{n,n-1}$ as $a_{n,n-1}/a_{n-1,n-1}$
  - redefine $a_{n,n-1}$ as $a_{n,n-1} - a_{n-1,n-1} * f_{n,n-1}$

Gaussian Elimination Technique

Step 2: Backwards Subsitution

- Solve Equations n through 1
  - Solve for x_n as $\frac{b_n}{a_{n,n}}$
  - Solve for x_{n-1} as $\frac{b_{n-1}-(a_{n-1,n}x_n)}{a_{n-1,n-1}}$
  - Solve for x_{n-2} as $\frac{b_{n-2}-(a_{n-2,n-1}x_{n-1})-(a_{n-2,n}x_n)}{a_{n-2,n-2}}$
  - .
  - .
  - .
  - Solve for x_1 as $\frac{b_1-(a_{12}x_2)-\ldots-(a_{1,n-1}x_{n-1})-(a_{1,n}x_n)}{a_{1,1}}$

A Generalized Algorithm

Step 1: Forward Elimination

for k *from* 1 *to* n-1

  for i *from* k+1 *to* n

    fact= $a_{i,k}/a_{k,k}$

      for j *from* k *to* n

        $a_{i,j} = a_{i,j} - fact \times a_{k,j}$

      end

    $b_i = b_i - fact \times b_k$

  end

end

Step 2: Backwards Substitution

$x_n = b_n/a_{n,n}$

for i *from* n-1 *to* 1

$$x_i = (b_i - \sum_{j=i+1}^{n} (a_{i,j}x_j))/a_{i,i}$$

end