Numerical Integration - Lecture 3

ME3001 - Mechanical Engineering Analysis

April 18, 2020

Solving Higher-Order Equations with ODE45

Lecture 3 - Solving Higher-Order Equations with ODE45:

- Review ODE45 Function
- A Homework Problem
- Solution Validation
- MATLAB Solution

Using the ODE45 Function

The **ode45** function is a MATLAB tool for solving differential equations. The equation(s) must be ___________-______________.

```
[TOUT,YOUT]=ode45(@ODEFUN,TSPAN,Y0,OPTS,P...);
```

ODEFUN - name of the function containing the model

TSPAN - time range for the initial value problem

Y0 - initial value of the dependent variable

OPTS - options defined by OPTIMSET function

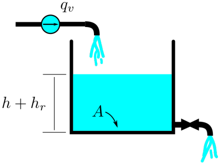
P... - additional parameters passed to ODEFUN

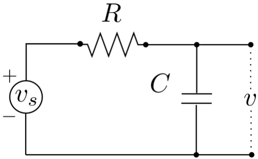
Euler's Method in MATLAB

```
% validate the solution with ODE45
opts=optimset('Display','none');
[t45,v45]=ode45(@vdot_model,time,v0,opts,F,m,c);

% show a graph of the solution
figure(1); hold on
plot(time,vel,'ro');pause %analytical solution
plot(t45,v45,'b*')  %numerical solution

% Inline function definitions go at the bottom
function [vdot]=vdot_model(tin,vin,F,m,c)
    vdot=(F-c*vin)/m;
end
```

First and Second Order Linear Systems

- First and second order linear models are frequently used in science and engineering
- Lets work one of the second order problems from the homework. Hopefully you see it could represent one of the physical systems.





Homework 5 - Problem 1 Part C plus a f(t)=6t

Solve the following ODE using the trial solution method.

$$3\ddot{x} + 12x = 6t \qquad x(0) = 2 \quad \dot{x}(0) = 2$$

Step 1: Find complementary part

Step 2: Find particular part

Step 3: Combine and Solve for Unknown Constants

Find Complementary Solution

Step 1: Find complementary part from the LHS=0 of the ODE.

Find Particular Solution

Step 2: Find particular part from the LHS=RHS of the ODE.

Combine and Solve for Unknown Constants

Step 3: The solution is the sum of the complementary and particular parts. Solve for the two remaining unknowns.

Analyitcal Solution

$$x(t) = 2cos(2t) + \frac{3}{4}sin(2t) + \frac{1}{2}t$$

As we have seen using ODE45 is not hard, but we have to setup the problem correctly. This is a re-occuring theme!!!

$3\ddot{x} + 12x = 6t \qquad x(0) = 2 \quad \dot{x}(0) = 2$

ODE45 can solve higher order equations but they must be written as a system of________ ___________

There are two derivatives so there are two ________ ___________ differential equations

x2 First Order from x1 Second Order

One second order ODE can be **decomposed** into *two* first order ODEs through a simple change of variables.

$$3\ddot{x} + 12x = 6t \qquad x(0) = 2 \quad \dot{x}(0) = 2$$

```
>> help ode45
 ode45  Solve non-stiff differential equations, medium order method.
    [TOUT,YOUT] = ode45(ODEFUN,TSPAN,Y0) with TSPAN = [T0 TFINAL] integrates
    the system of differential equations y' = f(t,y) from time T0 to TFINAL
    with initial conditions Y0. ODEFUN is a function handle. For a scalar T
    and a vector Y, ODEFUN(T,Y) must return a column vector corresponding
    to f(t,y). Each row in the solution array YOUT corresponds to a time
    returned in the column vector TOUT.  To obtain solutions at specific
    times T0,T1,...,TFINAL (all increasing or all decreasing), use TSPAN =
    [T0 T1 ... TFINAL].

    [TOUT,YOUT] = ode45(ODEFUN,TSPAN,Y0,OPTIONS) solves as above with default
    integration properties replaced by values in OPTIONS, an argument created
    with the ODESET function. See ODESET for details. Commonly used options
    are scalar relative error tolerance 'RelTol' (1e-3 by default) and vector
    of absolute error tolerances 'AbsTol' (all components 1e-6 by default).
    If certain components of the solution must be non-negative, use
    ODESET to set the 'NonNegative' property to the indices of these
    components.
```

Part 1 - Program Setup

```
clear variables;close all;clc

% create an array of time values
dt=.001;tstop=10;
t=0:dt:tstop;

% compute solution from derived equation
x_ex=2*cos(2*t)+3/4*sin(2*t)+1/2*t;
```

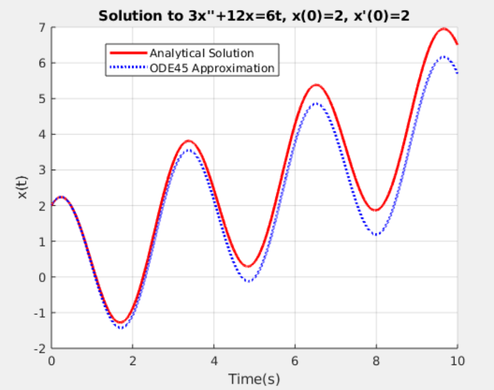
Part 2 - Euler's Method

```
% validate solution with ODE45
iv=[2 2];      % intitial vals for dependent var

opts=odeset('Stats','off');
[t_45,x_45]=ode45(@ode_sys,t,iv,opts);

% a function to use with ODE45
function [Zdot]=ode_sys(T,Z)
    Zdot=zeros(2,1);
    Zdot(1)=Z(2);
    Zdot(2)=(5*T-12*Z(1))/3;
end
```

Part 3 - Graph the Solutions

```
% plot the results of the method
% plot the results of the method
figure(1);hold on
plot(t,x_ex,'r-','LineWidth',2)
plot(t_45,x_45(:,1),'b:','LineWidth',2)

grid on
str=sprintf('Solution to 3x''''+12x=6t, x(0)=2, x''(0
title(str)
legend('Analytical Solution','ODE45 Approximation')
xlabel('Time(s)');ylabel('x(t)')
```

Do you believe the results?



The graphs are close but they are not exactly the same!