

## Lecture Module - Non-Linear Equations

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

### Module 2 - Non-Linear Equations

## Module 2 - Non-Linear Equations

- Topic 1 - Solving Non-Linear Equations
- Topic 2 - The Newton-Raphson Method, Secant Method
- Topic 3 - The Bisection Method

## Topic 1 - Solving Non-Linear Equations

- What is a Non-Linear Equation ?
- Solving Non-linear Equations
- Analytical vs. Numerical Methods
- Example

# What is a Non-Linear Equation ?

## Different Types of Non-Linear Equations

- Polynomials (excluding first order)
- Transcendentals

" a transcendental function "transcends" algebra in that it cannot be expressed in terms of a finite sequence of the algebraic operations of addition, multiplication, and root extraction. Examples of transcendental functions include the exponential function, the logarithm, and the trigonometric functions. "

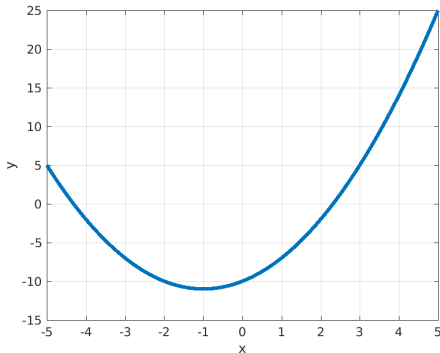
- Exponentials
- Logarithms
- Trigonometrics

# What is a Non-Linear Equation ?

# Solving Non-linear Equations

**Example:** Solve the following equation.

$$y = x^2 + 2x - 10$$



# Solving Non-linear Equations

## Defintion of **Solution**

• -

• -

• -

# Analytical vs. Numerical Methods

## Analytical

- solution to a problem that can be written in **closed form**
- solution in terms of known functions, constants, etc.
- gives an **exact answer**

## Numerical

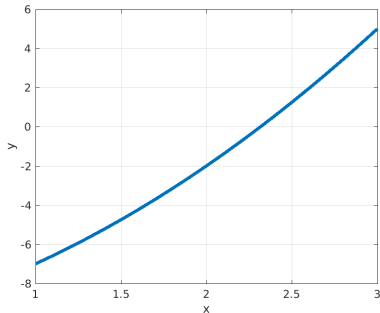
- an **approximation** to the solution of a mathematical equation
- iterative procedure or algorithm
-



## Example

We are looking for where the line crosses the x-axis, so how can we tell where this happens?

$$y = x^2 + 2x - 10$$



## Topic 2 - The Newton-Raphson Method, Secant Method

- Classification of Methods
- Taylor Series Derivation
- The Newton Raphson Method
- The Finite Difference
- Modified Newton-Raphson, Secant Method
- Algorithm Comparison

# Classification of Methods

## Theoretical/Analytical Solution Techniques

- solving the equation using exact mathematics
- leads to an exact or *analytical* solution

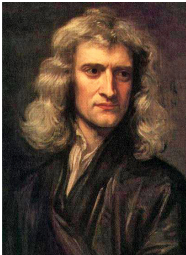
## Numerical Solution Techniques

- approximating the solution to the equation using varying methods, or *algorithms*
- leads to a approximate solution
- a.k.a. *Numerical Method*

## Classification of Methods

These scientists changed the world forever.

- Isaac Newton, mathematician and physicist, 1642-1727
- Joseph Raphson, mathematician, 1648-1715
- Brook Taylor, mathematician, 1685-1731



# Classification of Methods

The Newton-Raphson method is a *shooting method*.



## Taylor Series Derivation

The method can be derived from the Taylor series.

$$f(x) \approx f(a) + f'(a)(x - a) + \frac{f''(a)}{2!}(x - a)^2 + \dots + \frac{f^{(n)}(a)}{n!}(x - a)^{(n)}$$

# Taylor Series Derivation

The 4 possible sign (+/-) cases are handled by the algorithm.  
Now you have much better hammer. However, must be used properly...



# The Newton Raphson Method

## Algorithm Summary:

- - Step 1:
- - Step 2:
- - Step 3:



# The Newton Raphson Method

**General Use Algorithm:** a much better hammer ...

## Pros:

- can be used for many different equations
- problem specific algebra not required to obtain value of solution
- execution of numerical method is routine and can be automated

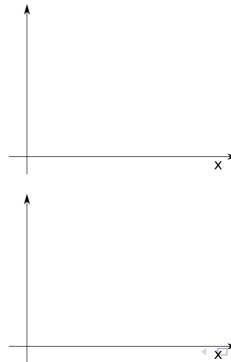
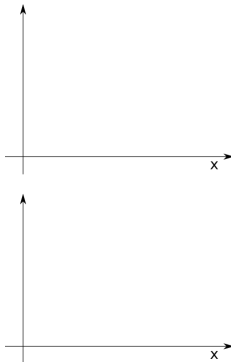
## Cons:

- problem definition can be difficult
- solution results and computation time dependent on initial estimation
- numerical solution must be computed for with defined equation parameters

# The Newton Raphson Method

## 4 General Use Cases

- The general problem of solving for the root of a non linear equation can be extended to 4 useful variants.
- With careful equation setup, all cases can be solved with the same systematic routine.



Solving Non-Linear Equations  
**The Newton-Raphson Method, Secant Method**  
The Bisection Method  
Mechanical Design Problem

Classification of Methods  
Taylor Series Derivation  
**The Newton Raphson Method**  
The Finite Difference  
Modified Newton-Raphson, Secant Method

# The Newton Raphson Method

# The Finite Difference

Our goal is to write a computer program to automate the Newton-Raphson method. We want our program to be (1) robust to different inputs and (2) user friendly.

# The Finite Difference

The Newton-Raphson method is not **purely numerical**, why?

- The Equation
- The Derivation

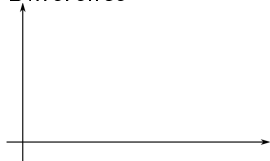
How can we avoid this issue?

*Hint:* Think about the title **secant** ...

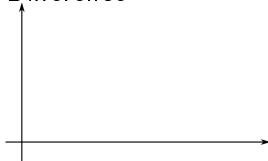
# The Finite Difference

This idea or technique is the foundation of a family of methods known as the *Finite Difference Methods*.

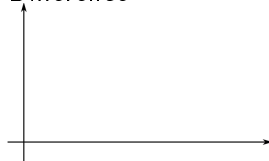
Forward  
Difference



Backwards  
Difference



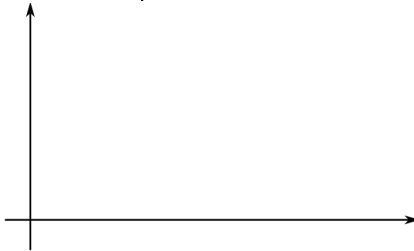
Central  
Difference



## Modified Newton-Raphson, Secant Method

The **secant** method is a modified version of the Newton-Raphson method which uses the Finite difference method to compute the slope values of the function to be solved.

Newton-Raphson



Secant



# Modified Newton-Raphson, Secant Method

## Algorithm Summary:

- - Step 1:
- - Step 2:
- - Step 3:



# Modified Newton-Raphson, Secant Method

**General Use Algorithm:** the secant method is a generalized, numerical tool for solving non linear equations

## Pros:

- -
- -
- -

## Cons:

- -
- -
- -

## Topic 3 - The Bisection Method

- Analytical vs. Numerical
- A Bracketing Method: Graphical Explanation
- Algorithm Description

# Analytical vs. Numerical

## Theoretical/Analytical Solution Techniques

- solving the equation using exact mathematics
- leads to an exact or *analytical* solution

## Numerical Solution Techniques

- approximating the solution to the equation using varying methods, or *algorithms*
- leads to a approximate solution
- a.k.a. *Numerical Method*

# Analytical vs. Numerical

# A Bracketing Method: Graphical Explanation

The Bisection method is a *bracketing method*.



# A Bracketing Method: Graphical Explanation



# A Bracketing Method: Graphical Explanation



# Algorithm Description

1

2

3

4



## Algorithm Description

See MATLAB example.

## Algorithm Description

**General Use Algorithm:** the bisection method can also be used to solve the general root finding problem

### Pros:

- -
- -
- -

### Cons:

- -
- -
- -

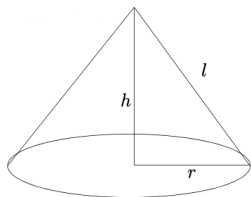
## Topic 3 - Mechanical Design Problem

- Problem Statement
- Mathematical Model
- Solution Approach
- Design

## Problem Statement

### A Mechanical Design Problem

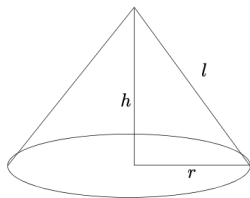
As an engineer you are asked to design a structure. The geometry of this structures is simple but certain properties are critical. Also you want to spend as little as possible on materials.



You are required to design is a cone with a surface area of exactly  $25m^2$  to a tolerance of  $0.1 m^2$  and a height of exactly  $1m$ . Your goal is to find the radius in meters.

## Problem Statement

What is the *mathematical model* of the cone?



surface area,  $s = \pi r l = \pi r \sqrt{h^2 + r^2}$

volume,  $v = \pi r^2 \frac{h}{3}$

# Problem Statement

How are you going to solve this problem?

## Problem Statement

How are you going to *design* the cone?

