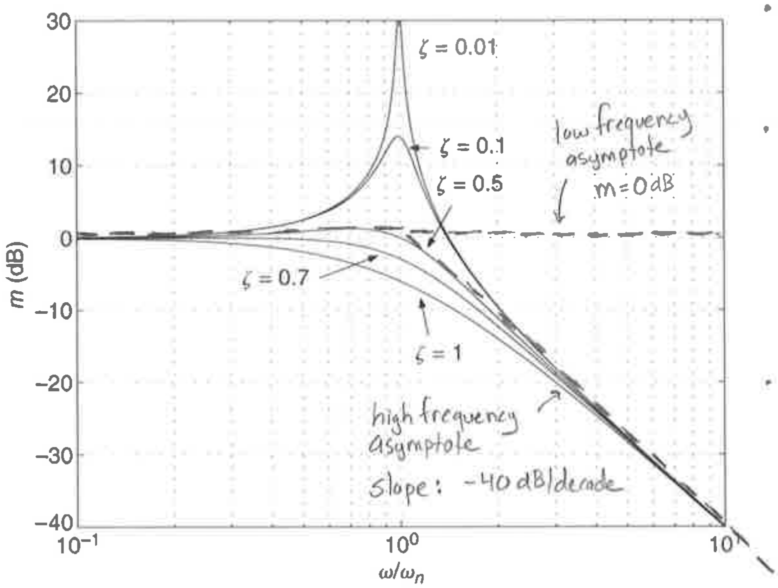
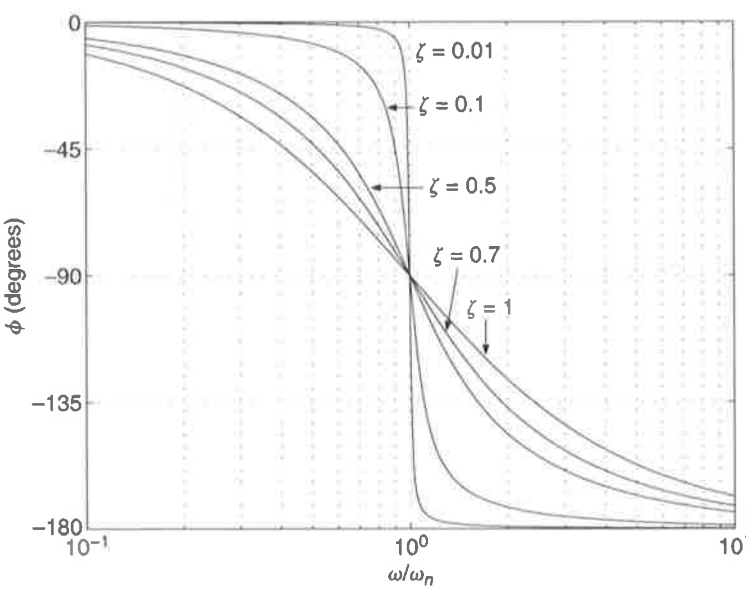
Hand Sketching 2nd Order Bode Plots

Bode Plots for $T(s) = \dfrac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$, $\zeta \le 1$ (underdamped)



(a)



(b)

Amplitude Ratio

Approximate m in three frequency ranges

- For $r \ll 1$ $(\omega \ll \omega_n)$, $m \approx -10\log(1) = 0$
  (low frequency asymptote)

- For $r \gg 1$ $(\omega \gg \omega_n)$, $m \approx -10\log(r^4 + 4\zeta^2 r^2)$
  $\approx -10\log r^4$
  $= -40\log r$

This gives a straight line with a slope of -40 dB/decade for the high frequency asymptote.

- For $r = 1$ $(\omega = \omega_n)$, we need to consider the phenomenon known as resonance. The resonance frequency is near ω_n and is given by:

$$\omega_r = \omega_n\sqrt{1-2\zeta^2} \qquad 0 \le \zeta \le 0.707$$

The peak value at the resonance frequency is

$$m_r = -20\log\left(2\zeta\sqrt{1-\zeta^2}\right) \qquad 0 \le \zeta \le 0.707$$

So for $\zeta \le 0.707$, one finds ω_r and m_r and adds the point to the plot. To either side of ω_r, the signal decays back to the low and high frequency asymptotes. For smaller values of the damping ratio, the peak is sharper.

We can also add the phase value at the resonance frequency to the phase plot by using the relation

$$\phi_r = -\tan^{-1}\left(\frac{\sqrt{1-2\zeta^2}}{\zeta}\right)$$

Phase

Approximate the phase in three frequency ranges

- For $r \ll 1$ $(\omega \ll \omega_n)$, $\phi \approx -\tan^{-1}\left(\frac{0}{1}\right) = 0°$
- For $r \gg 1$ $(\omega \gg \omega_n)$, $\phi \approx -\tan^{-1}\left(-\frac{r}{r^2}\right) = -\tan^{-1}\left(-\frac{1}{r}\right) = -180°$ (2nd order terms result in a 180° phase shift)
- For $r = 1$ $(\omega = \omega_n)$, $\phi = -\tan^{-1}\left(\frac{2\zeta}{0}\right) = -90°$

Note: the smaller the damping ratio, the steeper the slope through -90° at the breakpoint frequency (sharper curve).