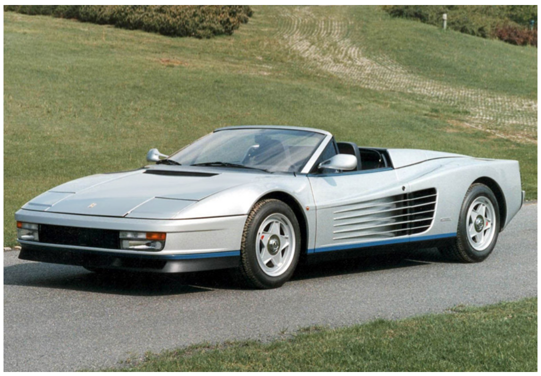
ME 3050 Lecture - Dynamic Modeling and Controls

Tristan W. Hill - Tennessee Technological University - Spring 2020

Ch. 8 - System Response in the Time Domain

- ### (8.1) Time Response of 1^{st} Order Systems

  - **Consider the model of the moving mass we derived.**



  - **The EOM is:**

$$m\dot{v} + cv = 0$$

  - **Solve for $v(t)$ using a method of your choice.**

  - **The method of Laplace Transforms is shown.**

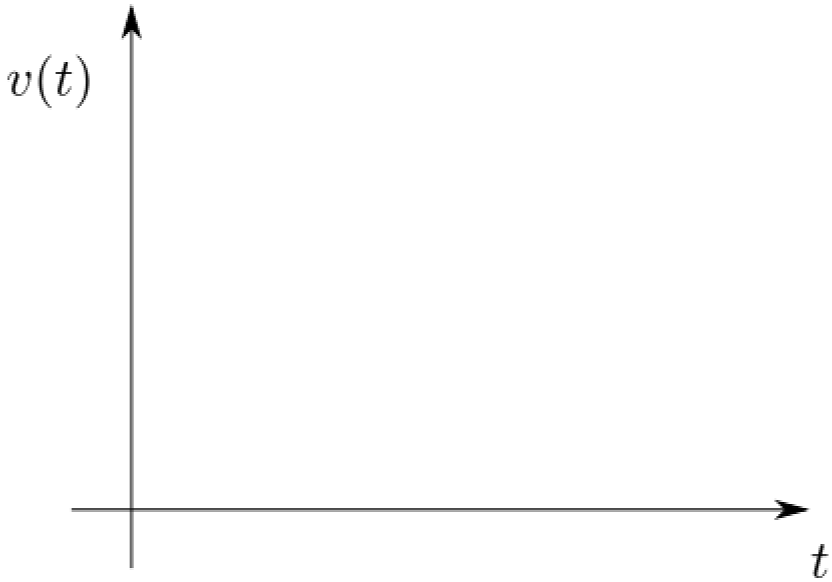
$$\mathcal{L}\{m\dot{v} + cv = 0\} \implies m[sV(s) - v(0)] + cV(s) = 0$$

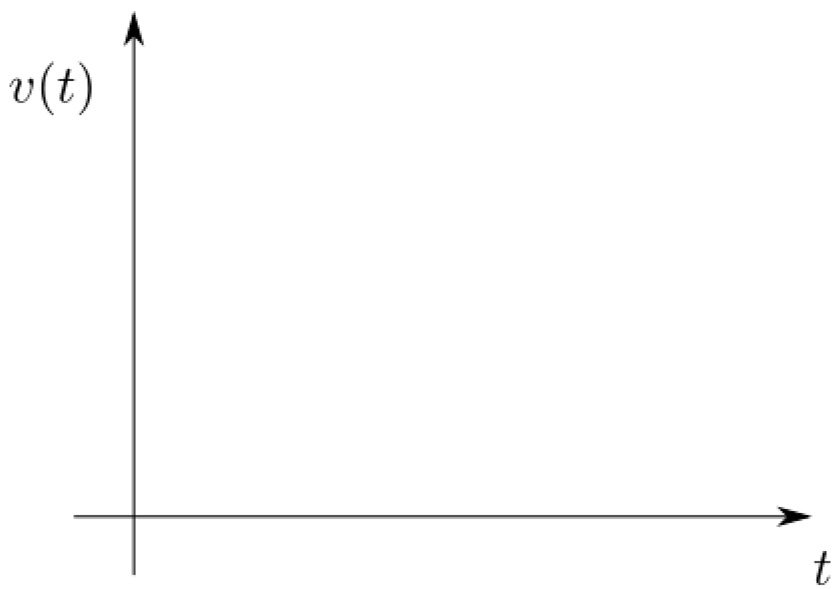
$$(ms + c)V(s) = \frac{mv(0)}{(ms+c)} = \frac{V(0)}{s+\frac{c}{m}}$$

  - **We can find the expected result from the table.**

$$v(t) = v(0)e^{-\frac{c}{m}t} = v(0)e^{-\frac{t}{\tau}} \qquad \text{with} \qquad \tau =$$

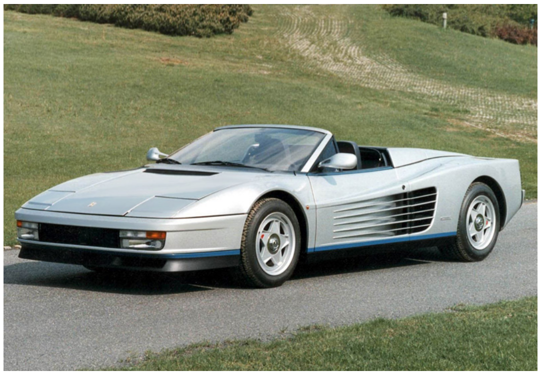
– **Sketch the System Response in the time Domain.**





– **Is this a stable system? What does that even mean?**

- **Consider the model subject to a Step Input, $f(t)$.**



$$m\dot{v} + cv = f(t)$$

$$f(t) = \begin{cases} 0 & t < 0 \\ F & t \geq 0 \end{cases}$$

- **The method of Laplace Transforms is shown.**

$$\mathcal{L}\{m\dot{v} + cv = F\} \implies m[sV(s) - v(0)] + cV(s) = \frac{F}{c}$$

$$(ms + c)V(s) = \frac{F}{s} + mv(0)$$

- **Partial Fraction Expansion leads to the following form.**

$$V(s) = \frac{F}{s(ms+c)} + \frac{mv(0)}{ms+c} \implies \frac{F}{s(ms+c)} = \frac{a}{s} + \frac{b}{ms+c}$$

- **'Cover up' to find the coefficients.**

$$a = \frac{F}{m\times 0+c} \qquad \text{and} \qquad b = \frac{F}{\frac{-c}{m}} = \frac{-Fm}{c}$$

- **This leads to a form that can be inverted with the table.**

$$V(s) = \frac{F}{c}\{\frac{1}{s} - \frac{1}{s+\frac{c}{m}}\} + \frac{v(0)}{s+\frac{c}{m}}$$

$$v(t) = \frac{F}{C}\{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

- **In these forms we can see the different components of the response.**

$$v(t) = \frac{F}{C}\{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

  – Forced Response

  – Free Response

  – Transient Response

  – Steady-State Response

- **Sketch the System Response in the time Domain.**