

Time Response - Lecture 4

ME3050 - Dynamics Modeling and Controls

April 15, 2020

Common Questions this Week

Lecture 4 - Common Questions this Week

- T1 - The Step Input
- T2 - Obtaining the Response Equations in Problem 1
- T3 - Using the Error function and the Time Constant
- T4 - Stability and the Roots

T1: The Step Input

The **step function** is a mathematical concept that represents an instant change.

Heavyside's Step Function



$$u_s(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$f_{step}(t) = F u_s(t) = \begin{cases} 0 & t < 0 \\ F & t \geq 0 \end{cases}$$

T2: Obtaining the Response Equations in Problem 1

You can see that each of the models in problem 1 is **linear** and **first order**. You do not have to re-derive (even though you could) the response equations but please reference where you found the equations you used. They are in the notes and in chapter 8.

T3: Finding the Time Value with Time Constant

In problem b) it asks for the time at which the response equation reaches a certain value. There are different ways to find this value.

- Plot the response curve and locate the value graphically. This might not be accurate.
- Use a 'root-finding' method to locate the value.
- Solve for the value with algebra. This is easy for this system.

T4: Stability of a Second Order System

Our model $m\ddot{x} + c\dot{x} + kx = 0$ is stable if the roots of the characteristic equation lie *to the right* of the imaginary axis of the complex plane (if the Real part of the root is positive). This makes sense because a positive α would cause the response to go to ∞ .

This is called the **Routh-Hurwitz stability conditions**

A second order model of the form $a_2s^2 + a_1s + a_0 = 0$

if a_2 , a_1 , and a_0 have the *same sign*.

This is in your reference handout and discussed on page 488 of System Dynamics, Palm III, Third Edition

T1: The Step Input

T2: Obtaining the Response Equations in Problem 1

T3: Finding the Time Value with Time Constant

T4: Stability of a Second Order System

T4: Stability of a Second Order System

References

- System Dynamics, Palm III, Third Edition - Chapter 8 - System Response in the Time Domain