

## Lecture Module - ODE Review

ME3050 - Dynamic Modeling and Controls

Mechanical Engineering

Tennessee Technological University

### Topic 3 - The Trial Solution Method

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- Exponential Assumption
- Complementary Solution
- Particular Solution
- Apply Initial Conditions

## Trial Solution Method

Use the **trial solution method** to solve the ODE.

This is an **analytical** method that you learned in calculus but it may have been called something different. In the Zill book it is called *Homogenous Linear ... Constant Coefficients* (4.3-4.4).

$$a_2y'' + a_1y' + a_0y = f(x)$$

Review  
**Exponential Assumption**  
Complementary Solution  
Particular Solution  
Apply Initial Conditions

# Exponential Assumption

## Complementary Solution

Step 1 - Find the **complementary part** of the solution from the left hand side of the ODE alone (LHS=0).

$$a_2 y'' + a_1 y' + a_0 y = f \quad \rightarrow \quad a_2 y'' + a_1 y' + a_0 y = 0$$

Assume an exponential solution for the complementary part.

$$y_{\text{complementary}} = y_c(x) =$$

Substitute this solution into the ODE (LHS=0).

Review  
Exponential Assumption  
**Complementary Solution**  
Particular Solution  
Apply Initial Conditions

## Complementary Solution

## Particular Solution

Step 2 - Find the **particular part** of the solution from the entire equation (LHS=RHS).

$$a_2 y'' + a_1 y' + a_0 y = f$$

The *form of the particular part* follows the RHS of the ODE.

$$y_{\text{particular}} = y_p(x) =$$

Substitute this solution into the ODE above and solve for any unknown constants in  $y_p(x)$ .

Review  
Exponential Assumption  
Complementary Solution  
**Particular Solution**  
Apply Initial Conditions

## Particular Solution



## Apply Initial Conditions

Step 3 - Now combine the **complementary** and **particular** solutions through *superposition*.

$$y(x) = y_c(x) + y_p(x) =$$

The ODE is first order and we have \_\_\_\_\_ unknown. Coincidence?

$$y(x) =$$

This **initial value problem** requires \_\_\_\_\_ initial condition.

## Apply Initial Conditions

$$y(x = 0) =$$

$$y'(x = 0) =$$

## Apply Initial Conditions

What does the solution look like this time?

$$y(x) =$$



## Summary - 3 Cases

If the differential equation is first and linear, the complementary solution takes the following form.

$$y(x) = Ae^{sx}$$

## Summary - 3 Cases

If the differential equation is second order and linear, the **complementary solution** takes one of the following forms.

Case 1:  $s_1, s_2 \in \mathbb{R}$  ,  $s_1 \neq s_2$

$$y(x) = c_1 e^{s_1 x} + c_2 e^{s_2 x}$$

Case 2:  $s_1, s_2 \in \mathbb{R}$  ,  $s_1 = s_2 = s$

$$y(x) = c_1 e^{sx} + c_2 x e^{sx}$$

Case 3:  $s_1, s_2 \notin \mathbb{R}$  ,  $s_1, s_2 = \alpha \pm \beta$

$$y(x) = e^{\alpha x} (c_1 \cos(\beta x) + c_2 \sin(\beta x))$$

## Summary - 3 Cases

The **particular solution** takes the form of the right hand side of the equation.

Example	Form	Particular Solution
$\dots = 10$	Constant	$y_p = B$
$\dots = 12x$	Linear	$y_p = Bx + C$
$\dots = 20e^{2x}$	Exponential	$y_p = Be^{2x}$
$\dots = a\cos(\beta x)$	Sinusoidal	$B\cos(\beta x) + C\sin(\beta x)$
$\dots = a\sin(\beta x)$	Sinusoidal	$B\cos(\beta x) + C\sin(\beta x)$