

Laplace Transform Pairs (Table 2.2.1)

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$X(s)$	$x(t), t \geq 0$
1. 1	$\delta(t)$, unit impulse
2. $\frac{1}{s}$	$u_s(t)$, unit step
3. $\frac{c}{s}$	constant, c
4. $\frac{e^{-sD}}{s}$	$u_s(t - D)$, shifted unit step
5. $\frac{n!}{s^{n+1}}$	t^n
6. $\frac{1}{s + a}$	e^{-at}
7. $\frac{1}{(s + a)^n}$	$\frac{1}{(n - 1)!} t^{n-1} e^{-at}$
8. $\frac{b}{s^2 + b^2}$	$\sin bt$
9. $\frac{s}{s^2 + b^2}$	$\cos bt$
10. $\frac{b}{(s + a)^2 + b^2}$	$e^{-at} \sin bt$
11. $\frac{s + a}{(s + a)^2 + b^2}$	$e^{-at} \cos bt$
12. $\frac{a}{s(s + a)}$	$1 - e^{-at}$
13. $\frac{1}{(s + a)(s + b)}$	$\frac{1}{b - a} (e^{-at} - e^{-bt})$
14. $\frac{s + p}{(s + a)(s + b)}$	$\frac{1}{b - a} [(p - a)e^{-at} - (p - b)e^{-bt}]$
15. $\frac{1}{(s + a)(s + b)(s + c)}$	$\frac{e^{-at}}{(b - a)(c - a)} + \frac{e^{-bt}}{(c - b)(a - b)} + \frac{e^{-ct}}{(a - c)(b - c)}$
16. $\frac{s + p}{(s + a)(s + b)(s + c)}$	$\frac{(p - a)e^{-at}}{(b - a)(c - a)} + \frac{(p - b)e^{-bt}}{(c - b)(a - b)} + \frac{(p - c)e^{-ct}}{(a - c)(b - c)}$
17. $\frac{b}{s^2 - b^2}$	$\sinh bt$
18. $\frac{s}{s^2 + b^2}$	$\cosh bt$
19. $\frac{a^2}{s^2(s + a)}$	$at - 1 + e^{-at}$
20. $\frac{a^2}{s(s + a)^2}$	$1 - (at + 1)e^{-at}$
21. $\frac{\omega_n^2}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$\frac{\omega_n}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \omega_n \sqrt{1 - \zeta^2} t$
22. $\frac{s}{s^2 + 2\zeta\omega_n s + \omega_n^2}$	$-\frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \left(\omega_n \sqrt{1 - \zeta^2} t - \phi \right), \phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$
23. $\frac{\omega_n^2}{s(s^2 + 2\zeta\omega_n s + \omega_n^2)}$	$1 - \frac{1}{\sqrt{1 - \zeta^2}} e^{-\zeta\omega_n t} \sin \left(\omega_n \sqrt{1 - \zeta^2} t + \phi \right)$
24. $\frac{1}{s[(s + a)^2 + b^2]}$	$\frac{1}{a^2 + b^2} \left[1 - \left(\frac{a}{b} \sin bt + \cos bt \right) e^{-at} \right], \phi = \tan^{-1} \frac{\sqrt{1 - \zeta^2}}{\zeta}$
25. $\frac{b^2}{s(s^2 + b^2)}$	$1 - \cos bt$
26. $\frac{b^3}{s^2(s^2 + b^2)}$	$bt - \sin bt$
27. $\frac{2b^3}{(s^2 + b^2)^2}$	$\sin bt - bt \cos bt$
28. $\frac{2bs}{(s^2 + b^2)^2}$	$t \sin bt$
29. $\frac{s^2 - b^2}{(s^2 + b^2)^2}$	$t \cos bt$
30. $\frac{s}{(s^2 + b_1^2)(s^2 + b_2^2)}$	$\frac{1}{b_2^2 - b_1^2} (\cos b_1 t - \cos b_2 t), \quad (b_1^2 \neq b_2^2)$
31. $\frac{s^2}{(s^2 + b^2)^2}$	$\frac{1}{2b} (\sin bt + bt \cos bt)$

Properties of the Laplace Transform (Table 2.2.2)

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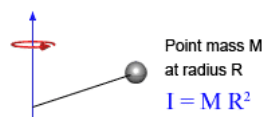
$x(t)$	$X(s) = \int_0^{\infty} f(t)e^{-st} dt$
1. $af(t) + bg(t)$	$aF(s) + bG(s)$
2. $\frac{dx}{dt}$	$sX(s) - x(0)$
3. $\frac{d^2x}{dt^2}$	$s^2X(s) - sx(0) - \dot{x}(0)$
4. $\frac{d^nx}{dt^n}$	$s^nX(s) - \sum_{k=1}^n s^{n-k}g_{k-1}$ $g_{k-1} = \left. \frac{d^{k-1}x}{dt^{k-1}} \right _{t=0}$
5. $\int_0^t x(t) dt$	$\frac{X(s)}{s} + \frac{g(0)}{s}$ $g(0) = \left. \int x(t) dt \right _{t=0}$
6. $x(t) = \begin{cases} 0 & t < D \\ g(t-D) & t \geq D \end{cases}$ $= u_s(t-D)g(t-D)$	$X(s) = e^{-sD}G(s)$
7. $e^{-at}x(t)$	$X(s+a)$
8. $tx(t)$	$-\frac{dX(s)}{ds}$
9. $x(\infty) = \lim_{s \rightarrow 0} sX(s)$	
10. $x(0+) = \lim_{s \rightarrow \infty} sX(s)$	

For Entries 2, 3, 4, and 5, if $x \neq 0$ for $t < 0$, then replace the initial conditions at $t = 0$ with the pre-initial conditions at 0^- .

Mass Moments of Inertia Table

Mass Moment of Inertia

Also called Moment of Inertia, Angular Mass



thin hoop or ring of radius R & mass M:	thick ring of inner radius R1, outer radius R2, and mass M:	solid cylinder or disc of radius R and mass M:	flat plate with sides of length A and B and mass M:
MR^2	$M(r_1^2 + r_2^2) / 2$	$MR^2 / 2$	$M(A^2 + B^2) / 12$
solid sphere of radius R and mass M:	thin-walled hollow sphere of radius R & mass M:	slender rod of length L and mass M, spinning around center:	slender rod of length L and mass M, spinning around end:
$(2/5)MR^2$	$(2/3)MR^2$	$ML^2 / 12$	$ML^2 / 3$