# Module 5 - Rotation Systems

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

## Topic 1 - Mass Moment of Inertia

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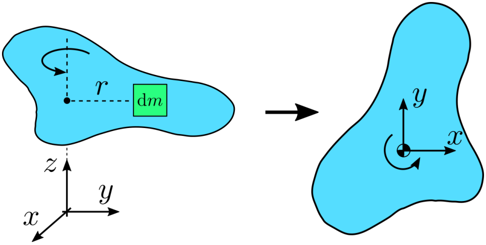
- Mathematical Defintion
- Table of Common Geometries
- The Parallel Axis Theorem
- Equivalent Inertia

# Mathematical Defintion

The mass moment of inertia is the combined resistance to angular acceleration from about an axis due to the mass of a body.




$I = \int_0^M r^2 dm$

This is done about an axis through the mass center which is also the geometric center for a uniform mass. For planar motion only rotation about the z-axis is considered.

Image: T. Hill

# Mathematical Defintion

If the body is considered as discrete point masses the mass moment of inertia can be easily found as summation. However we need it for a continuous rigid bodies.

$$I = \Sigma m_i r_i^2$$
$$= m_1 r_1^2 + m_1 r_1^2 + ... + m_i r_i^2 + ... + m_n r_n^2$$

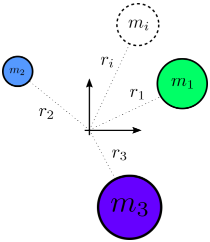
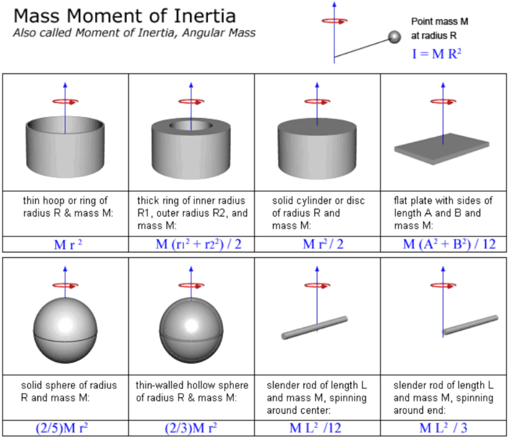



Image: T. Hill

# Table of Common Geometries

In situation where the point mass assumption is not appropriate the mass moment of inertia of common geometries is tabulated.

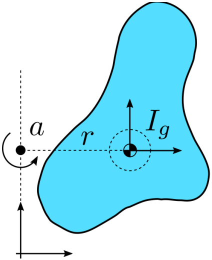



Images:

# The Parallel Axis Theorem

Further the object may be rotation about an axis that is removed from the geometrical center. In this situation the moment of inertia about the new axis is found using the parallel axis theorem.

$I_a = I_g + mr_{ag}^2$



# Equivalent Inertia

Some systems composed of translating and rotating parts whose motions are directly coupled can be modeled as as a purely translational or as a purely rotational system, by using the concepts of equivalent mass and inertia. These models can be derived using kinetic energy equivalence.

The textbook discusses several examples we will not discuss this further in ME3050.

Text: System Dynamics, 3rd Edition , Palm