Module 10 - The Laplace Transform

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 2 - Laplace Transforms Method

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- Step 1 - Apply Laplace Transform
- Step 2 - Solve for $X(s)$
- Step 3 - Rearrange to Find Invertable Form
- Step 4 - Invert for Final Answer

Step 1 - Apply Laplace Transform

Example:
Solve the first order differential equation using the Laplace Transforms Method with the initial condition given. =

$$4\dot{x} = sin\left(t\right) \qquad with \qquad x\left(t=0\right) = x_0$$

Apply the Laplace Transform to both sides of the differential equation.

Step 2 - Solve for $X(s)$

This step can seem open ended...

$$X(s) = \frac{1}{4s\left(s^2+1\right)} + \frac{x_0}{s}$$

Step 3 - Rearrange to Find Invertable Form

Write $X(s)$ in a form that can be inverted using the table of Laplace transform pairs. This typically involves partial fraction decomposition.

$$\frac{1}{4s\left(s^2+1\right)} = \frac{1/4}{s\left(s^2+1\right)} = \frac{a}{s} + \frac{bs+c}{s^2+1}$$

Mulitply through by the denominator $4s\left(s^2+1\right)$:

$$1 = 4as\left(s^2+1\right) + 4s\left(bs+c\right) = 4\left(a+b\right)s^2 + 4cs + 4a$$

Solve for the coefficients by *equating coefficients*.

$$\left(a+b\right) = 0 \quad c = 0 \quad a = \frac{1}{4} \implies a = \frac{1}{4} \quad b = -\frac{1}{4} \quad c = 0$$

Step 4 - Invert for Final Answer

Substitute the coefficients into $X(s)$,

$$X(s) = \frac{x_0}{s} + \frac{1}{4s} - \frac{s}{4\left(s^2+1\right)}$$

and use the inverse transform to solve for $x(t)$. Use the Table.

$$\mathcal{L}^{-1}\left(X\left(s\right)\right) = x(t) =$$

$$= x_0 + \frac{1}{4} - \frac{1}{4} cos\left(t\right) = x_0 + \frac{1}{4}\left(1 - cos\left(t\right)\right)$$

This method works for complex problems but it can get messy...