Module 13 - Higher Order Systems

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 1 - Deriving the 2DOF Model

Topic 1 - Deriving the 2DOF Model

- Motivation - Physical Models
- Model Description
- Newton's Approach
- Equations of Motion

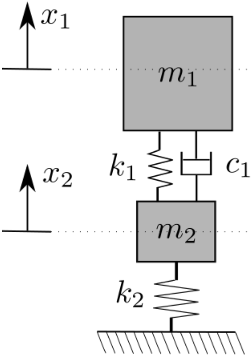
Motivation - Physical Models

Higher Order Models - Mechanical systems involve the interactions between multiple rigid bodies. This can be seen in many examples.

- Automobile Suspension
- Beam Deflection (FEA)
- Tether Based Space Travel
- Virtually Everything!

Motivation - Physical Models

Automobile Suspension - This is a common approximation of a typical automobile suspension known as the *quarter car model*.

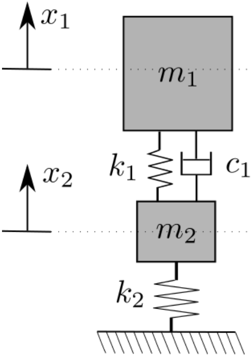


Is this valid?

Why? Why Not?

What does the response look like? How can you find out?

Model Description



List Assumptions:

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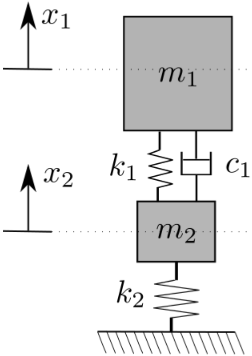
-
-

Newton's Approach

Draw one free body diagram for each body.



Newton's Approach

Write Newton's Second Law for each body.

$$\underline{+\uparrow \sum F_{x_1} = ma_1}$$

$$\underline{+\uparrow \sum F_{x_2} = ma_2}$$

Equations of Motion

Equations of Motion

Equation of Motion for Mass 1:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0$$

Equation of Motion for Mass 2:

$$m_2\ddot{x}_2 + k_2x_2 - c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = 0$$

It is common to write the equations of motion as a matrix equation. If you are unsure if you have the correct form just multiply it out and should match.

Equations of Motion

Re-write the equations of motion in matrix form.

$$\boldsymbol{M}\ddot{\boldsymbol{x}} + \boldsymbol{C}\dot{\boldsymbol{x}} + \boldsymbol{K}\boldsymbol{x} = \boldsymbol{F}(\boldsymbol{t})$$

References

- System Dynamics, Palm III, Third Edition - Chapter 4 - Spring and Damper Elements in Mechanical Systems - pg. 208