Lecture Module - ODE Review

ME3050 - Dynamic Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 3 - The Trial Solution Method

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- Exponential Assumption
- Complementary Solution
- Particular Solution
- Apply Initial Conditions

Trial Solution Method

Use the **trial solution method** to solve the ODE.

This is an **analytical** method that you learned in calculus but it may have been called something different. In the Zill book it is called *Homogenous Linear ... Constant Coefficients (4.3-4.4)*.

$$a_2 y'' + a_1 y' + a_0 y = f(x)$$

Exponential Assumption

Complementary Solution

Step 1 - Find the **complementary part** of the solution from the left hand side of the ODE alone (LHS=0).

$$a_2 y'' + a_1 y' + a_0 y = f \qquad \rightarrow \qquad a_2 y'' + a_1 y' + a_0 y = 0$$

Assume an exponential solution for the complementary part.

$$y_{complementary} = y_c(x) =$$

Substitute this solution into the ODE (LHS=0).

Complementary Solution

Particular Solution

Step 2 - Find the **particular part** of the solution from the entire equation (LHS=RHS).

$$a_2 y'' + a_1 y' + a_0 y = f$$

The *form of the particular part* follows the RHS of the ODE.

$$y_{particular} = y_p(x) =$$

Substitute this solution into the ODE above and solve for any unknown constants in $y_p(x)$.

Particular Solution

Apply Initial Conditions

Step 3 - Now combine the **complementary** and **particular** solutions through *superposition*.

$$y(x) = y_c(x) + y_p(x) =$$

The ODE is first order and we have ______ unknown. Coincidence?

$$y(x) =$$

This **initial value problem** requires ______ intial condition.

Apply Initial Conditions

$y(x = 0) =$

$y'(x = 0) =$

Apply Initial Conditions

What does the solution look like this time?

$$y(x) =$$

Summary - 3 Cases

If the differential equation is first and linear, the complementary solution takes the following form.

$$y(x) = Ae^{sx}$$

Summary - 3 Cases

If the differential equation is second order and linear, the **complementary solution** takes one of the following forms.

Case 1: $s_1, s_2 \in \mathbb{R}$, $s_1 \neq s_2$

$$y(x) = c_1 e^{s_1 x} + c_2 e^{s_2 x}$$

Case 2: $s_1, s_2 \in \mathbb{R}$, $s_1 = s_2 = s$

$$y(x) = c_1 e^{sx} + c_2 x e^{sx}$$

Case 3: $s_1, s_2 \notin \mathbb{R}$, $s_1, s_2 = \alpha \pm \beta$

$$y(x) = e^{\alpha x} \left(c_1 cos\left(\beta x\right) + c_2 sin\left(\beta x\right)\right)$$

Summary - 3 Cases

The **particular solution** takes the form of the right hand side of the equation.

Example	Form	Particular Solution
$... = 10$	Constant	$y_p = B$
$... = 12x$	Linear	$y_p = Bx + C$
$... = 20e^{2x}$	Exponential	$y_p = Be^{2x}$