

Module 4 - Energy Methods

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 1 - The Conservation of Energy

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- Brief History
- Second Law Derivation
- Non-Conservative Forces
- Engineering Applications

Brief History

... the law of conservation of energy states that the total energy of an isolated system remains constant; it is said to be conserved over time. This law, first proposed and tested by Émilie du Châtelet, means that energy can neither be created nor destroyed; rather, it can only be transformed or transferred from one form to another.



Text: Wikipedia, Image: Wikipedia

Brief History

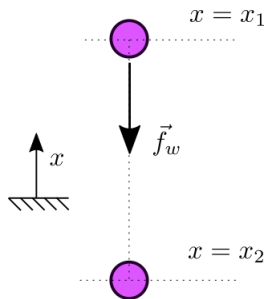


Text: Wikipedia

... however, special relativity showed that mass is related to energy and vice versa by $E = mc^2$, and science now takes the view that mass-energy as a whole is conserved. Theoretically, this implies that any object with mass can itself be converted to pure energy, and vice versa, though this is believed to be possible only under the most extreme of physical conditions ...

Second Law Derivation

A mass moves in the x direction with only the force of gravity acting on it. Newton's Second Law combined with the definition of differential work done by a force through a distance gives a relation between kinetic energy and work done by the external force.



$$\mathbf{f} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

$$dW = \mathbf{f} d\mathbf{x}$$

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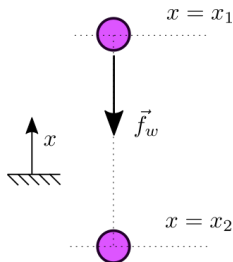
$$\rightarrow dW = m v_x dv_x$$

$$W_{12} = \int_{x_1}^{x_2} m v_x dv_x = \frac{1}{2} m v_x^2 \Big|_{x_1}^{x_2}$$

$$W_{12} = \frac{1}{2} m v_{x_2}^2 - \frac{1}{2} m v_{x_1}^2 = KE_1 - KE_2$$

Second Law Derivation

Alternativley, substitute the distance traveled into the differential work relation. Now you are left with the familiar relation between work done by gravity and the potential energy in the system.



$$dW = \mathbf{f} d\mathbf{x}$$

$$W_{12} = \int_{x_1}^{x_2} \mathbf{f} d\mathbf{x} = (-mg)|_{x_1}^{x_2} = (-mg)(x_2 - x_1)$$

$$W_{12} = -mg(x_2 - x_1)$$

Now we can see that

$$W_{12} = -mg(x_2 - x_1) = \frac{1}{2}mv_{x_2}^2 - \frac{1}{2}mv_{x_1}^2$$

$$\Delta KE + \Delta PE = 0$$

Second Law Derivation

The resulting equation clearly shows the conservation of energy for a 1DOF particle moving in the direction of gravity. This derivation is often shown in more generalized form but this version is easy to understand and will work for our purposes.

$$\Delta KE + \Delta PE = 0 \quad \text{or} \quad KE + PE = \text{Constant}$$

with $KE = \frac{1}{2}mv^2$ and $PE = mgh$

While deriving dynamics (EOMs) for many systems the alternate form shown the right will be used.

Non-Conservative Forces

Lastly, this derivation relies on the fact that f is a conservative force that can be derived from a function $V(x)$ which is defined as follows.

$$V(x) = \int dV = -f(x)dx$$

In this class we will only use the energy based methods in systems with no non conservative forces. You discussed this idea in your physics class.

Engineering Applications

The conservation of mechanical energy can be used to derive the equations of motion for many dynamic systems.