ME 3050 Lecture - Laplace Transform and Properties

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• The Laplace Transform is an Integral Transform:

Given a function $x(t)$ in the time domain where $t \geq 0$,
the Laplace Transform is defined as follows:

$$X(s) = \mathcal{L}\left\{x(t)\right\} = \int_0^\infty x(t)e^{-st}dt$$

And its inverse is similarly defined as:

$$\mathcal{L}^{-1}\left\{X(s)\right\} = x(t)$$

The Laplace Domain variable s is a complex number: $s = \sigma + j\omega$

It is useful to find the laplace transform of the derivative of a function:

$$\begin{aligned}
\mathcal{L}\left\{\tfrac{d}{dt}(x(t))\right\} = \mathcal{L}\left\{\dot{x}(t)\right\} &= s\,\mathcal{L}\left\{x(t)\right\} - x(t=0) \\
&= sX(s) - x(t=0) \\
&= sX(s) - x_0
\end{aligned}$$

$$\begin{aligned}
\mathcal{L}\left\{\tfrac{d^2}{dt^2}(x(t))\right\} = \mathcal{L}\left\{\ddot{x}(t)\right\} &= s^2\,\mathcal{L}\left\{x(t)\right\} - sx(t=0) - \dot{x}(t=0) \\
&= s^2X(s) - sx(t=0) - \dot{x}(t=0) \\
&= s^2X(s) - sx_0 - \dot{x}_0
\end{aligned}$$