Module 10 - The Laplace Transform

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 1 - Definition of the Laplace Transform

Topic 1 - Definition of the Laplace Transform

- An Integral Transform
- —
- —
- —

The Laplace Transform is an Integral Transform:

Given a function $x(t)$ in the time domain where $t \geq 0$, the Laplace Transform is defined as follows:

$$X(s) = \mathcal{L}\{x(t)\} = \int_0^\infty x(t)e^{-st}dt$$

And its inverse is similarly defined as:

$$\mathcal{L}^{-1}\{X(s)\} = x(t)$$

The Laplace Domain variable s is a complex number:

$$s = \sigma + j\omega$$

It is useful to find the laplace transform of the derivative of a function:

$$\mathcal{L}\{\frac{d}{dt}(x(t))\} = \mathcal{L}\{\dot{x}(t)\} = s\mathcal{L}\{x(t)\} - x(t=0)$$

$$= sX(s) - x(t=0)$$

$$= sX(s) - x_0$$

$$\mathcal{L}\{\frac{d^2}{dt^2}(x(t))\} = \mathcal{L}\{\ddot{x}(t)\} = s^2\mathcal{L}\{x(t)\} - sx(t=0) - \dot{x}(t=0)$$