ME 3050 Lecture - Laplace Transforms

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- ## Partial Fraction Expansion leads to a General Form:

$$X(s) = \frac{N(s)}{D(s)} = \frac{b_m s^m + b_{m-1} + ... + b_1 s + b_0}{s^n + a_{n-1} s^{n-1} + ... + a_1 s + a_0} \quad n \geq m$$

- ## Case 1 - Distinct Roots: n roots are real and distinct

The general form is factored:

$$X(s) = \frac{N(s)}{(s+r_1)(s+r_2)...(s+r_n)}$$

The fraction will expand to:

$$X(s) = \frac{C_1}{(s+r_1)} + \frac{C_2}{(s+r_2)} + ... + \frac{C_n}{(s+r_n)}$$

Where:

$$C_i = \lim_{s \to -r_i} \{X(s)(s+r_i)\}$$

And this leads to a solution:

$$x(t) = C_1 e^{-r_1 t} + C_2 e^{-r_2 t} + ... + C_n e^{-r_n t}$$

- **Case 2 - Repeated Roots:**
p number of roots have the same value $(s=-r)$ and remaining roots are distinct and real distinct

$$X(s) = \frac{N(s)}{(s+r_1)^p(s+r_{p+1})(s+r_{p+2})...(s+r_n)}$$

The fraction will expand to:

$$X(s) = \frac{C_1}{(s+r_1)^p} + \frac{C_2}{(s+r_1)^{p-1}} + ...$$

$$+\frac{C_p}{(s+r_1)} + \frac{C_{p+1}}{(s+r_{p+1})} + ... + \frac{C_n}{(s+r_n)}$$

Coefficients for the repeated root are:

$$C_1 = \lim_{s\to -r_i} \{X(s)(s+r_i)^p\}$$

$$C_2 = \lim_{s\to -r_i} \{\frac{d}{ds}X(s)(s+r_i)^p\}$$

$$C_i = \lim_{s\to -r_i} \{\frac{1}{(i-1)!}\frac{d^{(i-1)}}{ds^{(i-1)}}X(s)(s+r_i)^p\}$$

Coefficients for the distinct roots are the same as in Case 1:

And this leads to a solution:

$$x(t) = C_1\frac{t^{p-1}}{(p-1)!}e^{-r_1 t} + C_2\frac{t^{p-2}}{(p-2)!}e^{-r_1 t} + ...$$

$$... + C_p e^{-r_1 t} + C_{p+1}e^{-r_{p+1}t} ... + C_n e^{-r_n t}$$