Frequency Response - Lecture 2

ME3050 - Dynamics Modeling and Controls

April 19, 2020

The Bode Diagram

Lecture 2 - The Bode Diagram

- Review Frequency Response
- The Bode Diagram
- Frequency Response in MATLAB

Harmonic Input Function

The term **frequency response** is used to describe a system's response to a periodic input. Frequency response analysis focuses on a system's response to *harmonic* input such as sines and cosines. The input (forcing) function is written below.

$$f(t) = A\sin(\omega t)$$

Amplitude of the Input,	A	(N)
Frequency of Input,	ω	$\left(\frac{rad}{s}\right)$

First Order Frequency Response

The steady state response we derived is shown. Remember, after some amount of time passes, the transient term will disappear leaving just the sinusoidal terms.

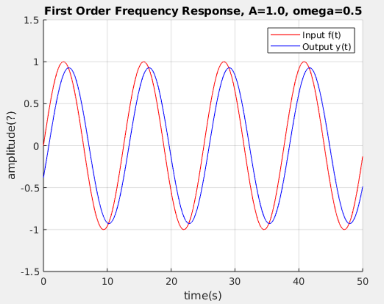
$$y_{ss}(t) = A|T(j\omega)|sin(\omega t + \angle T(j\omega)) = MAsin(\omega t + \phi)$$

The magintude ratio and phase shift can be found from $T(j\omega)$.

$$M(\omega) = |T(j\omega)| = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

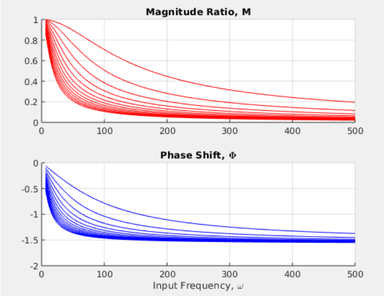
$$\phi(\omega) = \angle T(j\omega) = -tan^{-1}(\omega\tau)$$

Graph of Frequency Response



The amplitude of the response is determined by the input frequency.

Dependence on Input Frequency



You can see that the magnitude ratio decreases as the input frequency increases. The individual curves represent systems with different time constants.

Review Properties of Logarithms

Basic Properties of Logarithms:

Multiplication $log(pq) = log(p) + log(q)$

Division $log(\frac{x}{y}) = log(x) - log(y)$

Power $log(x^n) = nlog(x)$

Units of Decibels for Magnitude:

$m(dB) = 10log(M^2) = 20log(M)$ convert back: $M = 10^{\frac{m(dB)}{20}}$

Decibel (dB), unit for expressing the ratio between two physical quantities, usually amounts of acoustic or electric power, or for measuring the relative loudness of sounds. One decibel (0.1 bel) equals 10 times the common logarithm of the power ratio. - Britannica.com

Magnitude Ratio on a Logarithmic Scale

These relationships are more useful shown on a logarithmic scale. We can make use of the properties of logarithms in our analysis.

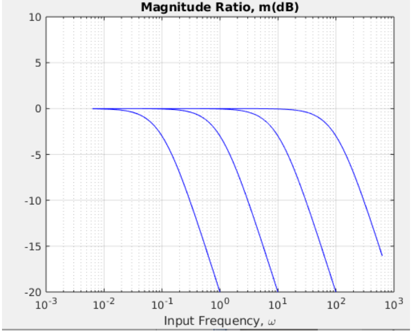
$$m\,(dB) = 20log\left(\frac{1}{\sqrt{1+\omega^2\tau^2}}\right) = 20\left(log\,(1) - log\sqrt{1+\omega^2\tau^2}\right)$$

$$m\,(dB) = 20log\,(1) - 10log\left(1+\omega^2\tau^2\right) = -10log\left(1+\omega^2\tau^2\right)$$

$$m\,(dB) = -10log\left(1+\omega^2\tau^2\right)$$

magnitude ratio in decibels

Magnitude Ratio on a Logarithmic Scale

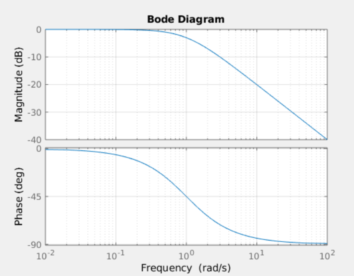


This is a Bode plot. It seems abstract but there is some very useful information shown.

Hendrik Bode (1905-1982)

Bode Plot in MATLAB

MATLAB has a built it tool for making Bode plots.




```
figure(1)
sys=tf(1,[tau(3) 1])
bode(sys);grid on
```

So what? What can you do with a Bode diagram?

References

- System Dynamics, Palm III, Third Edition - Chapter 9 - System Response in the Frequency Domain