# Lecture Module - ODE Review

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

## Module 2 - ODE Review

## Module 2 - ODE Review

- Topic 1 - ODE Review
- Topic 2 - Separation of Variables
- Topic 3 - The Trial Solution Method

## Topic 1 - ODE Review

- Definitions and Classification
- Engineering Applications
- Example

# What is a Differential Equation?

*Definition:*

A **differential equation** is an equation which describes a function

and one or more of its ______________________________ of the

______________________________ ______________________________

with respect to the ____________________________________.

# Standard Form of an ODE

Ordinary Differential Equations are written in the following form.

$$a_n \frac{dy^{(n)}}{d^{(n)}x} + a_{n-1}\frac{dy^{(n-1)}}{d^{(n-1)}x} + \ldots + a_2 \frac{dy^2}{d^2x} + a_1 \frac{dy}{dx} + a_0 y = f(x)$$

The apostrophe is commonly used for the derivative.

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_2 y'' + a_1 y' + a_0 y = f(x)$$

If time is the independent variable the equation changes slightly.

# Is the differential equation ordinary or partial?

An **ordinary** differential equation has __________ independent variable and __________ dependent variable.

A **partial** differential equation has _________________________ independent variable __________ dependent variable.

# What is the order of the equation?

The **order** of a differential equation is the

________________________ ________________________

present in the equation.

# What is the degree of the equation?

The **degree** of a differential equation is the _______________

of its highest derivative, after the equation has been made rational

and integral in all of its derivatives.

# Is the differential equation linear or non-linear?

An ordinary differential equation is ________________ if the following statements are true.

1. *The dependent variable and its derivatives are of the first degree.*
2. *The coefficients are constants or dependent on the independent variable.*

If either rule is broken, the equation is _____-________________.

# Engineering Applications

Differential equations are used to describe physical systems in many areas of engineering. An equation that represents a physical (or theoretical) system is known as a ________________________________ ________________________________.

- Solid Mechanics
- Kinematics and Dynamics
- Heat Transfer and Thermodynamics
- Fluid Mechanics

# Engineering Applications

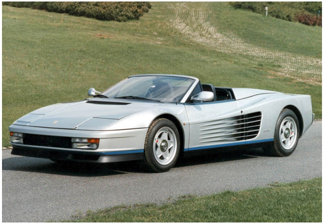
# Example

Newton's Second Law

$\Sigma F = ma$

leads to an *equation of motion.*

$\dot{y} + \frac{c}{m} y = f(t)$



## Topic 2 - Separation of Variables

- Review
- Separation of Variables
- Example

# What is a Differential Equation? Solution?

A **differential equation** is an equation which describes a function and one or more of its ________________________ of the

________________________ ________________________

with respect to the _____________________________.

The **solution** to a differential equation describes the

____________________ ____________________ as a function

of the ____________________ ____________________.

# Separation of Variables

Separation of Variables: **analytical** for solving differential equations

- Step 1 - Separate
- Step 2 - Integrate
- Step 2 - Solve for Unknowns

# Separation of Variables

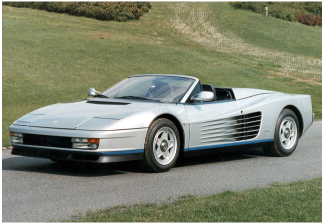
Alternative methods to find solution:

- -
- -
- -

# Problem Statement

Remember our example from the previous lecture?

$$\dot{v} + \frac{c}{m}v = f(t)$$



We are going to find an **analytical solution** to this problem.

# Separation of Variables

Assume the external force $f(t)$ is zero. Use separation of variables to find the solution $v(t)$.

$$\dot{v} + \frac{c}{m}v = 0$$

# Solution

The solution $v(t)$ has been found. What does it mean? What do we do next?

$$v(t) =$$

# Solution

The solution $v(t)$ has been found. What does it mean? What do we do next?

$$v(t) =$$

# Graph of Solution

What does the solution look like?

$$v(t) = v_0 e^{-\frac{c}{m}t}$$

## Topic 3 - The Trial Solution Method

- Exponential Assumption
- Complementary Solution
- Particular Solution
- Apply Initial Conditions
- Summary - 3 Cases

# Trial Solution Method

Use the **trial solution method** to solve the ODE.

This is an **analytical** method that you learned in calculus but it may have been called something different. In the Zill book it is called *Homogenous Linear ... Constant Coefficients (4.3-4.4)*.

$$a_2 y'' + a_1 y' + a_0 y = f(x)$$

# Exponential Assumption

# Complementary Solution

Step 1 - Find the **complementary part** of the solution from the left hand side of the ODE alone (LHS=0).

$$a_2 y'' + a_1 y' + a_0 y = f \qquad \rightarrow \qquad a_2 y'' + a_1 y' + a_0 y = 0$$

Assume an exponential solution for the complementary part.

$$y_{complementary} = y_c(x) =$$

Substitute this solution into the ODE (LHS=0).

# Particular Solution

Step 2 - Find the **particular part** of the solution from the entire equation (LHS=RHS).

$$a_2 y'' + a_1 y' + a_0 y = f$$

The *form of the particular part* follows the RHS of the ODE.

$$y_{particular} = y_p(x) =$$

Substitute this solution into the ODE above and solve for any unknown constants in $y_p(x)$.

# Apply Initial Conditions

Step 3 - Now combine the **complementary** and **particular** solutions through *superposition*.

$$y(x) = y_c(x) + y_p(x) =$$

The ODE is first order and we have _____ unknown. Coincidence?

$$y(x) =$$

This **initial value problem** requires _____ intial condition.

# Apply Initial Conditions

$$y(x = 0) =$$

$$y'(x = 0) =$$

# Apply Initial Conditions

What does the solution look like this time?

$$y(x) =$$

# Apply Initial Conditions

# Summary - 3 Cases

If the differential equation is first and linear, the complementary solution takes the following form.

$$y(x) = Ae^{sx}$$

# Summary - 3 Cases

If the differential equation is second order and linear, the **complementary solution** takes one of the following forms.

Case 1: $s_1, s_2 \in \mathbb{R}$ , $s_1 \neq s_2$

$$y(x) = c_1 e^{s_1 x} + c_2 e^{s_2 x}$$

Case 2: $s_1, s_2 \in \mathbb{R}$ , $s_1 = s_2 = s$

$$y(x) = c_1 e^{sx} + c_2 x e^{sx}$$

Case 3: $s_1, s_2 \notin \mathbb{R}$ , $s_1, s_2 = \alpha \pm \beta$

$$y(x) = e^{\alpha x} \left( c_1 cos\left(\beta x\right) + c_2 sin\left(\beta x\right)\right)$$

# Summary - 3 Cases

The **particular solution** takes the form of the right hand side of the equation.

| Example | Form | Particular Solution |
|---|---|---|
| $\ldots = 10$ | Constant | $y_p = B$ |
| $\ldots = 12x$ | Linear | $y_p = Bx + C$ |
| $\ldots = 20e^{2x}$ | Exponential | $y_p = Be^{2x}$ |
| $\ldots = acos\left(\beta x\right)$ | Sinusoidal | $Bcos\left(\beta x\right) + Csin\left(\beta x\right)$ |
| $\ldots = asin\left(\beta x\right)$ | Sinusoidal | $Bcos\left(\beta x\right) + Csin\left(\beta x\right)$ |