(Ch.3) Dynamics Review - Modeling Mechanical Systems

Translational Motion

Position: $x(t)$ Velocity: $v(t) = \frac{dx}{dt} = \dot{x}(t)$ Acceleration: $a(t) = \frac{dv}{dt} = \frac{d^2x}{dt^2} = \ddot{x}(t)$

Rotational Motion

Angular Position: $\theta(t)$ Angular Velocity: $\omega(t) = \frac{d\theta}{dt} = \dot{\theta}(t)$

Angular Acceleration: $\alpha(t) = \frac{d\omega}{dt} = \frac{d^2\theta}{dt^2} = \ddot{\theta}(t)$

Determining Equation of Motion

1. Draw FBD(s)
2. Apply Newton's 2nd Law or use an Energy Method

Newton's 2nd Law Approach

Translational: $\Sigma F = ma$ m = mass

Rotational: $\Sigma M_o = I_o \alpha$ I_o = mass moment of inertia about rotation point, O.
M_o = moment about O of all external forces + couples.

Energy Method

Kinetic Energy: Translational: $T = \frac{1}{2} m v^2$ Rotational: $T = \frac{1}{2} I_o \omega^2$

Potential Energy: Gravity: $V = mgh$ (g = accel. of gravity, h = vertical displacement)

Translational Spring: $V = \frac{1}{2} k x^2$ k = linear spring stiffness

Rotational Spring: $V = \frac{1}{2} k_t \theta^2$ k_t = torsional spring stiffness

If only conservative forces are applied (i.e. no damping, friction, etc.), then

$$T + V = \text{constant} \quad \text{(conservation of energy)}$$

$$\Delta T + \Delta V = 0 \quad \text{or} \quad T_2 + V_2 = T_1 + V_1$$

$$\Delta T = T_2 - T_1 = \tfrac{1}{2} m (v_2^2 - v_1^2)$$

$$\Delta V = V_2 - V_1 = \tfrac{1}{2} k (x_2^2 - x_1^2) \quad \text{- for a spring}$$

EOM can be found by realizing that

$$\frac{d}{dt}(T + V) = \frac{d}{dt}(\text{const}) = 0$$

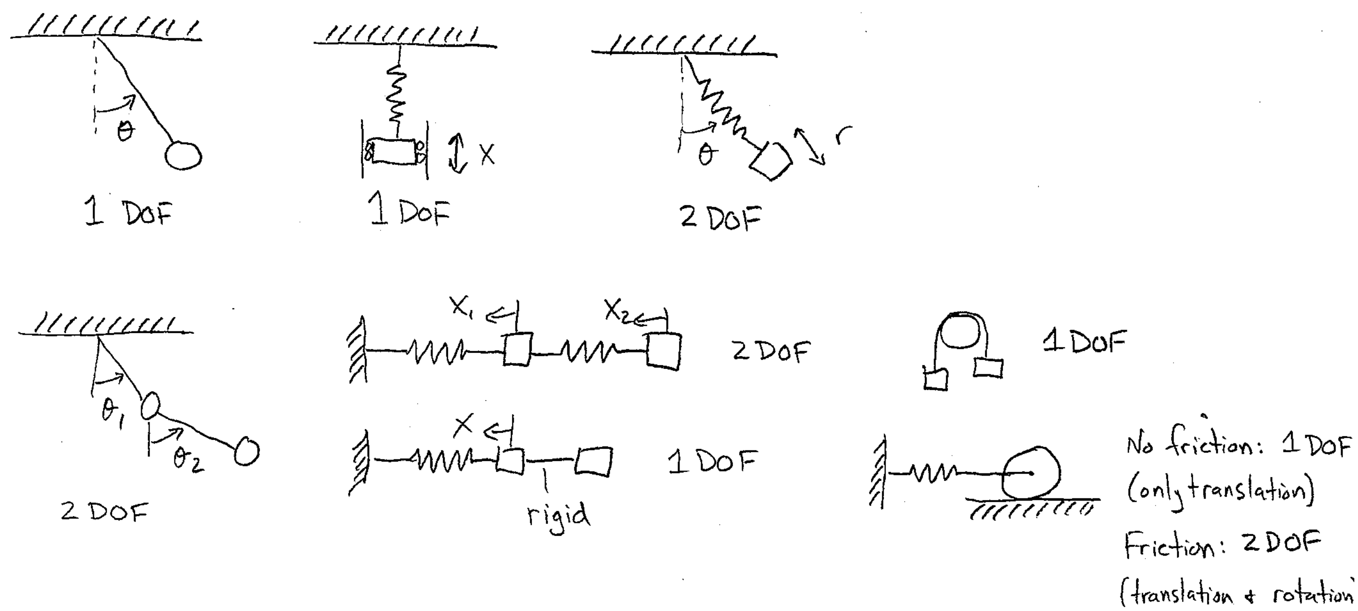
If nonconservative forces are present (i.e. damping, friction), then

$$T_2 + V_2 = T_1 + V_1 + W_{1\to2}$$ $W_{1\to2}$ is work of nonconservative forces.

This leads to a more complex method called the Lagrange Method. We won't cover in this course.

Degrees of Freedom

DOF is the number of coordinates required to characterize a system

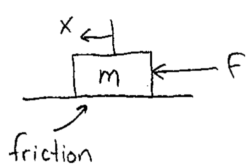


Need one equation of motion per DOF to describe system

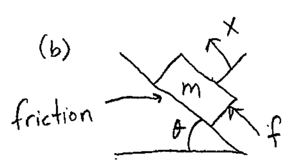
Example (Ex 3.1.2)

Find EOM for following systems

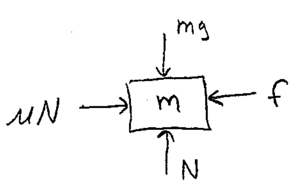
(a)



(b)

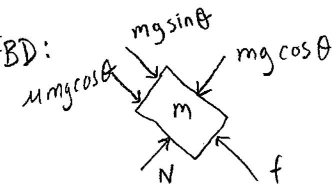


(a) FBD:



$$\Sigma F_x = m\ddot{x} = f - \mu \overset{(N)}{mg} \Rightarrow m\ddot{x} + \mu mg = f$$

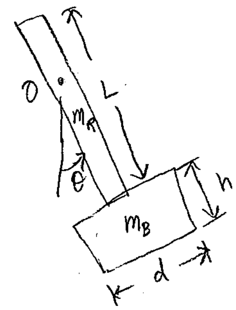
(b) FBD:



$$\Sigma F_x = m\ddot{x} = f - mg\sin\theta - \mu mg\cos\theta$$

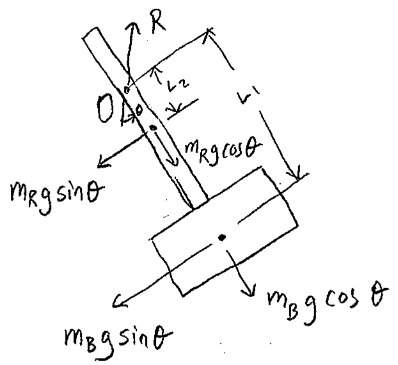
$$m\ddot{x} + mg(\sin\theta + \mu\cos\theta) = f$$

Example (P 3.38)



Pendulum
Both rod + block have mass, m_R, m_B
L_1, L_2 $\Rightarrow$ distances from centers of mass to pivot point, O.

FBD:



Newton's Method

$$\Sigma M_o = I_o \ddot{\theta} = (I_{Ro} + I_{Bo})\ddot{\theta}$$

Parallel Axis Theorem: $I = \underset{\substack{\uparrow \\ \text{inertia about symmetry axis}}}{I_s} + md^2$ (p. 123)

Rod: $I_s = \frac{1}{12} m_R L^2 \Rightarrow I_{Ro} = \frac{1}{12} m_R L^2 + m_R L_2$

Block: $I_s = \frac{1}{12} m_B (h^2 + d^2) \Rightarrow I_{Bo} = \frac{1}{12} m_B (h^2 + d^2) + m_B L_1$

$$(I_{Ro} + I_{Bo})\ddot{\theta} = -m_R g \sin\theta L_2 - m_B g \sin\theta L_1$$

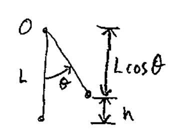
$$(I_{Ro} + I_{Bo})\ddot{\theta} + (m_R g L_2 + m_B g L_1)\sin\theta = 0$$

Energy Method

$$T + V = cst$$

Recall: $T = \frac{1}{2} I_o \omega^2 = \frac{1}{2} I_o \dot{\theta}^2$

Potential Energy:



$h = L - L\cos\theta$
$h = L(1-\cos\theta)$

$$\frac{1}{2}(I_{Ro} + I_{Bo})\dot{\theta}^2 + m_R g L_2 (1-\cos\theta) + m_B g L_1 (1-\cos\theta) = cst$$

* Differentiate w.r.t. time: use chain rule $\Rightarrow \frac{d}{dt}\dot{\theta}^2 = 2\dot{\theta}\ddot{\theta}$, $\frac{d}{dt}\cos\theta = -\sin\theta\dot{\theta}$

$$\cancel{2}\cancel{\tfrac{1}{2}}(I_{Ro} + I_{Bo})\cancel{\dot{\theta}}\ddot{\theta} + m_R g L_2 \sin\theta\cancel{\dot{\theta}} + m_B g L_1 \sin\theta\cancel{\dot{\theta}} = 0$$

$$(I_{Ro} + I_{Bo})\ddot{\theta} + (m_R g L_2 + m_B g L_1)\sin\theta = 0$$