Module 13 - Higher Order Systems

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 2 - State Space Representations

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- State Space Models
- The State Space Equation
- The Output Equation
- Simulation and Control

State Space Models

- *commonly used* for system models
- useful for *numerical simulation*
- used in the area of *automatic control*
- an ODE system has an equivalent State Space Model representation

State Space Models

Higher Order Differential Equations - An N^{th} order differential equation can be decomposed into a system of N first order differential equations. The resulting system is equivalent to the original equation.

The State Space Equation

The State Equation After the system of differential equations consists of first order equations only, these equations form the state equation.

The State Space Equation

The State Equation

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

- there are n *state variables* or *states* called $x_1 - x_n$
- there are m *inputs* called $u_1 - u_m$
- the *state vector* $\boldsymbol{x}$ is a collumn vector with n rows
- the *system matrix* $\boldsymbol{A}$ is a square matrix n rows and n columns.
- the *input vector* $\boldsymbol{u}$ is a column vector with m rows.
- the *control or input matrix* $\boldsymbol{B}$ is a matrix with n rows and m columns.

The Output Equation

The Output Equation

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}$$

- the *output vector* $\boldsymbol{y}$ is a collumn vector with p rows
- the *output matrix* $\boldsymbol{C}$ is a square matrix p rows and n columns.
- the *control matrix* $\boldsymbol{D}$ is a matrix with p rows and m columns.

The Output Equation

You can choose any combination of dependent variables or derived quantities for the output equation for your individual purposes.

This makes the state space model very useful to the designer.

Simulation and Control

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- useful for *numerical simulation*
- used in the area of *automatic control*
- an ODE system has an equivalent State Space Model representation

References

- System Dynamics, Palm III, Third Edition - Chapter 4 - Spring and Damper Elements in Mechanical Systems - pg. 208