Module 13 - Higher Order Systems

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 4 - Example - 2DOF Mass Spring System

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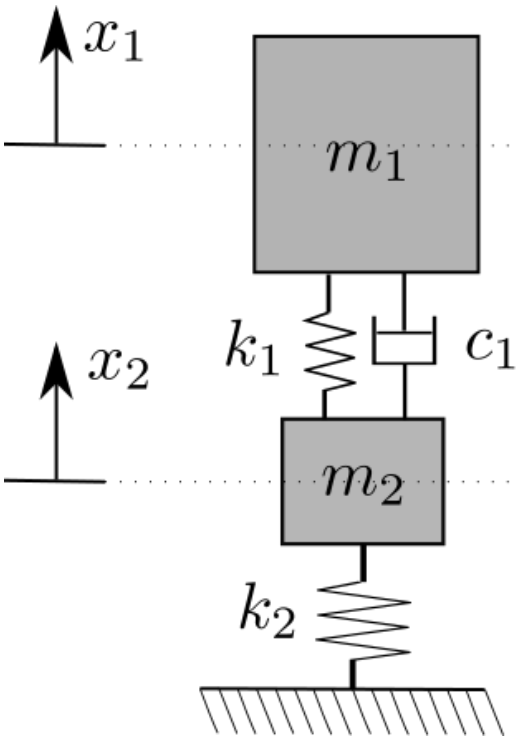
- Equation Decomposition
- State Space Equation
- Output Equation
- —

Equation Decomposition

Begin with the model and EOM we derived previously.



Equation of Motion for Mass 1:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0$$

Equation of Motion for Mass 2:

$$m_2\ddot{x}_2 + k_2x_2 - c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = 0$$

Equation Decomposition

Make a substitution for each dependent variable and its first derivative then re-write the system of equations as four first order differential equations.

let $z_1 = x_1$ and $z_2 = x_2$

also $z_3 = \dot{x}_1$ and $z_4 = \dot{x}_2$

You are choosing the *states* of your model during this process.

State Space Equation

Write the the State Equation.

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

State Space Equation

Output Equation

Write the the Output Equation.

$$\boldsymbol{y} = \boldsymbol{Cx} + \boldsymbol{Du}$$

Output Equation

References

- System Dynamics, Palm III, Third Edition - Section 8.1 - Response of First Order Systems - pg. 475