Frequency Response - Lecture 1

ME3050 - Dynamics Modeling and Controls

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Frequency Response of First Order Systems

Lecture 1- Frequency Response of First Order Systems

- Introduction to Chapter 9
- Review Complex Numbers
- Frequency Response of First Order Systems
- The Bode Plot

Harmonic Input Function

The term **frequency response** is used to describe a system's response to a periodic input. Frequency response analysis focuses on a system's response to *harmonic* input such as sines and cosines. The input (forcing) function is written below.

$$f(t) = A sin(\omega t)$$

Amplitude of the Input, A (N)

Frequency of Input, ω $\left(\frac{rad}{s}\right)$

Why Study Frequency Response?

Why do we care about the way a system responds to harmonic excitation? Why is **frequency analysis** important?

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What causes **harmonic** (or sinusoidal) excitation in the real world?

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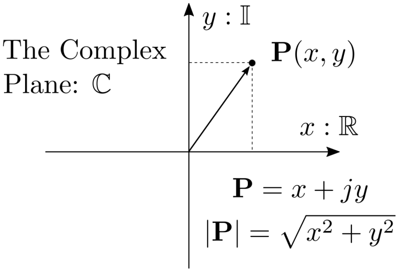
Frequency Response and Transfer Function

A linear, time-invariant (LTI) system has a **transfer function** $T(s)$ that describes the **input-output** relationship of the system. Under sinusoidal excitation (input) with frequency ω if the system is stable the transient affects in the response (output) will eventually dissappear leaving the **steady state sinusoidal response** of the same frequency as the input but with a phase shift w.r.t. the input.

The Complex Plane

In an underdamped system the roots of the characteristic polynomial are complex. Before we proceed we need to review some rules of arithmetic and complex numbers.



Cartesian Reprentation:

$\mathbf{P} = x + jy$

Polar Reprentation:

$\mathbf{P} = |\mathbf{P}|\angle\theta$

Exponential Reprentation:

$\mathbf{P} = |\mathbf{P}|e^{j\theta} = |\mathbf{P}|\left(cos\theta + jsin\theta\right)$

Complex Number Algebra

Consider two points $\mathbf{P_1}$ and $\mathbf{P_2}$ on the complex plane.

$\mathbf{P_1} = x_1 + jy_1$ and $\mathbf{P_2} = x_2 + jy_2$

Addition: $$\mathbf{P_1} + \mathbf{P_2} = (x_1 + x_2) + j(y_1 + y_2)$$

Multiplication: $$\mathbf{P_1}\mathbf{P_2} = |\mathbf{P_1}\mathbf{P_2}|\angle(\theta_1 + \theta_2)$$

Divsion: $$\frac{\mathbf{P_1}}{\mathbf{P_2}} = (x_1 + x_2) + j(y_1 + y_2)$$

Frequency Response of First Order Systems

Consider our $1^{\underline{st}}$ order mass damper system.

$m\dot{v} + cv = f(t)$ with a **time constant** $\tau = \frac{m}{c}$

The system is commonly re-written as shown below.

$m\dot{v} + cv = f(t) \;\rightarrow\; \tau\dot{y} + y = f(t)$

Obtain the Transfer Function

$\tau \dot{y} + y = f(t)$

Take the Laplace transform of the ODE.

$\mathcal{L}\{\tau \dot{y} + y\} = \mathcal{L}\{f(t)\}$

$\tau\left(sY(s) + y_0\right)) + Y(s) = F(s)$ The initial conditions are zero.

$\boxed{T(s) = \frac{Y(s)}{F(s)} = \frac{1}{\tau s+1} \quad \text{First Order Transfer Function}}$

This considers a *generalized* input function $f(t)$ and zero ICs.

T4: Stability of a Second Order System

Our model $m\ddot{x} + c\dot{x} + kx = 0$ is stable is the roots of the characteristic equation lie *to the right* of the imaginary axis of the complex plane (if the Real part of the root is positive). This makes sense because a positive α would cause the response to go to ∞.

This is called the **Routh-Hurwitz stability conditions**

A second order model of the form $a_2 s^2 + a_1 s + a_0 = 0$

if a_2, a_1, and a_0 have the *same sign*.

This is in your reference handout and discussed on page 488 of System Dynamics, Palm III, Third Edition

References

- System Dynamics, Palm III, Third Edition - Chapter 8 - System Response in the Time Domain