Module 3 - Newton's Approach

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 2 - Equations of Motion

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- Modeling Rigid Body Mechanical Systems
- Derive Equations of Motion
- Newton's Second Law Approach
- Convervation of Energy Method

Modeling Rigid Body Mechanical Systems

When modeling the motion of objects, the bending or twisting of the object is often negligible, and we can model the object as a rigid body. We begin this chapter by reviewing Newton's laws of motion and applying them to rigid bodies where the object;s motion is relatively uncomplicated, simple translations and simple rotations about a fixed axis. System Dynamics, Third Edition, Palm - Pg. 117

Derive Equations of Motion

To analyze the dynamics (motion and forces) of a system we start by deriving the equations of motion (EOM). This is typically done in one of two ways.

- Newton's Second Law Approach
- Conversation of Energy Method

The equations of motion are ordinary differential equations describing the dynamic relationships between the bodies in the system. One equation of motion is required for each DOF.

Newton's Second Law Approach

To derive the equations of motion with Newton's Second Law:

1. Draw a free body diagram (FBD) for each body in the system. Identify all forces acting on the bodies in the system.
2. Apply Newton's Second Law for each DOF. Typically the acceleration term is replaced and external forces are placed on the right hand side of the equation.

Translation: $\Sigma \mathbf{F} = m\mathbf{a}$ $\quad$ $\Sigma F_x = ma_x$

$$\Sigma F_y = ma_y$$

$$\Sigma F_z = ma_z$$

Rotation: $\Sigma \mathbf{M} = I_o \alpha$ $\quad$ $\Sigma M_o = I_o \alpha_z$

Convervation of Energy Method

Alternatively with Conservation of Energy:

1. Draw a free body diagram (FBD) for each body in the system. Identify all kinetic and potential energies present in the system.
2. Write the total energy equation as the sum of all energies for a particular body.
3. If no non-conservative forces are considered, Conservation of Energy states the change in the total energy must equal zero. Take the derivative of total energy equation and set it equal to zero. This relation is used to the derive the EOM(s).

Convervation of Energy Method

Note:

Either method can be used and for some problem types one or the other is preferred. We will look at several examples of both but the course will primarily focus on the Newton's Second Law approach.

The Conservation of Energy method, which is based upon Newton's law, is commonly used in advanced dynamics courses.