Module 4 - Energy Methods

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 1 - The Conservation of Energy

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- Brief History
- Second Law Derivation
- Non-Conservatve Forces

Brief History

... the law of conservation of energy states that the total energy of an isolated system remains constant; it is said to be conserved over time. This law, first proposed and tested by Émilie du Châtelet, means that energy can neither be created nor destroyed; rather, it can only be transformed or transferred from one form to another.

Text: Wikipedia, Image: Wikipedia

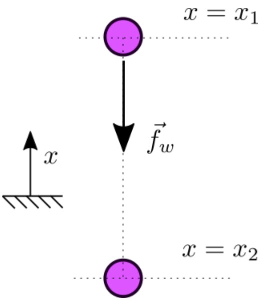
Brief History

Text, Images: Wikipedia

... however, special relativity showed that mass is related to energy and vice versa by $E = mc^2$, and science now takes the view that massenergy as a whole is conserved. Theoretically, this implies that any object with mass can itself be converted to pure energy, and vice versa, though this is believed to be possible only under the most extreme of physical conditions ...

Second Law Derivation

A mass moves in the x direction with only the force of gravity acting on it. Newton's Second Law combined with the definition of differential work done by a force through a distance gives a relation between kinetic energy and work done by the external force.

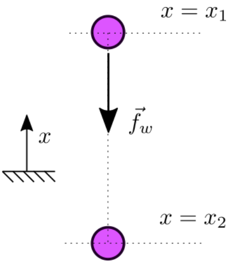


$$\mathbf{f} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$
$$dW = \mathbf{f}\,d\mathbf{x}$$
$$dW = \mathbf{f}\,d\mathbf{x} = m\mathbf{a}d\mathbf{x} = md\mathbf{v}\frac{d\mathbf{x}}{dt} = md\mathbf{v}\mathbf{v}$$
$$\rightarrow dW = mv_x dv_x$$
$$W_{12} = \int_{x_1}^{x_2} mv_x dv_x = \frac{1}{2}mv_x^2\Big|_{x_1}^{x_2}$$
$$W_{12} = \frac{1}{2}mv_{x_2}^2 - \frac{1}{2}mv_{x_1}^2 = KE_1 - KE_2 \qquad (1)$$

Images: T.Hill

Second Law Derivation

Now, substitute the distance traveled into the differential work relation and you are left with the familar relation between work done by gravity and the potential energy in the system.



$$dW = \mathbf{f} d\mathbf{x}$$

$$W_{12} = \int_{x_1}^{x_2} \mathbf{f} d\mathbf{x} = (-mg)|_{x_1}^{x_2}$$

$$W_{12} = -mg(x_2 - x_1) \qquad (2)$$

Combine (1) and (2).

$$W_{12} = -mg(x_2 - x_1) = \frac{1}{2} m v_{x_2}^2 - \frac{1}{2} m v_{x_1}^2$$

Images: T.Hill

Second Law Derivation

The resulting equation clearly shows the conservation of energy for a 1DOF particle moving in the direction of gravity. This derivation is often shown in more generalized form but this version is easy to understand and will work for our purposes.

$\Delta KE + \Delta PE = 0$ or $KE + PE = Constant$

with $KE = \frac{1}{2}mv^2$ and $PE = mgh$

While deriving dynamics (EOMs) for many systems the alternate form shown the right will be used.

Non-Conservatve Forces

Lastly, this derivation relies on the fact that f is a conservative force that can be derived from a function $V(x)$ which is defined as follows. (see page 119)

$V(x) = \int dV = -f(x)dx$

In this class we will only use the energy based methods in systems with no non conservative forces. You discussed this idea in your physics class.