Lecture Module - ODE Review

ME3050 - Dynamic Modeling and Controls

Mechanical Engineering
Tennessee Technological University

Topic 3 - The Trial Solution Method

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- Exponential Assumption
- Complementary Solution
- Particular Solution
- Apply Initial Conditions

Trial Solution Method

Use the trial solution method to solve the ODE.

This is an **analytical** method that you learned in calculus but it may have been called something different. In the Zill book it is called *Homogenous Linear* ... Constant Coefficients (4.3-4.4).

$$a_2y'' + a_1y' + a_0y = f(x)$$

Exponential Assumption

Complementary Solution

<u>Step 1</u> - Find the **complementary part** of the solution from the left hand side of the ODE alone (LHS=0).

$$a_2y'' + a_1y' + a_0y = f$$
 \rightarrow $a_2y'' + a_1y' + a_0y = 0$

Assume an exponential solution for the complementary part.

$$y_{complementary} = y_c(x) =$$

Substitute this solution into the ODE (LHS=0).

Complementary Solution

Particular Solution

Step 2 - Find the **particular part** of the solution from the entire equation $\overline{(LHS=RHS)}$.

$$a_2y'' + a_1y' + a_0y = f$$

The form of the particular part follows the RHS of the ODE.

$$y_{particular} = y_p(x) =$$

Substitute this solution into the ODE above and solve for any unknown constants in $y_p(x)$.

Particular Solution

Apply Initial Conditions

Step 3 - Now combine the **complementary** and **particular** solutions through *superposition*.

$$y(x) = y_c(x) + y_p(x) =$$

The ODE is first order and we have _____ unknown. Coincidence?

$$y(x) =$$

This **initial value problem** requires intial condition.

Apply Initial Conditions

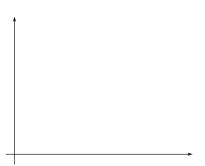
$$y(x=0) =$$

$$y(x=0) =$$
$$y'(x=0) =$$

Apply Initial Conditions

What does the solution look like this time?

$$y(x) =$$





Summary - 3 Cases

If the differential equation is first and linear, the complementary solution takes the following form.

$$y(x) = Ae^{sx}$$

Summary - 3 Cases

If the differential equation is second order and linear, the **complementary solution** takes one of the following forms.

Case 1:
$$s_1, s_2 \in \mathbb{R}$$
 , $s_1 \neq s_2$

$$y(x) = c_1 e^{s_1 x} + c_2 e^{s_2 x}$$

Case 2:
$$s_1, s_2 \in \mathbb{R}$$
, $s_1 = s_2 = s$

$$y(x) = c_1 e^{sx} + c_2 x e^{sx}$$

Case 3:
$$s_1, s_2 \notin \mathbb{R}$$
, $s_1, s_2 = \alpha \pm \beta$

$$y(x) = e^{\alpha x} \left(c_1 \cos(\beta x) + c_2 \sin(\beta x) \right)$$

Summary - 3 Cases

The **particular solution** takes the form of the right hand side of the equation.

| Example | Form | Particular Solution |
|--------------|-------------|---------------------|
| = 10 | Constant | $y_p = B$ |
| = 12x | Linear | $y_p = Bx + C$ |
| = $20e^{2x}$ | Exponential | $y_p = Be^{2x}$ |