Introduction to Chapter 9
Review Complex Numbers
T3: Finding the Time Value with Time Constant
T4: Stability of a Second Order System

Frequency Response - Lecture 1

ME3050 - Dynamics Modeling and Controls

April 19, 2020

Frequency Response of First Order Systems

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Lecture 1- Frequency Response of First Order Systems

- Introduction to Chapter 9

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- Frequency Response of First Order Systems

- The Bode Plot

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Harmonic Input Function
Why Study Frequency Response?

Harmonic Input Function

The term **frequency response** is used to describe a system's response to a periodic input. Frequency response analysis focuses on a system's response to *harmonic* input such as sines and cosines. The input (forcing) function is written below.

$$f(t) = A \sin(\omega t)$$

Amplitude of the Input, $\quad A \quad (N)$

Frequency of Input, $\quad \omega \quad (\frac{rad}{s})$

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Harmonic Input Function
Why Study Frequency Response?

Why Study Frequency Response?

Why do we care about the way a system responds to harmonic excitation? Why is **frequency analysis** important?

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What causes **harmonic** (or sinusoidal) excitation in the real world?

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The Complex Plane
Complex Number Algebra

The Complex Plane

In an underdamped system the roots of the characteristic polynomial are complex. Before we proceed we need to review some rules of arithmetic and complex numbers.
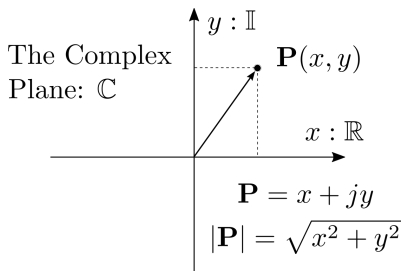
The Complex Plane: $\mathbb{C}$

Cartesian Reprentation:

$$\mathbf{P} = x + jy$$

Polar Reprentation:

$$\mathbf{P} = |\mathbf{P}|\angle\theta$$

Exponential Reprentation:

$$\mathbf{P} = |\mathbf{P}|e^{j\theta} = |\mathbf{P}|(cos\theta + jsin\theta)$$

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The Complex Plane
Complex Number Algebra

Complex Number Algebra

Consider two points $\mathbf{P_1}$ and $\mathbf{P_2}$ on the complex plane.

$\mathbf{P_1} = x_1 + jy_1$ and $\mathbf{P_2} = x_2 + jy_2$

Addition: $\mathbf{P_1} + \mathbf{P_2} = (x_1 + x_2) + j(y_1 + y_2)$

Multiplication: $\mathbf{P_1}\mathbf{P_2} = |\mathbf{P_1}\mathbf{P_2}|\angle(\theta_1 + \theta_2)$

Divsion: $\frac{\mathbf{P_1}}{\mathbf{P_2}} = (x_1 + x_2) + j(y_1 + y_2)$

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T3: Finding the Time Value with Time Constant

T3: Finding the Time Value with Time Constant

In problem b) it asks for the time at which the response equation reaches a certain value. There are different ways to find this value.

- Plot the response curve and locate the value graphically. This might not be accurate.

- Use a 'root-finding' method to locate the value.

- Solve for the value with algebra. This is easy for this system.

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Our model $m\ddot{x} + c\dot{x} + kx = 0$ is stable is the roots of the characteristic equation lie *to the right* of the imaginary axis of the complex plane (if the Real part of the root is positive). This makes sense because a positive α would cause the response to go to ∞.

This is called the **Routh-Hurwitz stability conditions**

A second order model of the form $a_2 s^2 + a_1 s + a_0 = 0$

if a_2, a_1, and a_0 have the *same sign*.

This is in your reference handout and discussed on page 488 of System Dynamics, Palm III, Third Edition

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References

- System Dynamics, Palm III, Third Edition - Chapter 8 - System Response in the Time Domain