Lecture Module - Alternate Model Forms

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 3 - State Space Representations

Topic 3 - State Space Representations

- State Space Models
- The State Space Equation
- The Output Equation
- Simulation and Control Applications

State Space Models

- Most linear differential equations can be written in state space form.
- A state space model is an equivalent representation of a dynamic system.
- This standard form allows us to use and share tools for analysis and design of complex systems.

State Space Models

Higher Order Differential Equations - An N^{th} order differential equation can be decomposed into a system of N first order differential equations. The resulting system is equivalent to the original equation.

The State Space Equation

After the system of differential equations consists of first order equations only, these equations form the state equation.

The State Space Equation

The State Equation

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

- there are n state variables or states: $x_1 - x_n$
- there are m inputs called $u_1 - u_m$
- the state vector $\boldsymbol{x}$ is a collumn vector with n rows
- the system matrix $\boldsymbol{A}$ is a square matrix n rows and n columns.
- the input vector $\boldsymbol{u}$ is a column vector with m rows.
- the control or input matrix $\boldsymbol{B}$ is a matrix with n rows and m columns.

The Output Equation

The Output Equation

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}$$

- the output vector $\boldsymbol{y}$ is a collumn vector with p rows
- the output matrix $\boldsymbol{C}$ is a square matrix p rows and n columns.
- the control matrix $\boldsymbol{D}$ is a matrix with p rows and m columns.

The designer chooses any combination of dependent variables or derived quantities for the output equation for your individual purposes. The number of outputs is *flexible*.

The Output Equation

Choose the outputs that you want to study and write the output equations as functions of the states and **not** their derivatives.

Simulation and Control Applications

- *commonly used* for system models
- useful for *numerical simulation*
- used in the area of *automatic control*
- This standard form allows us to use and share tools for analysis and design of complex systems.

References

- System Dynamics, Palm III, Third Edition - Chapter 4 - Spring and Damper Elements in Mechanical Systems - pg. 208