# State Space - Lecture 2

ME3050 - Dynamics Modeling and Controls

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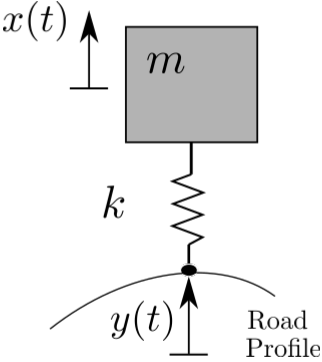
**State Space Models with Displacement Inputs**

## Lecture 2 - State Space Models with Displacement Inputs

- Mass Spring System with Displacement Input
- Mass Spring System Damper with Displacement Input
- State Space Model with Derivative Input

# Mass Spring Model

Consider the mass-spring system without damping.




The EOM is:

$$m\ddot{x} + k(x - y) = 0$$

# State Space Model

The mass-spring system equation of motion can easily be written as a state space system.

Equation of Motion:

$m\ddot{x} + k(x - y) = 0$

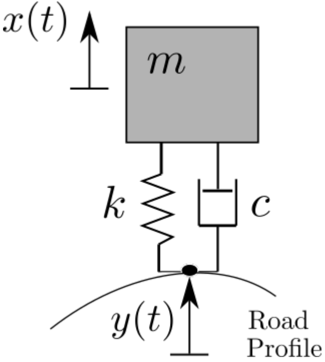
$z_1 = x$ and $z_2 = \dot{x}$

$\dot{z}_2 = -\frac{k}{m}z_1 + \frac{k}{m}y \quad and \quad \dot{z}_1 = z_2$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & 0 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{k}{m} \end{bmatrix} y(t)$$

# Mass Spring Damper Model

Consider the mass-spring system with damping now.




The EOM is:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

# State Space Model - Problem!

The mass-spring system equation of motion cannot be written as a state space system easily this time.

Equation of Motion:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$z_1 = x \quad \text{and} \quad z_2 = \dot{x}$$

$$\dot{z}_2 = -\frac{c}{m}z_2 + \frac{c}{m}\dot{y} - \frac{k}{m}z_1 + \frac{k}{m}y \quad and \quad \dot{z}_1 = z_2$$

The $\dot{y}$ term is a problem!

# State Space Model - Fixed

If we make a very clever substitution we can avoid this issue.

Equation of Motion:

$$m\ddot{x} + c(\dot{x} - \dot{y}) + k(x - y) = 0$$

$$z_1 = x \quad \text{and} \quad z_2 = \dot{x} - \frac{c}{m}y$$

$$\dot{z}_2 = -\frac{c}{m}(z_2 + \frac{c}{m}y) - \frac{k}{m}z_1 + \frac{k}{m}y \quad \textit{and} \quad \dot{z}_1 = z_2 + \frac{c}{m}y$$

$$\begin{bmatrix} \dot{z}_1 \\ \dot{z}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ -\frac{k}{m} & -\frac{c}{m} \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} \frac{c}{m} \\ -(\frac{c}{m})^2 + \frac{k}{m} \end{bmatrix} y(t)$$

# State Space Model - Output Equation

It is important that you keep the same substitutions when writing the output equations.

Output 1 - Position: $y_{O1} = x = z_1$

Output 2 - Velocity: $y_{O2} = \dot{x} = \dot{z}_1 = z_2 + \frac{c}{m} y$

$$\begin{bmatrix} \dot{y}_{O1} \\ \dot{z}_{O2} \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} z_1 \\ z_2 \end{bmatrix} + \begin{bmatrix} 0 \\ \frac{c}{m} \end{bmatrix} y(t)$$

# References

- System Dynamics, Palm III, Third Edition - Section 5.? - State Variable Models