Module 10 - The Laplace Transform

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 2 - Laplace Transforms Method

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- Step 1 - Apply Laplace Transform

- Step 2 - Solve for $X(s)$

- Step 3 - Rearrange to Find Invertable Form

- Step 4 - Invert for Final Answer

Step 1 - Apply Laplace Transform

Example:
Solve the first order differential equation using the Laplace Transforms Method with the initial condition given.

$$4\dot{x} = \sin(t) \qquad with \qquad x(t = 0) = x_0$$

Apply the Laplace Transform to both sides of the differential equation.

$$4(sX(s) - x_0) = \frac{1}{s^2 + 1}$$

Step 2 - Solve for $X(s)$

This step can seem open ended...

$$X(s) = \frac{1}{4s\left(s^2 + 1\right)} + \frac{x_0}{s}$$

Step 3 - Rearrange to Find Invertable Form

Write $X(s)$ in a form that can be inverted using the table of
Laplace transform pairs. This typically involves partial fraction
decomposition.

$$\frac{1}{4s\left(s^2+1\right)} = \frac{1/4}{s\left(s^2+1\right)} = \frac{a}{s} + \frac{bs+c}{s^2+1}$$

Mulitply through by the denominator $4s\left(s^2+1\right)$:

$$1 = 4as\left(s^2+1\right) + 4s\left(bs+c\right) = 4\left(a+b\right)s^2 + 4cs + 4a$$

Solve for the coefficients by *equating coefficients*.

$$(a+b) = 0 \quad c = 0 \quad a = \frac{1}{4} \implies a = \frac{1}{4} \quad b = -\frac{1}{4} \quad c = 0$$

Step 4 - Invert for Final Answer

Substitute the coefficients into $X(s)$,

$$X(s) = \frac{x_0}{s} + \frac{1}{4s} - \frac{s}{4(s^2 + 1)}$$

and use the inverse transform to solve for $x(t)$. Use the Table.

$$\mathcal{L}^{-1}(X(s)) = x(t) =$$

$$= x_0 + \frac{1}{4} - \frac{1}{4}\cos(t) = x_0 + \frac{1}{4}(1 - \cos(t))$$

This method works for complex problems but it can get messy...