

## Time Response - Lecture 4

ME3050 - Dynamics Modeling and Controls

April 15, 2020

**Common Questions this Week**

## Lecture 4 - Common Questions this Week

- T1 - The Step Input
- T2 - Obtaining the Response Equations in Problem 1
- T3 - Using the Error function and the Time Constant
- T4 - Stability and the Roots

# T1: The Step Input

The **step function** is a mathematical concept that represents an instant change.

## Heavyside's Step Function



$$u_s(t) = \begin{cases} 0 & t < 0 \\ 1 & t \geq 0 \end{cases}$$

$$f_{step}(t) = Fu_s(t) = \begin{cases} 0 & t < 0 \\ F & t \geq 0 \end{cases}$$

$u_s$  is the **unit step function**

$f_{step}$  is a step function of magnitude  $F$

## T2: Obtaining the Response Equations in Problem 1

You can see that each of the models in problem 1 is **linear** and **first order**. You do not have to re-derive (even though you could) the response equations but please reference where you found the equations you used. They are in the notes and in chapter 8.

## T3: Finding the Time Value with Time Constant

In problem b) it asks for the time at which the response equation reaches a certain value. There are different ways to find this value.

- Plot the response curve and locate the value graphically. This might not be accurate.
- Use a 'root-finding' method to locate the value.
- Solve for the value with algebra. This is easy for this system.

## T4: Stability of a Second Order System

Our model  $m\ddot{x} + c\dot{x} + kx = 0$  is stable if the roots of the characteristic equation lie *to the left* of the imaginary axis of the complex plane (if the Real part of the root is positive). This makes sense because a positive  $\alpha$  would cause the response to go to  $\infty$ .

This is called the **Routh-Hurwitz stability conditions**

A second order model of the form  $a_2s^2 + a_1s + a_0 = 0$

if  $a_2$ ,  $a_1$ , and  $a_0$  have the *same sign*.

This is in your reference handout and discussed on page 488 of System Dynamics, Palm III, Third Edition

## References

- System Dynamics, Palm III, Third Edition - Chapter 8 - System Response in the Time Domain