Module 11 - First Order Time Response

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

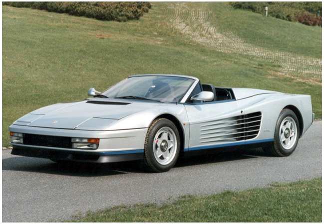
Topic 1 - First Order Free Response

Topic 1 - First Order Free Response

- Model and EOM
- Solution with Laplace Transforms Method
- The Critically Damped Case
- The Underdamped Case

Model and EOM

Consider the model of the moving mass we derived.



The EOM is:

$$m\dot{v} + cv = 0$$

Solution with Laplace Transforms Method

$$\mathcal{L}\{m\dot{v} + cv = 0\} \implies m[sV(s) - v(0)] + cV(s) = 0$$

$$(ms + c)V(s) = \frac{mv(0)}{(ms+c)} = \frac{V(0)}{s+\frac{c}{m}}$$

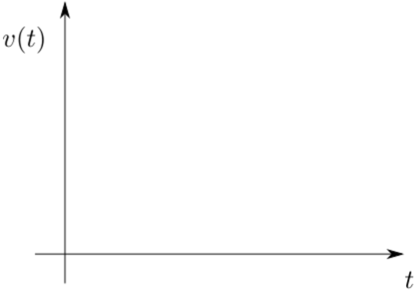
We can find the expected result from the table.

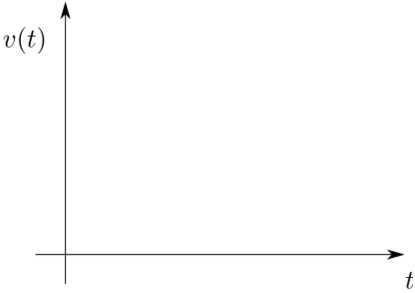
$$v(t) = v(0)e^{-\frac{c}{m}t} = v(0)e^{-\frac{t}{\tau}} \qquad \text{with} \qquad \tau =$$

Sketch Response Equation

Sketch the System Response in the time Domain.

$$v(t) = v(0)e^{-\frac{t}{\tau}}$$

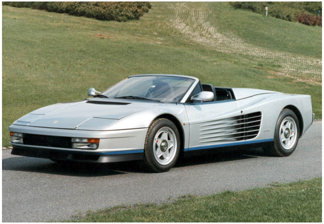




Is this a stable system? What does that mean?

Step Input Function

Consider the model subject to a Step Input, $f(t)$.



$$m\dot{v} + cv = f(t)$$

$$f(t) = \begin{cases} 0 & t < 0 \\ F & t \geq 0 \end{cases}$$

Solution with Laplace Transforms Method - Step 1

The method of Laplace Transforms is shown.

$$\mathcal{L}\{m\dot{v} + cv = F\} \implies m[sV(s) - v(0)] + cV(s) = \tfrac{F}{c}$$

$$(ms + c)V(s) = \tfrac{F}{s} + mv(0)$$

Solve for $V(s)$.

$$V(s) = \tfrac{F}{s(ms+c)} + \tfrac{mv(0)}{ms+c}$$

Solution with Laplace Transforms Method - Step 2

Expand $V(s)$as a partial fraction.

$$V(s) = \frac{F}{s(ms+c)} + \frac{mv(0)}{ms+c} \implies \frac{F}{s(ms+c)} = \frac{a}{s} + \frac{b}{ms+c}$$

'Cover up' to find the coefficients.

$$a = \frac{F}{m \times 0 + c} \qquad \text{and} \qquad b = \frac{F}{\frac{-c}{m}} = \frac{-Fm}{c}$$

This leads to a form that can be inverted with the table.

$$V(s) = \frac{F}{c}\left\{\frac{1}{s} - \frac{1}{s+\frac{c}{m}}\right\} + \frac{v(0)}{s+\frac{c}{m}}$$

Solution with Laplace Transforms Method - Step 3

Can you find these terms in the Table of Laplace Transforms?

$$V(s) = \frac{F}{c}\{\frac{1}{s} - \frac{1}{s+\frac{c}{m}}\} + \frac{v(0)}{s+\frac{c}{m}}$$

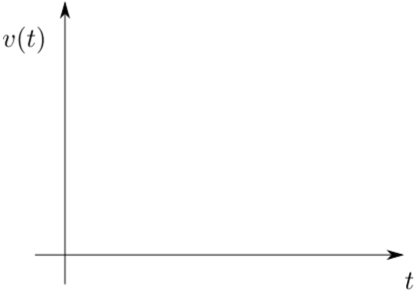
The inverse Laplace transform of $V(s)$ gives the time response.

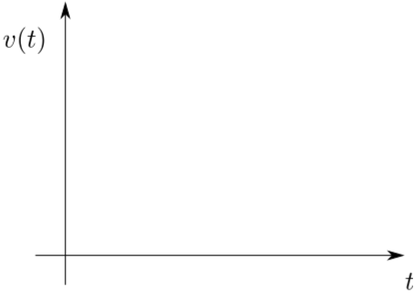
$$v(t) = \frac{F}{C}\{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

Sketch Response Equation

Sketch the System Response in the time Domain.

$$v(t) = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$





Is this a stable system?

Components of the Response

In these forms we can see the different components of the response.

$$v(t) = \frac{F}{C}\{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

- Forced Response
- Free Response
- Transient Response
- Steady-State Response

References

- System Dynamics, Palm III, Third Edition - Section 8.1 - Response of First Order Systems - pg. 475