Lecture Module - Introduction

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Module 1 - Introduction

Module 1 - Introduction

- Topic 1 - ODE Review
- Topic 2 - Separation of Variables
- Topic 3 -

Topic 1 - ODE Review

- Definitions and Classification
- Engineering Applications
- Example

What is a Differential Equation?

Definition:

A **differential equation** is an equation which describes a function and one or more of its _____ of the _____ _____ with respect to the _____.

Standard Form of an ODE

Ordinary Differential Equations are written in the following form.

$$a_n \frac{dy^{(n)}}{d^{(n)}x} + a_{n-1} \frac{dy^{(n-1)}}{d^{(n-1)}x} + \ldots + a_2 \frac{dy^2}{d^2x} + a_1 \frac{dy}{dx} + a_0 y = f(x)$$

The apostrophe is commonly used for the derivative.

$$a_n y^{(n)} + a_{n-1} y^{(n-1)} + \ldots + a_2 y'' + a_1 y' + a_0 y = f(x)$$

If time is the independent variable the equation changes slightly.

Is the differential equation ordinary or partial?

An **ordinary** differential equation has _____ independent variable and _____ dependent variable.

A **partial** differential equation has _____ independent variable _____ dependent variable.

What is the order of the equation?

The **order** of a differential equation is the

_____ _____

present in the equation.

What is the degree of the equation?

The **degree** of a differential equation is the _____

of its highest derivative, after the equation has been made rational

and integral in all of its derivatives.

Is the differential equation linear or non-linear?

An ordinary differential equation is _____ if the following statements are true.

1. *The dependent variable and its derivatives are of the first degree.*
2. *The coefficients are constants or dependent on the independent variable.*

If either rule is broken, the equation is _____-_____.

Engineering Applications

Differential equations are used to describe physical systems in many areas of engineering. An equation that represents a physical (or theoretical) system is known as a _____ _____.

- Solid Mechanics
- Kinematics and Dynamics
- Heat Transfer and Thermodynamics
- Fluid Mechanics

Engineering Applications

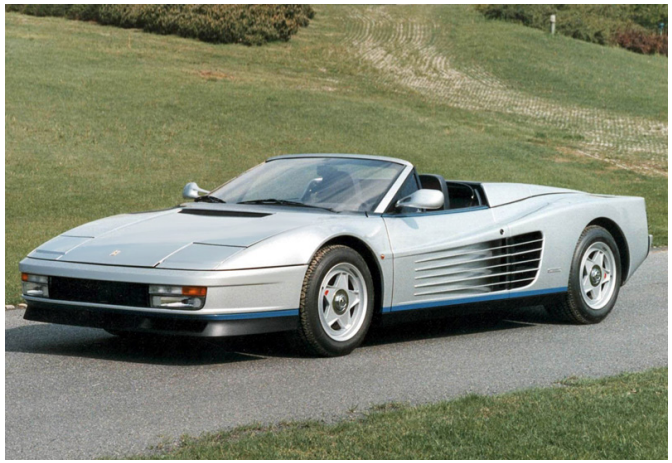
Example

Newton's Second Law

$\Sigma F = ma$

leads to an *equation of motion*.

$\dot{y} + \frac{c}{m} y = f(t)$



Topic 2 - Separation of Variables

- Review
- Separation of Variables
- Example

What is a Differential Equation? Solution?

A **differential equation** is an equation which describes a function and one or more of its _____ of the _____ _____ with respect to the _____.

The **solution** to a differential equation describes the _____ _____ as a function of the _____ _____.

Separation of Variables

Separation of Variables: **analytical** for solving differential equations

- Step 1 - Separate
- Step 2 - Integrate
- Step 2 - Solve for Unknowns

Separation of Variables

Alternative methods to find solution:

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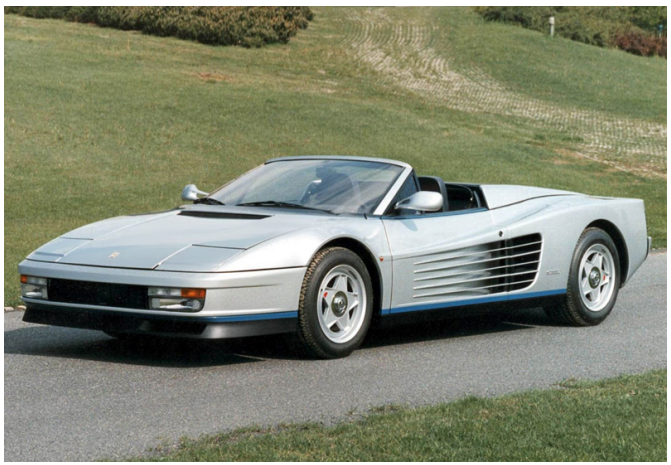
Problem Statement

Remember our example from the previous lecture?

$$\dot{v} + \frac{c}{m} v = f(t)$$



We are going to find an **analytical solution** to this problem.

Separation of Variables

Assume the external force $f(t)$ is zero. Use separation of variables to find the solution $v(t)$.

$$\dot{v} + \frac{c}{m} v = 0$$

Solution

The solution $v(t)$ has been found. What does it mean? What do we do next?

$$v(t) =$$

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Graph of Solution

What does the solution look like?

$$v(t) = v_0 e^{-\frac{c}{m}t}$$

Topic 3 -

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