Frequency Response - Lecture 2

ME3050 - Dynamics Modeling and Controls

April 19, 2020

The Bode Diagram

Lecture 2 - The Bode Diagram

- Review Frequency Response

- Magnitude Ratio in Decibels

- The Bode Diagram

- Graph of Frequency Response in MATLAB

Review Frequency Response
Magnitude Ratio in Decibels
Magnitude Ratio in Decibels

Harmonic Input Function
First Order Frequency Response
Graph of Frequency Response

Harmonic Input Function

The term **frequency response** is used to describe a system's response to a periodic input. Frequency response analysis focuses on a system's response to *harmonic* input such as sines and cosines. The input (forcing) function is written below.

$$f(t) = A\sin(\omega t)$$

Amplitude of the Input, $\quad A \quad (N)$

Frequency of Input, $\quad \omega \quad (\frac{rad}{s})$

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First Order Frequency Response

The steady state response we derived is shown. Remember, after some amount of time passes, the transient term will disappear leaving just the sinusoidal terms.
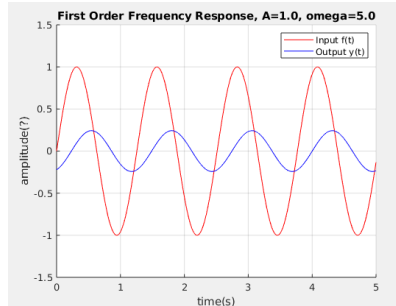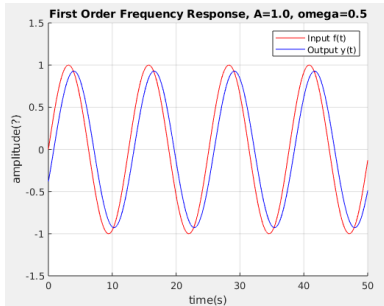
$$y_{ss}\left(t\right) = A|T\left(j\omega\right)|sin\left(\omega t + \angle T\left(j\omega\right)\right) = MAsin\left(\omega t + \phi\right)$$

The magnitude ratio and phase shift can be found from $T(j\omega)$.

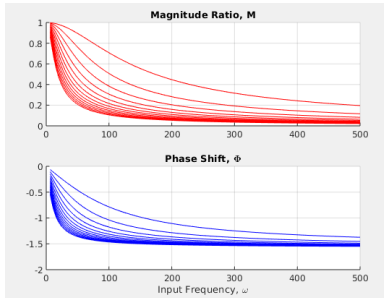$$M(\omega) = |T(j\omega)| = \frac{1}{\sqrt{1+\tau^2\omega^2}}$$

$$\phi(\omega) = \angle T\left(j\omega\right) = -tan^{-1}\left(\omega\tau\right)$$

Review Frequency Response
Magnitude Ratio in Decibels
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Graph of Frequency Response

The amplitude of the response is determined by the input frequency.

Dependence on Input Frequency

You can see that the magnitude ratio decreases as the input frequency increases. The indivdual curves represent systems different time constants.

Review Properties of Logarithms

Basic Properties of Logarithms:

Multiplication $\quad log(pq) = log(p) + log(q)$

Division $\qquad log(\frac{x}{y}) = log(x) - log(y)$

Power $\qquad\qquad log(x^n) = nlog(x)$

Units of Decibels for Magnitude:

$m(dB) = 10log(M^2) = 20log(M)$ $\qquad$ convert back: $\qquad M = 10^{\frac{m(dB)}{20}}$

Magnitude Ratio on a Logarithmic Scale

These relationships are more useful shown on a logarithmic scale. We can make use of the properties of logorithms in our analysis.

$$m\,(dB) = 20log\left(\frac{1}{\sqrt{1+\omega^2\tau^2}}\right) = 20\left(log\,(1) - log\sqrt{1+\omega^2\tau^2}\right)$$

$$m\,(dB) = 20log\,(1) - 10log\left(1+\omega^2\tau^2\right) = -10log\left(1+\omega^2\tau^2\right)$$

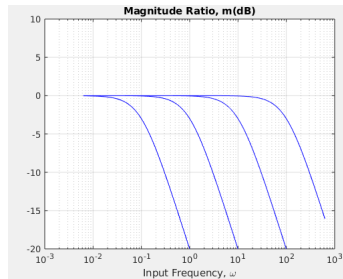$$m\,(dB) = -10log\left(1+\omega^2\tau^2\right)$$

magnitude ratio in decibels

Magnitude Ratio on a Logarithmic Scale

This is called a Bode plot.

Hendrik Bode (1905-1982)

References

- System Dynamics, Palm III, Third Edition - Chapter 9 - System Response in the Frequency Domain