

## Module 13 - Higher Order Systems

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

### Topic 2 - State Space Representations

## Topic 2 - State Space Representations

- State Space Models
- The State Space Equation
- The Output Equation
- Simulation and Control Applications

# State Space Models

- Most linear differential equations can be written in state space form.
- A state space model is an equivalent representation of a dynamic system.
- This standard form allows us to use and share tools for analysis and design of complex systems.

## State Space Models

**Higher Order Differential Equations** - An  $N^{th}$  order differential equation can be **decomposed** into a system of  $N$  first order differential equations. The resulting system is equivalent to the original equation.

# The State Space Equation

After the system of differential equations consists of first order equations only, these equations form [the state equation](#).

# The State Space Equation

## The State Equation

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

- there are  $n$  state variables or states:  $x_1 - x_n$
- there are  $m$  inputs called  $u_1 - u_m$
- the state vector  $\mathbf{x}$  is a column vector with  $n$  rows
- the system matrix  $\mathbf{A}$  is a square matrix  $n$  rows and  $n$  columns.
- the input vector  $\mathbf{u}$  is a column vector with  $m$  rows.
- the control or input matrix  $\mathbf{B}$  is a matrix with  $n$  rows and  $m$  columns.

# The Output Equation

## The Output Equation

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

- the **output vector**  $\mathbf{y}$  is a column vector with  $p$  rows
- the **output matrix**  $\mathbf{C}$  is a square matrix  $p$  rows and  $n$  columns.
- the **control matrix**  $\mathbf{D}$  is a matrix with  $p$  rows and  $m$  columns.

The designer chooses any combination of dependent variables or derived quantities for the output equation for your individual purposes. The number of outputs is *flexible*.

## The Output Equation

Choose the outputs that you want to study and write the output equations as functions of the states and **not** their derivatives.



# Simulation and Control Applications

- *commonly used* for system models
- useful for *numerical simulation*
- used in the area of *automatic control*
- This standard form allows us to use and share tools for analysis and design of complex systems.

## References

- System Dynamics, Palm III, Third Edition - Chapter 4 - Spring and Damper Elements in Mechanical Systems - pg. 208