

Lecture Module - Electrical Systems

ME3050 - Dynamic Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 4 - Example: DC Motor

Electrical Systems

- Brushed DC Motor
- Model Derivation
- State Space Form
- Transfer Functions
- Simulated Response

Model Derivation

Armature Controlled Brushed DC Motor

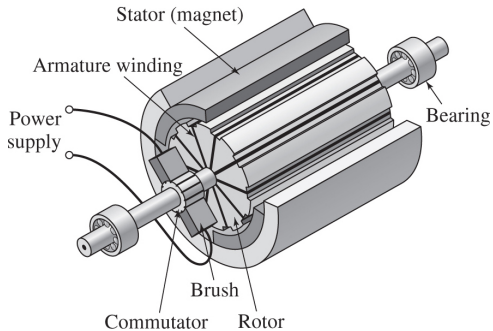
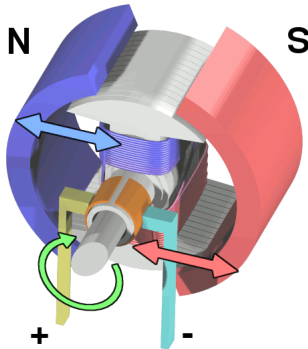


Image: System Dynamics, Palm, 4th, Pg. 376-378

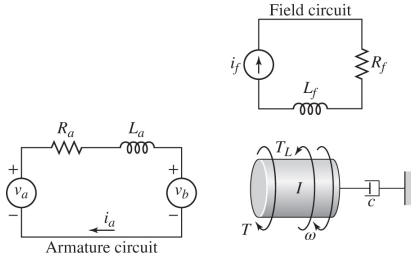
Brushed DC Motor



[Animation on Web](#)

Model Derivation

Armature Controlled Brushed DC Motor



v_a : armature voltage (input)

R_a : armature resistance

Torque on armature

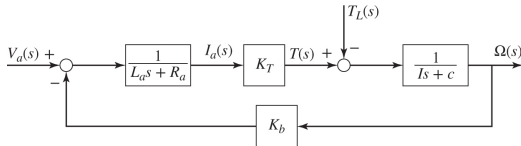
$$T = (nBLi_a)r = (nBLr)i_a = K_T i_a$$

Back EMF (electromotive force)
 voltage

$$v_b = nBLv = (nBLr)\omega = K_b \omega$$

Model Derivation

Armature Controlled Brushed DC Motor



Kirchoff's Voltage Law

$$v_a - R_a i_a - L_a \frac{di_a}{dt} - K_b \omega = 0$$

Newtons's Second Law

$$J \frac{d\omega}{dt} = T - c\omega - T_L = K_T i_a - c\omega - T_L$$

State Space Form

State-Variable (State-Space) form

$$\frac{di_a}{dt} = \dot{x}_1 = \frac{1}{L_a} (v_a - R_a i_a - K_b \omega) = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \end{bmatrix} \begin{bmatrix} v_a \\ T_L \end{bmatrix}$$

$$\frac{d\omega}{dt} = \dot{x}_2 = \frac{1}{J} (K_T i_a - c\omega - T_L) = \begin{bmatrix} -\frac{K_T}{J} & -\frac{c}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} v_a \\ T_L \end{bmatrix}$$

Write the state equation in matrix form with states $x_1 = i_a$, and $x_2 = \omega$

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} -\frac{R_a}{L_a} & -\frac{K_b}{L_a} \\ \frac{K_T}{J} & -\frac{c}{J} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} + \begin{bmatrix} \frac{1}{L_a} & 0 \\ 0 & -\frac{1}{J} \end{bmatrix} \begin{bmatrix} v_a \\ T_L \end{bmatrix}$$

Transfer Functions

The input-output relationships can be represented by the following transfer functions.

Armature Current to Armature Voltage

$$\frac{I_a(s)}{V_a(s)} = \frac{Is + c}{L_a Is^2 + (R_a I + cL_a)s + cR_a + K_b K_T}$$

Armature Current to External Load

$$\frac{I_a(s)}{T_L(s)} = \frac{K_b}{L_a Is^2 + (R_a I + cL_a)s + cR_a + K_b K_T}$$

Transfer Functions

Armature Angular Velocity to Armature Voltage

$$\frac{\Omega(s)}{V_a(s)} = \frac{K_T}{L_a I s^2 + (R_a I + c L_a) s + c R_a + K_b K_T}$$

Armature Angular Velocity to External Load

$$\frac{\Omega(s)}{T_L(s)} = \frac{L_a s + R_a}{L_a I s^2 + (R_a I + c L_a) s + c R_a + K_b K_T}$$

Simulated Response

The following MATLAB code defines a state space system object and simulates the system response to various inputs.

```
% DC Motor Example (System Dynamics 4th ed., Palm, Pg  
376-378)
```

```
clear; close all; clc
```

```
% define system parameters
```

```
KT=0.05; % (N*m/A)
```

```
Kb=KT; % (N*m/A)
```

```
c=10e-4; % (N*m*s/rad)
```

```
Ra=0.5; % (Ohm)
```

```
La=2e-3; % (H)
```

```
I=9e-5; % (kg*m^2)
```

Simulated Response

```
% define components of the state equation
A=[-Ra/La -Kb/La
    KT/I -c/I];
% B matrix is 2x2 because u vector is 2x1
B=[1/La 0
    0 -1/I];

% use first two states as outputs
C=[1 0
    0 1];
% the D matrix shape of B matrix
D=[0 0
    0 0];
```

Simulated Response

```
% create a state space model object  
sys1=ss(A,B,C,D);  
  
% simulate a step response  
figure(1)  
step(sys1)  
grid on; xlabel('time(s)')
```
