

# ME 3050 Lecture - Laplace Transforms

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- **Partial Fraction Expansion leads to a General Form:**

$$X(s) = \frac{N(s)}{D(s)} = \frac{b_ms^m + b_{m-1}s^{m-1} + \dots + b_1s + b_0}{s^n + a_{n-1}s^{n-1} + \dots + a_1s + a_0} \quad n \geq m$$

- **Case 1 - Distinct Roots:** n roots are real and distinct

The general form is factored:

$$X(s) = \frac{N(s)}{(s+r_1)(s+r_2)\dots(s+r_n)}$$

The fraction will expand to:

$$X(s) = \frac{C_1}{(s+r_1)} + \frac{C_2}{(s+r_2)} + \dots + \frac{C_n}{(s+r_n)}$$

Where:

$$C_i = \lim_{s \rightarrow -r_i} \{X(s)(s + r_i)\}$$

And this leads to a solution:

$$x(t) = C_1e^{-r_1t} + C_2e^{-r_2t} + \dots + C_ne^{-r_nt}$$

- **Case 2 - Repeated Roots:**  
**p** number of roots have the same value ( $s = -r$ ) and remaining roots are distinct and real distinct

$$X(s) = \frac{N(s)}{(s+r_1)^p(s+r_{p+1})(s+r_{p+2})...(s+r_n)}$$

The fraction will expand to:

$$X(s) = \frac{C_1}{(s+r_1)^p} + \frac{C_2}{(s+r_1)^{p-1}} + ...$$

$$+ \frac{C_p}{(s+r_1)} + \frac{C_{p+1}}{(s+r_{p+1})} + ... + \frac{C_n}{(s+r_n)}$$

Coefficients for the repeated root are:

$$C_1 = \lim_{s \rightarrow -r_i} \{X(s)(s + r_i)^p\}$$

$$C_2 = \lim_{s \rightarrow -r_i} \left\{ \frac{d}{ds} X(s)(s + r_i)^p \right\}$$

$$C_i = \lim_{s \rightarrow -r_i} \left\{ \frac{1}{(i-1)!} \frac{d^{(i-1)}}{ds^{(i-1)}} X(s)(s + r_i)^p \right\}$$

Coefficients for the distinct roots are the same as in Case 1:

And this leads to a solution:

$$x(t) = C_1 \frac{t^{p-1}}{(p-1)!} e^{-r_1 t} + C_2 \frac{t^{p-2}}{(p-2)!} e^{-r_1 t} + ...$$

$$... + C_p e^{-r_1 t} + C_{p+1} e^{-r_{p+1} t} ... + C_n e^{-r_n t}$$