Module 4 - Energy Methods

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 1 - The Conservation of Energy

Topic 1 - The Conservation of Energy

- Brief History
- Second Law Derivation
- Non-Conservatve Forces
- Engineering Applications

Brief History

... the law of conservation of energy states that the total energy of an isolated system remains constant; it is said to be conserved over time. This law, first proposed and tested by Émilie du Châtelet, means that energy can neither be created nor destroyed; rather, it can only be transformed or transferred from one form to another.

Text: Wikipedia, Image: Wikipedia

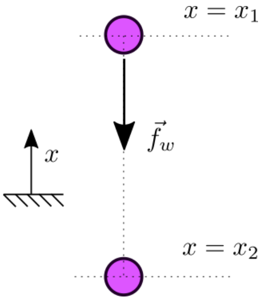
Brief History

… however, special relativity showed that mass is related to energy and vice versa by $E = mc^2$, and science now takes the view that massenergy as a whole is conserved. Theoretically, this implies that any object with mass can itself be converted to pure energy, and vice versa, though this is believed to be possible only under the most extreme of physical conditions …

Text: Wikipedia

Second Law Derivation

A mass moves in the x direction with only the force of gravity acting on it. Netwon's Second Law combined with the definition of differential work done by a force through a distance gives a relation between kinetic energy and work done by the external force.



$$\mathbf{f} = m\mathbf{a} = m\frac{d\mathbf{v}}{dt}$$

$$dW = \mathbf{f}\,d\mathbf{x}$$

$$dW = \mathbf{f}\,d\mathbf{x} = m\mathbf{a}d\mathbf{x} = md\mathbf{v}\frac{d\mathbf{x}}{dt} = md\mathbf{v}\mathbf{v}$$

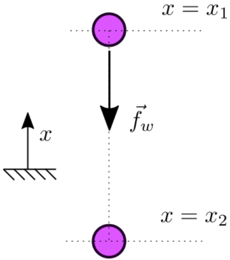
$$\rightarrow dW = mv_x dv_x$$

$$W_{12} = \int_{x_1}^{x_2} mv_x dv_x = \frac{1}{2}mv_x^2\Big|_{x_1}^{x_2}$$

$$W_{12} = \frac{1}{2}mv_{x_2}^2 - \frac{1}{2}mv_{x_1}^2 = KE_1 - KE_2$$

Second Law Derivation

Alternativley, subsitute the distance traveled into the differential work relation. Now you are left with the familar relation between work done by gravity and the potential energy in the system.



$$dW = \mathbf{f}\,d\mathbf{x}$$

$$W_{12} = \int_{x_1}^{x_2} \mathbf{f}\,d\mathbf{x} = (-mg)|_{x_1}^{x_2} = (-mg)(x_2 - x_1)$$

$$W_{12} = -mg(x_2 - x_1)$$

Now we can see that

$$W_{12} = -mg(x_2 - x_1) = \frac{1}{2}mv_{x_2}^2 - \frac{1}{2}mv_{x_1}^2$$

$$\Delta KE + \Delta PE = 0$$

Non-Conservatve Forces

Engineering Applications