

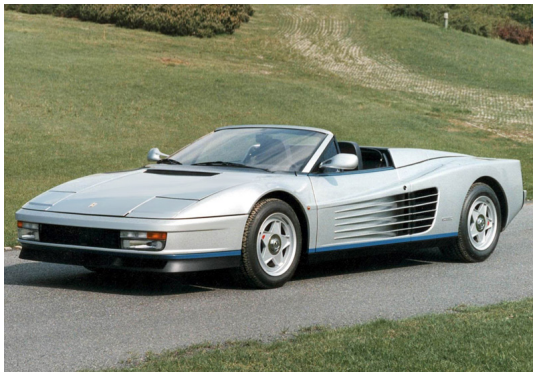
ME 3050 Lecture - Dynamic Modeling and Controls

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Ch. 8 - System Response in the Time Domain

- (8.1) Time Response of 1st Order Systems

- Consider the model of the moving mass we derived.



- The EOM is:

$$m\dot{v} + cv = 0$$

- Solve for $v(t)$ using a method of your choice.

- The method of Laplace Transforms is shown.

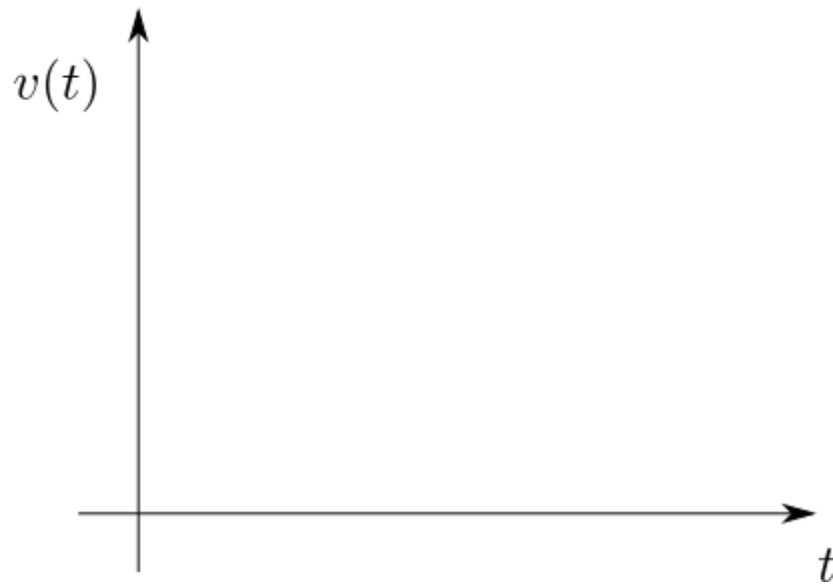
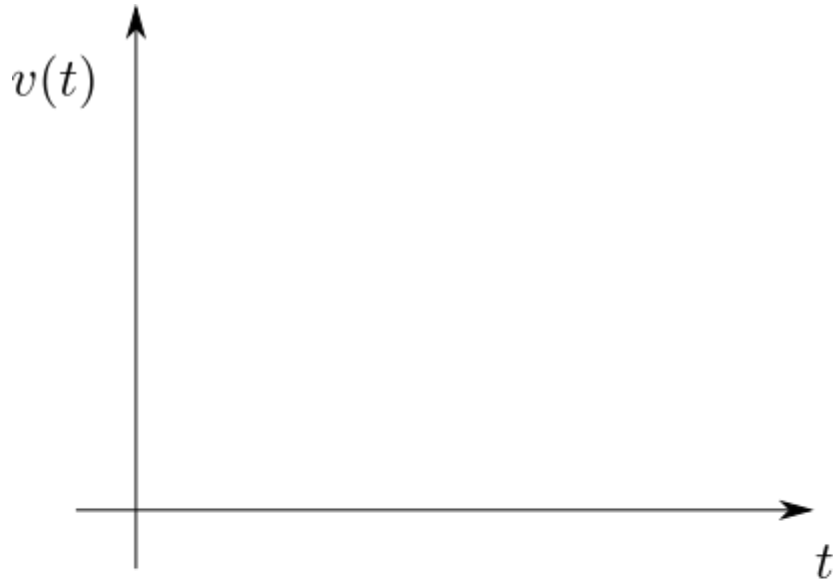
$$\mathcal{L}\{m\dot{v} + cv = 0\} \implies m[sV(s) - v(0)] + cV(s) = 0$$

$$(ms + c)V(s) = \frac{mv(0)}{(ms+c)} = \frac{V(0)}{s+\frac{c}{m}}$$

- We can find the expected result from the table.

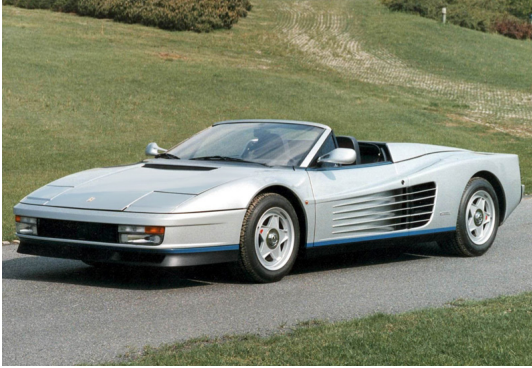
$$v(t) = v(0)e^{-\frac{c}{m}t} = v(0)e^{-\frac{t}{\tau}} \quad \text{with} \quad \tau =$$

- Sketch the System Response in the time Domain.



- Is this a stable system? What does that even mean?

- Consider the model subject to a Step Input, $f(t)$.



$$m\dot{v} + cv = f(t)$$

$$f(t) = \begin{cases} 0 & t < 0 \\ F & t \geq 0 \end{cases}$$

- The method of Laplace Transforms is shown.

$$\mathcal{L}\{m\dot{v} + cv = F\} \implies m[sV(s) - v(0)] + cV(s) = \frac{F}{s}$$

$$(ms + c)V(s) = \frac{F}{s} + mv(0)$$

- Partial Fraction Expansion leads to the following form.

$$V(s) = \frac{F}{s(ms+c)} + \frac{mv(0)}{ms+c} \implies \frac{F}{s(ms+c)} = \frac{a}{s} + \frac{b}{ms+c}$$

- 'Cover up' to find the coefficients.

$$a = \frac{F}{m \times 0 + c} \quad \text{and} \quad b = \frac{F}{\frac{-c}{m}} = \frac{-Fm}{c}$$

- This leads to a form that can be inverted with the table.

$$V(s) = \frac{F}{c} \left\{ \frac{1}{s} - \frac{1}{s + \frac{c}{m}} \right\} + \frac{v(0)}{s + \frac{c}{m}}$$

$$v(t) = \frac{F}{c} \{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

- In these forms we can see the different components of the response.

$$v(t) = \frac{F}{C}\{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

- Forced Response
- Free Response
- Transient Response
- Steady-State Response

- Sketch the System Response in the time Domain.

