Frequency Response - Lecture 3

ME3050 - Dynamics Modeling and Controls

April 25, 2020

Frequency Response of 2^{nd} Order Systems

Lecture 3 - Frequency Response of 2^{nd} Order Systems

- Review Transfer Functions
- Frequency Response of Overdamped Systems
- Frequency Response of Underdamped Systems
- MATLAB code for Bode Plots

Equivalent System Representations

The**Transfer Function** is the input-output relationship in the frequency domain and can be found from the equation of motion of the system.

$$T(s) = \frac{X(s)}{F(s)}$$

The Transfer Function is an equivalent representation of the system.

E.O.M $\leftrightarrow$ T(s) $\leftrightarrow$ Block Diagram

Transfer Function of 2^{nd} Order System

$$m\ddot{x} + c\dot{x} + kx = f(t) \qquad \text{with} \qquad f(t) = Asin(\omega t)$$

The transfer function can easily be found by taking the Laplace transform of the equation of motion.

$$\boxed{T(s) = \frac{X(s)}{F(s)} = \frac{1}{ms^2+cs+k} \qquad \text{Second Order Transfer Function}}$$

The Overdamped System

In an overdamped system, both roots are real and distinct.

The transfer function is shown below in terms of the system parameters

$T(s) = \frac{1/k}{\left(\frac{m}{k}\right)s^2+\left(\frac{c}{k}\right)s+1} = \frac{1/k}{(\tau_1 s+1)(\tau_2 s+1)}$ $\quad \tau_1, \tau_2$ - time constants of the roots

Substitute $s = j\omega$ into the transfer function.

$T(s) \to T(j\omega) = \frac{1/k}{(\tau_1 j\omega+1)(\tau_1 j\omega+1)}$

Now find the magnitude ratio anf phase angle. Convert to nunits of decibels and use log rules to expand.

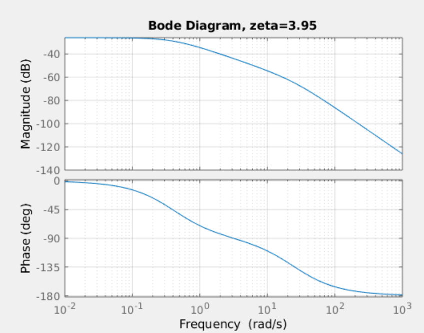
$M(\omega) = |T(j\omega)| = \frac{|1/k|}{|\tau_1 j\omega+1||\tau_2 j\omega+1|}$

$m(\omega) = 20logM(\omega) = 20log|1/k| - 20log|\tau_1\omega j + 1| - 20log|\tau_2\omega j + 1|$

The Bode Diagram

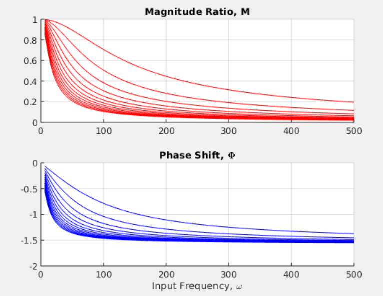
These three terms can be seen on the Bode diagram.

$$m(\omega) = 20logM(\omega) = 20log|1/k| - 20log|\tau_1\omega j + 1| - 20log|\tau_2\omega j + 1|$$



This shows that the magnitude ratio of the system across different regions of the input frequency.

Dependence on Input Frequency



You can see that the magnitude ratio decreases as the input frequency increases. The individual curves represent systems with different time constants.

Review Properties of Logarithms

Basic Properties of Logarithms:

Multiplication $log(pq) = log(p) + log(q)$

Division $log(\frac{x}{y}) = log(x) - log(y)$

Power $log(x^n) = nlog(x)$

Units of Decibels for Magnitude:

$m(dB) = 10log(M^2) = 20log(M)$ convert back: $M = 10^{\frac{m(dB)}{20}}$

Magnitude Ratio on a Logarithmic Scale

These relationships are more useful shown on a logarithmic scale. We can make use of the properties of logarithms in our analysis.

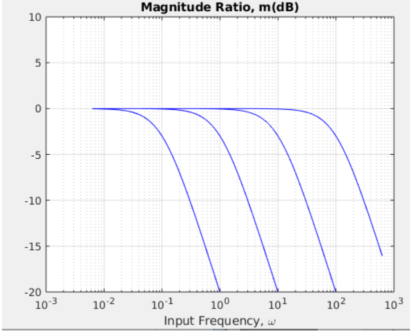
$$m\left(dB\right) = 20log\left(\frac{1}{\sqrt{1+\omega^2\tau^2}}\right) = 20\left(log\left(1\right) - log\sqrt{1+\omega^2\tau^2}\right)$$

$$m\left(dB\right) = 20log\left(1\right) - 10log\left(1+\omega^2\tau^2\right) = -10log\left(1+\omega^2\tau^2\right)$$

$$m\left(dB\right) = -10log\left(1+\omega^2\tau^2\right)$$

magnitude ratio in decibels

Magnitude Ratio on a Logarithmic Scale

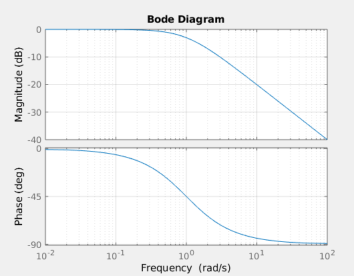


This is a Bode plot. It seems abstract but there is some very useful information shown.

Hendrik Bode (1905-1982)

Bode Plot in MATLAB

MATLAB has a built it tool for making Bode plots.




```
figure(1)
sys=tf(1,[tau(3) 1])
bode(sys);grid on
```

References

- System Dynamics, Palm III, Third Edition - Chapter 9 - System Response in the Frequency Domain