ME 3050 Lecture - State Space Models

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- **We have been studying very simple models:**

$$m\dot{v} + cv = f(t)$$

and

$$m\ddot{x} + c\dot{x} + kx = f(t)$$

- **These accurately describe all mechanical systems ... right?**

- **No, but we can improve them by adding complexity. How?**

Improvements/Additions to the model:

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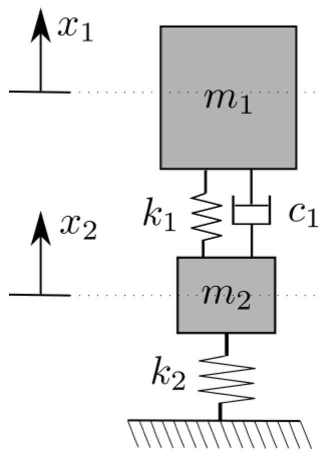
- **Higher Order Models** - Mechanical systems involve the interactions between multiple rigid bodys. This can be seen in many examples.
  - Automobile Suspension
  - Beam Deflection (FEA)
  - Tether Based Space Travel
  - Virtually Everything!



- **Higher Order EOMs** - There is one equation of motion for Each body. In class we derived the following EOMs for the suspension model shown.

Equation of Motion for Mass 1:

$$m_1\ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0$$

Equation of Motion for Mass 1:

$$m_2\ddot{x}_2 + k_2x_2 - c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = 0$$

We want to find a solution to this system of differential equations.
Find $x_1(t)$ and $x_2(t)$ due to given initial condtions x_{1o}, x_{2o}, v_{1o} , and v_{2o}.

- **State Space Model Representation (textbook 5.2)** -
  - *commonly used* for system models
  - useful for *numerical simulation*
  - used in the area of *automatic control*
  - an ODE system has an equivalent State Space Model representation

- **The State Equation** - Standard Form

$$\dot{\boldsymbol{x}} = \boldsymbol{A}\boldsymbol{x} + \boldsymbol{B}\boldsymbol{u}$$

  - there are n *state variables* or *states* called $x_1 - x_n$
  - there are m *inputs* called $u_1 - u_m$
  - the *state vector* $\boldsymbol{x}$ is a collumn vector with n rows
  - the *system matrix* $\boldsymbol{A}$ is a square matrix n rows and n columns.
  - the *input vector* $\boldsymbol{u}$ is a column vector with m rows.
  - the *control or input matrix* $\boldsymbol{B}$ is a matrix with n rows and m columns.

- **The Output Equation** - Standard Form

$$\boldsymbol{y} = \boldsymbol{C}\boldsymbol{x} + \boldsymbol{D}\boldsymbol{u}$$

  - the *output vector* $\boldsymbol{y}$ is a collumn vector with p rows
  - the *output matrix* $\boldsymbol{C}$ is a square matrix p rows and n columns.
  - the *control matrix* $\boldsymbol{D}$ is a matrix with p rows and m columns.

- **Example** - Let's do a simple example before we do the more complex suspension model. You can use this for any system of *linear* ODEs.