

ME 3050 Lecture - State Space Models

Tristan W. Hill - Tennessee Technological University - Spring 2020

- We have been studying very simple models:

$$m\dot{v} + cv = f(t)$$

and

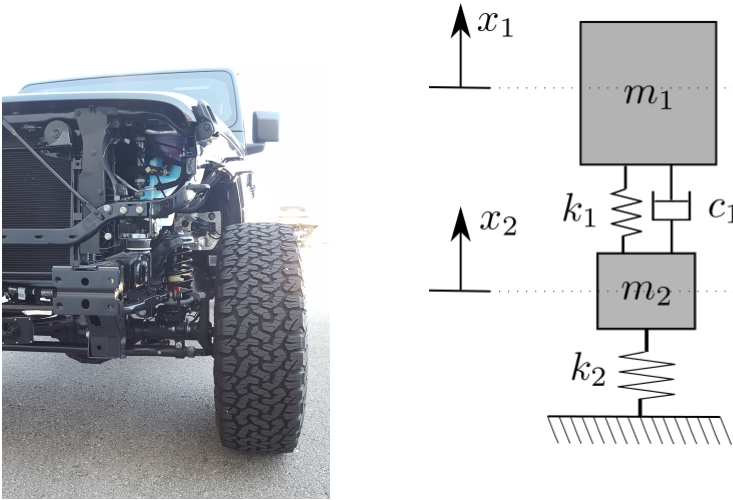
$$m\ddot{x} + c\dot{x} + kx = f(t)$$

- These accurately describe all mechanical systems ... right?
- No, but we can improve them by adding complexity. How?

Improvements/Additions to the model:

—
—
—

- **Higher Order Models** - Mechanical systems involve the interactions between multiple rigid bodies. This can be seen in many examples.
 - Automobile Suspension
 - Beam Deflection (FEA)
 - Tether Based Space Travel
 - Virtually Everything!



- **Higher Order EOMs** - There is one equation of motion for Each body. In class we derived the following EOMs for the suspension model shown.

Equation of Motion for Mass 1:

$$m_1 \ddot{x}_1 + c_1(\dot{x}_1 - \dot{x}_2) + k_1(x_1 - x_2) = 0$$

Equation of Motion for Mass 2:

$$m_2 \ddot{x}_2 + k_2 x_2 - c_1(\dot{x}_1 - \dot{x}_2) - k_1(x_1 - x_2) = 0$$

We want to find a solution to this system of differential equations.

Find $x_1(t)$ and $x_2(t)$ due to given initial conditions x_{1o}, x_{2o}, v_{1o} , and v_{2o} .

- **State Space Model Representation (textbook 5.2)** -
 - *commonly used* for system models
 - useful for *numerical simulation*
 - used in the area of *automatic control*
 - an ODE system has an equivalent State Space Model representation
- **The State Equation - Standard Form**

$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$$

- there are n *state variables* or *states* called $x_1 - x_n$
- there are m *inputs* called $u_1 - u_m$
- the *state vector* \mathbf{x} is a column vector with n rows
- the *system matrix* \mathbf{A} is a square matrix n rows and n columns.
- the *input vector* \mathbf{u} is a column vector with m rows.
- the *control or input matrix* \mathbf{B} is a matrix with n rows and m columns.

- **The Output Equation - Standard Form**

$$\mathbf{y} = \mathbf{C}\mathbf{x} + \mathbf{D}\mathbf{u}$$

- the *output vector* \mathbf{y} is a column vector with p rows
- the *output matrix* \mathbf{C} is a square matrix p rows and n columns.
- the *control matrix* \mathbf{D} is a matrix with p rows and m columns.

- **Example** - Let's do a simple example before we do the more complex suspension model. You can use this for any system of *linear* ODEs.

