Module 5 - Rotation Systems

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 3 - A Better Pendulum

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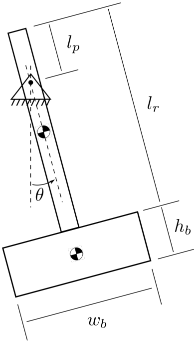
- Example Problem - Pendulum Model
- Mathematical Modeling
- Newton's Second Law Approach
- Derived Equations of Motion

Example Problem - Pendulum Model



length of rod, $l_r = 1.0$ (m)

length to pivot, $l_p = 0.25$ (m)

height of block, $h_b = 0.25$ (m)

width of block, $w_b = 0.5$ (m)

mass of rod, $m_r = 5$ (kg)

mass of block, $m_b = 25$ (kg)

Problem Statement - Derive the equations of motion using Newton's Second Law for the swinging pendulum shown.

Mathematical Modeling

First, describe the model and list any important assumptions.

- The pendulum rigid and is composed a rod and a block. It rotates about the pin.
- Theta is positive in the counter-clockwise direction from the zero point (pointing down).
- There is no friction in the system. The pin joint is perfect and there is no air resistance.
- Gravity acts on the two center of gravity locations shown.

Newton's Second Law Approach

Newton's Second Law Approach

1. Draw a Free Body Diagram
2. Make an assumption of motion
3. Determine all forces acting on the system and their directions.
4. Write Newton's second law for the appropriate DOF.
5. Re-write the ODE in the standard form of a system equation.

Newton's Second Law Approach

Identify the forces acting on the rigid body. Because this a rotational problem we can ignore the reaction forces at the pin, but keep in mind the do exist and we could solve for them. Also, the radially forces will affect the moment, so they too can be ignored. We are left with the tangential forces.

Newton's Second Law Approach

We need the perpendiculur moment arms from the forces to the pin. You have to be careful with the link lengths here. Define to new lengths l_1 as the lengths from the CG of the rod to pin and l_2 as the length from the CG of the block to the pin.

$$l_1 = \frac{1}{2}l_r - l_p$$
$$l_2 = \frac{1}{2}l_r + \frac{1}{2}l_b + l_1$$

Newton's Second Law Approach

Now, determine the mass moment of inertia for the rigid body.

The intertia for the rod about the CG is define as:

$$I_{r,CG} = \frac{1}{12} m_r l^2$$

The intertia for the block about the CG is define as:

$$I_{b,CG} = \frac{1}{12} m_b (h_b^2 + w_b^2)$$

They both must be translated to the pin joint using the *parallel axis theorem* so they can be summed.

$$I_{r,o} = \frac{1}{12} m_r l^2 + m_r l_1^2$$

$$I_{b,o} = \frac{1}{12} m_b (h_b^2 + w_b^2) + m_b l_2^2$$

$$I_o = I_{r,o} + I_{b,o} = \frac{1}{12} m_r l^2 + m_r l_1^2 + \frac{1}{12} m_b (h_b^2 + w_b^2) + m_b l_2^2$$

Newton's Second Law Approach

Now we can write Newton's Second Law for rotation about a fixed point. The sum of the moments equals the mass moment of inertia times the angular acceleration.

$$\Sigma M_o = I_o \times \alpha = I_o \ddot{\theta}$$

$$\Sigma M_o = -w_{r,tangential} \times l_1 - w_{b,tangential} \times l_2 = I_o \times \alpha = I_o \ddot{\theta}$$

$$\Sigma M_o = -w_r \sin\theta \times l_1 - w_b \sin\theta \times l_2 = I_o \times \alpha = I_o \ddot{\theta}$$

Rearrange into the standard form of the *equation of motion*.

Derived Equations of Motion

with,

$$l_1 = (1/2)l_r - l_p$$

$$l_2 = (1/2)l_r + (1/2)l_b + l_1$$

and

$$I_o = (1/12)m_r l^2 + m_r l_1^2 + (1/12)m_b(h_b^2 + w_b^2) + m_b l_2^2$$