Module 11 - First Order Time Response

ME3050 - Dynamics Modeling and Controls

Mechanical Engineering

Tennessee Technological University

Topic 1 - First Order Free Response

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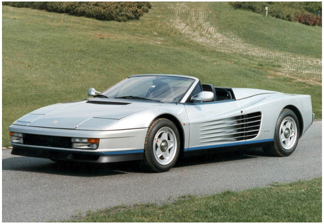
- Model and EOM
- Solution with Laplace Transforms Method
- The Critically Damped Case
- The Underdamped Case

Model and EOM

Consider the model of the moving mass we derived.



The EOM is:

$$m\dot{v} + cv = 0$$

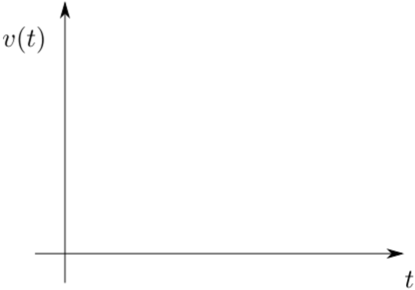
Solution with Laplace Transforms Method

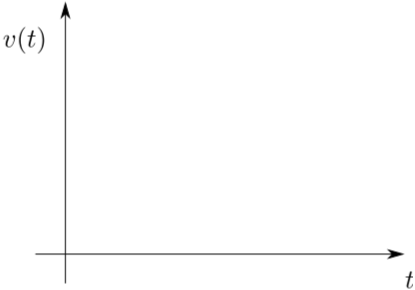
$$\mathcal{L}\{m\dot{v} + cv = 0\} \implies$$

We can find the expected result from the table.

Sketch Response Equation

Sketch the System Response in the time Domain.

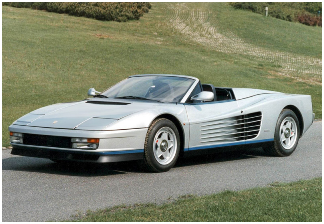




Is this a stable system? What does that mean?

Step Input Function

Consider the model subject to a Step Input, $f(t)$.



$$m\dot{v} + cv = f(t)$$

$$f(t) = \begin{cases} 0 & t < 0 \\ F & t \geq 0 \end{cases}$$

Solution with Laplace Transforms Method - Step 1

The method of Laplace Transforms is shown.

Solve for $V(s)$.

Solution with Laplace Transforms Method - Step 2

Expand $V(s)$as a partial fraction.

'Cover up' to find the coefficients.

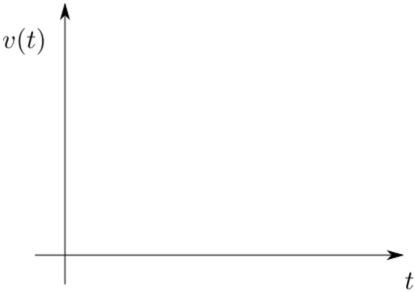
This leads to a form that can be inverted with the table.

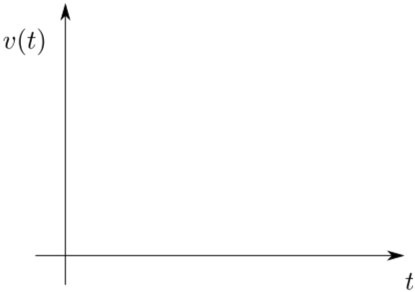
Solution with Laplace Transforms Method - Step 3

Can you find these terms in the Table of Laplace Transforms?
The inverse Laplace transform of $V(s)$ gives the time response.

Sketch Response Equation

Sketch the System Response in the time Domain.





Is this a stable system?

Components of the Response

In these forms we can see the different components of the response.

$$v(t) = \frac{F}{C}\{1 - e^{-\frac{t}{\tau}}\} + v(0)e^{-\frac{t}{\tau}} = \{v(0) - \frac{F}{c}\}e^{-\frac{t}{\tau}} + \frac{F}{c}$$

- Forced Response
- Free Response
- Transient Response
- Steady-State Response

References

- System Dynamics, Palm III, Third Edition - Section 8.1 - Response of First Order Systems - pg. 475