Ordinary Differential Equations - Lecture 5

ME3001 - Mechanical Engineering Analysis

Mechanical Engineering

Tennessee Technological University

Trial Solution for Second Order ODEs

Lecture 5 - Trial Solution for Second Order ODEs:

- Trial Solution Method
- Complementary Solution
- Particular Solution
- Apply Initial Conditions

Trial Solution Method

Use the **trial solution method** to solve the ODE.

This is an **analytical** method that you learned in calculus but it may have been called something different. In the Zill book it is called *Homogenous Linear ... Constant Coefficients (4.3-4.4)*.

$$a_2 y'' + a_1 y' + a_0 y = f(x)$$

Trial Solution Method

Complementary Solution

Step 1 - Find the **complementary part** of the solution from the left hand side of the ODE alone (LHS=0).

$$a_2 y'' + a_1 y' + a_0 y = f \quad \rightarrow \quad a_2 y'' + a_1 y' + a_0 y = 0$$

Assume an exponential solution for the complementary part.

$$y_{complementary} = y_c(t) =$$

Substitute this solution into the ODE (LHS=0).

Complementary Solution

Particular Solution

Step 2 - Find the **particular part** of the solution from the entire equation (LHS=RHS).

$$a_2 y'' + a_1 y' + a_0 y = f$$

The *form of the particular part* follows the RHS of the ODE.

$$y_{particular} = y_p(t) =$$

Substitute this solution into the ODE above and solve for any unknown constants in $y_p(t)$.

Particular Solution

Apply Initial Conditions

Step 3 - Now combine the **complementary** and **particular** solutions through *superposition*.

$$y(x) = y_c(x) + y_p(x) =$$

The ODE is first order and we have ______ unknown. Coincidence?

$$y(x) =$$

This **initial value problem** requires ______ intial condition.

Apply Initial Conditions

$$y(x = 0) =$$

$$y'(x = 0) =$$

Apply Initial Conditions

What does the solution look like this time?

$$y(x) =$$