ME 3050 Lecture - Dynamic Modeling and Controls

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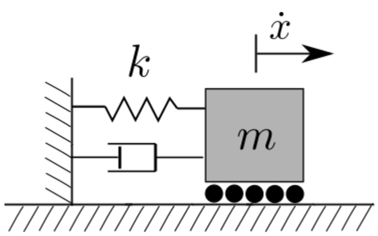
Ch. 8 - System Response in the Time Domain

- **(8.2) Time Response of 2^{nd} Order Systems**

  - **Now consider our mass-spring-damper system.**



  - **The EOM is:**

$$m\ddot{x} + kx = 0 \quad \text{with} \quad x(t=0) = x_0, \quad \text{and} \quad v(t=0) = v_0$$

  - **You have practiced solving for $x(t)$. Look at the solution.**

$$x(t) = \frac{v_0}{\omega_n} sin(\omega_n t) + x_0 cos(w_n t) \quad \text{with} \quad \omega_n = \sqrt{\frac{k}{m}}$$

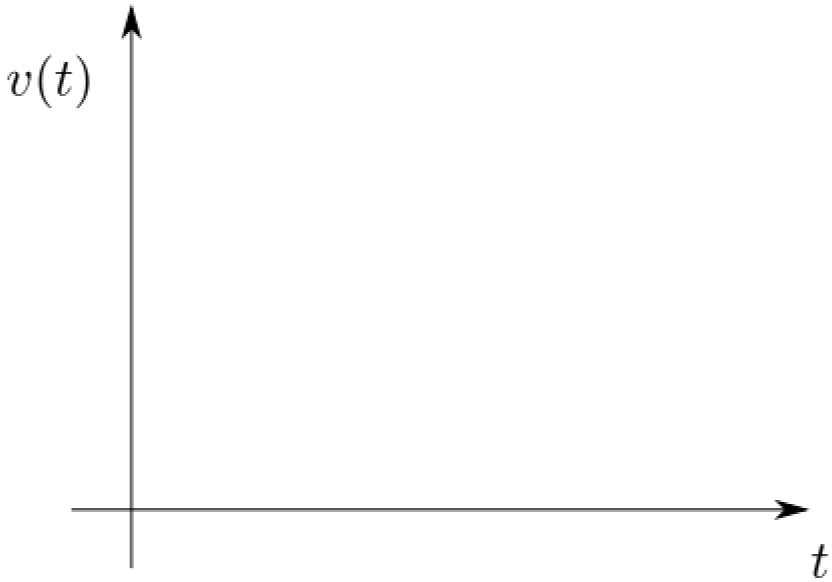
  - **The solution is more commmonly used in the following form. The phase shift ϕ has been introduced.**

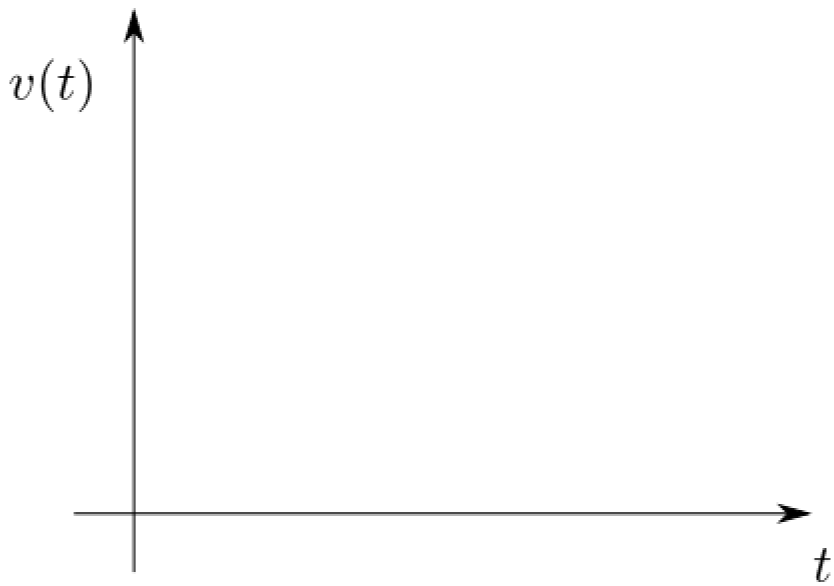
$$x(t) = Acos(\omega_n t - \phi) \quad A = \sqrt{x_0^2 + [\frac{v_0}{\omega_n}]^2} \quad \phi = tan^{-1}(\frac{v_0}{x_0 \omega_n})$$

  - **Or we could use sine instead**

$$x(t) = Asin(\omega_n t + \phi) \quad A = \sqrt{x_0^2 + [\frac{v_0}{\omega_n}]^2} \quad \phi = tan^{-1}(\frac{x(0)\omega_n}{v_0})$$

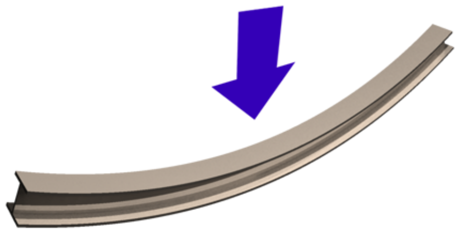
– **Sketch the System Response in the time Domain.**





– **Is this a stable system? What does that even mean?**

- **Now bring the damper back.**



$$m\ddot{x} + c\dot{x} + kx = 0$$

- **The trial solution method is shown.**

$$m\ddot{x} + c\dot{x} + kx = 0 \implies (mr^2 + cr + k)Ae^{rt} = 0$$

- **You can see the Characteristic Equation becomes:**

$$(mr^2 + cr + k) = 0$$

- **Now you can solve for the roots. In system dynamics they are called** $s_{1,2}$

$$r_{1,2} = s_{1,2} = \frac{-c \pm \sqrt{c^2 - 4mk}}{2m} = -\frac{c}{2m} \pm \sqrt{(\frac{c}{2m})^2 - \frac{k}{m}}$$

- **The roots of the system determine the nature of the behaviour.**

$$c^2 - 4mk = 0 \quad \text{and} \quad c = 2\sqrt{mk}$$

- **This value of c is called the critical damping value.**

if $\quad c < 2\sqrt{mk} \quad$ the system will oscillate

if $\quad c \geq 2\sqrt{mk} \quad$ the system will NOT oscillate

- **Now we want to quatify <u>how much</u> damping there is in the system.**

The damping ratio ζ is the ratio of actual damping, c, to critical damping.

$\zeta = \frac{c}{2\sqrt{mk}}$

- **We can now re-write the roots with this new quantity.**

$$s_{1,2} = -\zeta\omega_n \pm \omega_n\sqrt{\zeta^2 - 1} = -\zeta\omega_n \pm j\omega_n\sqrt{1 - \zeta^2}$$

- **We define one more important new quantity, <u>damped natural frequency</u>.**

$$\omega_d = \omega_n\sqrt{1 - \zeta^2}$$

- **We can now re-write the roots one more time.**

$$s_{1,2} = \zeta\omega_n \pm j\omega_d$$