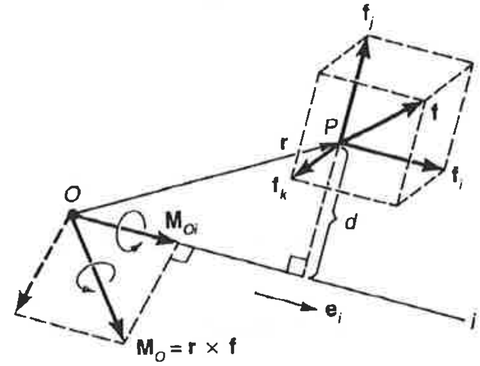


Figure 7.7

Although the moment about the axis can be computed according to this definition, an alternative form is often easier to use and provides a different interpretation. Substitution of Eq. (7.10) into Eq. (7.12) and use of the vector identity $\mathbf{a} \bullet (\mathbf{b} \times \mathbf{c}) = (\mathbf{a} \times \mathbf{b}) \bullet \mathbf{c}$ leads to $\mathbf{M}_{Oi} = [\mathbf{e}_i \times \mathbf{r}) \bullet \mathbf{f}\,]\mathbf{e}_i$

Now if $\mathbf{f}$ is resolved into a component $\mathbf{f}_i$ parallel to the axis Oi, a component $\mathbf{f}_j$ perpendicular to O_i and in the plane of $\mathbf{r}$ and O_i, and a component $\mathbf{f}_k$ perpendicular to this plane, several important facts become apparent. Referring to Fig. 7.7, note that because both $\mathbf{f}_i$ and $\mathbf{f}_j$ are perpendicular to $\mathbf{e}_i \times \mathbf{r}$, neither of these components contributes to $\mathbf{M}_{Oi'}$. Also, because $\mathbf{e}_i \times \mathbf{r}$ has the magnitude $d = |\mathbf{r}| \sin \angle \frac{\mathbf{r}}{\mathbf{i}}$, we can express the magnitude of the moment component as

$$|\mathbf{M}_{Oi}| = df_k$$

where d is the perpendicular distance from point P to the axis Oi.

The *sense* of $\mathbf{M}_{Oi}$ (that is, whether it is directed in the positive or negative i-direction) is readily determined from the direction of $\mathbf{f}_k$ and the right-hand rule. Alternatively, the sense may be determined by the sign of the factor $\mathbf{e}_i \bullet \mathbf{M}_O$ in Eq. (7.12). Note that the same value of d would be obtained regardless of where the point O is on the i-axis, and recall that the position vector $\mathbf{r}$ in the definition $\mathbf{M}_O = \mathbf{r} \times \mathbf{f}$ can be from O to any point on the line of action of $\mathbf{f}$. This means that Eq. (7.12) will yield the value $\mathbf{M}_{Oi}$ with $\mathbf{r}$ as a position vector from *any* point on the axis O_i to *any* point on the line of action of $\mathbf{f}$.

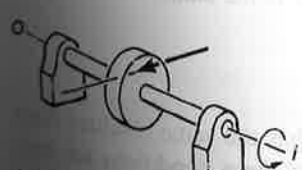


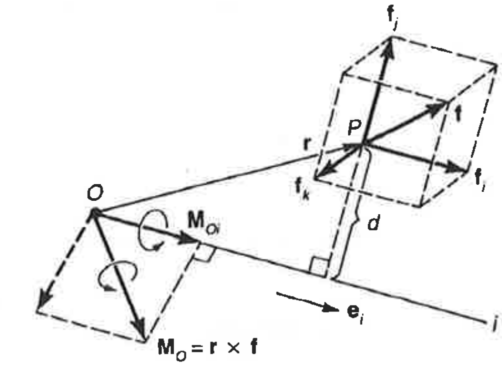
Figure 7.8

The moment about an axis is a measure of the tendency of the force(s) to cause rotation about the axis. For example, if a rotor is mounted in bearings and subjected to a set of forces, the moment of these forces about the axis of the bearings is found to be directly related to the rate of change of rotational speed. Neither forces parallel to the axis nor forces with lines of action passing through the axis will affect the rotation.

Resultant Forces and Moments

If there are several forces $\mathbf{f}_1, \mathbf{f}_2, \ldots, \mathbf{f}_n$, each with its own line of action, the **resultant force** is defined as

$$\mathbf{f} = \sum_{i=1}^{n} \mathbf{f}_i$$



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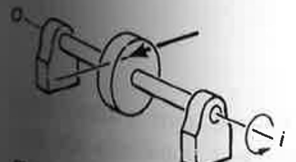

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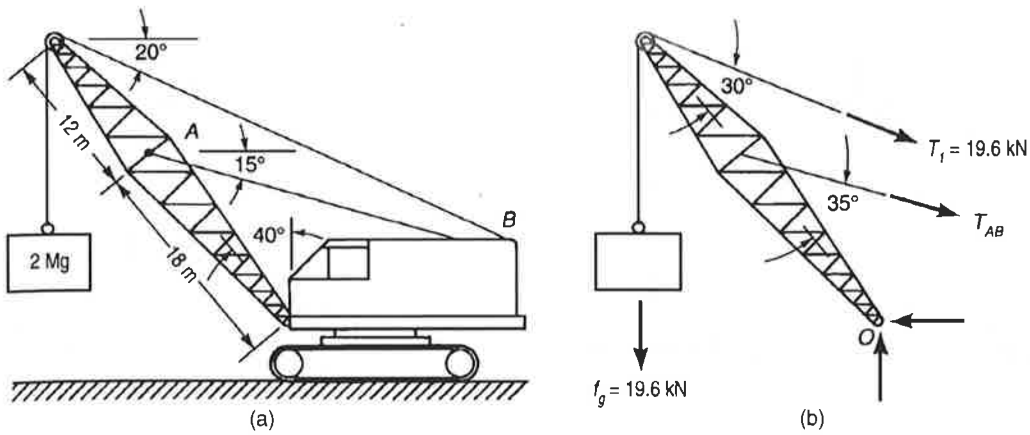

Exhibit 6

Solution

The boundary of the free-body passes through the two upper cable segments and the support point, labeled O in Exhibit 6(b). By considering moment equilibrium of a system consisting of the pulley and a portion of the cable, including the section that is in contact with the pulley (not shown), we find that $T_1 = f_g = 19.6$ kN.

To avoid introducing the unknown reaction at O into the analysis, consider moments of forces about this point. With the radius of the pulley denoted as r, the equation of moments about O is

$$f_g[(30\text{ m})\sin 40° + r] - T_1[(30\text{ m})\sin 30° + r] - T_{AB}\sin 35°(18\text{ m}) = 0$$

With the value of $T_1 = f_g$ substituted, this is readily solved for the tension in the supporting cable:

$$T_{AB} = \frac{(19.6\text{ kN})(30\text{ m})(\sin 40° - \sin 30°)}{(18\text{ m})\sin 35°} = 8.13\text{ kN}$$

Example 7.8

Gravity forces on the structural members are negligible compared with P and Q in Exhibit 7(a). Evaluate all the forces acting on each of the three members in the A-frame.

Solution

Free-bodies of the entire frame and of each individual member are shown in Exhibit 7(b)–(e). The roller support at D means that no horizontal force can be transmitted from the ground at that point. From the free-body in view (b) we can consider moments about point E,

$$(3a)Q + (3a\tan 30°)P - (6a\tan 30°)R_D = 0$$

horizontal forces,

$$R_{Ex} - Q = 0$$

The following examples provide further illustration of the use of the basic laws of static equilibrium.

Example 7.6

Neglecting gravity forces in Exhibit 5(a), except those on the 300-kg load, determine the forces in the cable and in the boom.

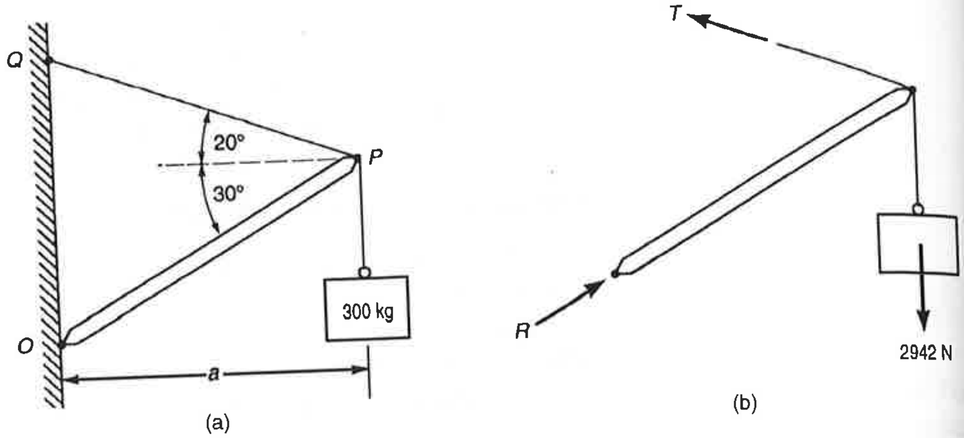

Exhibit 5

Solution

First, a free-body diagram is drawn (Exhibit 5(b)), with the system boundary passing through the support point O and through the cable segment PQ. Vanishing of moment about point P requires that the reaction at O must be directed along the boom. The equation of moments about O can be expressed as

$$(T \sin 20° - 2942 \text{ N})a + (T \cos 20°)(a \tan 30°) = 0$$

from which

$$T = \frac{2942 \text{ kN}}{\sin 20° + \cos 20° \tan 30°} = 3.33 \text{ kN}$$

Equilibrium of horizontal forces,

$$R \cos 30° - T \cos 20° = 0$$

leads to the magnitude of the reaction at O:

$$R = \frac{(3.33 \text{ N})\cos 20°}{\cos 30°} = 3.61 \text{ kN}$$

Example 7.7

Neglecting gravity forces except those on the 2-Mg load shown in Exhibit 6(a), determine the tension in cable AB, which is holding up the crane boom.