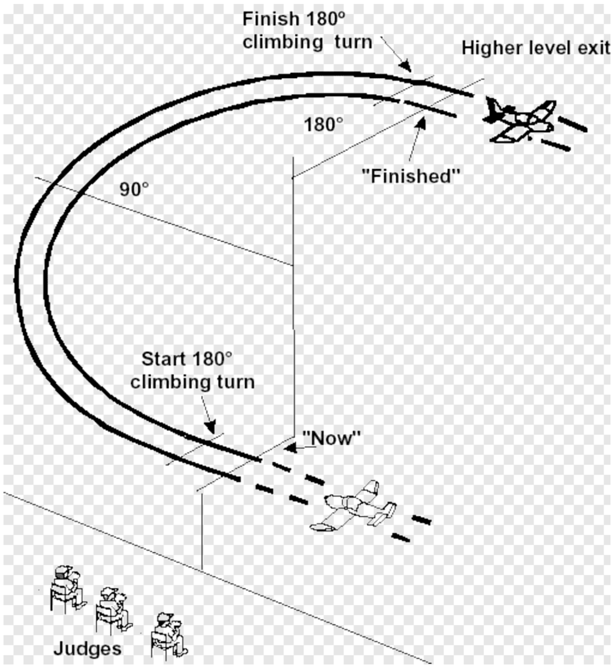
FE Review - Solid Mechanics - Dynamics

Review of Particle and Rigid Body Dynamics



Outline:

- Kinematics of a Particle
- Rigid Body Kinematics
- Newtons Laws of Motion
- Work and Energy Methods
- Kinetics of Rigid Bodies

- Kinematics of a Particle

$$\vec{v} = \frac{d\vec{r}}{dt} = v\hat{e_t}$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\hat{e_t} + \vec{v}\frac{ds}{dt}\frac{d\hat{e_t}}{ds} = \dot{v}\hat{e_t} + \frac{\nu^2}{\rho}\hat{e_n}$$

$$\frac{d\hat{e_t}}{ds} = \kappa\hat{e_n}$$

Distance Velocity and the Tangential Component of Acceleration

$$\frac{dv}{dt} = a_t \qquad \text{or} \qquad v = v_0 + \int a_t dt$$

$$\frac{ds}{dt} = v \qquad \text{or} \qquad s = s_0 + \int v dt$$

$$v\frac{dv}{ds} = a_t \qquad \text{or} \qquad v^2 = v_0^2 + 2\int a_t ds$$

Constant Tangential Acceleration

$$v = v_0 + a_t t$$

$$s = s_0 + v_0 t + \frac{1}{2}a_t t^2$$

$$v^2 = v_0^2 + 2a_t s$$

- Rigid Body Kinematics

  Constraint of Rigidity

  $$\frac{d}{dt}|r_{pq}|^2 = \frac{d}{dt}(\vec{r}_{pq} \cdot \vec{r}_{pq}) = 2\vec{r}_{pq} \cdot \frac{d\vec{r}_{pq}}{dt} = 0$$

  Instantaneous Zero Velocity

- Newtons Laws of Motion

  $$\vec{f} = m\vec{a}$$

  $$\vec{p} = \Sigma_i m_i v_i$$

  $$\frac{d\vec{p}}{dt} = \Sigma_i m_i a_i$$

- Work and Energy Methods
- Kinetics of Rigid Bodies

- **Example 8.8**: Piston, Crank and Connecting Rod

As the crank OQ in Exhibit 7 rotates clockwise at 200 rad/s, the piston P moves vertically. What will be the velocity of the piston at the instant when the angle θ is 50 degrees?

Solution

Since point Q must follow a circular path, its speed may be determined from Eq. (8.15): $v_Q = (0.075 \text{ m})(200 \text{ s}^{-1}) = 15$ m/s, with the direction of $\mathbf{v}_Q$ as indicated in the figure. Because the cylinder wall constrains the piston, its velocity is vertical. The connecting rod PQ is rigid, so the velocities of the points P and Q must

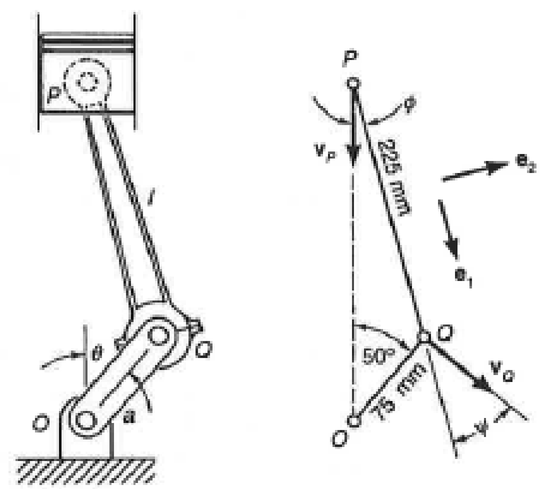


Exhibit 7

satisfy $v_P \cos\phi = v_Q \cos\psi$. The trigonometric rule of sines, applied to the triangle OPQ, gives

$$\sin\phi = \frac{a}{l}\sin\theta = \frac{75}{225}\sin 50^\circ$$

which yields $\phi = 14.8°$. The other required angle is then $\psi = 90° - \theta - \phi = 25.2°$. Once these angles are determined, the constraint equation yields the speed of the piston:

$$v_P = \frac{\cos\psi}{\cos\phi} v_Q = 14.04 \text{ m/s}$$

• **Example 8.9**: Piston, Crank and Connecting Rod

What will be the angular velocity of the connecting rod in Example 8.8, at the instant when the angle θ is 50 degrees?

Solution

Referring to Exhibit 7 for the definition of $\mathbf{e}_2$, we see that

$$\begin{aligned}\omega &= \frac{v_Q \sin\psi + v_Q \sin\phi}{l} \\ &= \frac{(15\ \text{m/s})\sin 25.2^\circ + (14.04\ \text{m/s})\sin 14.8^\circ}{0.225\ \text{m}} = 44.3\ \text{rad/s}\end{aligned}$$

The positive value indicates that the rotation is counterclockwise at this instant.

• **Example 8.10**: Piston, Crank and Connecting Rod

What is the location of the instantaneous center C' of the connecting rod in Examples 8 and 9? Use this to verify the previously-determined values of the angular velocity of the connecting rod and the velocity of point P.

Solution

The velocity of any point of the connecting rod must be perpendicular to the line from C' to that point. Hence C' must lie at the point of intersection of the

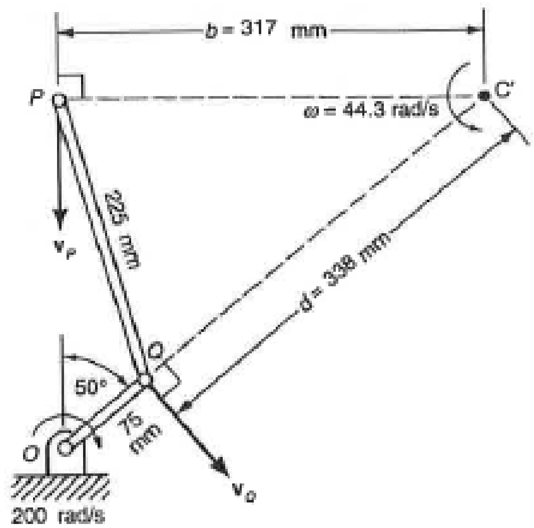


Exhibit 8

horizontal line through P and the line through Q perpendicular to $\mathbf{v}_Q$ (i.e., on the line through O and Q), as shown in Exhibit 8. The pertinent distances can be found as follows:

$$OP = (75 \text{ mm}) \cos 50^\circ + (225 \text{ mm}) \cos 14.8^\circ = 266 \text{ mm}$$
$$PC' = OP \tan 50^\circ = 317 \text{ mm}$$
$$QC' = OP \sec 50^\circ - 75 \text{ mm} = 338 \text{ mm}$$

The angular velocity of the connecting rod is then

$$\omega = \frac{v_Q}{QC'} = \frac{15 \text{ m/s}}{0.339 \text{ m}} = 44.3 \text{ rad/s}$$

and the velocity of P is then

$$v_P = PC'\omega = (0.317 \text{ m})(44.\ 3 \text{ s}^{-1}) = 14.04 \text{ m/s}$$

in agreement with values the previously obtained.

- **Example 8.12**: Piston, Crank and Connecting Rod

If the speed of the crank in Examples 8–10 is constant, what are the acceleration $\mathbf{a}_P$ of the piston and the angular acceleration α of the connecting rod at the instant when the angle θ is 50 degrees (Exhibit 10)?

Solution

When the crank speed is constant, the acceleration of Q is entirely centripetal, of magnitude

$$a_Q = (0.075 \text{ m})(200 \text{ s}^{-1})^2 = 3000 \text{ m/s}^2$$

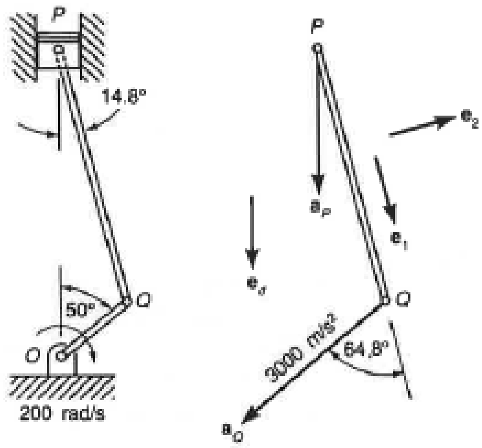


Exhibit 10

and directed toward the center of curvature O of the path of Q. The acceleration of P is vertically upward or downward. To determine the direction, we define a downward unit vector $\mathbf{e}_d$ and let $\mathbf{a}_p = a_p\mathbf{e}_d$ (see Exhibit 10). A positive value of a_P then indicates a downward acceleration and a negative value an upward acceleration. These expressions for a_Q and $\mathbf{a}_P$ are substituted into Eq. (8.25), along with the previously determined angular velocity of the rod, giving

$$(3000 \text{ m/s}^2) \cos 64.8° = a_P \cos 14.8° - (0.225 \text{ m})(44.3 \text{ s}^{-1})^2$$

which yields

$$a_P = 1779 \text{ m/s}^2$$

The angular acceleration α of the rod can then be determined from Eq. (8.26);

$$\alpha = \frac{(3000 \text{ m/s}^2)\cos 154.8° - (1779 \text{ m/s}^2)\cos 104.8°}{0.225 \text{ m}} = -10\ 050 \text{ rad/s}^2$$

The negative value indicates that the angular acceleration is clockwise; that is, the 44.3-rad/s counterclockwise angular velocity is rapidly decreasing at this instant.