FE Exam Review

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Chapter 8 Dynamics - Lecture 1

Chapter 8 Dynamics - Lecture 1

- Introduction to Dynamics
- Kinematics of a Particle
- Rigid Body Kinematics
- Newtons Laws of Motion
- Examples

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Also in Ch. 8

- Work and Energy Methods
- Kinetics of Rigid Bodies

Introduction to Dynamics

What is Dynamics?

- **Dynamics** is a subset of mechanics focused on the motion of bodies and the forces that affect them.
- .. fundamental to many disciplines of engineering.
- .. essential in **mechanical engineering** and **design**.

Kinematics of a Particle

We begin with a particle (point mass) moving in space.

$$\vec{v} = \frac{d\vec{r}}{dt} = v\hat{e}_t$$

$$\vec{a} = \frac{d\vec{v}}{dt} = \dot{v}\hat{e}_t + \vec{v}\frac{ds}{dt}\frac{d\hat{e}_t}{ds} = \dot{v}\hat{e}_t + \frac{\nu^2}{\rho}\hat{e}_n$$

$$\frac{d\hat{e}_t}{ds} = \kappa\hat{e}_n$$

Kinematics of a Particle

Distance Velocity and the Tangential Component of Acceleration

$$\frac{dv}{dt} = a_t \qquad or \qquad v = v_0 + \int a_t dt$$

$$\frac{ds}{dt} = v \qquad or \qquad s = s_0 + \int v dt$$

$$v\frac{dv}{ds} = a_t$$

$$v^2 = v_0^2 + 2\int a_t ds$$

Kinematics of a Particle

Constant Tangential Acceleration

$$v = v_0 + a_t t$$

$$s = s_0 + v_0 t + \frac{1}{2} a_t t^2$$

$$v^2 = v_0^2 + 2 a_t s$$

Rigid Body Kinematics

Rigid Body Kinematics

Constraint of Rigidity

$$\frac{d}{dt}|r_{pq}|^2 = \frac{d}{dt}(\vec{r}_{pq} \cdot \vec{r}_{pq}) = 2\vec{r}_{pq} \cdot \frac{d\vec{r}_{pq}}{dt} = 0$$

Instantaneous Zero Velocity

Newtons Laws of Motion

Newtons Laws of Motion

Every object persists in its state of rest or uniform motion in a straight line unless it is compelled to change that state by forces impressed on it.

Force is equal to the change in momentum (mV) per change in time. For a constant mass, force equals mass time acceleration ($F = ma$).

For every action, there is an equal and opposite re-action.

Newtons Laws of Motion

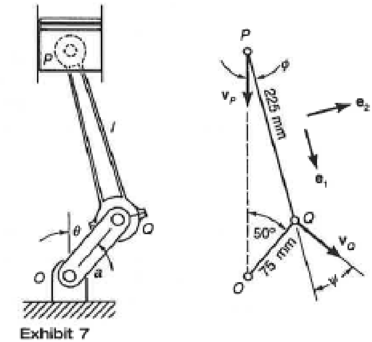
$$\vec{f} = m\vec{a}$$

$$\vec{p} = \Sigma_i m_i v_i$$

$$\frac{d\vec{p}}{dt} = \Sigma_i m_i a_i$$

Examples

Example 1 (Ex. 8.8):



As the crank OQ in Exhibit 7 rotates clockwise at 200 rad/s, the piston P moves vertically. What will be the velocity of the piston at the instant when the angle θ is 50 degrees?

Examples

Example 1 (cont.):

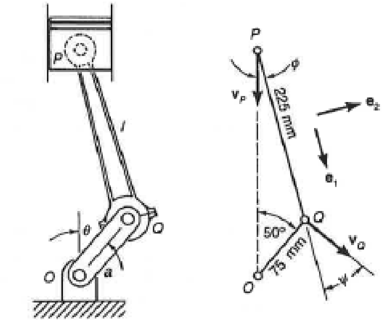


Exhibit 7

Examples

Example 2 (Problem 8.9):

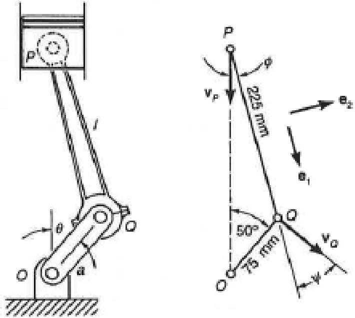


Exhibit 7

What will be the angular velocity of the connecting rod in Example 8.8, at the instant when the angle θ is 50 degrees?

Examples

Example 2 (cont.):

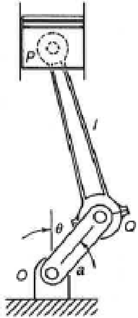
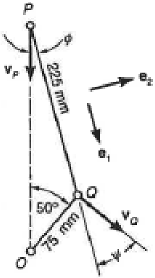


Exhibit 7



Examples

Additional Examples (cont.):

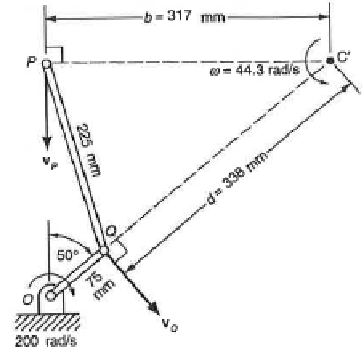


Exhibit 8

What is the location of the instantaneous center C' of the connecting rod in Examples 8 and 9? use this to verify the previously-determined values of the angular velocity of the connecting rod and the velocity of point P.

Examples

Example 3:

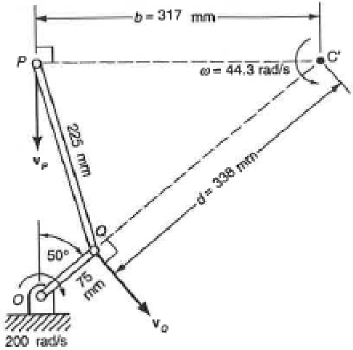


Exhibit 8

Examples

Solutions to Example 1 (Ex. 8.8):

As the crank OQ in Exhibit 7 rotates clockwise at 200 rad/s, the piston P moves vertically. What will be the velocity of the piston at the instant when the angle θ is 50 degrees?

Solution

Since point Q must follow a circular path, its speed may be determined from Eq. (8.15): $v_Q = (0.075 \text{ m})(200 \text{ s}^{-1}) = 15$ m/s, with the direction of $\mathbf{v}_Q$ as indicated in the figure. Because the cylinder wall constrains the piston, its velocity is vertical. The connecting rod PQ is rigid, so the velocities of the points P and Q must

Examples

Solutions to Example 1 (Ex. 8.8) (cont.):

satisfy $v_P \cos\phi = v_Q \cos\psi$. The trigonometric rule of sines, applied to the triangle *OPQ*, gives

$$\sin\phi = \frac{a}{l}\sin\theta = \frac{75}{225}\sin 50^\circ$$

which yields $\phi = 14.8^\circ$. The other required angle is then $\psi = 90^\circ - \theta - \phi = 25.2^\circ$. Once these angles are determined, the constraint equation yields the speed of the piston:

$$v_P = \frac{\cos\psi}{\cos\phi} v_Q = 14.04 \text{ m/s}$$

Examples

Solutions to Example 2 (Ex. 8.9):

What will be the angular velocity of the connecting rod in Example 8.8, at the instant when the angle θ is 50 degrees?

Solution

Referring to Exhibit 7 for the definition of $\mathbf{e}_2$, we see that

$$\omega = \frac{v_Q \sin\psi + v_Q \sin\phi}{l}$$

$$= \frac{(15 \text{ m/s})\sin 25.2° + (14.04 \text{ m/s})\sin 14.8°}{0.225 \text{ m}} = 44.3 \text{ rad/s}$$

The positive value indicates that the rotation is counterclockwise at this instant.

Examples

Solutions to Example 3 (Ex. 8.10):

What is the location of the instantaneous center C' of the connecting rod in Examples 8 and 9? Use this to verify the previously-determined values of the angular velocity of the connecting rod and the velocity of point P.

Solution

The velocity of any point of the connecting rod must be perpendicular to the line from C' to that point. Hence C' must lie at the point of intersection of the

Examples

Solutions to Example 3 (Ex. 8.10) (cont.):

horizontal line through P and the line through Q perpendicular to $\mathbf{v}_Q$ (i.e., on the line through O and Q), as shown in Exhibit 8. The pertinent distances can be found as follows:

$$OP = (75 \text{ mm}) \cos 50^\circ + (225 \text{ mm}) \cos 14.8^\circ = 266 \text{ mm}$$
$$PC' = OP \tan 50^\circ = 317 \text{ mm}$$
$$QC' = OP \sec 50^\circ - 75 \text{ mm} = 338 \text{ mm}$$

The angular velocity of the connecting rod is then

$$\omega = \frac{v_Q}{QC'} = \frac{15 \text{ m/s}}{0.339 \text{ m}} = 44.3 \text{ rad/s}$$

and the velocity of P is then

$$v_P = PC'\omega = (0.317 \text{ m})(44.3 \text{ s}^{-1}) = 14.04 \text{ m/s}$$

Examples

Solutions to Example (Ex. 8.10) (cont.):

horizontal line through P and the line through Q perpendicular to $\mathbf{v}_Q$ (i.e., on the line through O and Q), as shown in Exhibit 8. The pertinent distances can be found as follows:

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