Module 3 - Calibration

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Topic 3 - Linear Regression

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- Motivation - Functional Relationship
- Least Squares Regression
- Using Software Packages
- Example: IR Sensor Calibration

Motivation - Functional Relationship

A measured variable is often a function of one or more independent variables that are controlled during the measurement. ... This is a common procedure used to document the relationship between the measured variable and an independent process variable. ...

We can use regression analysis to establish a functional relationship between the dependent variable and the independent variable. This discussion pertains directly to polynomial curve fits.

Other functions such as exponential and log fits can also be used.

Text: T.HIll, Theory and Design of Mechanical Measurements, 5th Edition

Least Squares Regression

Consider the graphs below. This is a calibration curve.

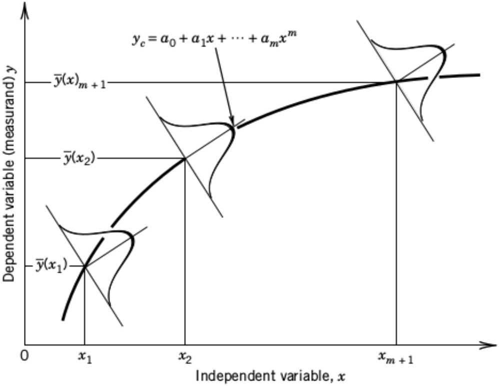


Figure 4.9 Distribution of measured value y about each fixed value of independent variable x. The curve y_c represents a possible functional relationship.

Image: Theory and Design of Mechanical Measurements, 5th Edition

Using Software Packages

- We are trying to find a polynomial of best fit for the data.

  $y_c(x) = a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m \qquad m \leq (n-1)$

- This is done by minimizing the quantity below.

  $D = \Sigma_{i=1}^{N}(y_i - y_{ci})^2 = \Sigma_{i=1}^{N}(y_i - a_0 + a_1 x + a_2 x^2 + \cdots + a_m x^m)^2$

- For a first order curve the coefficients become:

  $a_0 = \frac{\Sigma x_i \Sigma x_i y_i - \Sigma x_i^2 \Sigma y_i}{(\Sigma x_i)^2 - N \Sigma x_i^2} \quad , \; a_1 = \frac{\Sigma x_i \Sigma y_i - N \Sigma x_i y_i}{(\Sigma x_i)^2 - N \Sigma x_i^2}$

Using Software Packages

Most spreadsheet and engineering software packages can perform a regression analysis on a data set. Examples will be shown throughout the course in MATLAB and EXCEL.

- MATLAB - curve fitting toolbox - *polyfit()*
- Excel - *linest()*
- Python - *numpy.linalg.lstsq()*
- Labview - *Linear Fit VI, Exponential Fit VI, etc.*
- and many more...

Sample Calibration Data

Sample #	Known Distance $x_i(m)$	Measured Voltage $y_i(V)$
1	6	1.67
2	9	1.17
3	12	0.89
4	15	0.72
5	18	0.61

Linear Least Squares Regression Coefficients:

$a_0 = 1.5640 \quad , \quad a_1 = 1.5640$

Functional Relationship:

$y = a_1 * x + a_0$

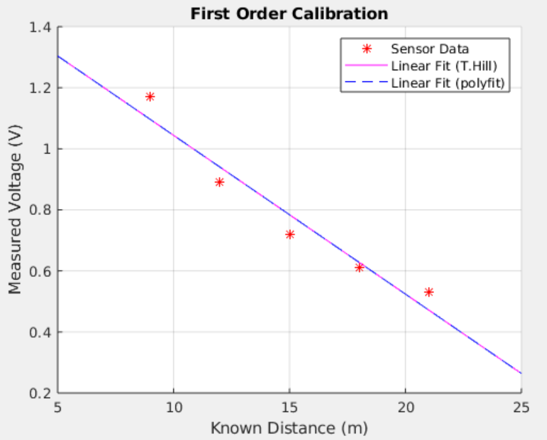
Linear Regression in MATLAB - Manual

```
clear variables;close all;clc

x=[9 12 15 18 21];
y=[1.17 0.89 0.72 0.61 0.53];
n=length(x);

% perform Linear Least Squares Regression
a0=(sum(x)*sum(x.*y)-sum(x.^2)*sum(y))/(sum(x)^2-n*sum(x.^2));
a1=(sum(x)*sum(y)-n*sum(x.*y))/(sum(x)^2-n*sum(x.^2));

x_f=5.0:1.0:25;
y_f=a1*x_f+a0;

% generate 1st order curve fit with polyfit()
P1=polyfit(x,y,1);

x_f1=5.0:1.0:25;
y_f1=P1(1)*x_f1+P1(2);
```

Linear Regression in MATLAB - Manual

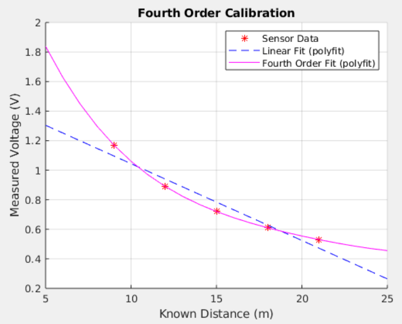
```
figure(1);hold on

plot(x,y,'r*')
plot(x_f,y_f,'m-')
plot(x_f1,y_f1,'b--')

title('First Order Calibration')
xlabel('Known Distance (m)')
ylabel('Measured Voltage (V)')
legend('Raw Data','Linear Fit (T.Hill)','Linear Fit (polyfit)')
grid on
```

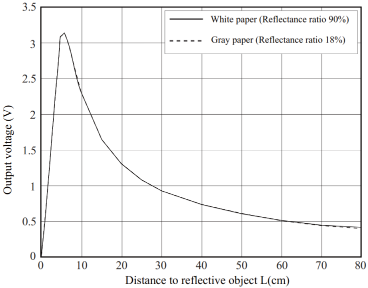
First Order Calibration Curve



Images: T.Hill

Fourth Order Calibration Curve





Images: T.Hill, Sharp