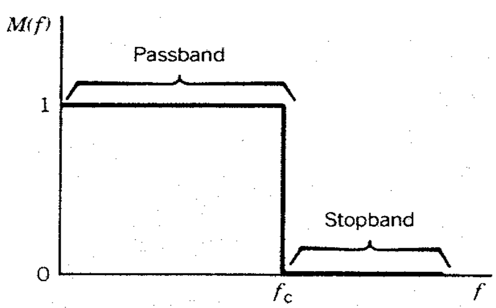
<u>Filter Classifications</u>

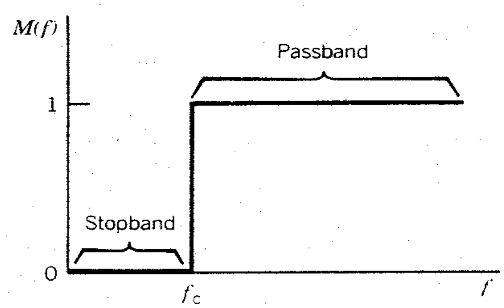
Electronic *filters* can often by applied to sensor signals to remove unwanted frequency components. Filters are classified based on the range of frequencies passed through. Four common types of filters are:

- *Low-pass* – Only frequencies below a selected cutoff frequency, f_c , are passed
- *High-pass* – Only frequencies above a selected cutoff frequency, f_c , are passed
- *Bandpass* – Only frequencies above cutoff frequency, f_{c1} , and below f_{c2} are passed
- *Notch* – Only frequencies below cutoff frequency, f_{c1} , and above f_{c2} are passed

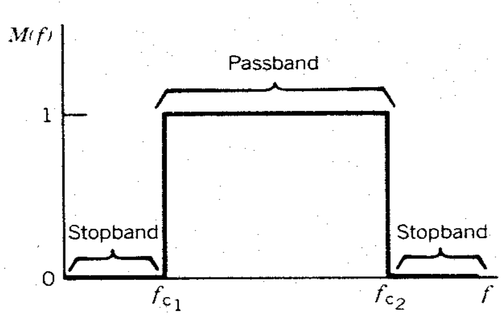
Filter Behavior – Ideal (not real! No filter can obtain this sharp cutoff)



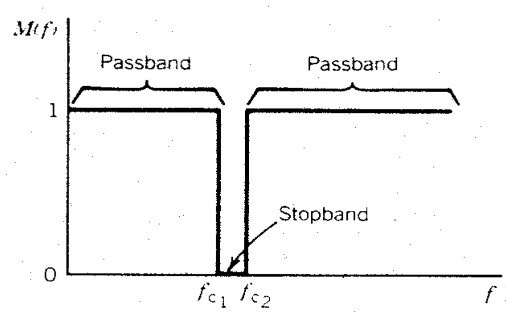
(a) Low-pass filter



(b) High-pass filter



(c) Bandpass filter



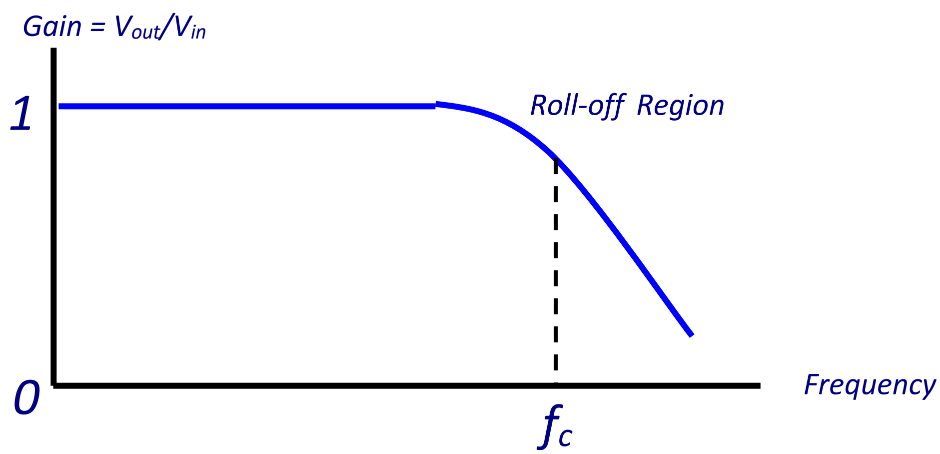
(d) Notch filter

Figure from *Theory and Design of Mechanical Measurements, R.S. Figliola and D.E. Beasley, 1991.*

Real Filters

The frequency response characteristics of an actual "real" filter can only approximate the ideal responses illustrated in the previous figure.

In particular, real filters display a *roll-off* at a limiting frequency instead of a sharp cut-off. The figure below demonstrates a real response from a low pass filter, compare this with the ideal low pass filter in the prior figure.



RC & RL FILTERS

Good results for low cost measurement systems, for example in an application where one wants to minimize noise at high frequencies, can often be obtained by the use of simple RC and RL circuits. See the Table below for mathematical expressions of the ratio of output voltage to input voltage as a function of the frequency of the input voltage.

Note:

The cutoff frequency for a filter built from an RC circuit is determined by $f_{cutoff} = 1 / (2\pi\ RC)$ Hz.

At the cutoff frequency, the response V_{out} equals to 0.707 V_{in}, this is the -3dB point.

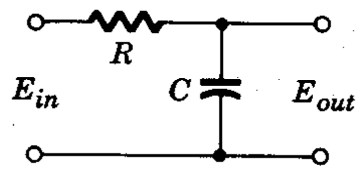
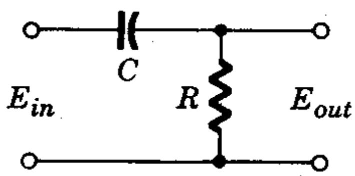
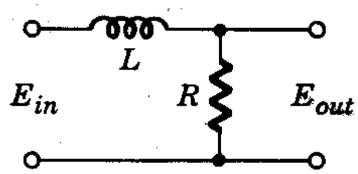
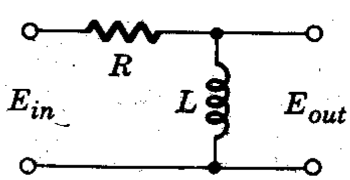
Diagram	*Type*	*Time constant or resonant freq.*	*Formula and approximation*
	**A** low-pass *RC*	$T = RC$	$\frac{E_{out}}{E_{in}} = \frac{1}{\sqrt{1+\omega^2T^2}} \approx \frac{1}{\omega T}$ $\phi_A = -\tan^{-1}(R\omega C)$
	**B** high-pass *RC*	$T = RC$	$\frac{E_{out}}{E_{in}} = \frac{1}{\sqrt{1+\frac{1}{\omega^2T^2}}} \approx \omega T$ $\phi_B = \tan^{-1}\frac{1}{R\omega C}$
	**C** low-pass *RL*	$T = \frac{L}{R}$	$\frac{E_{out}}{E_{in}} = \frac{1}{\sqrt{1+\omega^2T^2}} \approx \frac{1}{\omega T}$ $\phi_C = -\tan^{-1}\frac{\omega L}{R}$
	**D** high-pass *RL*	$T = \frac{L}{R}$	$\frac{E_{out}}{E_{in}} = \frac{1}{\sqrt{1+\frac{1}{\omega^2T^2}}} \approx \omega T$ $\phi_D = \tan^{-1}\frac{R}{\omega L}$

Table from *"Experimental Methods for Engineers," J.P. Holman, 1966, p. 132.*

A little bit of math… and a little bit of MATLAB

The differential equation for a Low-Pass RC circuit is

$$RC\frac{dv}{dt}+v=v_s$$

What is time constant, τ?

And when we consider an input voltage to the circuit of $v_s(t)$ = A sin(ωt), a sinusoidal input,

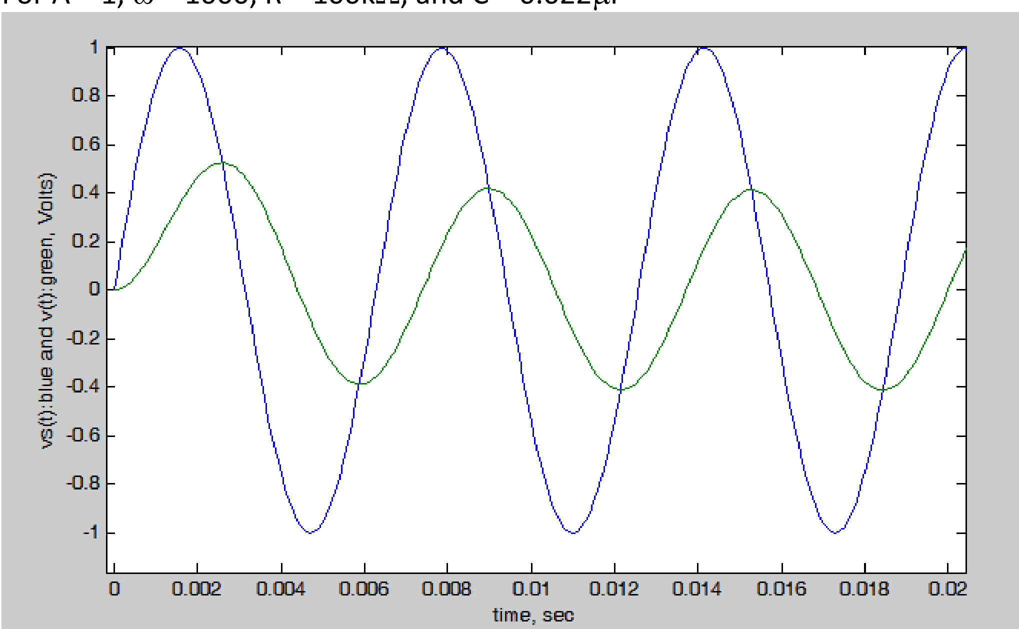
The output voltage will be … (applying our best differential equation solving!)

$$v(t)=\frac{A\omega\tau}{1+\omega^2\tau^2}\left(e^{-t/\tau}-\cos\omega t+\frac{1}{\omega\tau}\sin\omega t\right)$$

Do you see a term in the solution that will become very small as time increases beyond 5 tau?

Plotting the time traces for input (blue) and output (green) from MATLAB…

For A = 1, ω = 1000, R = 100kΩ, and C = 0.022μF




```
omega=1000;
A=1;
R=100e3;
C=0.022e-6;
tau=R*C;
fc = 1/tau
t = 0:0.0001:1;
vs=A*sin(omega*t);
v=( (A*omega*tau) / (1+omega^2*tau^2) )*(exp(-t/tau) - cos(omega*t) +
(1/(omega*tau))*sin(omega*t));
plot(t,vs,t,v);
xlabel('time, sec'),ylabel('vs(t):blue and v(t):green, Volts)')
```

Another way to examine the behavior of the RC filter is to see what happens for **all** values of input frequency, ω. Looking ahead to ME3050 and ME3060, you will learn about *Transfer Functions and Bode plots.*

The *transfer function,* in terms of the Laplace transform, s, for this differential equation is

$$T(s) = \frac{1}{\tau s + 1}$$

and if we let the Laplace variable, s, be equal to s = jω in order to see the effect of driving the system at different frequencies, we get a complex function,

$$T(j\omega) = \frac{1}{\tau(j\omega) + 1}$$

Now we consider the magnitude and phase of the complex transfer function

$$M = \frac{1}{\sqrt{1+\omega^2\tau^2}} \qquad and \qquad \phi = -\tan^{-1}(\omega\tau)$$

These values can be plotted in MATLAB using the *bode* function, which will show the GAIN, M in untis of dB. Note a 0 value of dB is equivalent to 1, so this is showing a low pass filter behavior.

```
num=[1];
den=[R*C 1];
sys = tf(num,den)
bode(sys)
```