

Module 5 - Strain Applications

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering
Tennessee Technological University

Topic 3 - Principle Strains

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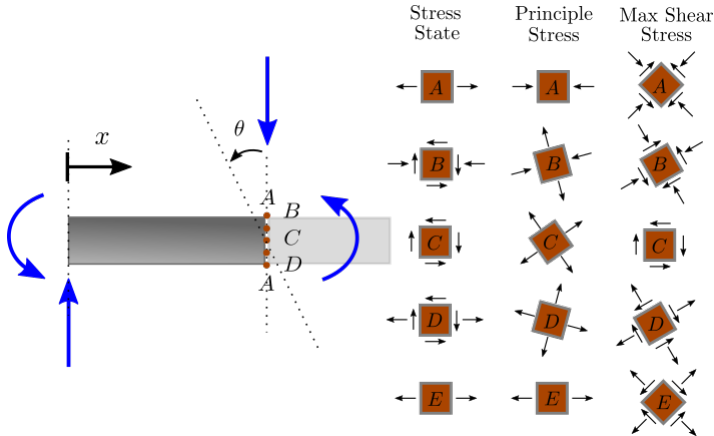
- Motivation
- Principle Stress and Strain
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- Example: A Pressure Vessel

Motivation

The design of load-carrying components for machines and structures requires information concerning the **distribution of forces within the particular component**. Proper design of devices such as shafts, pressure vessels, and support structures must consider **load-carrying capacity and allowable deflections**. Mechanics of materials provides a basis for predicting these essential characteristics of a mechanical design, and provides the fundamental understanding of the behavior of load-carrying parts. However, theoretical analysis is often not sufficient, and **experimental measurements** are required to achieve a final design.

Text: Theory and Design of Mechanical Measurements

Principle Stress and Strain



Principle Stress and Strain

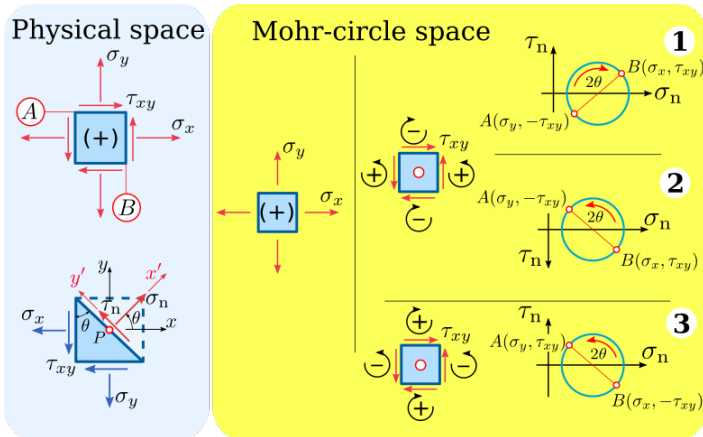


Image: Wikipedia

Determining Principle Stresses

A strain gauge rosette can be mounted on a physical component to measure the principle strains and their directions. Depending on what is known about the stress state this may be done in one of four ways.

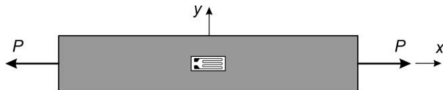
- Case 1: Uniaxial Stress
- Case 2: Isotropic Stress
- Case 3: Pure Torsional Stress
- Case 4: Biaxial Stress
 - with known principle directions
 - or with unknown principle directions

Reference: METU

Determining Principle Stresses

Case 1: Uniaxial Stress

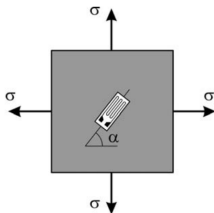
$$\sigma_{xx} = E\epsilon_{xx}$$



Case 2: Isotropic Stress

$$\sigma_{xx} = \sigma_{yy} = \sigma_1 = \sigma_2 = \sigma = \frac{E\nu}{1-\nu}$$

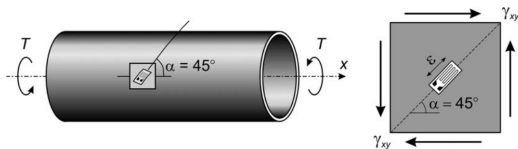
$$\tau_{xy} = 0$$



Images, Reference: METU

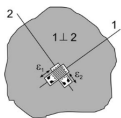
Determining Principle Stresses

Case 3: Pure Torsional Stress



$$\tau_{xy} = \tau_{max} = G\gamma_{xy} \quad \text{with} \quad \gamma = 2\epsilon$$

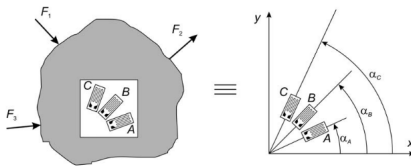
Case 4a: Biaxial Stress - The principle directions are known.



$$\sigma_1 = \frac{E}{1-\nu^2} (\epsilon_1 + \nu\epsilon_2) \quad \text{and} \quad \sigma_2 = \frac{E}{1-\nu^2} (\epsilon_2 + \nu\epsilon_1)$$

Determining Principle Stresses

Case 4b: Biaxial Stress - The principle directions are not known.



$$\epsilon_A = \epsilon_{xx} \cos^2 \alpha_A + \epsilon_{yy} \sin^2 \alpha_A + \gamma_{xy} \cos \alpha_A \sin \alpha_A$$

$$\epsilon_B = \epsilon_{xx} \cos^2 \alpha_B + \epsilon_{yy} \sin^2 \alpha_B + \gamma_{xy} \cos \alpha_B \sin \alpha_B$$

$$\epsilon_C = \epsilon_{xx} \cos^2 \alpha_C + \epsilon_{yy} \sin^2 \alpha_C + \gamma_{xy} \cos \alpha_C \sin \alpha_C$$

$$\epsilon_1 = \frac{1}{2}(\epsilon_{xx} + \epsilon_{yy}) + \frac{1}{2}\sqrt{(\epsilon_{xx} - \epsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\epsilon_2 = \frac{1}{2}(\epsilon_{xx} + \epsilon_{yy}) - \frac{1}{2}\sqrt{(\epsilon_{xx} - \epsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\epsilon_{xx} - \epsilon_{yy}}$$

$$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1 + \nu\epsilon_2) \quad \text{and} \quad \sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2 + \nu\epsilon_1)$$

Images, Reference: METU

Example: A Pressure Vessel

A practical example of this can be seen [here](#) in which a strain gauge is used to measure the pressure in a tank from a strain reading alone.

Which case from above are do they use?



Image: Wikipedia