

## Module 3 - Calibration

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

### Topic 3 - Linear Regression

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- Motivation - Functional Relationship
- Least Squares Regression
- Using Software Packages
- Example: IR Sensor Calibration
- Activity: IR Sensor Calibration

## Motivation - Functional Relationship

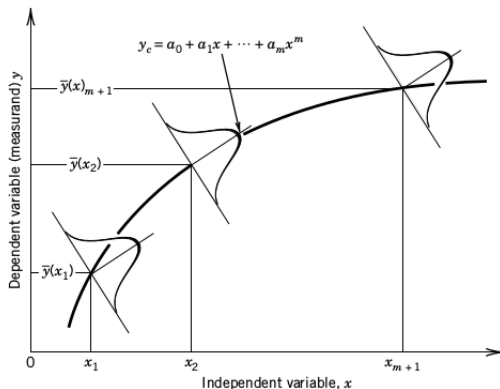
A measured variable is often a function of one or more independent variables that are controlled during the measurement. ... This is a common procedure used to document the relationship between the measured variable and an independent process variable. ...

We can use **regression** analysis to establish a **functional relationship** between the **dependent** variable and the **independent** variable. This discussion pertains directly to polynomial curve fits.

Other functions such as exponential and log fits can also be used.

# Least Squares Regression

Consider the graphs below. This is a calibration curve.



**Figure 4.9** Distribution of measured value  $y$  about each fixed value of independent variable  $x$ . The curve  $y_c$  represents a possible functional relationship.

Image: Theory and Design of Mechanical Measurements, 5th Edition

## Using Software Packages

- We are trying to find a polynomial of best fit for the data.

$$y_c(x) = a_0 + a_1x + a_2x^2 + \cdots + a_mx^m \quad m \leq (n - 1)$$

- This is done by minimizing the quantity below.

$$D = \sum_{i=1}^N (y_i - y_{ci})^2 = \sum_{i=1}^N (y_i - a_0 + a_1x + a_2x^2 + \cdots + a_mx^m)^2$$

- For a first order curve the coefficients become:

$$a_0 = \frac{\sum x_i \sum x_i y_i - \sum x_i^2 \sum y_i}{(\sum x_i)^2 - N \sum x_i^2} \quad , \quad a_1 = \frac{\sum x_i \sum y_i - N \sum x_i y_i}{(\sum x_i)^2 - N \sum x_i^2}$$

## Using Software Packages

Most spreadsheet and engineering software packages can perform a **regression analysis** on a data set. Examples will be shown throughout the course in MATLAB and EXCEL.

- MATLAB - curve fitting toolbox - *polyfit()*
- Excel - *linest()*
- Python - *numpy.linalg.lstsq()*
- Labview - *Linear Fit VI, Exponential Fit VI, etc.*
- and many more...

## Sample Calibration Data

Sample #	Known Distance $x_i(m)$	Measured Voltage $y_i(V)$
1	6	1.67
2	9	1.17
3	12	0.89
4	15	0.72
5	18	0.61

Linear Least Squares Regression Coefficients:

$$a_0 = 1.5640, \quad a_1 = 1.5640$$

Functional Relationship:

$$y = a_1 * x + a_0$$

# Linear Regression in MATLAB - Manual

```
clear variables;close all;clc

x=[];
y=[];
n=length(x);

% perform Linear Least Squares Regression
a0=
a1=

x_f=5.0:1.0:25;
y_f=

% generate 1st order curve fit with polyfit()
P1=polyfit(x,y,1);

x_f1=5.0:1.0:25;
y_f1=
```



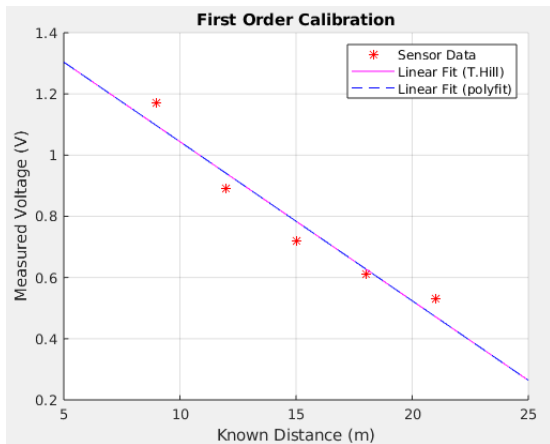
# Linear Regression in MATLAB - Manual

```
figure(1);hold on

plot(x,y,'r*')
plot(x_f,y_f,'m-')
plot(x_f1,y_f1,'b--')

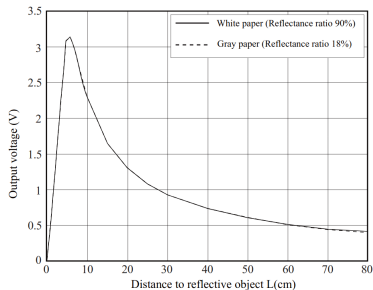
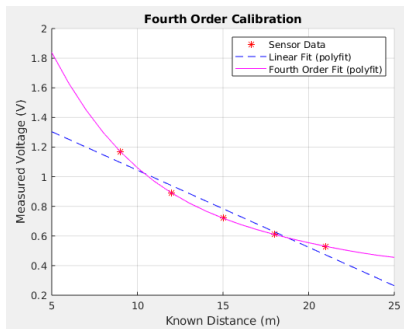
title('First Order Calibration')
xlabel('Known Distance (m)')
ylabel('Measured Voltage (V)')
legend('Raw Data','Linear Fit (T.Hill)','Linear Fit (polyfit)')
grid on
```

# First Order Calibration Curve



Images: T.Hill

## Fourth Order Calibration Curve



Images: T.Hill, Sharp

## Activity: Sensor Calibration - IR Ranger

**Groups of up to 2** - At least one computer is required.

### Instructions:

- Use the sample calibration data provided to calculate the **static sensitivity** and **zero offset** of a 1st order regression line using a MATLAB program.
- Show the calibration data and calibration curve on the same graph. Label the graph and show a legend.
- Show how the calibration curve could be used for a measurement application. Show two(2) example data points.
- Modify your program to calculate an improved calibration curve using a different mathematical model. Discuss the qualities of each curve. Are there benefits and tradeoffs?