Lecture Module - Time Varying Circuits

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Module 4 - Time Varying Circuits

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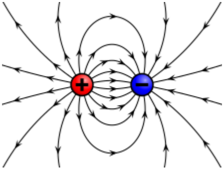
- Topic 1 - The Dynamics of Circuits
- Topic 2 - First Order Systems
- Topic 3 - —

Topic 1 - The Dynamics of Circuits

- Review Electrical Quantities
- Resistance and Impedance
- Capacitance and Inductance
- Example: RC Circuit

Review Electrical Quantities

- Charge - the physical property of matter that causes it to experience a force when placed in an electromagnetic field.



- Voltage - the difference in electric potential between two points ... can be caused by electric charge, by electric current through a magnetic field, by time-varying magnetic fields, or some combination of these three.
- Current - the rate of flow of electric charge past a point or region. An electric current is said to exist when there is a net flow of electric charge through a region.

Review Electrical Quantities

- Resistance - a measure of a components opposition to the flow of electric current. The inverse quantity is electrical conductance, and is the ease with which an electric current passes.
- Capacitance - the ratio of the change in electric charge of a system to the corresponding change in its electric potential (voltage).
- Inductance - the tendency of an electrical conductor to oppose a change in the electric current flowing through it. The flow of electric current creates a magnetic field around the conductor. The field strength depends on the magnitude of the current, and follows any changes in current.

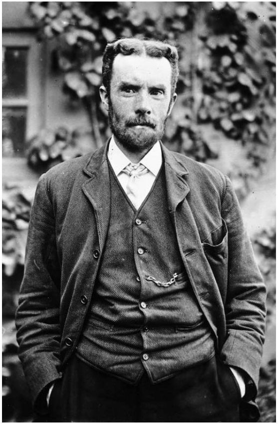
Resistance and Impedance

... electrical impedance is the measure of the opposition that a circuit presents to a current when a voltage is applied.

The term impedance was coined by Oliver Heaviside in July 1886. Arthur Kennelly was the first to represent impedance with complex numbers in 1893.

$$Z = \frac{V}{I} = \frac{|V|}{|I|} e^{j(\phi_V - \phi_I)}$$

$$R = \frac{V}{I}$$



Wikipedia

Resistance and Impedance

In addition to resistance as seen in DC circuits, impedance in AC circuits includes the effects of the induction of voltages in conductors by the magnetic fields (inductance), and the electrostatic storage of charge induced by voltages between conductors (capacitance). The impedance caused by these two effects is collectively referred to as reactance and forms the imaginary part of complex impedance whereas resistance forms the real part.

Wikipedia

Capacitance and Inductance

In many DC applications the transient, or dynamic, behavior of the circuit must be considered. This may be counter-intuitive.

Capacitance - the ratio of the change in electric charge of a system to the corresponding change in its electric potential (voltage).

$$i_C(t) = C\frac{dv_C(t)}{dt}$$

$$v_C = \frac{1}{C}\int_{t_1}^{t_2} i_C(t)dt$$

Capacitance and Inductance

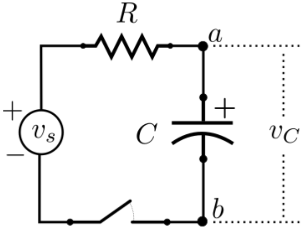
Inductance - the tendency of an electrical conductor to oppose a change in the electric current flowing through it. The flow of electric current creates a magnetic field around the conductor. The field strength depends on the magnitude of the current, and follows any changes in current.

$$v_L(t) = L\frac{d}{dt}i_L(t) = L\frac{di_L}{dt}$$

$$i_L(t) = \frac{1}{L}\int_{t_1}^{t_2} v_L\,(t)\,dt$$

Example: RC Circuit

The basic RC circuit is a simple but important example that I assume you saw in your circuits course. This circuit demonstrates a fundamental concept and has several practical uses in mechanical measurements.



Example: RC Circuit

Topic 2 - First Order Systems

- General System Model
- Mechanical-Electrical Analogies
- Example: RC Circuit
- Activity: Bulb Thermometer

General System Model

The behavior of a circuit is dependent on time, and many common circuits can be represented by a *linear ordinary differential equation* which can be written in the following standard form.

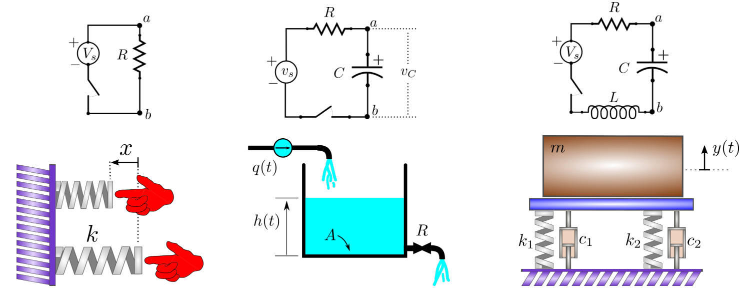
$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = f(t)$$

General System Model

Mechanical-Electrical Analogies

Many mechanical systems are also time dependent, or *dynamic* and a mechanical-electrical analog is often draw between the two.

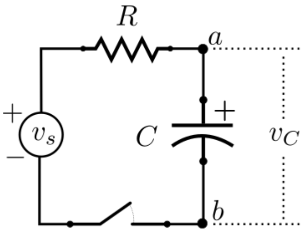
Zero Order | First Order | Second Order



This concept *was* used for analysis and simulation.

Example: RC Circuit

The RC circuit is a first order system. The response to a step input v_s is exponential which is described a single parameter the time constant τ.



First Order Model

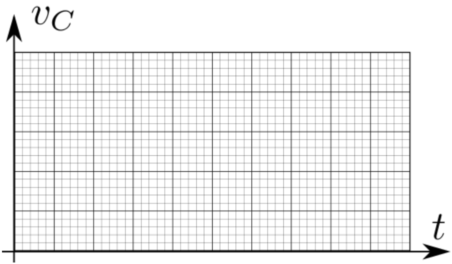
$$RC\dot{v}_C + v_C = v_S$$

Response Equation

$$v_C(t) = v_s\left(1 - e^{-\frac{t}{RC}}\right)$$

Example: RC Circuit

$time(s)$	$response(V)$



Example: RC Circuit

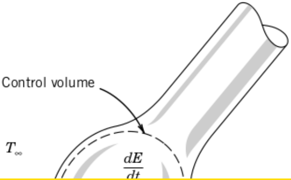
Example: RC Circuit

Activity: Bulb Thermometer

Activity: This should be completed by each student as an individual. You are encouraged to discuss with your peers.

Consider the bulb thermometer shown which can be modeled as a first order system. Write the system model and the expected reponse.

$$\frac{dE}{dt} = \dot{Q} \qquad \frac{dE}{dt} = mc_v \frac{T(t)}{dt} \qquad \dot{Q} = hA_s \Delta T$$



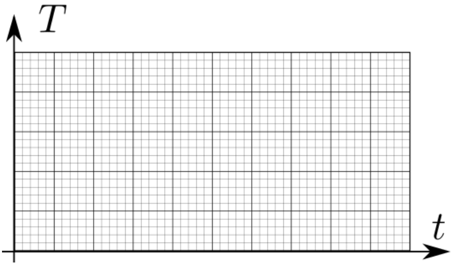
First Order Model:

$$mc_v \frac{T(t)}{dt} + hA_s T(t) = hA_s T_\infty$$

Activity: Bulb Thermometer

Activity: Fill out the table below and sketch a graph to represent the data.

$time(s)$	$response(^\circ C)$
0	
1τ	
2τ	
3τ	
4τ	



Activity: Answer the following questions.

(hint: Think about the *general system model*.)

What is the time constant of the bulb thermometer system?

$\tau =$

What is the static sensitivity? What are the units?

$K =$

Topic 3 -