

Module 5 - Time Varying Circuits

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Topic 2 - First Order Systems

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- General System Model
- Mechanical-Electrical Analogies
- Example: RC Circuit
- Example: Bulb Thermometer

General System Model

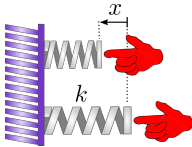
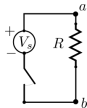
The behavior of a circuit is dependent on time, and many common circuits can be represented by a *linear ordinary differential equation* which can be written in the following standard form.

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \dots + a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = f(t)$$

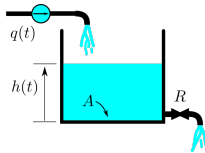
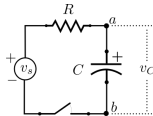
Mechanical-Electrical Analogies

Many mechanical systems are also time dependent, or *dynamic* and a mechanical-electrical analog is often drawn between the two.

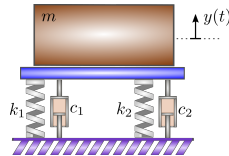
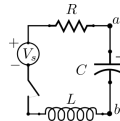
Zero Order



First Order



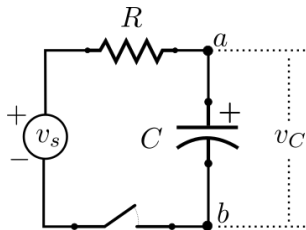
Second Order



This concept was used for analysis and simulation.

Example: RC Circuit

The RC circuit is a first order system. The response to a step input v_s is exponential which is described a single parameter the time constant τ .



First Order Model

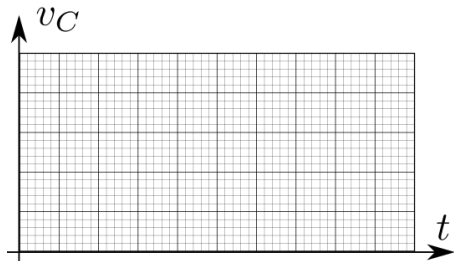
$$RC\dot{v}_C + v_C = v_s$$

Response Equation

$$v_C(t) = v_s \left(1 - e^{-\frac{t}{RC}}\right)$$

Example: RC Circuit

<i>time(s)</i>	<i>response(V)</i>



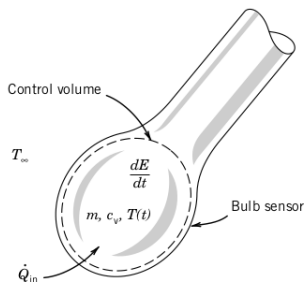
Example: Bulb Thermometer

Consider the bulb thermometer shown which can be modeled as a first order system. Where does the *model* come from?

$$\frac{dE}{dt} = \dot{Q}$$

$$\frac{dE}{dt} = mc_v \frac{T(t)}{dt}$$

$$\dot{Q} = hA_s \Delta T$$



First Order Model

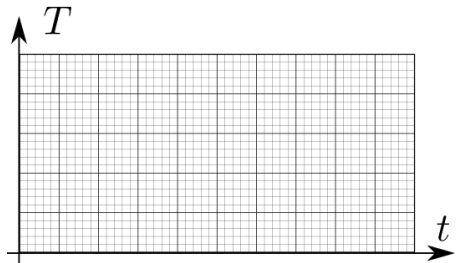
$$mc_v \frac{T(t)}{dt} + hA_s T(t) = hA_s T_\infty$$

Response Equation

$$T(t) = T_\infty + [T(0) - T_\infty] e^{-\frac{t}{\tau}}$$

Example: Bulb Thermometer

<i>time(s)</i>	<i>response($^{\circ}\text{C}$)</i>



Example: Bulb Thermometer

Think about the *general system model*.

What is the time constant of the bulb thermometer system?

$\tau =$

What is the static sensitivity? What are the units?

$K =$