Lecture Module - Time Varying Circuits

Mechanical Engineering

Tennessee Technological University

Topic 2 - First Order Systems

Topic 2 - First Order Systems

- General System Model
- Mechanical-Electrical Analogies
- Example: RC Circuit
- Example: Bulb Thermometer

General System Model

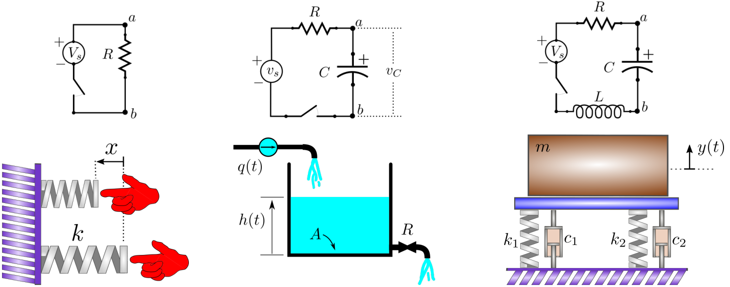
The behavior of a circuit is dependent on time, and many common circuits can be represented by a *linear ordinary differential equation* which can be written in the following standard form.

$$a_n \frac{d^n x}{dt^n} + a_{n-1} \frac{d^{n-1} x}{dt^{n-1}} + \ldots + a_2 \frac{d^2 x}{dt^2} + a_1 \frac{dx}{dt} + a_0 x = f(t)$$

Mechanical-Electrical Analogies

Many mechanical systems are also time dependent, or *dynamic* and a mechanical-electrical analog is often draw between the two.

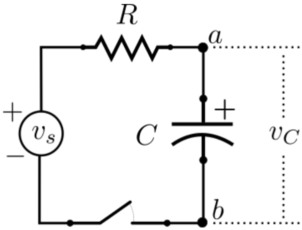
<u>Zero Order</u> <u>First Order</u> <u>Second Order</u>



This concept *was* used for analysis and simulation.

Example: RC Circuit

The RC circuit is a first order system. The response to a step input v_s is exponential which is described a single parameter the time constant τ.



First Order Model

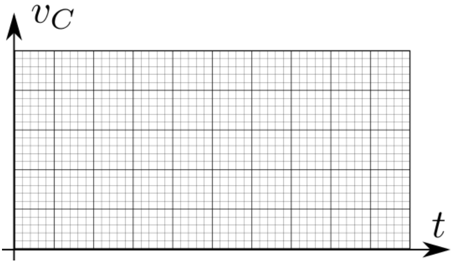
$$RC\dot{v}_C + v_C = v_S$$

Response Equation

$$v_C\left(t\right) = v_s\left(1 - e^{-\frac{t}{RC}}\right)$$

Example: RC Circuit

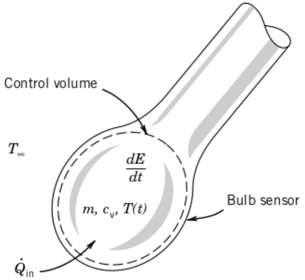
time(*s*)	*response*(*V*)



Example: Bulb Thermometer

Consider the bulb thermometer shown which can be modeled as a first order system. Where does the *model* come from?

$$\frac{dE}{dt} = \dot{Q} \qquad \frac{dE}{dt} = mc_v \frac{T(t)}{dt} \qquad \dot{Q} = hA_s \Delta T$$



First Order Model

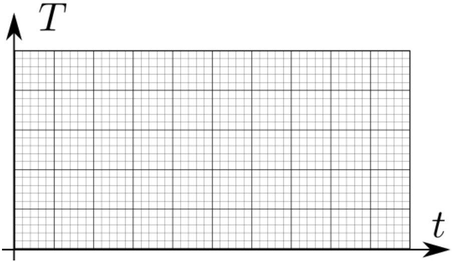
$$mc_v \frac{T(t)}{dt} + hA_s T(t) = hA_s T_\infty$$

Response Equation

$$T(t) = T_\infty + [T(0) - T_\infty] e^{-\frac{t}{\tau}}$$

Example: Bulb Thermometer

$time(s)$	$response(^\circ C)$



Example: Bulb Thermometer

Think about the *general system model*.

What is the time constant of the bulb thermometer system?

$\tau =$

What is the static sensitivity? What are the units?

$K =$