Lecture Module - Strain Applications

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Module 9 - Strain Applications

Module 9 - Strain Applications

- Topic 1 - Beam Models
- Topic 2 - Multiple Gauge Bridge
- Topic 3 - Principal Strains

Topic 1 - Beam Models

- Euler–Bernoulli Beam Theory
- Force-Deflection Model
- Cantilevered Beam
- Deflection and Strain

Euler–Bernoulli Beam Theory

Euler–Bernoulli beam theory (also known as engineer's beam theory or classical beam theory)[1] is a simplification of the linear theory of elasticity which provides a means of calculating the load-carrying and deflection characteristics of beams. It covers the case for small deflections of a beam that are subjected to lateral loads only.

Text: Wikipedia

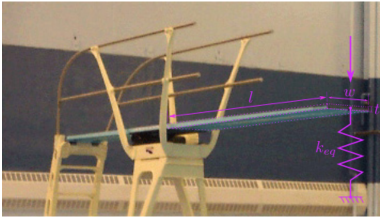
Euler–Bernoulli Beam Theory

It is thus a special case of Timoshenko beam theory. It was first enunciated circa 1750,[2] but was not applied on a large scale until the development of the Eiffel Tower and the Ferris wheel in the late 19th century. Following these successful demonstrations, it quickly became a cornerstone of engineering and an enabler of the Second Industrial Revolution.

Additional mathematical models have been developed such as plate theory, but the simplicity of beam theory makes it an important tool in the sciences, especially structural and mechanical engineering.

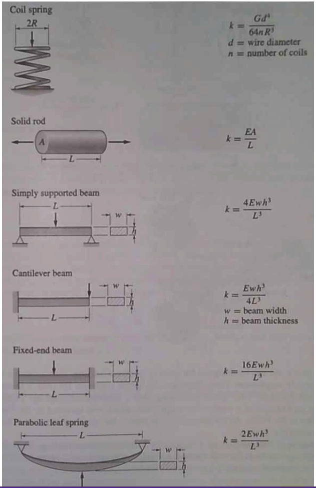
Text: Wikipedia

Force-Deflection Model



These stiffness equations come from the beam deflection equations you have and will study.

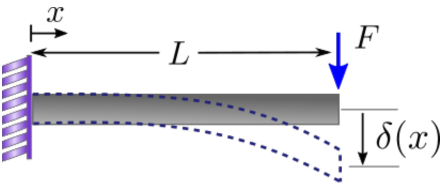
You can see them here or look in your copy of Shigleys, and here is a good section on beam analysis analysis.



Force-Deflection Model

Cantilevered Beam

The beam equations relate internal moment and shear as well as deflection along the length of the beam to the given beam geometry and loading.



$$\delta(x) = -\frac{Fx^2}{6EI}(3L - x)$$

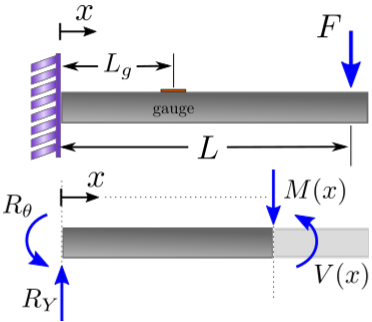
$$\delta_{max} = \delta|_{x=L} = -\frac{FL^3}{3EI}$$

$$\theta(x) = -\frac{Fx}{2EI}(2L - x)$$

$$\theta_{max} = \theta|_{x=L} = -\frac{FL^2}{2EI}$$

Cantilevered Beam

The shear and moment are both given as a function of x, the direction along the beam.

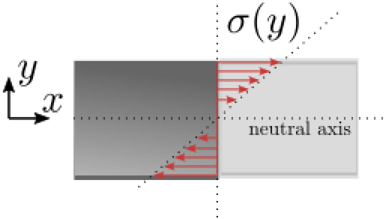


$$M(x) = -F(L - x)$$

$$M_{max} = M|_{x=0} = -FL$$

Cantilevered Beam

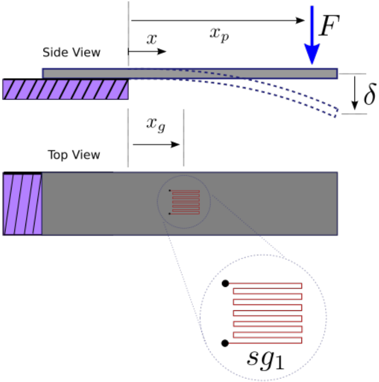
The internal stress is given as a function of y, the distance from the neutral axis.



$$\sigma = \frac{Mc}{I}$$

Deflection and Strain

Now, with the beam equations you can relate measured strain at a known location to deflection at the end of the beam.These equations are available for many different beam types and loading conditions. Beam Eq's



Deflection and Strain

Deflection and Strain

Topic 2 - Multiple Gauge Bridge

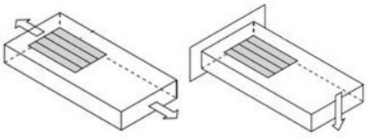
- The Quarter Bridge
- Using Multiple Gauges
- Apparent Strain and Temperature
- Example: Digital Scale

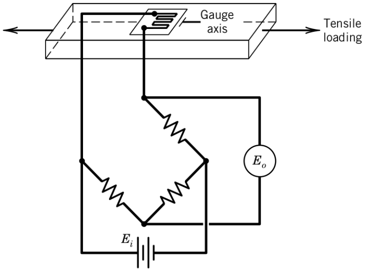
The Quarter Bridge

Hooke's law describes the linear relationship between stress and strain of an elastic member with modulus of elasticity E.

$\sigma = E\epsilon$

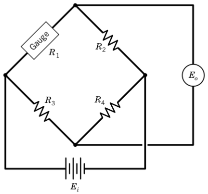
If the loading is known the beam models can be applied.





The Quarter Bridge

Consider the quarter bridge case in which the gauge is R_1 and $R_1 = R_2 = R_3 = R_4$ in a condition of zero strain.



$$E_{out} = E_{in} \times \left(\frac{R_1}{R_1+R_2} - \frac{R_3}{R_3+R_4} \right) \implies E_{out} = E_{in} \frac{(R_1 R_4 - R_2 R_3)}{(R_1+R_2)(R_3+R_4)}$$

Now, R_1 changes by δR causing E_{out} to change by δE_{out}.

$$E_{out} + \delta E_{out} = E_{in} \frac{((R_1+\delta R)R_4 - R_2 R_3)}{((R_1+\delta R)+R_2)(R_3+R_4)}$$

The Quarter Bridge

In practice the bridge is balanced in a condition of zero strain which gives an output voltage of $E_{out} = 0v$. Then any additional strain causes a δE_{out} which is measured.

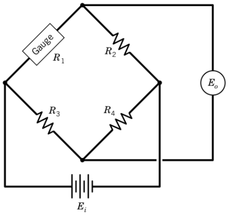
The previous equation is commonly written in the following practical form.

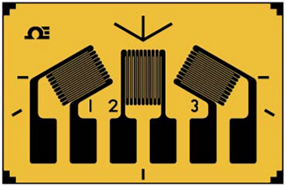
$$\frac{\delta E_{out}}{E_{in}} = \frac{\delta R/R}{4+2(\delta R/R)} \approx \frac{\delta R/R}{4} \quad \text{or} \quad \frac{\delta E_{out}}{E_{in}} = \frac{GF\epsilon}{4+2GF\epsilon} \approx \frac{GF\epsilon}{4}$$

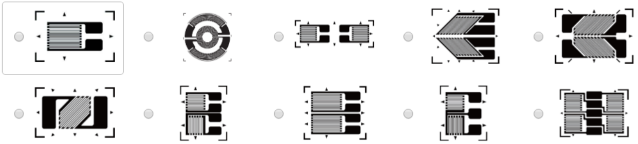
with Gage Factor defined as $GF \equiv \frac{\delta R/R}{\delta L/L} = \frac{\delta R/R}{\epsilon}$

Using Multiple Gauges

An improved system uses a set of gauges known as a rosette with the wheatstone bridge in a half or full bridge configuration.

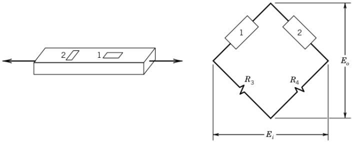






Using Multiple Gauges

If multiple gauges are used the relationship between strain and output voltage is derived with a similar process (see page 479).



The result gives an expression relating the output voltage, the gauge factor, and all four measured strains.

$$\frac{\delta E_{out}}{E_{in}} = \frac{GF}{4}\left(\epsilon_1 - \epsilon_2 + \epsilon_4 - \epsilon_3\right)$$

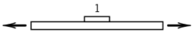
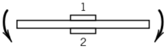
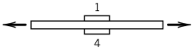
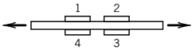
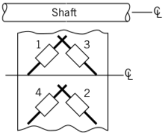
Apparent Strain and Temperature

Equation 11.22 shows that for a bridge containing one or more strain gauges, equal strains on opposite bridge arms sum, whereas equal strains on adjacent arms of the bridge cancel. These characteristics can be used to increase the output of the bridge, to provide temperature compensation, or to cancel unwanted components of strain.

Text: Theory and Design of Mechanical Measurements

This summarizes the reasons to use a wheatstone instead of a operational amplifier when measuring strain with a metallic strain gauge.

Apparent Strain and Temperature

	Arrangement	Compensation Provided	Bridge Constant κ
	Single gauge in uniaxial stress	None	κ = 1
	Two gauges sensing equal and opposite strain—typical bending arrangement	Temperature	κ = 2
	Two gauges in uniaxial stress	Bending only	κ = 2
	Four gauges with pairs sensing equal and opposite strains	Temperature and bending	κ=4
	One axial gauge and one Poisson gauge		κ= 1 +ν
	Four gauges with pairs sensing equal and opposite strains—sensitive to torsion only; typical shaft arrangement.	Temperature and axial	κ = 4

For more details read this document from NI.

Apparent Strain and Temperature

Example: Digital Scale

Have you ever wondered how a digital scale works?

Does it matter where you place item? Why or why not?

Example: Digital Scale

Topic 3 - Principal Strains

- Motivation
- Principal Stress and Strain
- Determining Principal Stresses
- Example: A Pressure Vessel

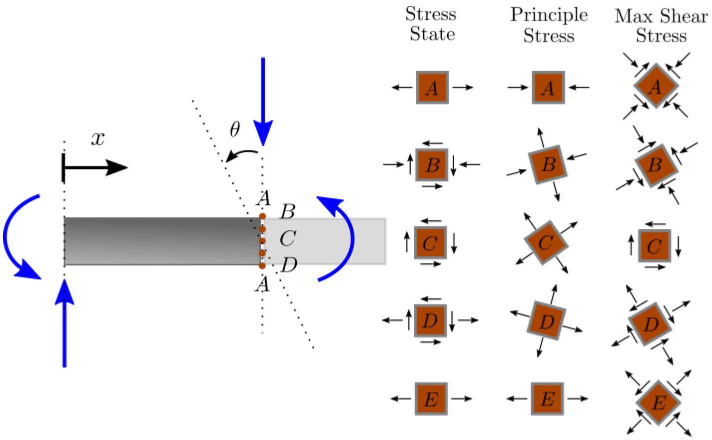
Motivation

The design of load-carrying components for machines and structures requires information concerning the distribution of forces within the particular component. Proper design of devices such as shafts, pressure vessels, and support structures must consider load-carrying capacity and allowable deflections. Mechanics of materials provides a basis for predicting these essential characteristics of a mechanical design, and provides the fundamental understanding of the behavior of load-carrying parts. However, theoretical analysis is often not sufficient, and experimental measurements are required to achieve a final design.

Text: Theory and Design of Mechanical Measurements

Motivation

Principal Stress and Strain



Principal Stress and Strain

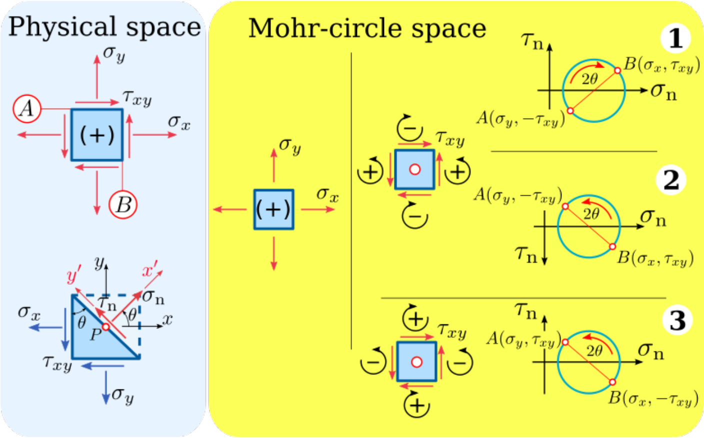


Image: Wikipedia

Principal Stress and Strain

Determining Principal Stresses

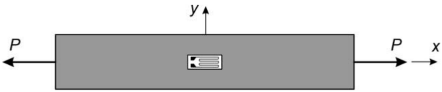
A strain gauge rosette can be mounted on a physical component to measure the principle strains and their directions. Depending on what is known about the stress state this may be done in one of four ways.

- Case 1: Uniaxial Stress
- Case 2: Isotropic Stress
- Case 3: Pure Torsional Stress
- Case 4: Biaxial Stress
  - with known principle directions
  - or with unknown principle directions

Reference: METU

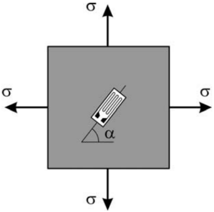
Determining Principal Stresses

Case 1: Uniaxial Stress



$\sigma_{xx} = E\epsilon_{xx}$

Case 2: Isotropic Stress



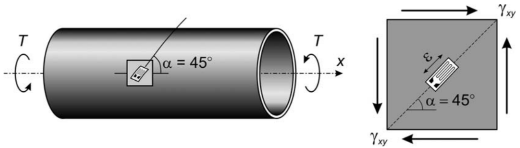
$\sigma_{xx} = \sigma_{yy} = \sigma_1 = \sigma_2 = \sigma = \frac{E\nu}{1-\nu}$

$\tau_{xy} = 0$

Images, Reference: METU

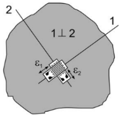
Determining Principal Stresses

Case 3: Pure Torsional Stress



$\tau_{xy} = \tau_{max} = G\gamma_{xy}$ with $\gamma = 2\epsilon$

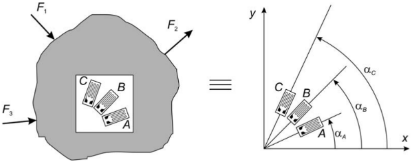
Case 4a: Biaxial Stress - The principle directions are known.



$\sigma_1 = \frac{E}{1-\nu^2}(\epsilon_1 + \nu\epsilon_2)$ and $\sigma_2 = \frac{E}{1-\nu^2}(\epsilon_2 + \nu\epsilon_1)$

Determining Principal Stresses

Case 4b: Biaxial Stress - The principle directions are not known.



$$\varepsilon_1 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) + \frac{1}{2}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) - \frac{1}{2}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\varepsilon_A = \varepsilon_{xx}\cos^2\alpha_A + \varepsilon_{yy}\sin^2\alpha_A + \gamma_{xy}\cos\alpha_A\sin\alpha_A$$

$$\varepsilon_B = \varepsilon_{xx}\cos^2\alpha_B + \varepsilon_{yy}\sin^2\alpha_B + \gamma_{xy}\cos\alpha_B\sin\alpha_B$$

$$\varepsilon_C = \varepsilon_{xx}\cos^2\alpha_C + \varepsilon_{yy}\sin^2\alpha_C + \gamma_{xy}\cos\alpha_C\sin\alpha_C$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$$

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \text{ and } \sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

Images, Reference: METU

Example: A Pressure Vessel

A practical example of this can be seen here in which a strain gauge is used to measure the pressure in a tank from a strain reading alone.

Which case from above are do they use?

Image: Wikipedia