ME 3050 - Solution

Homework 3 Newton's Method - 1DOF EOMS

1. For each of the following systems shown use Newton's Approach to derive an equation of motion describing the dynamics of the system. Include and clearly label the following for each system.

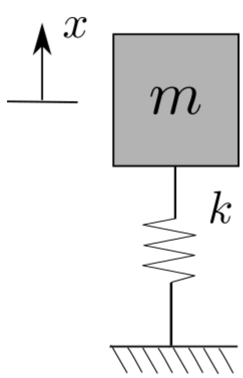
   (a) Important Modeling Assumptions

   (b) A Free Body Diagram for each Body Involved

   (c) Netwon's Second Law

   (d) The resulting EOM in Standard Form

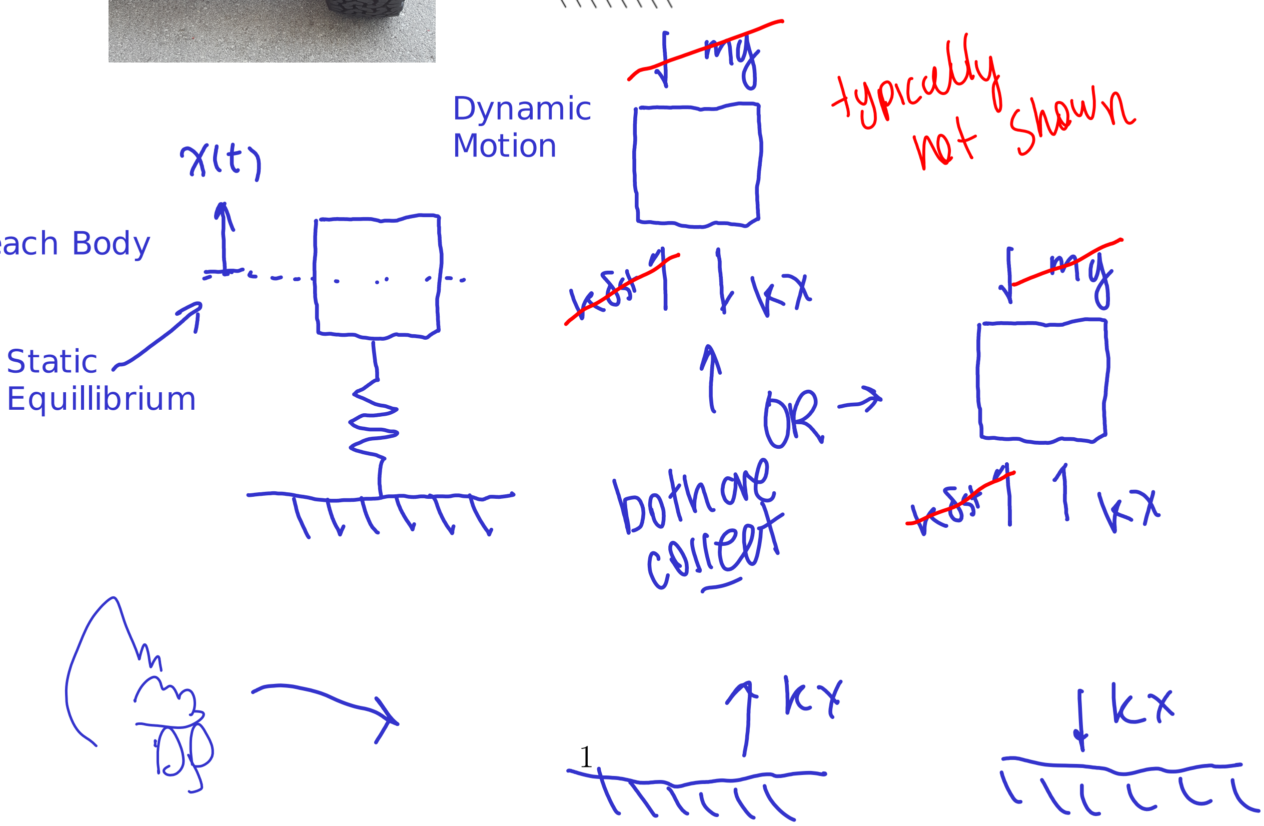
   (a) System 1 - Automobile Suspension - 1DOF Tranlational Mass-Spring Sytem



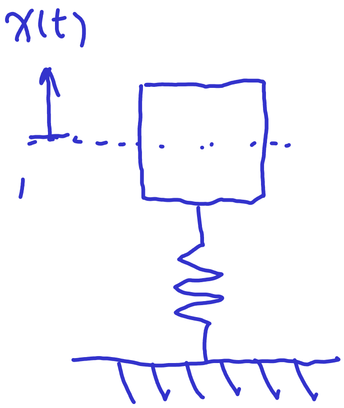
(a) Important Modeling Assumptions

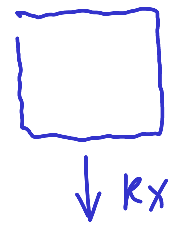
* The motion is 1DOF and vertical
* The tire and spring act with single k
* There is no friction modeled

(b) FBD of each Body



System 1 Continued





$$\frac{dx}{dt} = v_x$$

$$\frac{dv_x}{dt} = a_x$$

(c) Write Newtons Second Law

$$x\uparrow \; \underline{\sum F} = m a_x$$

$$-kx = m a_x$$

$$-kx = m\frac{dv_x}{dt} = m\frac{d^2x}{dt^2} = m\ddot{x}$$

$$-kx = m\ddot{x}$$

$$\boxed{m\ddot{x} + kx = 0}$$

(d) Write EOM in standard form. this is $a_2\ddot{x} + a_1\dot{x} + a_0 x = f(t)$ ✓

w/ $a_2 = m$, $a_1 = 0$, $a_0 = k$, $f(t) = 0$

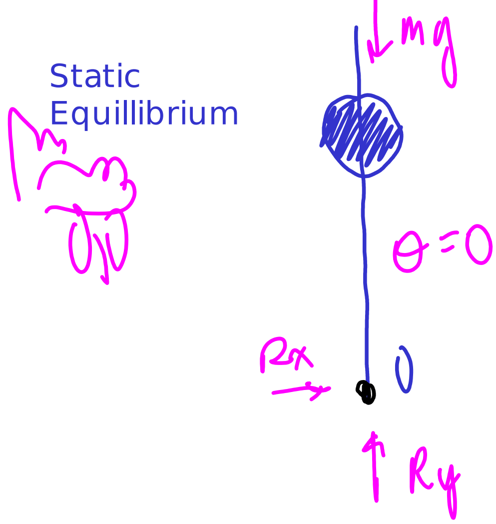
Important Note: You the designer must choose the reference frame to derive your equations in. This requires choosing a positive direction and zero point (reference). For most problems it is convienent to choose the zero energy point as the reference. Here theta=0 is vertical and CW is positive. The resulting EOM is not dependent on the coordinates used.

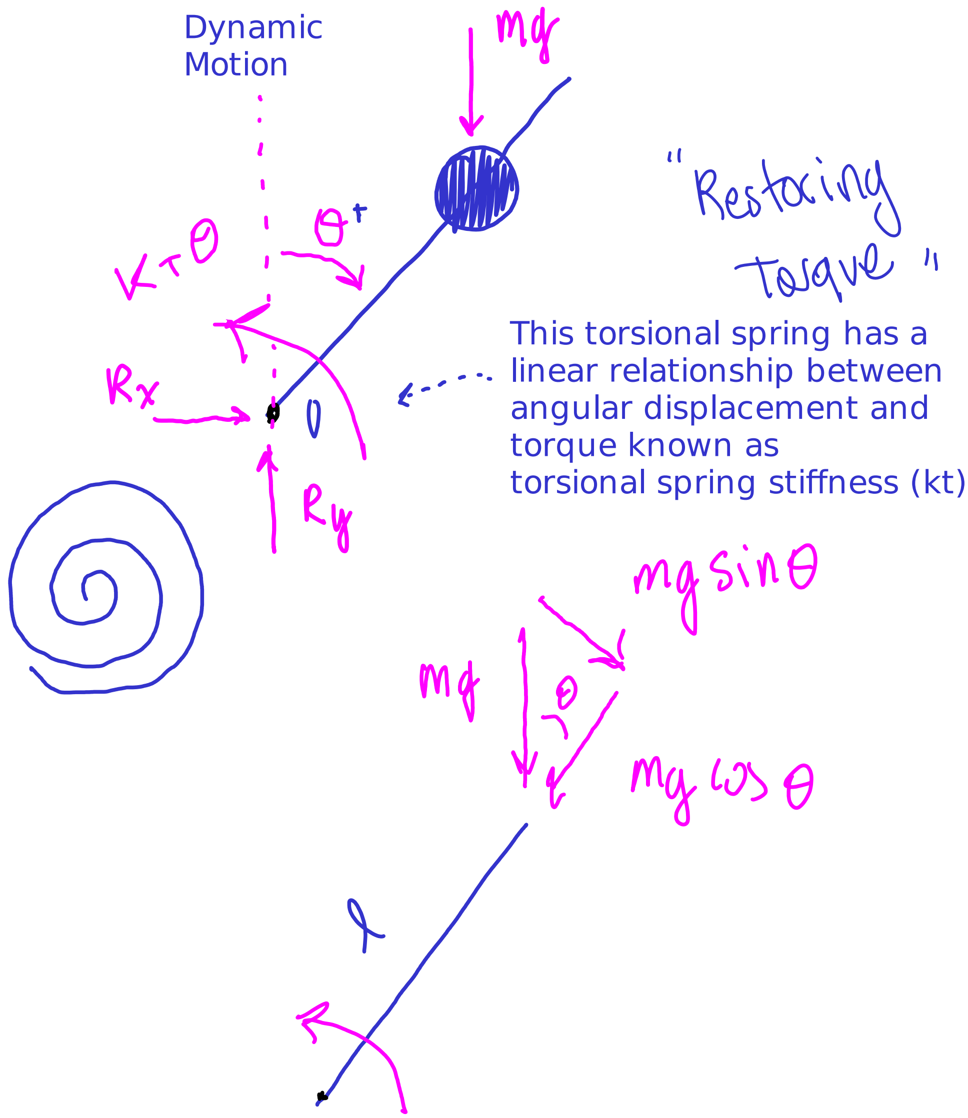
(b) System 2 - An Inverted Pendulum Metronome - 1DOF Rotational Mass Spring

(a) Important Modeling Assumptions

* The motion is 1DOF and rotational
* There is a linear torsional spring inside that is not shown in the this image
* There is no friction modeled
* gravity acts in the vertical direction
* The mass acts as a point mass.
* The link is massless and is rigidly attached to the mass.

(b) FBD of each Body
note: the massless link does not need it own diagram because it is attached rigidly to the mass





aligned w/ reference

System 2 Continued

(c) Write Newtons Second Law

$$+\circlearrowright \quad \sum M_0 = I\alpha = I_0\ddot{\theta}$$

$$T_{mg} - k_T\theta = I_0\ddot{\theta}$$

$I_0 = ml^2$

point mass @ distance l

$$mg\sin\theta \cdot l - k_T\theta = I_0\ddot{\theta}$$

(d) Write EOM in standard form.

$$ml^2\ddot{\theta} + k_T\theta - mgl\sin\theta = 0$$

no external forces ? ? ?

Restoring Force

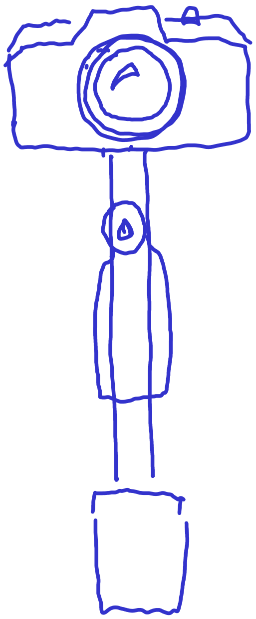
+

Non-restoring Force

-

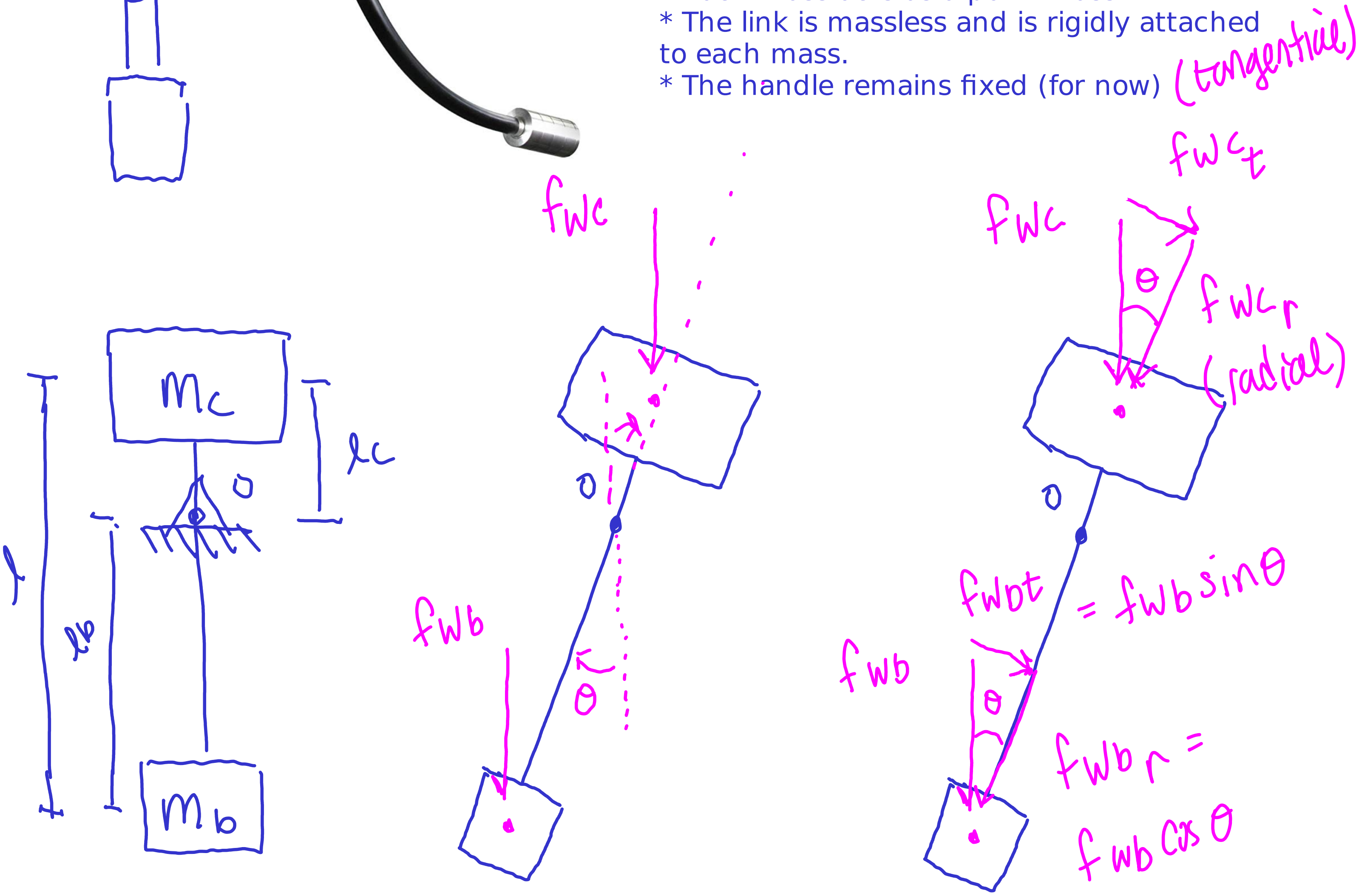
Note: This problem is very sinilar to the previous problem after it has been reduced to 1DOF

(c) System 3 - A Passive Gimbal Mount for Camera - 1DOF (assumed) Rotational System



(a) Important Modeling Assumptions

* The motion is simplified and assumed to be 1DOF and rotational (3 rotations could be modeled)
* There is frictionless bearing between the handle and the camera
* There is no friction modeled
* gravity acts in the vertical direction up two separate masses (camera and balance)
* Each mass acts as a point mass.
* The link is massless and is rigidly attached to each mass.
* The handle remains fixed (for now)



$$I_o = I_{co} + I_{bo}$$
$$= m_c l_c^2 + m_b l_b^2$$

find the moment of inertia for each about o then add

this could be more complex but i have assumed that they are both point masses.. the length is more influential that the individual gemetry

Note: The geometry must be described carefully in order for the model to be valid. Choose a convention that you can follow. I have shown l, lc and lb in the drawing above.

again for rotation

(c) Write Newtons Second Law

$$+\circlearrowleft \sum M_o = I_o \alpha = I_o \ddot{\theta}$$

$$-f_{wb_t} \cdot l_b + f_{wc_t} \cdot l_c = I_o \ddot{\theta}$$

$$-f_{wb} \cdot \sin\theta \cdot l_b + f_{wc} \cdot \sin\theta \cdot l_c = I_o \ddot{\theta}$$

$$-m_b g \sin\theta \, l_b + m_c g \sin\theta \, l_c = I_o \ddot{\theta}$$

$$I_o \ddot{\theta} + (m_b l_b g - m_c l_c g) \sin\theta = 0$$

(d) Write EOM in standard form.

The ODE shown above in non-linear in terms of theta because of the sin(). If you use the small angle approximation the ODE becomes linear and is in the standard form. However the model is only valid for a small range of theta is this is done.

if small angle identity is used $\quad \sin\theta \to \theta$, $\cos\theta \to 1$

✓

$$I_o \ddot{\theta} + (m_b l_b g - m_c l_c g)\,\theta = 0$$

I_o defined previously

2. Choose one of the systems above and use an Conservation of Energy approach to validate the your answer from part one. Clearly shown all of your work.

system 1 is done in "Energy Methods Module"

System 3, is also shown in or something very similar

Stay tuned for more...