BACKGROUND

The theoretical explanation for Least Squares Regression Analysis can be found in the Figliola text, Chapter 4 (1, p121). To fit a linear model, $y = a_0 + a_1 x$, to the selected experimental data sets (x_1, y_1), (x_2, y_2), …, (x_N, y_N) we can use the outcomes of the Example 4.9 in the text

The Equations for Linear Least Squares Regression $y = a_0 + a_1 x$

$$
\begin{aligned}
a_0 &= \frac{\sum x_i \sum x_i y_i - \sum x_i^2 \sum y_i}{\left(\sum x_i\right)^2 - N \sum x_i^2} \\
a_1 &= \frac{\sum x_i \sum y_i - N \sum x_i y_i}{\left(\sum x_i\right)^2 - N \sum x_i^2}
\end{aligned}
\tag{4.40}
$$

To determine the “goodness of fit” we can use the correlation coefficient, r, or the coefficient of determination, r^2. The equation for the correlation coefficient is given below. Once r is calculated, the square of r, r^2, is also known.

Correlation Coefficient

$$
r = r_{xy} = \frac{N \sum_{i=1}^{N} x_i y_i - \sum_{i=1}^{N} x_i \sum_{i=1}^{N} y_i}{\sqrt{N \sum_{i=1}^{N} x_i^2 - \left(\sum_{i=1}^{N} x_i\right)^2} \sqrt{N \sum_{i=1}^{N} y_i^2 - \left(\sum_{i=1}^{N} y_i\right)^2}} \tag{4.41}
$$

Note on units: for our Challenge 6, considering the data we are using from the calibration of the LVDT, the units for each coefficient will be

- Units for slope, a_1 (mV/V) / cm
- Units for intercept, a_0 (mV/V)

BACKGROUND, cont'd

Physical characteristics and sensitivity values for typical commercial LVDTs are shown below (2, p 395).

Table 11.4 Typical Variable Differential Transformer Specifications

Linear Range, Inches	Transformer Size (OD × Length), Inches	Core Size (Diameter × Length), Inches	Sensitivity, mV/0.001 in./V Input into High-Impedance Load — Excitation Frequency, Hz: *60*	*400*	*2000*	*5000*	*10,000*
± 0.005	$\frac{3}{8} \times \frac{9}{16}$	0.10×0.20	0.40		1.9		
± 0.050	$\frac{7}{8} \times 1\frac{1}{8}$	$0.25 \times \frac{7}{8}$	0.70	3.00	3.7	3.7	3.75
± 0.020	$\frac{1}{2} \times \frac{5}{8}$	$0.10 \times \frac{1}{4}$	0.85		3.5		
± 0.200	$\frac{7}{8} \times 2\frac{1}{2}$	$0.25 \times 1\frac{7}{8}$	1.4	2.5	2.5	2.3	2.3
± 0.400	$\frac{7}{8} \times 4\frac{3}{8}$	$0.25 \times 3\frac{1}{8}$	0.8	1.0	1.0	0.5	0.5
± 1.0	$\frac{7}{8} \times 6\frac{5}{8}$	$0.25 \times 4\frac{1}{4}$	0.1	0.3	0.4	0.4	0.3
± 5.0	$\frac{7}{8} \times 18$	0.25×6	0.05	0.15	0.15	0.15	0.15

References:

1. R.S. Figliola and D.E. Beasley, Theory and Design for Mechanical Measurements, 7th Ed., Hoboken: Wiley, 2019.

2. T.G. Beckwith and R.D. Marangoni, *Mechanical Measurements*, 4th Ed., T, New York: Addison Wesley, 1990.