

Module 3 - Calibration

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Topic 3 - Linear Regression

Topic 3 - Linear Regression

- Motivation - Functional Relationship
- Least Squares Regression
- Using Software Packages
- Example: IR Sensor Calibration
- Activity: IR Sensor Calibration

Motivation - Functional Relationship

A measured variable is often a function of one or more independent variables that are controlled during the measurement. ... This is a common procedure used to document the relationship between the measured variable and an independent process variable. ...

We can use **regression** analysis to establish a **functional relationship** between the **dependent** variable and the **independent** variable. This discussion pertains directly to polynomial curve fits.

Other functions such as exponential and log fits can also be used.

Text: T.Hill, Theory and Design of Mechanical Measurements, 5th Edition

Least Squares Regression

Consider the graphs below. This is a calibration curve.

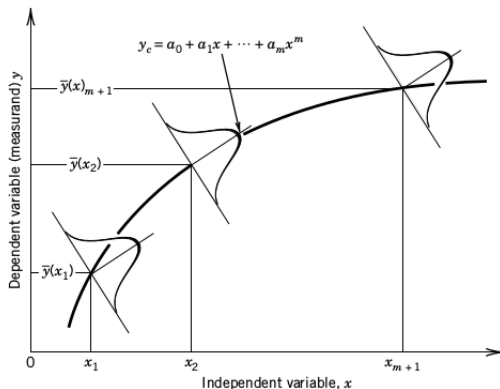


Figure 4.9 Distribution of measured value y about each fixed value of independent variable x . The curve y_c represents a possible functional relationship.

Image: Theory and Design of Mechanical Measurements, 5th Edition

Using Software Packages

- We are trying to find a polynomial of best fit for the data.

$$y_c(x) = a_0 + a_1x + a_2x^2 + \cdots + a_mx^m \quad m \leq (n - 1)$$

- This is done by minimizing the quantity below.

$$D = \sum_{i=1}^N (y_i - y_{ci})^2 = \sum_{i=1}^N (y_i - a_0 + a_1x + a_2x^2 + \cdots + a_mx^m)^2$$

- For a first order curve the coefficients become:

$$a_0 = \frac{\sum x_i \sum x_i y_i - \sum x_i^2 \sum y_i}{(\sum x_i)^2 - N \sum x_i^2} \quad , \quad a_1 = \frac{\sum x_i \sum y_i - N \sum x_i y_i}{(\sum x_i)^2 - N \sum x_i^2}$$

Using Software Packages

Most spreadsheet and engineering software packages can perform a **regression analysis** on a data set. Examples will be shown throughout the course in MATLAB and EXCEL.

- MATLAB - curve fitting toolbox - *polyfit()*
- Excel - *linest()*
- Python - *numpy.linalg.lstsq()*
- Labview - *Linear Fit VI, Exponential Fit VI, etc.*
- and many more...

Sample Calibration Data

Sample #	Known Distance $x_i(m)$	Measured Voltage $y_i(V)$
1	6	1.67
2	9	1.17
3	12	0.89
4	15	0.72
5	18	0.61

Linear Least Squares Regression Coefficients:

$$a_0 = 1.5640, \quad a_1 = 1.5640$$

Functional Relationship:

$$y = a_1 * x + a_0$$

Linear Regression in MATLAB - Manual

```
clear variables;close all;clc

x=[9 12 15 18 21];
y=[1.17 0.89 0.72 0.61 0.53];
n=length(x);

% perform Linear Least Squares Regression
a0=(sum(x)*sum(x.*y)-sum(x.^2)*sum(y))/(sum(x)^2-n*sum(x.^2));
a1=(sum(x)*sum(y)-n*sum(x.*y))/(sum(x)^2-n*sum(x.^2));

x_f=5.0:1.0:25;
y_f=a1*x_f+a0;

% generate 1st order curve fit with polyfit()
P1=polyfit(x,y,1);

x_f1=5.0:1.0:25;
y_f1=P1(1)*x_f1+P1(2);
```

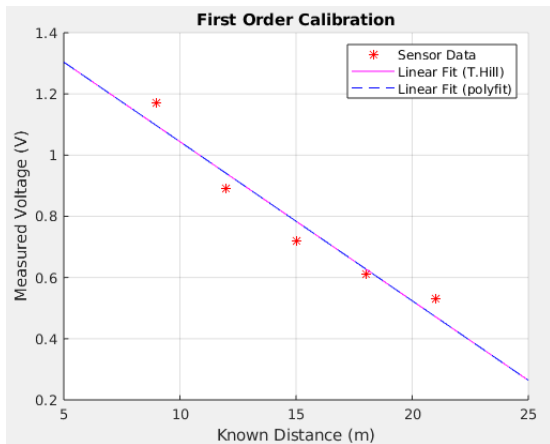

Linear Regression in MATLAB - Manual

```
figure(1);hold on

plot(x,y,'r*')
plot(x_f,y_f,'m-')
plot(x_f1,y_f1,'b--')

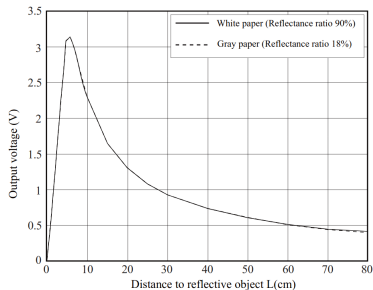
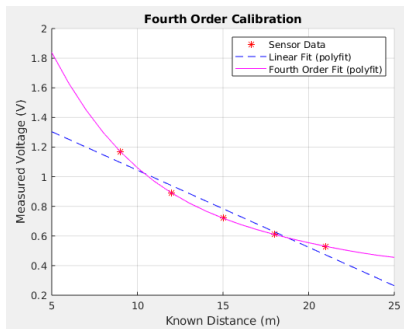
title('First Order Calibration')
xlabel('Known Distance (m)')
ylabel('Measured Voltage (V)')
legend('Raw Data','Linear Fit (T.Hill)','Linear Fit (polyfit)')
grid on
```

First Order Calibration Curve



Images: T.Hill

Fourth Order Calibration Curve



Images: T.Hill, Sharp

Activity: Sensor Calibration - IR Ranger

Groups of up to 2 - At least one computer is required.

Instructions:

- 1 Use the sample calibration data provided to calculate the **static sensitivity** and **zero offset** of a 1st order regression line using a MATLAB program.
- 2 Show the calibration data and calibration curve on the same graph. Label the graph and show a legend.
- 3 Show how the calibration curve could be used for a measurement application. Show two(2) example data points.
- 4 Modify your program to calculate an improved calibration curve using a different mathematical model. Discuss the qualities of each curve. Are there benefits and tradeoffs?

Submission:

- Your code should calculate the required values and generate the required graphs.
- Submit your MATLAB code to the appropriate ilearn folder. Include all names on the code.
- Each group member must submit the complete assignment to receive credit.