Module 5 - Strain Applications

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Topic 1 - Beam Models

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- Euler–Bernoulli Beam Theory
- Force-Deflection Model
- Cantilevered Beam
- Deflection and Strain

Euler–Bernoulli Beam Theory

Euler–Bernoulli beam theory (also known as engineer's beam theory or classical beam theory)[1] is a simplification of the linear theory of elasticity which provides a means of calculating the load-carrying and deflection characteristics of beams. It covers the case for small deflections of a beam that are subjected to lateral loads only.

Text: Wikipedia

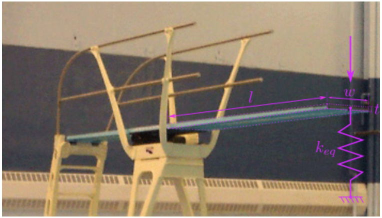
Euler–Bernoulli Beam Theory

It is thus a special case of Timoshenko beam theory. It was first enunciated circa 1750,[2] but was not applied on a large scale until the development of the Eiffel Tower and the Ferris wheel in the late 19th century. Following these successful demonstrations, it quickly became a cornerstone of engineering and an enabler of the Second Industrial Revolution.

Additional mathematical models have been developed such as plate theory, but the simplicity of beam theory makes it an important tool in the sciences, especially structural and mechanical engineering.

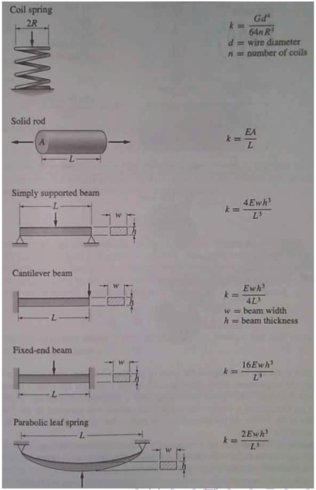
Text: Wikipedia

Force-Deflection Model



These stiffness equations come from the beam deflection equations you have and will study.

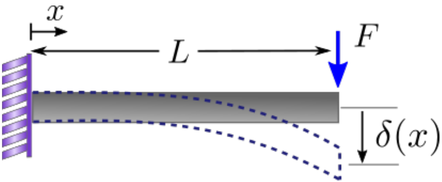
You can see them here or look in your copy of Shigleys, and here is a good section on beam analysis analysis.



Coil spring	$k = \frac{Gd^4}{64nR^3}$ d = wire diameter n = number of coils
Solid rod	$k = \frac{EA}{L}$
Simply supported beam	$k = \frac{4Ewh^3}{L^3}$
Cantilever beam	$k = \frac{Ewh^3}{4L^3}$ w = beam width h = beam thickness
Fixed-end beam	$k = \frac{16Ewh^3}{L^3}$
Parabolic leaf spring	$k = \frac{2Ewh^3}{L^3}$

Force-Deflection Model

The beam equations relate internal moment and shear as well as deflection along the length of the beam to the given beam geometry and loading.



$$\delta(x) = -\frac{Fx^2}{6EI}(3L - x)$$

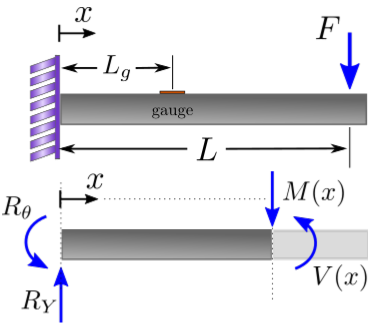
$$\delta_{max} = \delta|_{x=L} = -\frac{FL^3}{3EI}$$

$$\theta(x) = -\frac{Fx}{2EI}(2L - x)$$

$$\theta_{max} = \theta|_{x=L} = -\frac{FL^2}{2EI}$$

Cantilevered Beam

The shear and moment are both given as a function of x, the direction along the beam.

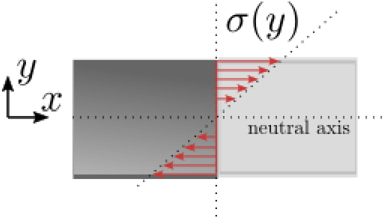


$$M(x) = -F(L - x)$$

$$M_max = M|_{x=0} = -FL$$

Cantilevered Beam

The internal stress is given as a function of y, the distance from the neutral axis.



$$\sigma = \frac{Mc}{I}$$

Deflection and Strain

Now, with the beam equations you can relate measured strain at a known location to deflection at the end of the beam. These equations are available for many different beam types and loading conditions. Beam Eq's