ME 410 - MECHANICAL ENGINEERING LABORATORY

EXPERIMENT 5:

STRESS ANALYSIS BY USING STRAIN GAGES

INTRODUCTION

A state of strain in 3-D (the 3-D strain tensor) may be characterized by its six Cartesian strain components:

ε_{xx} , ε_{yy} , ε_{zz} (Normal Strains)

γ_{xy} , γ_{yz} , γ_{zx} (Shearing Strains)

$$\begin{bmatrix} \varepsilon_{xx} & \gamma_{xy} & \gamma_{xz} \\ \gamma_{xy} & \varepsilon_{yy} & \gamma_{yz} \\ \gamma_{xz} & \gamma_{yz} & \varepsilon_{zz} \end{bmatrix} \text{(Symmetric Strain Tensor in matrix form)}$$

or, equally well, by its three principal strain components, ε_1 , ε_2 , ε_3 and their three associated principal directions, θ_1 , θ_2 , θ_3 .

The symbol ε is used for normal strains, and ε_{xx} , for instance, is defined as the change in length of a line segment parallel to the x axis divided by its original length. Similarly, the symbol γ is used to designate shearing strains, and γ_{xy} , for instance, is defined as the change in the right angle formed by line segments parallel to the x and y axes.

For the most part, strain gage applications are limited for the strain measurements on free surfaces of a body. The two dimensional state of stress existing on a 2-D surface can be expressed in terms of three Cartesian strains ε_{xx} , ε_{yy} and γ_{xy} .

A strain gage cannot measure the stresses directly; it senses the deformations on the surface of structural parts. Since stresses are related to deformations through Hooke's law for linear elastic materials in the elastic range, results of a strain measurement can be used to predict the stresses in a specimen.

BASIC CHARACTERISTICS OF A STRAIN GAGE

Historically, the development of strain gages has followed many different paths, and various methods have been developed based on mechanical, optical, electrical, acoustic and pneumatic principles. In spite of the very wide variations in the strain gage designs, they all have four basic and common characteristics. These are gage length, gage sensitivity, measuring range, and, accuracy and reproducibility.

Gage Length: Strains cannot be measured at a point with any type of gage, and as a consequence non-linear strain fields and local high strains are measured with some degree of error being introduced. In these cases, the error will definitely depend on the gage length L_o. In selecting a gage for a given application, gage length is one of the most important considerations.

Gate Sensitivity: Sensitivity is the smallest value of strain which can be read on the scale associated with the strain gage.

Range: It represents the maximum strain which can be recorded without resetting or replacing the strain gage.

Accuracy: Accuracy is the closeness to an accepted standard value or set of values, and is numerically equal to the referred error value.

Reproducibility: Reproducibility is the closeness or agreement between two or more measurements of the same quantity taken at different times

IDEAL GAGE CHARACTERISTICS

1- The calibration constant for the gage should be stable and it should not vary with either time or temperature.
2- The gage should have the capability of measuring strains with an accuracy of $\pm 1\ \mu\varepsilon$ (*i. e.* 10^{-6} mm/mm).

3- Gage size should be as small as possible to adequately estimate the strain at a point.
4- It should be portable to read the gage either on location or remotely.
5- Changes in temperature should not influence the signal output from the gage during the readout period.
6- Installation and operation of the gage system should be simple.
7- The gage should exhibit a linear response to strain.
8- The gage should be suitable for use as the sensing element in other transducer systems where an unknown quantity such as pressure is sensed by changes in strain.

The electrical resistance strain gages very closely meet the requirements stated above.

BRIEF HISTORY

The electrical resistance strain gage in the basic form known today was first used in 1936. The discovery of the principle upon which electrical resistance strain gage is based was made in 1856 by Lord Kelvin, who loaded copper and iron wires in tension and noted that their resistance increased with the applied strain to the wire. Furthermore, he observed that the iron wire showed a greater increase in resistance than the copper wire when they were both subjected to the same strain. Lord Kelvin also employed the *Wheatstone bridge* technique to measure the resistance change. In that classical experiment, he established three very important facts which helped further development of electrical resistance strain gages;

1. The resistance of the wire changes as a function of strain
2. Different materials have different sensitivities
3. Wheatstone bridge can be employed to accurately measure these resistance changes

There are different types of commercial strain gages; these are:

1. Unbonded wire gages
2. Bonded wire gages

3. Bonded foil gages
4. Piezo-resistive gages
5. Semi-conductive gages

The first three of these types are very similar and they are based on Lord Kelvin's findings. The major differences between them are based on the design concepts rather than principles. The last two are entirely new concepts and are based on the use of a semiconductor as the strain sensing element. Bonded foil gages (Fig. 1) is the very common gage type.

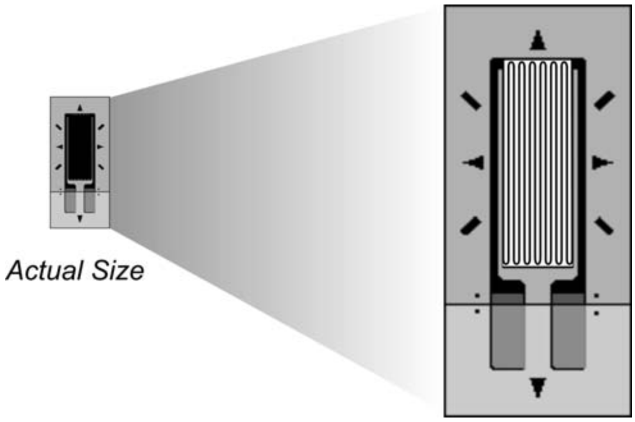

Fig.1. Bonded foil strain gages

STRAIN GAGE ARRANGEMENTS

The strain gage is one of the important electrical measurement technique applied to the measurement of mechanical quantities. As their name indicates they are used for the measurement of strain. The strain of a body is always caused by an external influence or internal effect such as force, pressure, moment, heat, structural change of the material etc.

The strain gage must be mounted on the surface of the specimen of which the stress shall be determined. This is normally done with the aid of special bonding agents or glues. The two dimensional state of stress existing on the surface of the specimen can be expressed in terms of three Cartesian strain components through Hooke’s law. In general, it is necessary to measure three strains at point to completely define the strain field. In certain special cases, the state of strain may be established with a single strain gage. For different states of stress, the gage arrangements may be as follows:

CASE 1: UNIAXIAL STATE OF STRESS

In this problem shown in Fig. 2, only σ_{xx} is present. In this case, a single element strain gage is placed with its axis coincident with the *x* axis. Then

$$\sigma_{xx} = E\,\varepsilon_{xx} \tag{1}$$

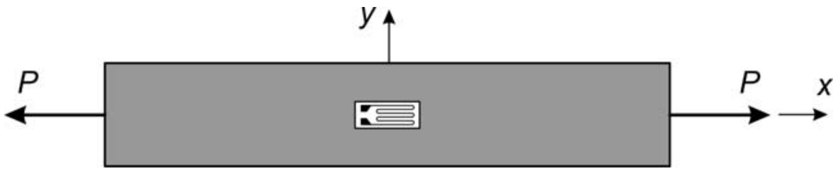

Fig. 2. Uniaxial state of stress

CASE 2: ISOTROPIC STATE OF STRESS

For this case, as shown in Fig. 3, the state of stress at a given point is given as $\sigma_{xx} = \sigma_{yy} = \sigma_1 = \sigma_2 = \sigma$ and $\tau_{xy} = 0$.

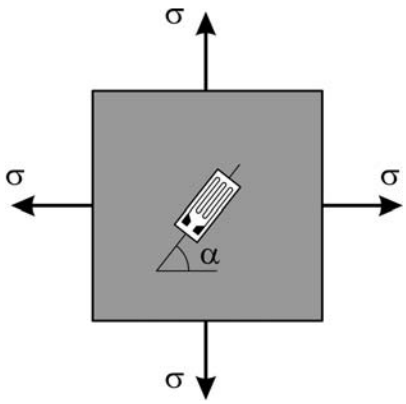

Fig. 3. Isotropic state of stress

In this case, to determine the state of stress at a given point a single element strain gage may be placed in any direction and the magnitude of stresses may be established from:

$$\sigma_{xx} = \sigma_{yy} = \sigma_1 = \sigma_2 = \sigma = \frac{E\,\varepsilon}{1-\nu} \tag{2}$$

CASE 3: PURE TORSION ($\sigma_{xx} = \sigma_{yy} = 0$ and only τ_{xy} is present)

In this case as shown in Fig. 4, a single strain gage is placed with its axis coincident with one of the principal stress direction (45° with the *x* axis). The maximum shearing stress is calculated from:

$$\tau_{xy} = \tau_{max} = G\,\gamma_{xy} \tag{3}$$

where $\gamma_{xy} = 2\varepsilon$ (ε is the strain measured strain from the gage)

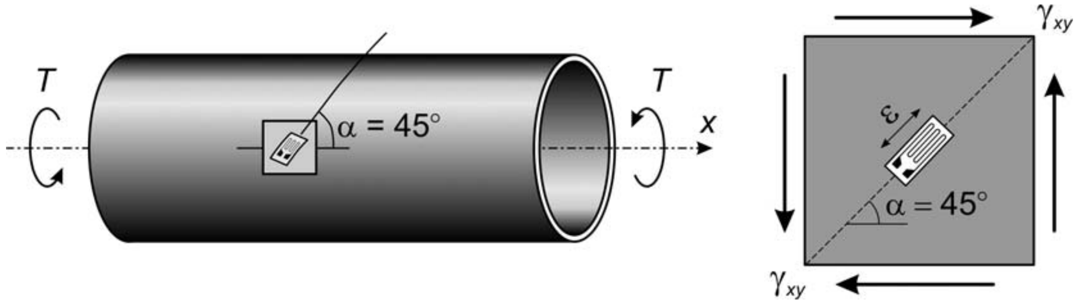

Fig. 4. Pure torsion loading and shearing strain

CASE 4: BIAXIAL STATE OF STRESS

a) If less is known about the state of stress, but directions of principal stresses are known, then two-element rectangular rosette is placed on the specimen with its axes coincident with principal directions. Two strains ε_1 and ε_2 are obtained from the gage and the corresponding principal stresses are as calculated as:

$$\sigma_1 = \frac{E}{1-\nu^2}\left(\varepsilon_1 + \nu\,\varepsilon_2\right) \text{ and } \sigma_2 = \frac{E}{1-\nu^2}\left(\varepsilon_2 + \nu\,\varepsilon_1\right) \tag{4}$$

These relations give the complete state of stress since the principal directions are known *a priori*.

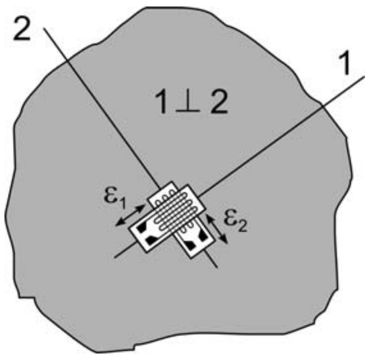

Fig. 5. Biaxial state of stress and 2-element strain gage

b) If knowledge of the stress field or principal directions is not available, then a three-element rosette is needed. To show that three measurements are sufficient, consider three strain gages placed along axes *A*, *B*, and *C*, as shown in Fig. 6.

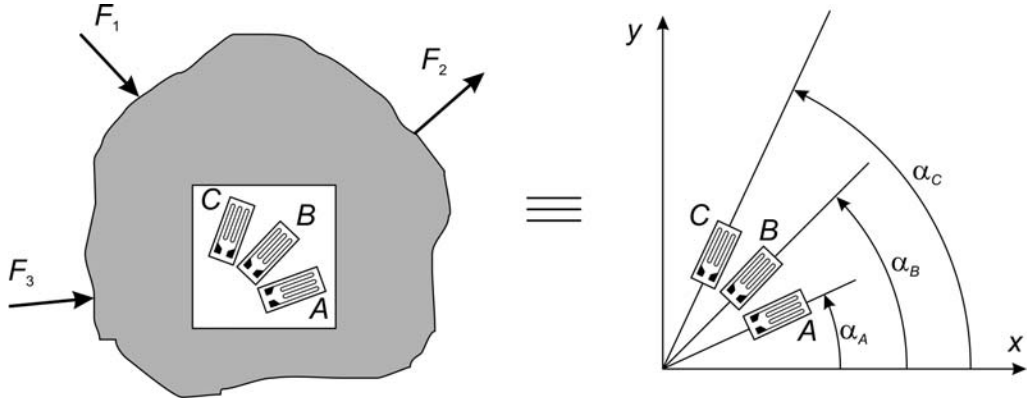

Fig. 6. Three-element rosette

Then the following transformation relations are valid for strain readings ε_A, ε_B and ε_C from gages *A*, *B* and *C*, respectively;

$$\begin{aligned}
\varepsilon_A &= \varepsilon_{xx}\cos^2\alpha_A + \varepsilon_{yy}\sin^2\alpha_A + \gamma_{xy}\cos\alpha_A\sin\alpha_A \\
\varepsilon_B &= \varepsilon_{xx}\cos^2\alpha_B + \varepsilon_{yy}\sin^2\alpha_B + \gamma_{xy}\cos\alpha_B\sin\alpha_B \\
\varepsilon_C &= \varepsilon_{xx}\cos^2\alpha_C + \varepsilon_{yy}\sin^2\alpha_C + \gamma_{xy}\cos\alpha_C\sin\alpha_C
\end{aligned} \tag{5}$$

The Cartesian components of strain, ε_{xx}, ε_{yy} and γ_{xy}, can be determined from a simultaneous solution of above equations. The principal strains and the principal directions may then be established by employing;

$$\varepsilon_1 = \frac{1}{2}\left(\varepsilon_{xx}+\varepsilon_{yy}\right) + \frac{1}{2}\sqrt{\left(\varepsilon_{xx}-\varepsilon_{yy}\right)^2+\gamma_{xy}^2} \tag{6a}$$

$$\varepsilon_2 = \frac{1}{2}\left(\varepsilon_{xx}+\varepsilon_{yy}\right) - \frac{1}{2}\sqrt{\left(\varepsilon_{xx}-\varepsilon_{yy}\right)^2+\gamma_{xy}^2} \tag{6b}$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_{xx}-\varepsilon_{yy}} \tag{7}$$

The solution of Eq. 7 yields two values for the angle θ. These are θ_1, which refers to the angle between the *x*-axis and the axis of the maximum principal strain ε_1, and θ_2, which is the angle between the *x*-axis and the axis of the minimum principal strain ε_2.

The principal stresses can then be calculated from the principal strains by utilizing Eqs. 4 as;

$$\sigma_1 = \frac{E}{1-\nu^2}\left(\varepsilon_1+\nu\,\varepsilon_2\right) \text{ and } \sigma_2 = \frac{E}{1-\nu^2}\left(\varepsilon_2+\nu\,\varepsilon_1\right) \tag{4}$$

In practice, three-element rosettes with fixed angles (that is, α_A, α_B, and α_C are fixed at specified values) are employed to provide sufficient data to completely define the strain and stress fields. These rosettes are defined by the fixed angles as the rectangular rosette, and the delta rosette.

THREE-ELEMENT RECTANGULAR ROSETTE

The three-element rectangular rosette employs gages placed at 0°, 45°, and 90° positions, as indicated in Fig. 7.

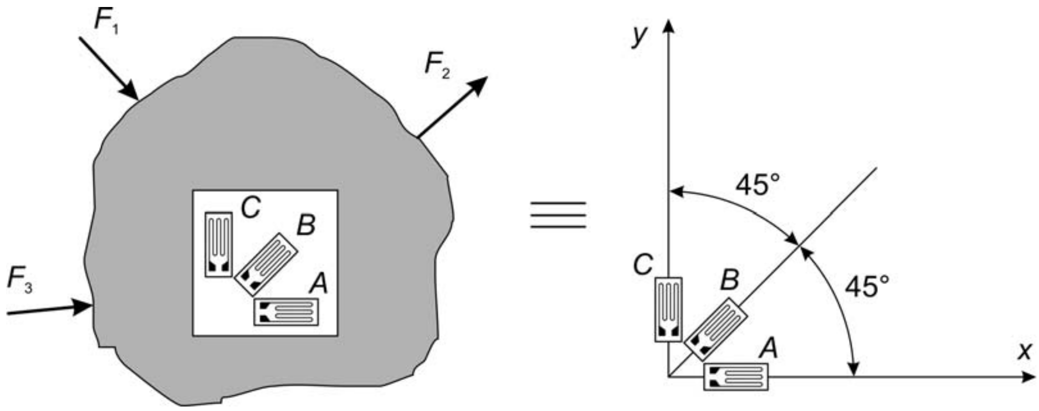

Fig. 7. Three-element rectangular rosette

For this particular rosette:

$\alpha_A = 0^\circ \qquad \cos\alpha_A = 1 \qquad \sin\alpha_A = 0$

$\alpha_B = 45^\circ \qquad \cos\alpha_B = 0.707 \qquad \sin\alpha_b = 0.707$

$\alpha_C = 90^\circ \qquad \cos\alpha_C = 0 \qquad \sin\alpha_C = 1$

It is clear from Eq. 5 that

$$\varepsilon_A = \varepsilon_{xx}, \; \varepsilon_B = \frac{1}{2}\left(\varepsilon_{xxB} + \varepsilon_{yy} + \gamma_{xy}\right) \text{ and } \varepsilon_C = \varepsilon_{yy} \tag{8}$$

From above equations:

$$\varepsilon_{xx} = \varepsilon_A, \; \varepsilon_{yy} = \varepsilon_C, \; \gamma_{xy} = 2\varepsilon_B - \varepsilon_A - \varepsilon_C \tag{9}$$

Thus, by measuring the strains, ε_A, ε_B and ε_C, the Cartesian components of strains ε_{xx}, ε_{yy} and γ_{xy} can be determined by using Eq. 9. Next, by utilizing Eqs. 6 and 7, the principal strain ε_1 and ε_2 and the principal angle θ can be calculated. Finally, the principal stresses occurring in the component can be solved by employing Eq. 4.

DELTA ROSETTE

The delta rosette employs three gages placed at the 0°, 120°, and 240° positions, as indicated in Fig. 8.

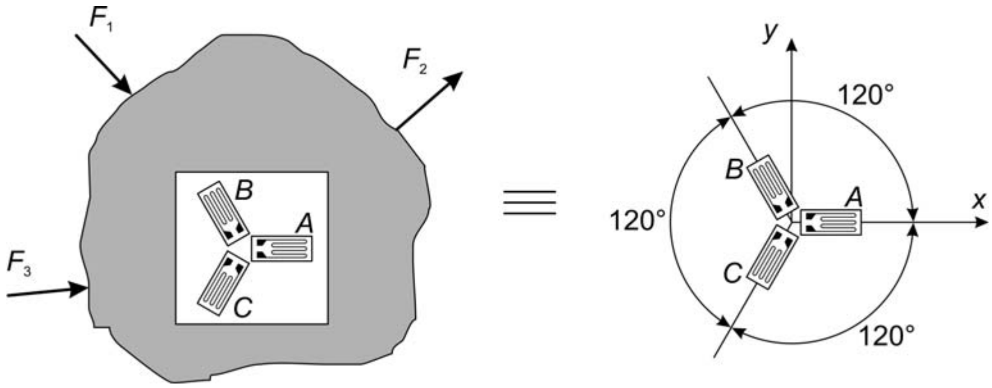

Fig. 8. Delta rosette for 2-D loading

For this particular rosette:

$\alpha_A = 0^\circ$ $\cos\alpha_A = 1$ $\sin\alpha_A = 0$

$\alpha_B = 120^\circ$ $\cos\alpha_B = -0.5$ $\sin\alpha_B = \sqrt{3}/2$

$\alpha_C = 240^\circ$ $\cos\alpha_C = -0.5$ $\sin\alpha_C = -\sqrt{3}/2$

It is clear from Eq. 5 that

$$\varepsilon_A = \varepsilon_{xx}, \; \varepsilon_B = \frac{1}{4}\varepsilon_{xx} + \frac{3}{4}\varepsilon_{yy} - \frac{\sqrt{3}}{4}\gamma_{xy}, \; \varepsilon_C = \frac{1}{4}\varepsilon_{xx} + \frac{3}{4}\varepsilon_{yy} + \frac{\sqrt{3}}{4}\gamma_{xy} \tag{10}$$

Solving Eq. 10 for ε_{xx}, ε_{yy} and γ_{xy} in terms of ε_A, ε_B and ε_C yields

$$\varepsilon_{xx} = \varepsilon_A, \; \varepsilon_{yy} = \frac{1}{3}\left[2\left(\varepsilon_B + \varepsilon_C\right) - \varepsilon_A\right], \; \gamma_{xy} = \frac{1}{\sqrt{3}}\left(\varepsilon_C - \varepsilon_B\right) \tag{11}$$

Next, by utilizing Eqs. 6 and 7, the principal strains ε_1 and ε_2 and the principal angle θ can be found. Finally, the principal stresses occurring in the component can be calculated by employing Eq. 4.

STRAIN GAGE CIRCUITS

The gage factor of a gage provided by the gage manufacturer is given as

$$S_g = \frac{\Delta R/R}{\varepsilon_{xx}} \tag{12}$$

where the gage axis coincides with the *x*-axis and $\varepsilon_{yy} = -\nu\,\varepsilon_{xx}$.

The resistance of an electrical resistance strain gage will change due to applied strain according to Eq. 12, which indicates

$$\frac{\Delta R}{R} = S_g \varepsilon_{xx} \tag{13}$$

In order to apply the electrical-resistance strain gage in any experimental stress analysis, the quantity $\Delta R/R$ must be measured and converted to strain which produced the resistance change. Two types of electrical circuits, potentiometer and Wheatstone bridge, are commonly employed to convert the value of $\Delta R/R$ to a voltage signal (denoted here as ΔE) which can be measured with a recording instrument.

POTENTIOMETER CIRCUITS

The potentiometer circuit is used in *dynamic strain gage applications* to convert $\Delta R/R$ to voltage signal ΔE for fixed values of R_1 and R_2 the output voltage E is

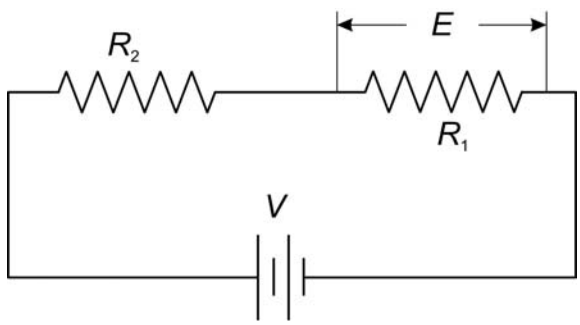

Fig. 9. Potentiometer circuit

$$E = \frac{R_1}{R_1 + R_2} V = \frac{V}{1 + r} \tag{14}$$

where V is the input voltage, and $r = R_2/R_1$.

If incremental changes ΔR_1 and ΔR_2 occur in the value of resistors R_1 and R_2, the change ΔE of the output voltage E can be computed by using Eq. 16 as follows:

$$\Delta E = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2}\right) V \tag{15}$$

WHEATSTONE BRIDGE

The Wheatstone bridge circuit shown in Fig. 10 may be used for both static and dynamic strain gage applications. Consider an initially balanced bridge with $R_1 R_3 = R_2 R_4$ then $E = 0$, and then change each value of resistance R_1, R_2, R_3 and R_4 by incremental amount ΔR_1, ΔR_2, ΔR_3 and ΔR_4. The voltage output ΔE of the bridge can be obtained as:

$$\Delta E = \frac{r}{(1+r)^2}\left(\frac{\Delta R_1}{R_1} - \frac{\Delta R_2}{R_2} + \frac{\Delta R_3}{R_3} - \frac{\Delta R_4}{R_4}\right) V \tag{16}$$

where $r = R_2/R_1$.

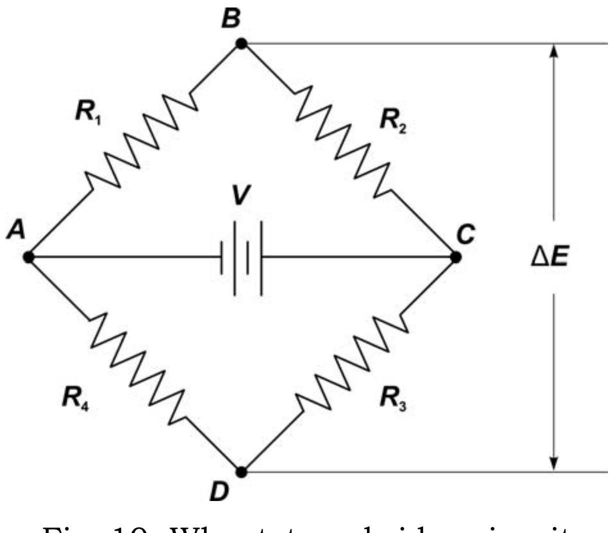

Fig. 10. Wheatstone bridge circuit

WHEATSTONE BRIDGE ARRANGEMENTS

CASE 1: QUARTER BRIDGE (One Active Gage)

This bridge circuit shown in Fig. 11, consists of a single active gage in position R_1 and is often employed for many dynamic and static strain measurements where temperature compensation in the circuit is not critical. The value of R_1 is, of course, equal to R_g, and from Eq. 14, we have:

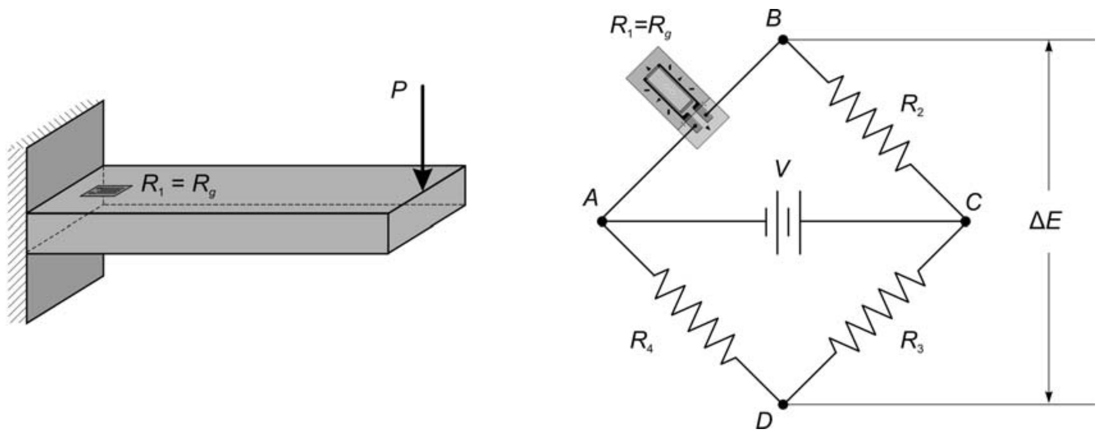

Fig. 11. Quarter bridge (one-active gage) configuration

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g} = S_g\,\varepsilon \tag{17}$$

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_3}{R_3} = \frac{\Delta R_4}{R_4} = 0 \tag{18}$$

By combining Eqs.16-19, the voltage output ΔE of the bridge can be obtained as:

$$\Delta E = \frac{r}{(1+r)^2}\, V S_g\,\varepsilon \tag{19}$$

Since $r = 1$ ($R_1 = R_2 = R_3 = R_4 = R_g$), Eq. 19 becomes

$$\Delta E = \frac{V}{4} S_g\,\varepsilon \tag{20}$$

CASE 2: HALF BRIDGE (One Active Gage and One Dummy Gage)

This bridge arrangement shown in Fig. 12, employs one active gage in arm R_1 and one dummy gage in R_2 which is utilized for temperature compensation.

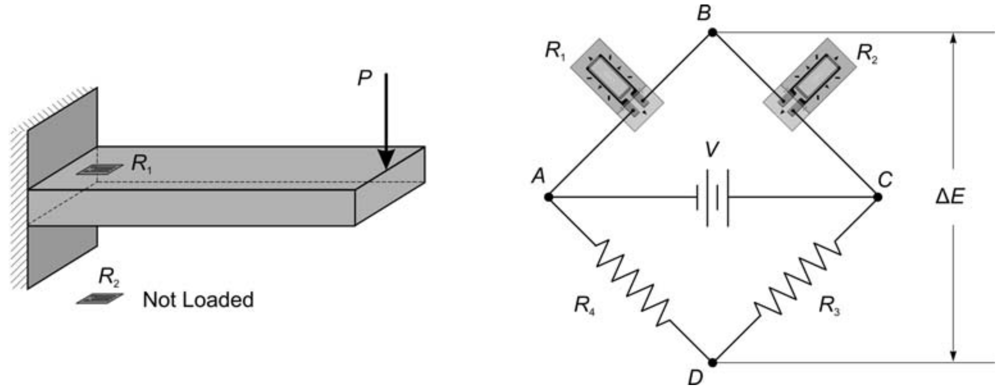

Fig. 12. Half bridge (one-active and one-dummy gage) configuration

From Eq. 13, for active gage (R_1) and dummy gage (R_2) we have:

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g} = S_g\,\varepsilon + S_g\,\varepsilon_{temp} \text{ and } \frac{\Delta R_2}{R_2} = S_g\,\varepsilon_{temp} \tag{21a}$$

And for the other two resistances

$$\frac{\Delta R_3}{R_3} = \frac{\Delta R_4}{R_4} = 0 \tag{21b}$$

By combining Eqs. 16 and 21 the voltage output ΔE of the bridge can be obtained as

$$\Delta E = \frac{V}{4} S_g\,\varepsilon \tag{22}$$

CASE 3: HALF BRIDGE (Two Active Gages)

The bridge arrangement in this case incorporates an active gage in the R_1 position and another active gage in the R_2 position. For example, if the gages are placed on a beam in bending, as shown in Fig.13, the signals from each of the two gages will add and the value of the bridge output will be:

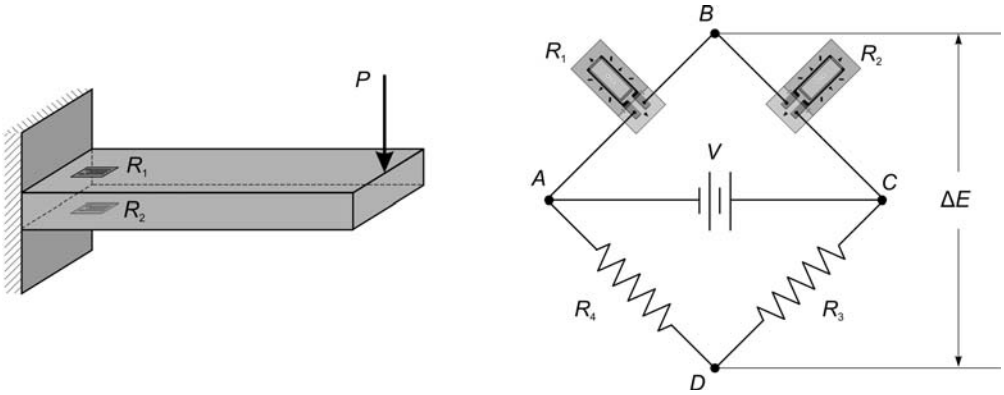

Fig. 13. Half bridge with two-active strain gages

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_g}{R_g} = S_g\,\varepsilon + S_g\,\varepsilon_{temp} \text{ and } \frac{\Delta R_2}{R_2} = \frac{\Delta R_g}{R_g} = -S_g\,\varepsilon + S_g\,\varepsilon_{temp} \qquad (23a)$$

For the other two resistances

$$\frac{\Delta R_3}{R_3} = \frac{\Delta R_4}{R_4} = 0 \qquad (23b)$$

Then Eq. 16 becomes:

$$\Delta E = \frac{V}{2} S_g\,\varepsilon \qquad (24)$$

CASE 4: FULL BRIDGE (Four Active Gages)

In this bridge arrangement, four active gages placed in the bridge, with one gage in each of the four arms. If the gages are placed on, say, a beam in bending, as shown in Fig. 14, the signals from each of the four gages will add and the value of the bridge output will be:

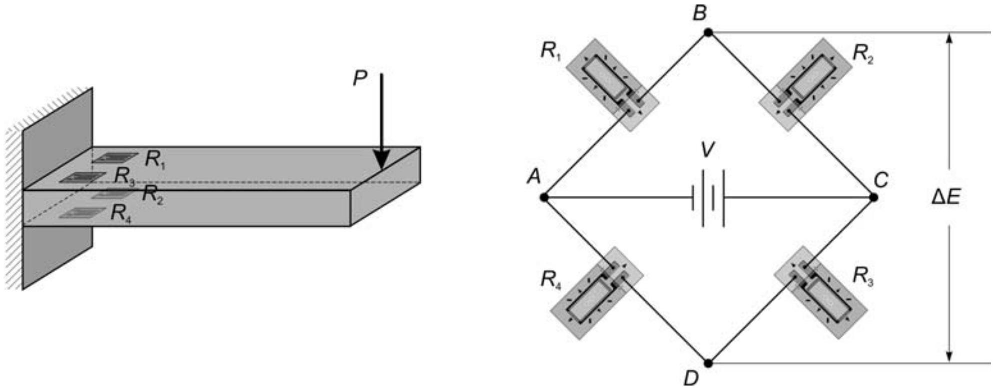

Fig. 14. Full bridge (four-active gages) configuration

$$\frac{\Delta R_1}{R_1} = \frac{\Delta R_3}{R_3} = \frac{\Delta R_g}{R_g} = S_g\,\varepsilon + S_g\,\varepsilon_{temp} \qquad (25a)$$

$$\frac{\Delta R_2}{R_2} = \frac{\Delta R_4}{R_4} = \frac{\Delta R_g}{R_g} = -S_g\,\varepsilon + S_g\,\varepsilon_{temp} \qquad (25b)$$

Then Eq. 16 becomes:

$$\Delta E = V\,S_g\,\varepsilon \qquad (26)$$

STRAIN GAGE INSTALLATION

Solvent Degreasing

- Performed to remove oils, greases, organic contaminants and soluble chemical residues

- Porous materials (like cast iron, cast aluminum and titanium) may require heating to drive off the absorbed contaminants
- Can be performed by hot vapor degreaser, ultasonically agitated liquid bath or aerosol type spray degreasers
- If possible degrease all specimen (if not at least 100 to 150 mm area around the gage)

Surface Abrading

- Scale, rust, paint and galvanized coatings must be removed
- Suitable surface texture for bonding should be obtained
- The optimum surface finish depends on application (see manufacturers recommendations)

Gage Location Layout Lines

- To locate the gages accurately, the axis along which strain measurement and a perpendicular line must be drawn
- The lines must be drawn with a tool that burnishes the surface rather than scoring or scribing (which detoriates surface quality)

Surface Conditioning

- Application of the appropriate conditioner and scrubbing the surface with cotton tipped applicators should be continued until a clean tip is not discolored
- The surface must always be kept wet and conditioner should never be allowed to dry on the surface
- After conditioning the surface must be cleaned with a fresh sponge with a singe stroke, twice in opposite directions

Neutralizing

- The surface must be at the pre-defined optimum alkalinity of strain gage adhesives (mostly 7.0 to 7.5 pH, refer to manufacturers directions)
- The surface must always be kept wet and neutralizer should never be allowed to dry on the surface
- After neutralizing the surface must be cleaned with a fresh sponge with a singe stroke, twice in opposite directions