Lecture Module - Probability and Statistics

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Topic 2 - Characterizing a Population of Data

Topic 2 - Characterizing a Population of Data

- A Population of Data
- Variance and Standard Deviation
- Using the Z Table
- Example: Using the Z Table

A Population of Data

Consider a manufacturing facility that makes ball bearings.

- How does the manufacturer ensure the quality of the product?
- How the does the seller communicate this to the buyer?
- How does the Engineer use this information?

A Population of Data

For a given set of measurements we want to quantify:

- a representative value that characterizes the average of the measured data set
- a representative value that provides a measure of the variation in the data set
- how well the average of the measured data set represents the average of the entire population

Variance and Standard Deviation

The true variance is:

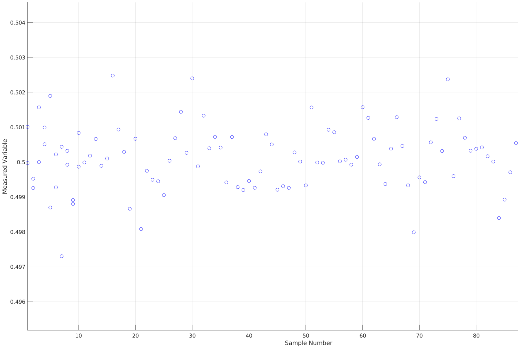
$$\sigma^2 = \int_{-\infty}^{\infty} (x - x')^2 p(x) dx$$

For discrete data this becomes:

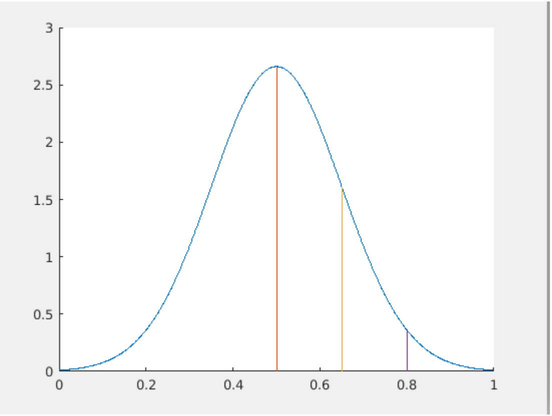
$$\sigma^2 = \lim_{N \to \infty} \frac{1}{N} \sum_{i=1}^{N} (x_i - x')^2$$

The square root of the variance is the standard deviation.

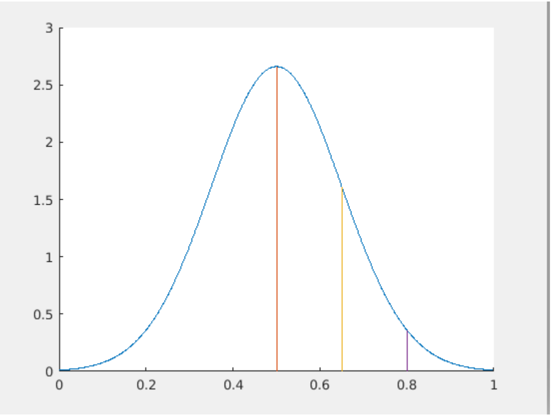
$$\sigma = \sqrt{\sigma^2}$$



Variance and Standard Deviation



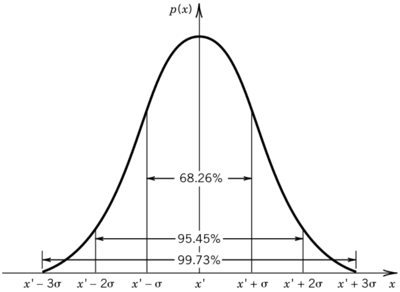
Variance and Standard Deviation



Using the Z Table

... The area under the portion of the probability density function curve, $p(x)$, defined by the interval $x' - z_1\sigma \leq x \leq x' + z_1\sigma$ provides the probability that a measurement will assume a value within that interval. Direct integration of $p(x)$ for a normal distribution between the limits $x' \pm z_1\sigma$ yields that for $z_1 = 1$, 68.26% of the area under $p(x)$ lies within $\pm 1\sigma$ x' . This means that there is a 68.26% chance that a measurement of x will have a value within the interval $x' \pm 1\sigma$.

Using the Z Table



$z_1 = 1$, 68.26% of the area under $p(x)$ lies within $\pm z_1\sigma$ of x'.
$z_1 = 2$, 95.45% of the area under $p(x)$ lies within $\pm z_1\sigma$ of x'.
$z_1 = 3$, 99.73% of the area under $p(x)$ lies within $\pm z_1\sigma$ of x'.

Using the Z Table

It follows directly that the representative value that characterizes a measure of the variation in a measured data set is the standard deviation. The probability that the ith measured value of x will have a value between $x' \pm z_1\delta$ is $2 \times P(z_1) \times 100 = P\%$.

This is written as

$$x_i = x' \pm z_1\sigma \quad (P\%)$$

Example: Using the Z Table

Table 4.3 Probability Values for Normal Error Function: One-Sided Integral Solutions for $p(z_1) = \frac{1}{(2\pi)^{1/2}} \int_0^{z_1} e^{-\beta^2/2} d\beta$

$z_1 = \frac{x_1 - x'}{\sigma}$	0.00	0.01	0.02	0.03	0.04	0.05	0.06	0.07	0.08	0.09
0.0	0.0000	0.0040	0.0080	0.0120	0.0160	0.0199	0.0239	0.0279	0.0319	0.0359
0.1	0.0398	0.0438	0.0478	0.0517	0.0557	0.0596	0.0636	0.0675	0.0714	0.0753
0.2	0.0793	0.0832	0.0871	0.0910	0.0948	0.0987	0.1026	0.1064	0.1103	0.1141
0.3	0.1179	0.1217	0.1255	0.1293	0.1331	0.1368	0.1406	0.1443	0.1480	0.1517
0.4	0.1554	0.1591	0.1628	0.1664	0.1700	0.1736	0.1772	0.1809	0.1844	0.1879
0.5	0.1915	0.1950	0.1985	0.2019	0.2054	0.2088	0.2123	0.2157	0.2190	0.2224
0.6	0.2257	0.2291	0.2324	0.2357	0.2389	0.2422	0.2454	0.2486	0.2517	0.2549
0.7	0.2580	0.2611	0.2642	0.2673	0.2794	0.2734	0.2764	0.2794	0.2823	0.2852
0.8	0.2881	0.2910	0.2939	0.2967	0.2995	0.3023	0.3051	0.3078	0.3106	0.3133
0.9	0.3159	0.3186	0.3212	0.3238	0.3264	0.3289	0.3315	0.3340	0.3365	0.3389
1.0	0.3413	0.3438	0.3461	0.3485	0.3508	0.3531	0.3554	0.3577	0.3599	0.3621
1.1	0.3643	0.3665	0.3686	0.3708	0.3729	0.3749	0.3770	0.3790	0.3810	0.3830
1.2	0.3849	0.3869	0.3888	0.3907	0.3925	0.3944	0.3962	0.3980	0.3997	0.4015
1.3	0.4032	0.4049	0.4066	0.4082	0.4099	0.4115	0.4131	0.4147	0.4162	0.4177
1.4	0.4192	0.4207	0.4292	0.4236	0.4251	0.4265	0.4279	0.4292	0.4306	0.4319
1.5	0.4332	0.4345	0.4357	0.4370	0.4382	0.4394	0.4406	0.4418	0.4429	0.4441
1.6	0.4452	0.4463	0.4474	0.4484	0.4495	0.4505	0.4515	0.4525	0.4535	0.4545
1.7	0.4554	0.4564	0.4573	0.4582	0.4591	0.4599	0.4608	0.4616	0.4625	0.4633
1.8	0.4641	0.4649	0.4656	0.4664	0.4671	0.4678	0.4686	0.4693	0.4699	0.4706
1.9	0.4713	0.4719	0.4726	0.4732	0.4738	0.4744	0.4750	0.4758	0.4761	0.4767
2.0	0.4772	0.4778	0.4803	0.4788	0.4793	0.4799	0.4803	0.4808	0.4812	0.4817
2.1	0.4821	0.4826	0.4830	0.4834	0.4838	0.4842	0.4846	0.4850	0.4854	0.4857
2.2	0.4861	0.4864	0.4868	0.4871	0.4875	0.4878	0.4881	0.4884	0.4887	0.4890
2.3	0.4893	0.4896	0.4898	0.4901	0.4904	0.4906	0.4909	0.4911	0.4913	0.4916
2.4	0.4918	0.4920	0.4922	0.4925	0.4927	0.4929	0.4931	0.4932	0.4934	0.4936
2.5	0.4938	0.4940	0.4941	0.4943	0.4945	0.4946	0.4948	0.4949	0.4951	0.4952
2.6	0.4953	0.4955	0.4956	0.4957	0.4959	0.4960	0.4961	0.4962	0.4963	0.4964
2.7	0.4965	0.4966	0.4967	0.4968	0.4969	0.4970	0.4971	0.4972	0.4973	0.4974
2.8	0.4974	0.4975	0.4976	0.4977	0.4977	0.4978	0.4979	0.4979	0.4980	0.4981
2.9	0.4981	0.4982	0.4982	0.4983	0.4984	0.4984	0.4985	0.4985	0.4986	0.4986
3.0	0.49865	0.4987	0.4987	0.4988	0.4988	0.4988	0.4989	0.4989	0.4989	0.4990