Module 5 - Strain Applications

ME3023 - Measurements in Mechanical Systems

Mechanical Engineering

Tennessee Technological University

Topic 3 - Principle Strains

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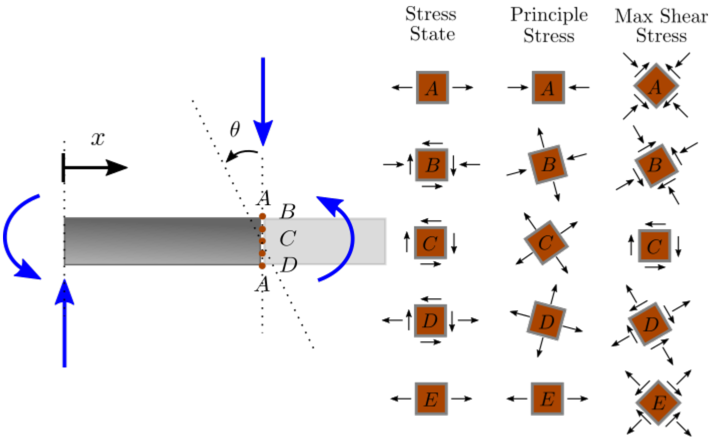
- Motivation
- Principle Stress and Strain
- Determining Principle Stresses
- Example: A Pressure Vessel

Motivation

The design of load-carrying components for machines and structures requires information concerning the distribution of forces within the particular component. Proper design of devices such as shafts, pressure vessels, and support structures must consider load-carrying capacity and allowable deflections. Mechanics of materials provides a basis for predicting these essential characteristics of a mechanical design, and provides the fundamental understanding of the behavior of load-carrying parts. However, theoretical analysis is often not sufficient, and experimental measurements are required to achieve a final design.

Text: Theory and Design of Mechanical Measurements

Principle Stress and Strain



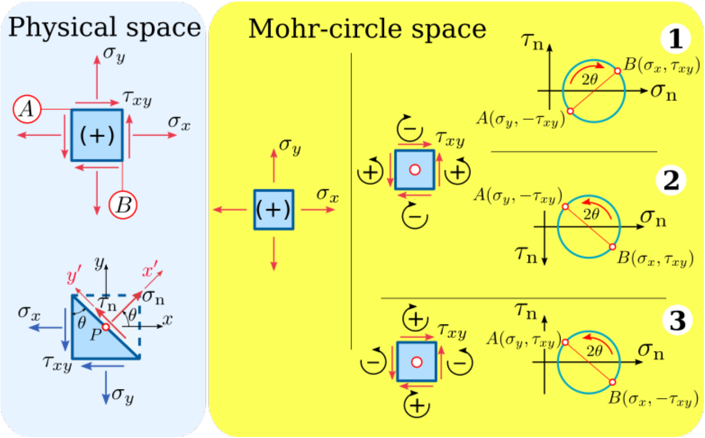


Image: Wikipedia

Determining Principle Stresses

A strain gauge rosette can be mounted on a physical component to measure the principle strains and their directions. Depending on what is known about the stress state this may be done in one of four ways.

- Case 1: Uniaxial Stress
- Case 2: Isotropic Stress
- Case 3: Pure Torsional Stress
- Case 4: Biaxial Stress
  - with known principle directions
  - or with unknown principle directions

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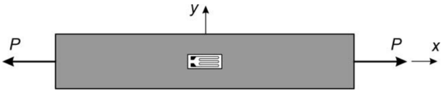
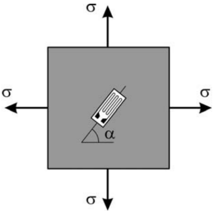
Determining Principle Stresses

Case 1: Uniaxial Stress



$$\sigma_{xx} = E\epsilon_{xx}$$

Case 2: Isotropic Stress

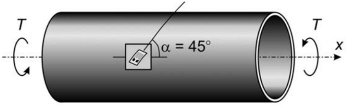


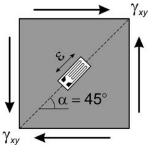
$$\sigma_{xx} = \sigma_{yy} = \sigma_1 = \sigma_2 = \sigma = \frac{E\nu}{1-\nu}$$

$$\tau_{xy} = 0$$

Determining Principle Stresses

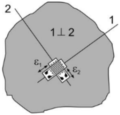
Case 3: Pure Torsional Stress





$\tau_{xy} = \tau_{max} = G\gamma_{xy}$ with $\gamma = 2\epsilon$

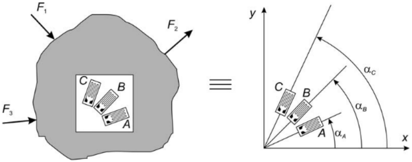
Case 4a: Biaxial Stress - The principle directions are known.



$\sigma_1 = \frac{E}{1-\nu^2}\left(\epsilon_1 + \nu\epsilon_2\right)$ and $\sigma_2 = \frac{E}{1-\nu^2}\left(\epsilon_2 + \nu\epsilon_1\right)$

Determining Principle Stresses

Case 4b: Biaxial Stress - The principle directions are not known.



$$\varepsilon_1 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) + \frac{1}{2}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\varepsilon_2 = \frac{1}{2}(\varepsilon_{xx} + \varepsilon_{yy}) - \frac{1}{2}\sqrt{(\varepsilon_{xx} - \varepsilon_{yy})^2 + \gamma_{xy}^2}$$

$$\varepsilon_A = \varepsilon_{xx}\cos^2\alpha_A + \varepsilon_{yy}\sin^2\alpha_A + \gamma_{xy}\cos\alpha_A\sin\alpha_A$$

$$\varepsilon_B = \varepsilon_{xx}\cos^2\alpha_B + \varepsilon_{yy}\sin^2\alpha_B + \gamma_{xy}\cos\alpha_B\sin\alpha_B$$

$$\varepsilon_C = \varepsilon_{xx}\cos^2\alpha_C + \varepsilon_{yy}\sin^2\alpha_C + \gamma_{xy}\cos\alpha_C\sin\alpha_C$$

$$\tan 2\theta = \frac{\gamma_{xy}}{\varepsilon_{xx} - \varepsilon_{yy}}$$

$$\sigma_1 = \frac{E}{1-\nu^2}(\varepsilon_1 + \nu\varepsilon_2) \text{ and } \sigma_2 = \frac{E}{1-\nu^2}(\varepsilon_2 + \nu\varepsilon_1)$$

Text:

Example: A Pressure Vessel

A practical example of this can be seen here in which a strain gauge is used to measure the pressure in a tank from a strain reading alone.

Which case from above are do they use?