1. **WORKPIECE LOCALIZATION ALGORITHMS**
   1. **Filtering w/ Bounding Box and Voxel**

A bounding box filter removes points outside of prescribed limits which is used to eliminate data outside of the expected location of the workpiece. This reduces the computational requirements of the algorithms used in the following workpiece localization process by reducing the number of elements in the pointclouds.

**Add math**

The space inside the bounding box is divided into a uniform three-dimensional grid of cells referred to as voxels. The input cloud points are sorted into the grid by location, and then the centroid of the points in each voxel is added to the filtered point cloud [15]. This reduces redundant data points and allows for the resolution to be set using the voxel grid size as a parameter.

* 1. **Segmentation with RANSAC**

Sample consensus represents a class of algorithms which can be used for fitting pointcloud data to various geometrical models such as lines, planes, and cylinders. The input points are sorted into inliers which fit the model and outliers which do not. Random sample consensus (RANSAC) involves repeated random sub-sampling to detect and segment pointcloud shapes [10][11][19]. This detection and segmentation of geometrical features in the pointcloud is useful in manufacturing applications such as welding in which the workpiece consists of well-defined geometrical shapes.

**Add math**

Although variations have been developed, the RANSAC algorithm is generally an iterative two-part process. The first part involves a hypothesis in which the first minimal sample set (MSSs) is selected at random from the input dataset (source pointcloud) then used as the basis for computing the model parameters. Following the selection of the minimal sample set, RANSAC checks which elements of the instantiated model are consistent with the entire dataset – these elements are referred to as the consensus set (CS) [10]. During each iteration, the instantiated model’s elements are compared to the original dataset. If the new MSS increases the number of correct correspondences compared to the best CS, it will then overwrite the previous CS. The algorithm continuously iterates and is terminated only when the CS reaches a certain threshold. When this threshold has been reached, the instantiated model based on model parameters of the newest MSS, the MSS is said to be consistent with the entire dataset.

improved performance [4] [6] [17] in the presence of outliers, and methods are available (used in this approach) for automatic rejection of non-corresponding outliers []. However, the approach in this work addresses the problem by reducing the reference point cloud to a subset of the LiDAR cloud which contains a portion of the workpiece without the surrounding table or clamps.

One of the key issues with standard ICP is that it may not reach the global minimum of convergence which can be due to things such as false correspondences. These false correspondences cause poor initial alignment and therefore increase the chance of getting stuck in a local minimum [7]. Outlier rejection based on Random Sample Consensus (RANSAC) is one of the several methods including distance-based rejection, or duplicate target point rejection, that reduces the number of outliers in pointclouds. Furthermore, RANSAC may also be utilized to provide a good initial guess for the transformation estimation in ICP [6].

“Classical ICP” uses the point-to-point error metric.

The ICP method as described in [12], [4] is summarized here. Let be the reference point cloud set for representing the workpiece to be aligned with the source point cloud set for where , and where each point corresponds to the point with the same index (*i=k*). The reference set will come from a CAD model while the source cloud set will be collected from a data scan of the real environment. The mean square objective function using the point-to-point error metric (or point-to-plane [4][6]) to be minimized is,

(1)

where is an array that projects onto and that translates onto . If the correct correspondences are known, the correct relative rotation/translation can be calculated in closed form. A closed form implementation that can be found in the PCL library [16] is briefly described. The center of mass for the reference point cloud set and source point cloud set respectively is calculated for each set as,

(2)

The reference point cloud set and source point cloud set are shifted by their center mass such that they are distributed around zero as,

(3)

(3)

A cross-covariance matrix, is defined as

(4)

A singular value decomposition (SVD) of is given as

(5)

where D is a diagonal matrix containing the singular values, , ordered such that and are the left and right singular vectors of . When the , the rotation and translation minimizing are unique and given by:

(6)

and

(7)

In practice, the correspondences assumed in Eq. (1) are not truly known. Therefore, this process is performed iteratively[], by assuming a correspondence between the reference and source point cloud set based on a minimum distance between points. The source is corrected and the process repeats until convergence of the source and reference point cloud set occurs as given in the error .