Dynamic Maximal Independent Sets

Presentation 1

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Outline

Introduction

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$$G = (V, E)$$
$$n := |V|$$
$$m := |E|$$
$$\Delta := max\{deg(v)\}$$

Independent Set (IS):

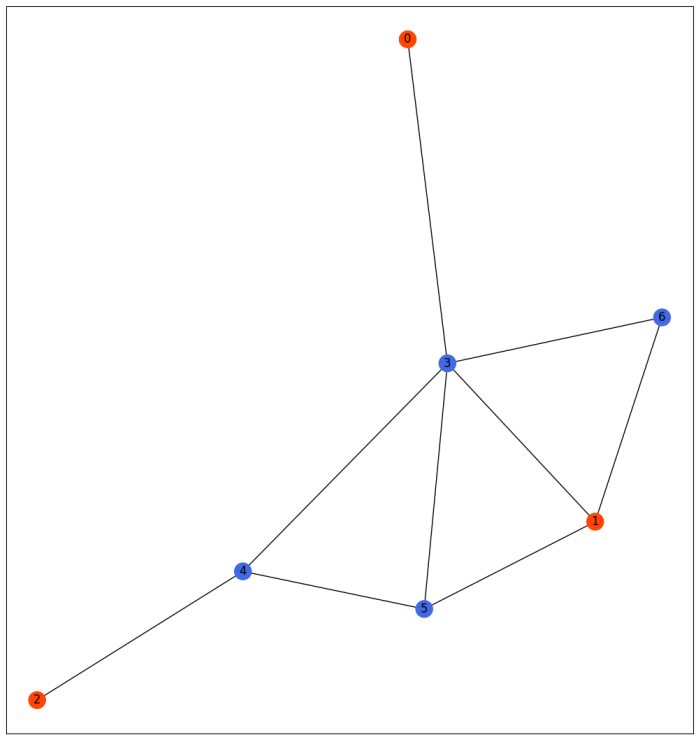
$$\mathcal{M} \subset V : \ \forall u, v \in \mathcal{M} : \ \{u, v\} \notin E$$

$\mathcal{M}$ is **maximal** if (MIS):

$$\nexists \ IS \ \mathcal{M}' : \mathcal{M} \subsetneq \mathcal{M}'$$

Example MIS

- Orange: in the MIS
- Blue: not in MIS



Intuitive Approach

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- Iterate over all vertices
- Check if vertex can be in MIS
- Complexity: $\mathcal{O}(m)$

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- Recompute after every update

Types of updates

- Node removal
- Node insertion
- Edge removal
- Edge insertion

- 1 node might leave $\mathcal{M}$
- up to Δ might enter $\mathcal{M}$

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  $\rightarrow$ Assign cost of next insertion to removal

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1. $\mathcal{A}_S$ for light vertices
2. $\mathcal{A}_T$ for heavy vertices
    - respect the IS on the light nodes!

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- $\mathcal{A}_T$ takes $\mathcal{O}(m_H) = \mathcal{O}((\frac{m}{\Delta_C})^2) = \mathcal{O}(m^{2/3})$

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- $\mathcal{A}_T$ takes $\mathcal{O}(m_H) = \mathcal{O}((\frac{m}{\Delta_C})^2) = \mathcal{O}(m^{2/3})$

- overall amortized complexity: $\mathcal{O}(min\{\Delta, m^{2/3}\})$
  $\rightarrow$ all nodes could be light

Implicit MIS Algorithm

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- The previous algorithms:
  - kept explicit copy of MIS
  - analyzed for amortized complexity
- Relax requirement to keep explicit copy of MIS

Implicit MIS $\mathcal{A}_I$

- Only maintain an IS not an MIS
- Only process removals of nodes

Implicit MIS $\mathcal{A}_I$

- Only maintain an IS not an MIS
- Only process removals of nodes
- Check if $v \in \mathcal{M}$ when asked
- Worst case complexity $\mathcal{O}(min\{\Delta, \sqrt{m}\})$

Realisation of $\mathcal{A}_I$

- Again **light** and **heavy** nodes
- $\Delta_C := \sqrt{m}$

Realisation of $\mathcal{A}_I$

- Again **light** and **heavy** nodes
- $\Delta_C := \sqrt{m}$
- Maintain count for heavy nodes only
- Compute status for light nodes when queried

Conclusion

- Code and slides at:
  github.com/thilofischer/dynamic_mis