Basic Linear algebra

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Linear Algebra is one of the most important basic areas in Mathematics, having at least as great an impact as Calculus, and indeed it provides a significant part of the machinery required to generalise Calculus to vector-valued functions of many variables. Unlike many algebraic systems studied in Mathematics or applied within or outwith it, many of the problems studied in Linear Algebra are amenable to systematic and even algorithmic solutions, and this makes them implementable on computers – this explains why so much calculational use of computers involves this kind of algebra and why it is so widely used. Many geometric topics are studied making use of concepts from Linear Algebra, and the idea of a linear transformation is an algebraic version of geometric transformation. Finally, much of modern abstract algebra builds on Linear Algebra and often provides concrete examples of general ideas.

These notes were originally written for a course at the University of Glasgow in the years 2006–7. They cover basic ideas and techniques of Linear Algebra that are applicable in many subjects including the physical and chemical sciences, statistics as well as other parts of mathematics. Two central topics are: the basic theory of *vector spaces* and the concept of a *linear transformation*, with emphasis on the use of matrices to represent linear maps. Using these, a geometric notion of *dimension* can be made mathematically rigorous leading its widespread appearance in physics, geometry, and many parts of mathematics.

The notes end by discussing *eigenvalues* and *eigenvectors* which play a rôle in the theory of *diagonalisation* of square matrices, as well as many applications of linear algebra such as in geometry, differential equations and physics.

There are some assumptions that the reader will already have met vectors in 2 and 3-dimensional contexts, and has familiarity with their algebraic and geometric aspects. Basic algebraic theory of matrices is also assumed, as well as the solution of systems of linear equations using Gaussian elimination and row reduction of matrices. Thus the notes are suitable for a secondary course on the subject, building on existing foundations.

There are very many books on Linear Algebra. The Bibliography lists some at a similar level to these notes. University libraries contain many other books that may be useful and there are some helpful Internet sites discussing aspects of the subject.