SUPERCOOLED PHASE TRANSITIONS IN THE VERY EARLY UNIVERSE

S.W. HAWKING and I.G. MOSS
Department of Applied Mathematics and Theoretical Physics, Cambridge, UK

Received 12 January 1982

The universe might have had a prolonged exponentially expanding phase caused by its being stuck in a metastable state of the grand unified phase transition. The only way that it could exit from this exponential expansion without introducing too much inhomogeneity or spatial curvature would be through a homogeneous "bubble" solution in which quantum tunnelling occured everywhere at the same time. This would produce more baryons than the conventional scenarios.

There has been considerable interest recently in the possibility that the universe might have undergone an exponentially expanding "inflationary" stage caused by the vacuum energy of a supercooled metastable state of a first-order phase transition associated with the breakdown of the grand unification symmetry [1–4]. Such an exponential expansion might provide the answer to a number of problems in cosmology such as

(1) Why are there so few magnetic monopoles [1,5].

(2) Why is the universe so spatially homogeneous and isotropic?

(3) Why is the universe so nearly flat, i.e. why is it expanding at almost exactly the critical rate to avoid recollapse?

The main difficulty with this inflationary scenario has been to find some way of getting out of the exponentially expanding stage without creating large-scale inhomogeneities that would be incompatible with observation. In his original paper Guth [2] suggested that small bubbles of the broken symmetry phase would form by vacuum tunelling [6] and then expand by a large factor before colliding with other bubbles. However, as he himself admitted, and others have confirmed [7], one would end up with a very inhomogeneous universe dominated by a few very large bubbles.

Most of the papers that have discussed the exit from the inflationary stage have treated it essentially as a phase transition problem in flat spacetime and have neglected the curvature and finite horizon size of the universe. The aim of this letter is to show that these effects play a very important role and can lead to a phase transition which occurs simultaneously at all points of space thus not creating any inhomogeneity or magnetic monopoles. If the adjustable parameters in the effective potential for the Higgs fields lie in a certain range, the spatial flatness of the universe would also be accounted for.

The effective potential of the Higgs fields has the general form

$$V(\phi) = \tfrac{1}{2}(m^2 + \xi R + cT^2)\phi^2 + \tfrac{1}{4}\alpha^2\phi^4(\log \phi^2/\phi_0^2 - \tfrac{1}{2} + m^2/\alpha^2\phi_0^2) + \tfrac{1}{8}\alpha^2\phi_0^4 - \tfrac{1}{4}m^2\phi_0^2\,, \tag{1}$$

where ϕ is some measure of the magnitude of the Higgs field and ϕ_0 is its expectation value at zero temperature in flat space with $V(\phi_0) = 0$. The quantities m^2 and ξ are renormalised parameters which can be given any value. $\xi = \frac{1}{6}$ is a natural choice because it corresponds to conformal invariance when $m = 0$ and it is not renormalized at one loop though it is at two loops. The quantity c is a numerical factor of order 1 which depends on the group and the representation. Neglecting small corrections arising from the ξR term, the energy density of the universe will be

$$\rho = (\pi^2/30)g(T)T^4 + V(\phi,T) , \tag{2}$$

where $g(T)$ is the effective number of spin states of relativistic particles. The rate of expansion will be given by

$$H^2 = (8\pi/3m_\rho^2)\rho - k/S^2 \tag{3}$$

where $H = \dot{S}/S$ and S is the scale factor of the universe.

At very high temperatures the universe will be in the symmetric phase $\phi = 0$. If $m^2 + \xi R \geqslant 0$, the symmetric phase will remain a local minimum of the potential as the temperature falls although it will cease to be a global minimum at some critical temperature T_c of the order of $\alpha\phi_0$. However, in realistic models the probability to tunnel to the broken symmetry phase with $\phi \approx \phi_0$ is very low [8]. The universe will therefore continue to expand in the symmetric phase until the thermal energy, the first term in eq. (2), becomes small compared to the vacuum energy $V(0, T)$. This will lead to an exponential expansion and the universe will rapidly approach a de Sitter state which depends only on the value of $V(0)$ through the relation

$$H = [8\pi V(0)/3m_p^2]^{1/2} \tag{4}$$

but which is otherwise independent of the initial conditions and the initial value of k. This is very similar to the way that a gravitational collapse rapidly approaches a stationary black-hole state which depends only on the mass and angular momentum but which is otherwise independent of the nature of the collapsing body. Following the black-hole analogy, we shall therefore assume a cosmological "no hair" theorem and treat the universe as a de Sitter model.

In the gravitational collapse case, the black hole has an effective temperature [9] $T_B = m_p^2(8\pi M)^{-1}$. Similarly, de Sitter space has an effective temperature [10] $T_s = H/2\pi$. In the case of grand unified phase transitions this will be of the order of 10^{11} GeV. In a de Sitter universe it is impossible to define a temperature smaller than T_s. It is therefore incorrect to use in the effective potential (1) the redshifted temperature of the universe when this falls below T_s. In fact one can regard the ξR term as representing the contribution of T_s to the effective potential.

In accordance with our viewpoint that de Sitter space has no hair, we shall calculate the probability of tunnelling from the local minimum at $\phi = 0$ to the global minimum by looking for solutions of the classical coupled scalar and gravitational field equations that are near the euclidean version of de Sitter space, i.e. a four-sphere of radius H^{-1}.

If $m^2 > 2H^2(1-6\xi)$, there are inhomogeneous "bubble" solutions as there are in the decay of the false vacuum in flat spacetime [6]. Gravity and the curvature of the universe do not have such effect except to make the bubble slightly smaller [11]. However, as mentioned earlier, this case would lead to unacceptably large inhomogeneities in the universe today.

If $2H^2(1 - 6\xi) \geqslant m^2 \geqslant -12H^2\xi$, the potential (1) would lead to inhomogenous bubble solutions in flat spacetime whose radius was greater than that of the de Sitter space, H^{-1}. In this case the only euclidean solution of the coupled equations, apart from $\phi = 0$, is the homogeneous solution $\phi = \phi_1$ where $V(\phi_1)$ is the local maximum of V on a four-sphere of radius H_1^{-1} where $H_1^2 = 8\pi V(\phi_1)/3m_p^2$. The tunnelling probability per unit four-volume is of the order of $(m^2 + \xi R)^2 e^{-B}$ where

$$B = \tfrac{1}{8}m_p^4 \,[1/V(0) - 1/V(\phi_1)] \tag{5}$$

is the difference between the combined gravitational and scalar field actions of the $\phi = \phi_1$, and the $\phi = 0$ solutions. Even though B may be very large and the tunnelling probability very small, this does not matter because the universe will continue in the essentially stationary de Sitter state until it makes a quantum transition everywhere to the $\phi = \phi_1$ solution.

The $\phi = \phi_1$ solution is unstable as is reflected by the fact that it has a negative perturbative mode $\phi = \phi_1 + \epsilon$ where ϵ is a constant on the four-sphere. One would therefore expect the solution in lorentzian spacetime to evolve according to the classical field equations with ϕ running downhill from the maximum at $\phi = \phi_1$ to the global minimum at $\phi = \phi_0$. The solution will presumably be spatially homogeneous but evolving in time so it can be represented by a Friedmann–Robertson–Walker metric with ϕ a functionof time only. One can introduce Robertson–Walker coordinates with $K = +1,0$ or -1 into de Sitter space though

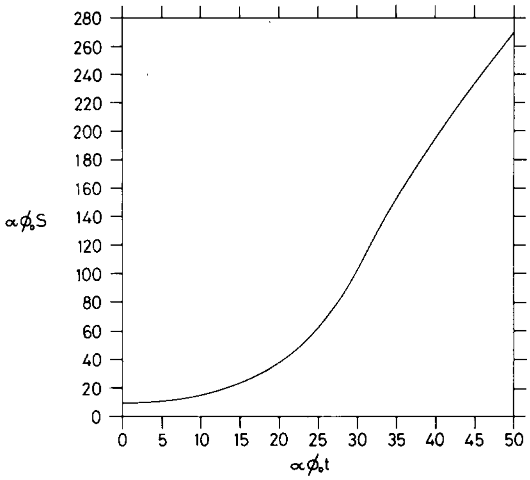

Fig. 1. The scale factor of the universe starting from a $k = +1$ time symmetric state with $\phi = \phi_1 + \epsilon$.

the coordinates cover the whole space only in the first case. It would seem reasonable to start the classical evolution of the solution on a spacelike surface whose curvature (positive or negative) is of the order of the radius H_1^{-1} of the de Sitter space. The most natural choice would seem to be the surface of time symmetry in the $K = +1$ coordinates but the conclusions in the $K = -1$ case are very similar.

While the scalar field remains near $\phi = \phi_1$, the expansion of the universe will continue to be nearly exponential or, in the $K = +1$ case, $S = H_1^{-1} \cosh H_1 t$ where S is the scale factor. The timescale for ϕ to run downhill from the maximum at $\phi = \phi_1$ is

$$3H_1/(m^2 + 12\xi H_1^2)\,. \tag{6}$$

When ϕ approaches the global minimum ϕ_0 it will start to perform damped harmonic osciallations around $\phi = \phi_0$ and the expansion of the universe will change to that of a model filled with non relativistic matter. Figs. 1 and 2 show the numerical solutions for S and ϕ in the case $K = +1, m^2 + 12\xi H_1^2 = 24H_1^2$.

One can think of the oscillations of ϕ about ϕ_0 as corresponding to a very large number of Higgs particles in a coherent state. Presumably they will decay into light gauge particles and fermions which will have an approximately thermal distribution at a temperature T_2 given by $(\pi^2/30)g(T_2)T_2^4 \approx V(\phi_1)$. The decay of the Higgs particles would probably give a much larger baryon asymmetry than in the normal scenario.

The radius of the curvature of the space sections of the universe at the time of thermalization will be of the order $H_1^{-1} \cosh H_1 t_2$ where t_2 is the time from the $\phi = \phi_1$ state to thermalization. If one assumes that this is a few times the timescale (6) for ϕ to run downhill, one finds that the universe would have been sufficiently spatially flat to expend to its present size with a nearly critical density provided that $m^2 + 12\xi H_1^2 < \frac{1}{12} H_1^2$.

This seems the only situation in which one can have a significant exponential expansion at the GUT era without introducing either too much inhomogeneity or too much spatial curvature.

Finally, if $m^2 + 12\xi H_1^2 < 0$, the symmetric state $\phi = 0$ will cease to be a local minimum of the potential before the temperature drops to T_s so there will not be a prolonged exponential expansion phase.

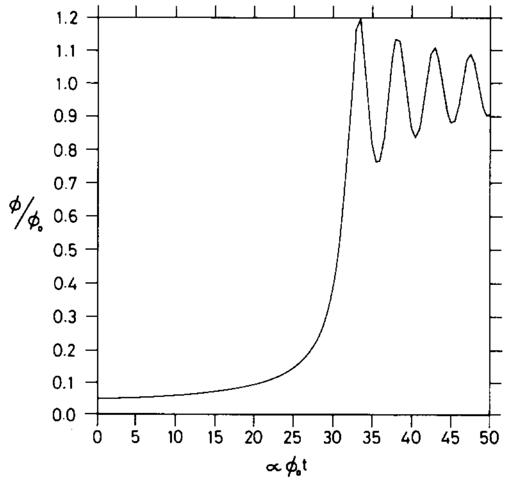

Fig. 2. The value of the scalar field in the same universe.

We are very grateful to A.D. Linde for discussions which gave rise to this paper.

References

[1] A.H. Guth and S.H. Tye, Phys. Rev. Lett. 44 (1980) 631, 963.
[2] A.H. Guth, Phys. Rev. D23 (1981) 347.
[3] A.H. Guth and E.J. Weinberg, Phys. Rev. D23 (1981) 876.
[4] A.D. Linde, Lebedev Institute preprint 229 (1981).
[5] Ya.B. Zeldovich and M.Yu Khlopov, Phys. Lett. 79B (1978) 239.

J. Preskill, Phys. Rev. Lett. 43 (1979) 1365.
[6] S. Coleman, Phys. Rev. D15 (1977) 2929;
C. Callan and S. Coleman, Phys. Rev. D16 (1977) 1762.
[7] S.W. Hawking, I.G. Moss and J.M. Stewart, Bubble collisions in the very early universe, Cambridge preprint (1981).
[8] A.D. Linde, Phys. Lett. 99B (1981) 391.
G.P. Cook and K.T. Mahanthappa, Phys. Rev. D23 (1981) 1321;
A. Billoire and K. Tamvakis, CERN preprint TH.3019 (1981)
[9] S.W. Hawking, Commun. Math. Phys. 43 (1975) 199.
[10] G.W. Gibbons and S.W. Hawking, Phys. Rev. D15 (1977) 2738.
[11] S. Coleman and F. De Luccia, Phys. Rev. D21 (1980) 3305.