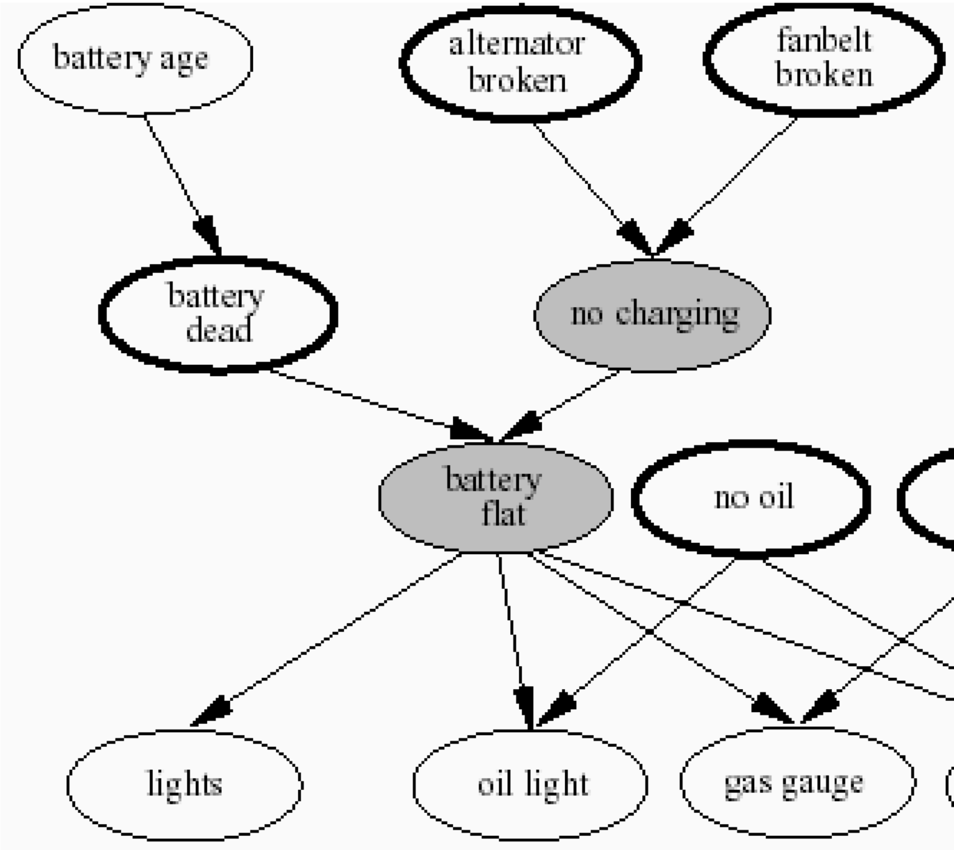
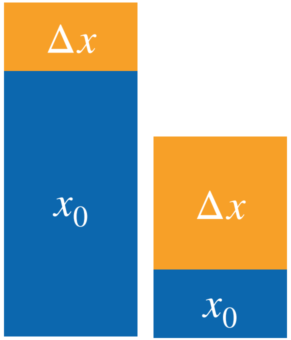
Dependent Random Variables



Relative Change



- Influence of variables

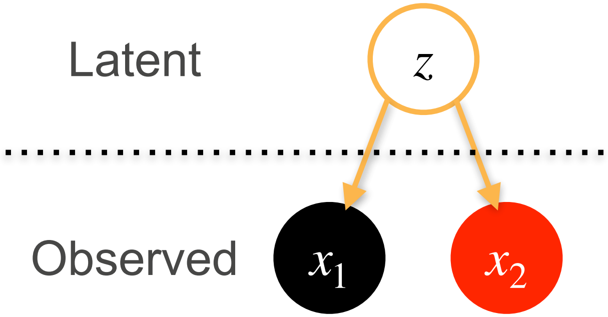
$$f(x) = w^\top x + b = \sum_{i=1}^{n} w_i x_i + b$$

- Influence is based on the *change* in value Δx
- We need to determine the reference value x_0 to assess whether a feature i matters.
- Data type dependent, but means are a good way to start (images, text, tabular data, audio, etc.)

Distributions, joint, marginal, expectations

- **Want to know the influence of x_i on $f(x)$**
  - Direct influence via function (that's what we're after)
  - Confounded by indirect influence on other features (from x_i on x_{-i})
- **Options**
  - Use conditional expectation $p(x_{-i} | x_i)$
  - Use marginal $p(x_{-i})$

Backdoor Problem



- x_1, x_2 are conditionally indep.
- Observed distribution

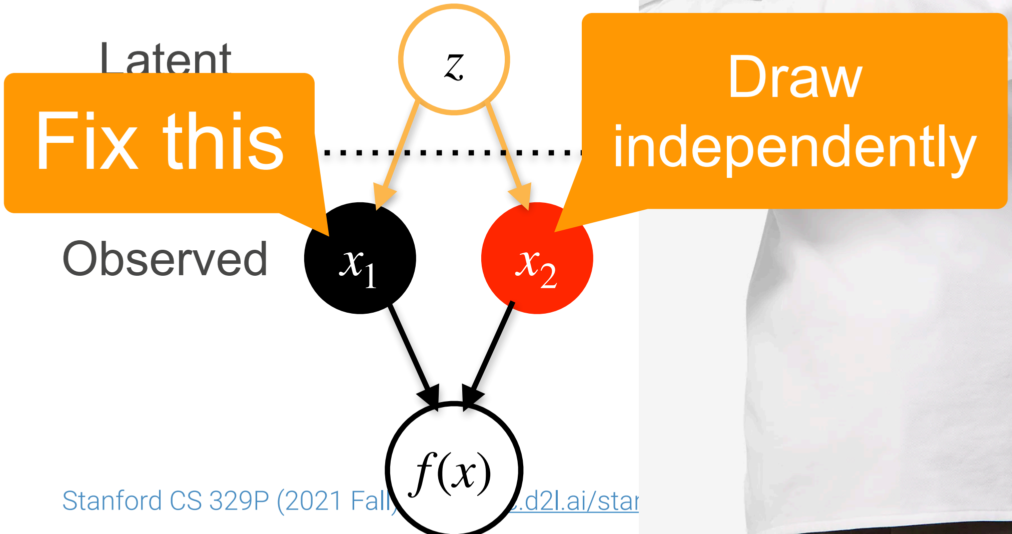
$$p(x_1, x_2) = \int dp(z)p(x_1 \mid z)p(x_2 \mid z)$$

- Conditional is dependent

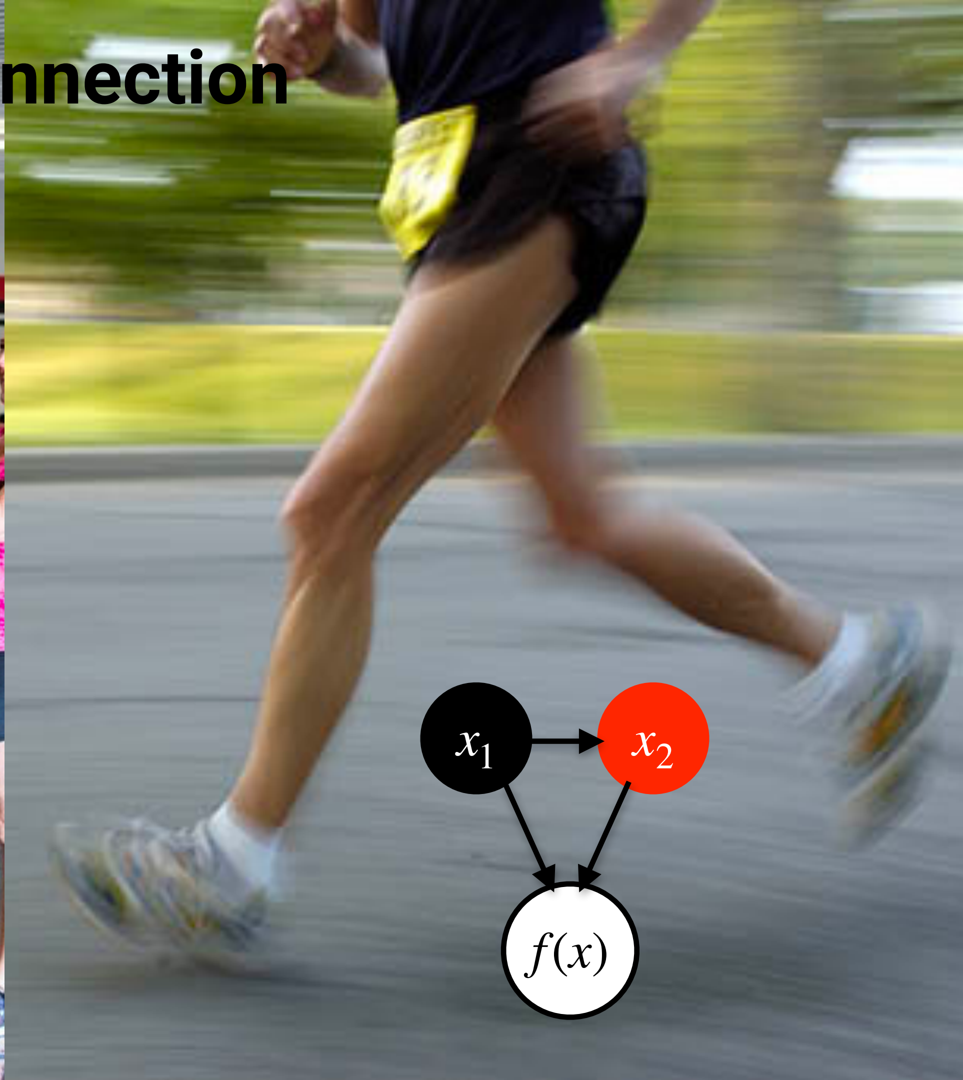
$$p(x_1, x_2) \neq p(x_1)p(x_2)$$

Approve Credit?

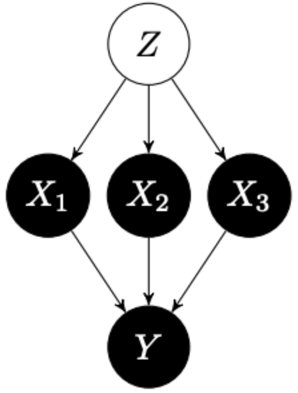
- Credit rating based on observed attributes
- Changing observed attributes will not change underlying condition.



Causal Connection



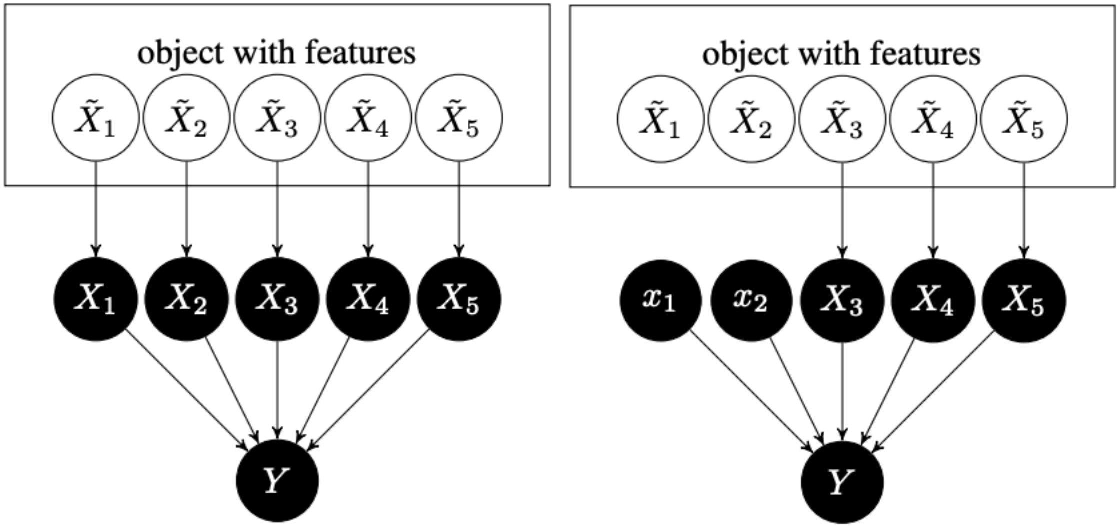
More Context



- Pearl's do operator

$$\mathbf{E}[y \mid \mathrm{do}(X_1 = x_1)] = \int dp(x_2, x_3)\mathbf{E}[y \mid x_1, x_2, x_3]$$

- Use samples from marginals rather than conditional distribution. This is easier and usually more correct.




AND NOW FOR SOMETHING
COMPLETELY DIFFERENT.

The Parliament of Micronesia

The Parliament of Micronesia

The parliament of Micronesia is made up of four political parties, A, B, C, and D, which have 45, 25, 15, and 15 representatives, respectively. They are to vote on whether to pass a $1M spending bill and **how much of this amount should be controlled by each of the parties**. A majority vote, that is, a minimum of 51 votes, is required in order to pass any legislation, and if the bill does not pass then every party gets zero to spend.

Winning coalitions are (A,B), (A,C), (A, D), (B, C, D) and any superset of these. Payoff function $v(S)$ for set S.

Why do we care?

- Replace parties with features
- Replace payoff with influence

Axioms

- **Symmetry**
  For any payoff function $v(S)$ not containing i or j with
  $$v(S \cup \{i\}) = v(S \cup \{j\}) \text{ for all } S$$
  we need the reward to satisfy $\phi(i) = \phi(j)$.
- **Dummy Player**
  For any S without i with $v(S \cup \{i\}) = v(S) + v(\{i\})$ we need the reward to satisfy $\phi(i) = v(\{i\})$.

Axioms

- **Additivity**
  For any payoff function $v = v_1 + v_2$ the rewards are also additive, i.e. $\phi(i) = \phi_1(i) + \phi_2(i)$

- **Shapley Value Theorem**
  The payoff function satisfying symmetry, dummy and additivity that divides the payoff the grand coalition (everyone contributes) is uniquely defined by the Shapley value of the game.

Shapley Value Function

- **Definition**

$$\phi(i, N) = \sum_{S \in N \backslash \{i\}} \frac{1}{|N|} \binom{|N| - 1}{|S|}^{-1} \left[v(S \cup \{i\}) - v(S) \right]$$

- **Properties**
  - Average over all $|N|$ set sizes and over all ways to choose $S \in N \backslash \{i\}$. Normalized by cardinality.
  - Add up all incremental contributions.
  - Easy to check that it satisfies the axioms. Uniqueness is a bit more work.

Back to Micronesia

- A (45), B (25), C (15), or D (15) have individual payoff 0.
- Any coalition including A succeeds
  (B, C, D are interchangeable)
- The only successful coalition without A is (B, C, D).

- Going over the definition yields payoffs

$$A = \frac{1}{2} \text{ and } B = C = D = \frac{1}{6}$$

Back to Feature Scoring - use this for $f(x)$