SET: A

AMERICAN INTERNATIONAL UNIVERSITY-BANGLADESH
Faculty of Science & Technology
Department of Mathematics
MAT2101: Complex Variables, Laplace and Z-transformations (Sections: All)

Final Examination SUMMER 2022-2023

Total Marks: 40 Time: 2 hours
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Instruction: Answer all the questions with the given conditions.

1. Answer **ANY FIVE** of the following questions: $(5 \times 2 = 10)$

(a) Separate real and imaginary parts of the complex valued function $w = 3z + i + e^{\frac{\pi i}{3}}$

(b) Evaluate $\mathcal{L}\{\delta(t-2) + 2\sinh 4t\}$.

(c) Evaluate $\mathcal{L}\{3t^2 + 6 + \cos 3t\}$.

(d) Evaluate $\mathcal{L}\{e^{3t} \sin 2t\}$.

(e) Evaluate $\mathcal{L}\{te^{3t}\}$.

(f) Evaluate $\mathcal{L}\{(t-2)^2\, u(t-2)\}$, where $u(t-2)$ is the unit step function.

(g) Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s-5} - \frac{8}{s^2-16}\right\}$.

(h) Evaluate $\mathcal{L}^{-1}\left\{\frac{s}{(s-2)^2+25}\right\}$.

2. Answer **ANY TWO** of the following questions: $(2 \times 5 = 10)$

(a) Let the rectangular region R in z-plane which is bounded by the lines $x = 3, y = 0$, $x = 4$ and $y = 5$. Determine the region R' of the w-plane into which R is mapped under the following transformations:

$$w = z - (1 + 3i)$$

(b) Evaluate the following improper integral using Cauchy's residue theorem (CRT):

$$\int_0^\infty \frac{x^2 dx}{(x^2+16)^2}.$$

(c) Expand $f(z) = \frac{5z}{(z-1)(2-z)}$ in a Laurent series valid for i) $|z| < 1$, ii) $1 < |z| < 2$.

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3. Answer **ANY TWO** of the following questions: $(2 \times 5 = 10)$

   a) Find Laplace transformation of the following functions:

      i) $f(t) = t\cos t - e^{4t}$, ii) $f(t) = (1-\cos t)^2$.

   b) Consider the following function:

$$f(t) = \begin{cases} 0, & 0 < t < 2 \\ t-2, & 2 < t < 3 \\ 4-t, & 3 < t < 4 \\ 0, & t > 4 \end{cases}.$$

      i) Sketch the above function.

      ii) Write $f(t)$ in terms of unit step function.

      iii) Find Laplace transformation of (ii).

   c) Find inverse Laplace transformation of the following function using partial fraction:

$$F(s) = \frac{s^2+4s+5}{(s-2)^2(s-3)}.$$

   d) Find inverse Laplace transformation of the following functions:

      i) $F(s) = \frac{2s-7}{s^2+4s+13}$, ii) $F(s) = \frac{e^{-3s}}{s^2+\pi^2} + \frac{e^{-2s}}{s^2}$.

4. Answer **ANY TWO** of the following questions: $(2 \times 5 = 10)$

   a) Solve the following linear differential equation using Laplace transformation:

$$\ddot{y}(t) - 2\dot{y}(t) - 8y(t) = 4e^{-3t}, y(0) = 0, \dot{y}(0) = 1.$$

   b) Solve the following linear differential equation using Laplace transformation:

$$\ddot{y}(t) + 3y(t) = 0, y(0) = 1, \dot{y}(0) = -2.$$

   c) Solve the following system of differential equations using Laplace transformation, where $x(t) \equiv x, y(t) \equiv y,\ \dot{y} \equiv \frac{dy(t)}{dt}$ and $\dot{x} \equiv \frac{dx(t)}{dt}$:

$$\begin{cases} \dot{x} = 4x + y \\ \dot{y} = 9x + 4y \end{cases} \text{subject to } x(0) = 3, y(0) = 2.$$

Important Formulae:
i. $\mathcal{L}\{y(t)\} = Y(s)$.
ii. $\mathcal{L}[\dot{y}(t)] = sY(s) - y(0)$.
iii. $\mathcal{L}[\ddot{y}(t)] = s^2Y(s) - sy(0) - \dot{y}(0)$.
where $\dot{y} \equiv \frac{dy(t)}{dt}$ and $\ddot{y} \equiv \frac{d^2y(t)}{dt^2}$.