Complex Variabl

Final term

SET: A

AMERICAN INTERNATIONAL UNIVERSITY-BANGLADESH
Faculty of Science & Technology
Department of Mathematics
MAT2101: Complex Variables, Laplace and Z-Transformations (Sections: All)

Final Examination FALL: 2022-2023

Total Marks: 40 Time: 2 hours

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**Instruction: Answer all the questions with the given conditions.**

1. Answer ANY FIVE of the following questions: $(5 \times 2 = 10)$

 (a) Separate real and imaginary parts of the complex valued function $w = 3z + i - 2$

 (b) Evaluate $\mathcal{L}\{\delta(t-5) + 2\}$.

 (c) Evaluate $\mathcal{L}\{e^{-t} - 2t + 6\}$.

 (d) Evaluate $\mathcal{L}\{e^{t} \sin 5t\}$.

 (e) Evaluate $\mathcal{L}\{t \cos 2t\}$.

 (f) Evaluate $\mathcal{L}[t\, u(t-8)]$, where $u(t-8)$ is the unit step function.

 (g) Evaluate $\mathcal{L}^{-1}\left\{\frac{1}{s+1} - \frac{4}{s^3}\right\}$.

 (h) Evaluate $\mathcal{L}^{-1}\left\{\frac{s-1}{(s-1)^2+9}\right\}$.

2. Answer ANY TWO of the following questions: $(2 \times 5 = 10)$

 (a) Let the rectangular region $R$ in $z$-plane which is bounded by the lines $x = 2, y = 0$, $x = 4$ and $y = 5$. Determine the region $R'$ of the $w$-plane into which $R$ is mapped under the transformation $w = 2z + 1 + i$ .

 (b) Evaluate the following improper integral using Cauchy's residue theorem (CRT):

$$\int_{-\infty}^{\infty} \frac{dx}{(x^2+9)^2}.$$

 (c) Expand $f(z) = \frac{4z}{(z-2)(3-z)}$ in a Laurent series valid for $|z| < 2$. Also sketch ROC.

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3. Answer ANY TWO of the following questions: $(2 \times 5 = 10)$

a) Find Laplace transformation of the following function using definition:

$$f(t) = \begin{cases} 2t\,; & 0 \le t \le 2 \\ 3\,; & t > 2 \end{cases}.$$

b) Consider the following function:

$$f(t) = \begin{cases} t^2; & 0 \le t < 4 \\ 2; & t \ge 4 \end{cases}.$$

i) Sketch the above function.

ii) Write $f(t)$ in terms of unit step function.

iii) Find Laplace transformation of (ii).

c) Find inverse Laplace transformation of the following function using partial fraction:

$$F(s) = \frac{s^2 - 4}{(s-1)^2\,(s-3)}.$$

d) Find inverse Laplace transformation of the following function and sketch $f(t)$:

$$F(s) = \frac{3(e^{-2s} - 4e^{-6s})}{s}.$$

4. Answer ANY TWO of the following questions: $(2 \times 5 = 10)$

a) Solve the following linear differential equation using Laplace transformation:

$$\ddot{y}(t) - 5\dot{y}(t) + 6y(t) = 0\,; \qquad y(0) = 1,\ \dot{y}(0) = -2.$$

b) Solve the following linear differential equation using Laplace transformation:

$$\frac{d^2y}{dt^2} + 5\frac{dy}{dt} - 24y = 3u_3(t)\,;\ y(0) = 0,\ \dot{y}(0) = 0.$$

c) Solve the following system of linear differential equation using Laplace transformation:

$$\begin{aligned} \frac{dx(t)}{dt} &= 5x(t) + y(t) \\ \frac{dy(t)}{dt} &= 4x(t) + 5y(t) \end{aligned}\ ;\ x(0) = 2,\ y(0) = 5.$$

| Important Formulae: |
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| (I) $\mathcal{L}\{y(t)\} = Y(s)$. |
| (i) $\mathcal{L}\{\dot{y}(t)\} = sY(s) - y(0)$. |
| (ii) $\mathcal{L}\{\ddot{y}(t)\} = s^2\,Y(s) - s\,y(0) - \dot{y}(0)$. |