Grade _____ Student Name _____ Score _____

Matsumoto Math Olympiad Club - Homework

Submit on Dec 12th and 14th, 2017
Course slides are online https://weidongshao.github.io/presos/cubes.html. YouTube Video Link https://youtu.be/sQ3FSFOEpLo

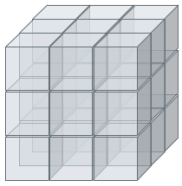
1. How many 1-cm cubes are needed to build a cube with an edge of 8-cm? What is the sum of lengths of all edges?

2. A rectangular box has 4cm in width, 12cm in height, and 8 in depth. What is the sum of lengths of all edges? How many 1-cm cubes are needed to build this box?

3. **Reference Problem:** A wooden cube that measures 3 cm along each edge is painted red. The painted cube is then cut into 1-cm cubes as shown in the diagram.

   (a) How many of 1-cm cubes do not have red paint on any face?

   (b) How many of 1-cm cubes that only have 1 face painted red?



Answer:

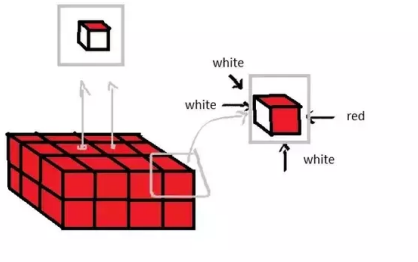
   (a) Only the 1-cm cube in the middle is not colored. All other 1-cm cubes have at least one face painted. So the answer is 1.

   (b) The middle 1-cm cube of each face has just one red face. There are 6 faces of the 3-cm cube. Hence 6 1-cm cubes have red paint on just 1 face.

Repeat the above questions for 4-cm edge length

4. Repeat the above problem for 5-cm edge length

5. A rectangular $4 \times 3 \times 2$ block has its surface painted red and is cut into cubes with each edge equaling 1 unit.

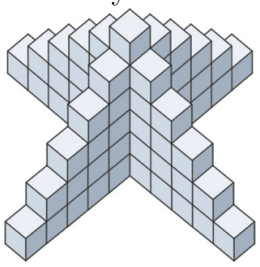


(a) How many cubes have exactly one face painted red?

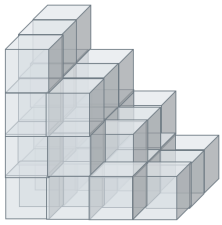
(b) How many cubes have exactly two faces painted red?

(c) How many cubes have exactly three faces painted red?

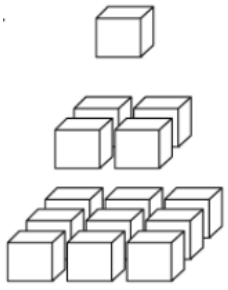
6. How many cubes are needed to build a tower like this, but 12 cubes high?



7. The builder wants to paint the staircase he has built. Each pint of paint can paint one cube on all faces. i.e. 6 faces. How many pints of paint does he need? (note you dont have to paint the bottom face).



8. Kelly is building a tower with cubes. He wants the top of the tower to have only one cube. The next layer down will have four cubes. The third layer from the top will have nine cubes. Kelly wants to make the tower 10 stories high. How many cubes does he need to use for the base of the tower? Hint: Each layer has an edge length increased by 1 from the layer above.



9. A rectangular prism can fit two spherical balls, each having radius r. Each ball touches 5 sides of the rectangular prism. What is the volume of the prism, in terms of r?

10. Each of the following shapes below can be folded into a cube (along the edges). Do you see it? In your mind, try to figure out how it happens.