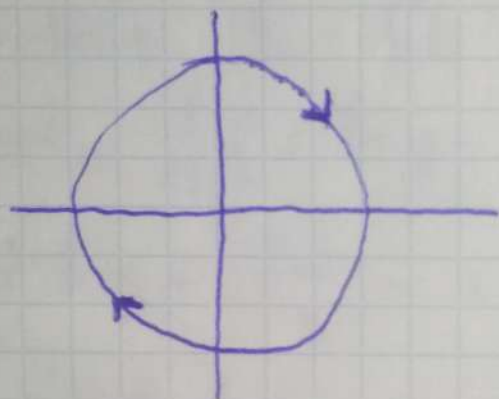


Контрольна робота

студента групи ПМ 1-22

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$$5) \begin{cases} \dot{x}_1 = x_2 \\ \dot{x}_2 = -x_1 \end{cases} \quad \begin{aligned} x_1(t) &= 0, \quad x_2(t) = 0 \\ x_1(t) &= A \sin t, \quad x_2(t) = A \cos t \end{aligned}$$
$$v(x_1, x_2) = (x_2, -x_1)$$



$$4) \begin{cases} \dot{x}_1 = y \\ \dot{x}_2 = -x_1 \end{cases} \Rightarrow \begin{cases} x_1 \dot{x}_1 = x_1 y \\ y \dot{x}_2 = -x_1 y \end{cases} \Rightarrow$$

$$\Rightarrow x_1 \dot{x}_1 + y \dot{x}_2 = 0 \Rightarrow \frac{1}{2} \frac{d}{dt} (x_1^2(t) + y^2(t)) = 0 \Rightarrow x_1^2(t) + y^2(t) = C \Rightarrow$$

$\Rightarrow U(x_1, y) = x_1^2 + y^2$ — перший інтеграл

$$3) \ a) \ \begin{cases} (x-1)^2 + y^2 - 4 = 0 \\ (x+1)^2 + y^2 - 4 = 0 \end{cases} \quad y^2 = 4 - (x+1)^2$$

$$(x-1)^2 + 4 - (x+1)^2 - 4 = 0$$

$$x^2 - 2x + 1 - x^2 - 2x - 1 = 0$$

$$-4x = 0$$

$$x = 0$$

$$y^2 = 3, \quad y_{1,2} = \pm \sqrt{3}$$

Особные точки: $(0, \sqrt{3}), (0, -\sqrt{3})$

(Дви)

$$5) \ \begin{cases} x - 3y - 2 = 0 \\ 6y - 2x + 4 = 0 \end{cases} \Rightarrow \begin{cases} x - 3y - 2 = 0 \\ -2x + 6y + 4 = 0 \end{cases}$$

$$0 = 0 \quad (2 + 3y, y), \quad y \in \mathbb{R}$$

Безмиз

$$6) \ \begin{cases} x = x - y \\ y = xy - 2x + 1 \end{cases}$$

$$\begin{cases} x - y = 0 \\ xy - 2x + 1 = 0 \end{cases}$$

$$\begin{cases} x = y \\ xy - 2x + 1 = 0 \end{cases}$$

$$y^2 - 2y + 1 = 0$$

$$y = 1, \quad x = 1$$

Одна особл. точка
 $(1, 1)$

$$1) \begin{cases} \ddot{x} = (x-1)(y-2) \\ \ddot{y} = (x-2)(y-1) \end{cases}$$

$$\begin{cases} (x-1)(y-2) = 0 \\ (x-2)(y-1) = 0 \end{cases}$$

$$(1; 1)$$

$$(2; 2)$$

$$(x-1)(y-2) = f_1(x_1, x_2) = xy - x \cdot 2 - y + 2$$

$$(x-2)(y-1) = f_2(x_1, x_2) = xy - x - 2y + 2$$

$$\frac{df_1}{dx_1} = y - 2$$

$$\frac{df_1}{dx_2} = x - 1$$

$$\frac{df_2}{dx_1} = y - 1$$

$$\frac{df_2}{dx_2} = x - 2$$

$$A_1 = \begin{pmatrix} -1 & 0 \\ 0 & -1 \end{pmatrix}$$

$$A_1: \begin{pmatrix} -1-\lambda & 0 \\ 0 & -1-\lambda \end{pmatrix} = (-1-\lambda)^2 = 0$$

$\lambda_1 = \lambda_2 = -1$ — стійкий дисипативний вузол

$$A_2 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$$

$$A_2: \begin{pmatrix} -1 & 1 \\ 1 & -1 \end{pmatrix} = 1^2 - 1 = 0$$

$$\lambda_1 = -1$$

$$\lambda_2 = 1$$

— cigno

$$2) a) \begin{cases} \dot{x}_1 = 3x_1 \\ \dot{x}_2 = 3x_2 \end{cases}$$

$$\begin{pmatrix} \lambda & \varepsilon \\ 0 & \lambda \end{pmatrix}$$

$$\begin{matrix} \lambda = 0 \\ \varepsilon = 1 \end{matrix}$$

$$A = \begin{pmatrix} 0 & 3 \\ 0 & 3 \end{pmatrix}$$

$$A^2 = A^2$$

Особл. точка $(0; 0)$

$$6) \begin{cases} \dot{x}_1 = 2x_1 + 3x_2 + 4e^t \\ \dot{x}_2 = x_1 - 4e^t \end{cases}$$

$$x_1(0) = 5$$

$$x_2(0) = -1$$

$$A = \begin{pmatrix} 2 & 3 \\ 1 & 0 \end{pmatrix}$$

$$b = \begin{pmatrix} 4e^x \\ -4e^x \end{pmatrix}$$

$$\det |A - \lambda I| = \lambda^2 - 2\lambda - 3 = 0$$

$$D = 4$$

$$\lambda_1 = 3$$

$$\lambda_2 = -1$$

$$\lambda_1 = 3$$

$$\begin{pmatrix} -1 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$-\alpha + 3\beta = 0$$

$$\alpha = 3\beta$$

$$h_1 = \begin{pmatrix} 3 \\ 1 \end{pmatrix}$$

$$\lambda_2 = -1: \begin{pmatrix} 3 & 3 \\ 1 & 1 \end{pmatrix} \begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$$

$$h_1 = \begin{pmatrix} -1 \\ 1 \end{pmatrix}$$

$$3\lambda - 3 = 0$$

Получим расщепление:

λ_1	λ_2	w	n	z
-1	3	1	0	0

$$\begin{cases} x = q_1(t)e^{w_1 t} \\ y_1 = q_2(t)e^{w_1 t} \end{cases} \quad \begin{cases} x_k = ae^t \\ y_k = be^t \end{cases}$$

$$\begin{cases} ae^t = 2ae^t + 3be^t + 4e^t \\ be^t = ae^t - 4e^t \end{cases}$$

$$\begin{cases} a = 2a + 3b + 4 \\ b = a - 4 \end{cases}$$

$$\begin{cases} a = 2 \\ b = -2 \end{cases}$$

$$x_0 = 2e^t$$

$$y_0 = -2e^t$$

$$x(t) = 3ce^{3t} - c_1 e^{-t} + 2e^t$$

$$y(t) = ce^{3t} + c_1 e^{-t} - 2e^t$$

Итого:

$$\begin{cases} 5 = 3c - c_1 + 2 \\ -1 = c + c_1 - 2 \end{cases}$$

$$\begin{cases} 3c - c_1 = 3 \\ c + c_1 = 1 \end{cases}$$

$$c = 1, \quad c_1 = 0$$

В-го: $3e^{3t} + 2e^t$
 $e^{3t} - 2e^t$