Д/З

№ 4

(421) $x^2 y'' = y'^2$

Нехай $y' = z(x)$

$x^2 z' = z^2$

$$\int \frac{dz}{z^2} = \int \frac{dx}{x^2} + C_1, \quad z = \frac{x}{1 - C_1 x} = y'$$

$$y = \int \frac{x\,dx}{1 - C_1 x} + C_2 = \begin{cases} -\frac{1}{C_1} x - \frac{1}{C_1^2} \ln|C_1 x - 1| + C_2, & \text{якщо } C_1 \neq \\ \frac{x^2}{2} + C_2, & \text{якщо } C_1 = 0 \\ C_2, & \text{якщо } C_1 = \infty \end{cases}$$

(423) $y^3 y'' = 1$

$y' = p$

$y'' = p \cdot p'$, то

$y^3 \cdot p \cdot p' = 1$

$p\,dp = \frac{dy}{y^3}$

$\frac{p^2}{2} = -\frac{1}{2y^2} + C$

$p^2 = -\frac{1}{y^2} + C$

$p = \pm \left(C - \frac{1}{y^2}\right)^{1/2}$

$y' = \pm \left(C - \frac{1}{y^2}\right)^{1/2}$

$$\frac{dy}{\pm\left(C - \frac{1}{y^2}\right)^{1/2}} = dx$$

$$\frac{y\,dy}{\pm(cy^2-1)^{1/2}} = dx$$

$$\frac{1}{2c}\frac{d(y^2-c)}{(cy^2-1)^{1/2}} = dx$$

$$\pm\frac{1}{2c}\frac{(cy^2-1)^{1/2}}{1/2} = x + \hat{c}$$

$$(cy^2-1)^{1/2} = cx + \hat{c}.$$

(426) $yy'' + 1 = y'^2$

$y' = p, \quad y'' = pp'$

$ypp' + 1 = p^2$

$$yp\frac{dp}{dy} = p^2 - 1$$

$$\frac{p\,dp}{p^2-1} = \frac{dy}{y}$$

$$\int\frac{p\,dp}{p^2-1} = \frac{1}{2}\int\frac{d(p^2-1)}{p^2-1} = \frac{1}{2}\ln|p^2-1| + c$$

$$\frac{1}{2}\ln|p^2-1| = \ln Cy$$

$$\sqrt{p^2-1} = Cy$$

$$p = \pm\sqrt{Cy^2+1}$$

$$\frac{dy}{dx} = \pm\sqrt{cy^2+1}$$

$$\pm \frac{dy}{\sqrt{cy^2+1}} = dx$$

1) $c < 0 \Rightarrow c = -c_1^2$

$$\pm \int \frac{dy}{\sqrt{-c^2y^2+1}} = \pm \frac{1}{c} \arcsin cy$$

$$\pm \frac{1}{c} \arcsin cy = x + C_1$$

$$\underline{cy = \pm \sin(cy + C_2)}$$

2) $c = 0$

$$\pm dy = dx$$

$$\underline{y = x \pm C}$$

3) $c > 0 \Rightarrow c = c_1^2$

$$\pm \int \frac{dy}{\sqrt{c^2y^2+1}} = \pm \frac{1}{c} \operatorname{arcsh} \frac{y}{c}$$

$$\pm \frac{1}{c} \operatorname{arcsh} cy = x + C_1$$

$$\underline{cy = \pm \operatorname{sh}(cy + C_2)}$$

427 $y''(e^x+1) + y' = 0$

$$y' = z$$

$$z'(e^x+1) + z = 0$$

$$\frac{dz}{dx}(e^x+1) = -z$$

$$\frac{dz}{z} = -\frac{dx}{e^x+1}$$

$$-\int \frac{dx}{e^x+1} = -\int \frac{de^x}{e^x(e^x+1)} = \int \frac{de^x}{e^x+1} =$$

$$= -\int \frac{de^x}{e^x} = \ln(e^x+1) - x$$

$$\ln(e^x+1) - x = \ln Cz$$

$$z = C\,\frac{e^x+1}{e^x}$$

$$y' = C\,\frac{e^x+1}{e^x}$$

$$y = \int C\,\frac{e^x+1}{e^x}\,dx = C\int(1+e^{-x})\,dx =$$

$$= C(x - e^{-x}) + C_1$$

464 $yy'' = (y')^2 + 15y^2\sqrt{x}$

$$y' = yz$$

$$y'' = y'z + yz' = yz^2 + yz'$$

$$y^2z^2 + y^2z' = y^2z^2 + 15y^2\sqrt{x}$$

$$y^2z' = 15y^2\sqrt{x}$$

$$y^2z' = 15y^2\sqrt{x}$$

1) $y^2 = 0 \Rightarrow y = 0$

2) $z' = 15\sqrt{x}$

$z = \int 15\sqrt{x}\,dx = 10x^{1{,}5} + C$

$y' = y(10x^{1{,}5} + C)$

$\frac{dy}{dx} = y(10x^{1{,}5} + C)$

$\frac{dy}{y} = (10x^{1{,}5} + C)\,dx$

$\ln|y| = \int (10x^{1{,}5} + C)\,dx = 4x^{2{,}5} + Cx + C_1$

$\ln|yC_2| = 4x^{2{,}5} + Cx$

В-дь: $y = 0$, $\ln|yC_2| = 4x^{2{,}5} + Cx$

501) $yy'' = 2xy'^2$, $y(2) = 2$, $y'(2) = 0{,}5$

$y' = yz(x)$: $z' = z^2(2x - 1)$

Інтегруємо: $z = \frac{1}{C_1 + x - x^2} = \frac{y'}{y}$

$C_1 = 6$

$\int \frac{dx}{(x+2)(3-x)} = \ln|y| - \ln C_2$, звідки

$y = C_2\sqrt[5]{\frac{2+x}{3-x}}$. Підставимо $x = 2$ і $y = 2$:

$C_2 = \sqrt[5]{8}$

В-дь: $y = \sqrt[5]{8\frac{2+x}{3-x}}$