511 $y'' + y' - 2y = 0$

$\lambda^2 + \lambda - 2$ — вигляд р-ня

$\lambda_1 = 1, \quad \lambda_2 = -2$

$y = C_1 e^x + C_2 e^{-2x}$

512 $y'' + 4y' + 3y = 0$

$\lambda^2 + 4\lambda + 3 = 0$

$\lambda_1 = -1 \qquad \lambda_2 = -3$

$y = C_1 e^{-x} + C_2 e^{-3x}$

513 $y'' - 2y' = 0, \quad y(0) = 0, \quad y'(0) = 2$

$\lambda^2 - 2\lambda = 0$

$y = C_1 + C_2 e^{2x}$

$y' = 2 C_2 e^{2x}$

$C_1 + C_2 = 0$

$2 C_2 = 2$

$C_1 = -1, \qquad C_2 = 1$

$y = e^{2x} - 1$

(514) $2y'' - 5y' + 2y = 0$

$2\lambda^2 - 5\lambda + 2 = 0$

$D = 25 - 16 = 9$

$\lambda_{1,2} = \dfrac{5 \pm \sqrt{9}}{4} = \dfrac{5 \pm 3}{4} = \dfrac{1}{2}; 2$

$y = C_1 e^{1/2 x} + C_2 e^{2x}$

(515) $y'' + 4y' + 5y = 0$

$\lambda^2 - 4\lambda + 5 = 0$

$\lambda_{1,2} = 2 \pm \sqrt{4-5} = 2 \pm i$

$y = C_1 e^{(2+i)x} + C_2 e^{(2-i)x} = C_1 e^{2x} e^{ix} +$

$+ C_2 e^{2x} e^{-ix} =$

$= C_1 e^{2x}(\cos x + i \sin x) + C_2 e^{2x}(\cos x -$

$- i \sin x) = e^{2x}(C_1 \cos x + C_2 \sin x)$

B-gb: $y = e^{2x}(C_1 \cos x + C_2 \sin x)$

(516) $y'' + 2y' + 10y = 0$

$\lambda^2 + 2\lambda + 10 = 0$

$D = 4 - 40 = -36$

$\lambda_{1,2} = \dfrac{-2 \pm 6i}{2} = -1 \pm 3i$

$y = C_1 e^{-x} \cos 3x + C_2 e^{-x} \sin 3x = e^{-x}(C_1 \cos 3x + C_2 \sin 3x)$

523 $4y'' + 4y' + y = 0$

$4\lambda^2 + 4\lambda + 1 = 0$

$(2\lambda + 1)^2 = 0$

$\lambda = -\frac{1}{2} \quad (2)$

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$y = C_1 e^{-\frac{1}{2}x} + C_2 x e^{-\frac{1}{2}x}$

524 $y^V - 6y^{IV} + 9y^{III} = 0$

$\lambda^5 - 6\lambda^4 + 9\lambda^3 = 0$

$\lambda^3(\lambda^2 - 6\lambda + 9) = 0$

$\lambda_{1,2,3} = 0, \quad \lambda_{4,5} = 3$

$y = C_1 + C_2 x + C_3 x^2 + C_4 e^{3x} + C_5 x e^{3x}$

525 $y^V - 10y^{III} + 9y' = 0$

$\lambda^5 - 10\lambda^3 + 9\lambda = 0$

$\lambda(\lambda^4 - 10\lambda^2 + 9) = 0$

$\lambda_1 = 0, \quad \lambda_{2,3} = \pm 1$

$\lambda_{3,4} = \pm 3$

$y(x) = C_1 + C_2 e^x + C_3 e^{-x} + C_4 e^{3x} + C_5 e^{-3x}$

(526) $y^{IV} + 2y'' + y = 0$

$\lambda^4 + 2\lambda^2 + 1 \equiv \cdot (\lambda^2 + 1)^2 = 0$, $\lambda_1 = \lambda_2 = i$

$y_1 = e^{ix}$, $y_2 = xe^{ix}$, $y_3 = e^{-ix}$, $y_4 = xe^{-ix}$

$y = C_1 e^{ix} + C_2 e^{-ix} + x(C_3 e^{ix} + C_4 e^{-ix})$

$y = C_1 \cos x + C_2 \sin x + x(C_3 \cos x + C_4 \sin x)$

(533) $y'' - 2y' - 3y = e^{4x}$, $y(0) = 1$, $y'(0) = 0$

$\lambda^2 - 2\lambda - 3 = 0$

$y = C_1 e^{-x} + C_2 e^{3x}$

$\tilde{y} = Q_0(x) e^{4x} \equiv a_0 e^{4x}$

$5 a_0 e^{4x} = e^{4x}$

$y = C_1 e^{-x} + C_2 e^{3x} + \frac{1}{5} e^{4x}$

$y = C_1 e^{-x} + C_2 e^{3x} + \frac{1}{5} e^{4x}$

$C_1 + C_2 + \frac{1}{5} = 1$, $\quad -C_1 + 3C_2 + \frac{4}{5} = 0$

$y = \frac{4}{5} e^{-x} + \frac{1}{5} e^{4x}$

(534) $y'' + y = 4xe^x$

$\lambda^2 + 1 = 0$

$\lambda = \pm i$

$y = C_1 \cos x + C_2 \sin x$

$y = (ax + b) e^x$

$y' = ae^x + (ax + b)e^x = e^x(ax + b + a)$

$y'' = ae^x + e^x(ax + a + b) = e^x(ax + 2a + b)$

$y'' + y' = 4xe^x$

$e^x(2ax + 2a + 2b) = 4e^x x$

$2ax + 2a + 2b = 4x$

$\begin{cases} 2a = 4 \\ 2a + 2b = 0 \end{cases}$ $\begin{aligned} a &= 2 \\ b &= -2 \end{aligned}$

$y = C_1 \cos x + C_2 \sin x + 2(x-1)e^x$

(535) $y'' - y = 2e^x - x^2$

$y = C_1 e^x + C_2 e^{-x}$

$y'' - y = 2e^x,$ $y'' - y = -x^2$

$\tilde{y}_1 = a_0 x e^x,$ $\tilde{y}_2 = b_0 x^2 + b_1 x + b_2$

$2a_0 e^x = 2e^x,$ $2b_0 - b_0 x^2 - b_1 x - b_2 = -x^2$

$a_0 = 1,$ $b_0 = 1,$ $-b_1 = 0,$ $b_2 = 2$

$$y^* = \bar{y}_1 + \bar{y}_2 = xe^x + x^2 + 2$$

$$y = C_1 e^x + C_2 e^{-x} + xe^x + x^2 + 2$$

536 $\quad y'' + y' - 2y = 3xe^x$

$$\lambda^2 + \lambda - 2 = 0$$

$$\lambda_1 = -2 \qquad \lambda_2 = 1$$

$$y_0 = C_1 e^{-2x} + C_2 e^x$$

$$y_z = e^x A x + x e^x B x$$

$$y' = e^x A x + e^x A + 2x e^x B + e^x x^2 B$$

$$y'' = e^x A x + e^x A + e^x A + 2 e^x B + 2x e^x B +$$

$$+ 2 x e^x B + e^x x^2 B = e^x A x + 2 e^x A +$$

$$+ 2 e^x B + 4 x e^x B + e^x x^2 B, \quad \text{то}$$

$$3A + 2B + 4xB + 2xB = 3xe$$

$$6Bx + 3A + 2B = 3x$$

$$\begin{cases} 6B = 3 \\ 3A + 2B = 0 \end{cases} \qquad \begin{aligned} B &= \frac{1}{2} \\ A &= -\frac{1}{3} \end{aligned}$$

$$y = e^x \left(\frac{x^2}{2} - \frac{x}{2} \right) + C_1 e^{-2x} + C_2 e^x.$$

537) $y'' - 3y' + 2y = 0$

$\lambda^2 - 3\lambda + 2 = 0$

$\lambda_1 = 1 \qquad \lambda_2 = 2$

$y = C_1 e^x + C_2 e^{2x}$

$y = a\cos x + b\sin x$

$y' = -a\sin x + b\cos x$

$y'' = -a\cos x - b\sin x$

$-a\cos x - b\sin x + 3a\sin x - 3b\cos x +$

$+ 2a\cos x + 2b\sin x = \sin x$

$\begin{cases} a - 3b = 0 \\ b + 3a = 1 \end{cases}$

$b = 0,1$

$a = 0,3$

$y = C_1 e^x + C_2 e^{2x} + 0,3\cos x + 0,1\sin x$

582) $y'' - 2y' + y = 0 \qquad , \quad y(2) = 1, \quad y'(2) = -2$

$\lambda^2 - 2\lambda + 1 = 0$

$(\lambda - 1)^2 = 0$

$\lambda_{1,2} = 1$

$y = (C_1 + C_2 x)e^x = C_1 e^x + x e^x C_2$

$$y' = C_1 e^x + C_1 e^x + x e^x C_2$$

$$1 = C_1 e^2 + 2 e^2 C_2$$

$$-2 = C_1 e^2 + C_2 e^2 + 2 e^2 C_2$$

$$3 = 2 e^2 C_2 - C_2 e^2 - 2 e^2 C_2 = - C_2 e^2$$

$$C_2 = -3 e^{-2}$$

$$C_1 e^2 = 1 + 2 e^2 3 e^{-2} = 1 + 6 = 7$$

$$C_1 = 7 e^{-2}$$

$$y = e^{-2} (7 - 3x) e^x = (7 - 3x) e^{x-2}$$

(583) $y'' + y = 4 e^x$

$$\lambda = \pm i$$

$$y = C_1 \cos x + C_2 \sin x$$

$$y_c = 2 e^x$$

$$y(\lambda) = C_1 \cos x + C_2 \sin x + 2 e^x$$

$$\begin{cases} C_1 + 2 = 4 \\ C_2 + 2 = -3 \end{cases} \Rightarrow \begin{aligned} C_1 &= 2 \\ C_2 &= -5 \end{aligned}$$

$$y = 2 \cos x - 5 \sin x + 2 e^x$$