У3
№ 7

1148 $\frac{dx}{y+z} = \frac{dy}{x+z} = \frac{dz}{x+y}$

1) $\frac{dx-dy}{y+z-x-z} = \frac{dy-dz}{x+z-x-y}$

$\frac{d(x-y)}{x-y} = \frac{d(y-z)}{y-z}$

$\ln|x-y| = \ln|y-z| + \ln C_1$

$x-y = C_1(y-z)$

2) $\frac{dx+dy+dz}{2(x+y+z)} = \frac{-dx-dy}{(x-y)}$

$\frac{d(x+y+z)}{2(x+y+z)} = -\frac{d(x-y)}{x-y}$

$\frac{1}{2}\ln|x+y+z| = -\ln|x-y| + \ln C_2$

$(x+y+z)(x-y)^2 = C_2$

В-дь: $x-y = C_1(y-z)$

$(x+y+z)(x-y)^2 = C_2$

1152 $\frac{dx}{z} = \frac{dy}{xz} = \frac{dz}{y}$

$$\frac{dx}{1} = \frac{dy}{x}$$

$$\frac{x^2}{2} = y + C_1$$

$$x^2 - 2y = C_1$$

$$y = \frac{x^2 - C_1}{2}, \quad \text{то}$$

$$\frac{dx}{z} = \frac{2\,dz}{x^2 - C_1}$$

$$(x^2 - C_1)\,dx = 2z\,dz$$

$$\frac{x^3}{3} - C_1 x = z^2 + C_2$$

$$\frac{x^3}{3} - (x^2 - 2y)\,x - z^2 = C_2,$$

$$-\frac{2}{3}x^2 + 2xy - z^2 = C_2$$

$$-2x^3 + 6xy - 3z^2 = C_2.$$

В-дь: $x^2 - 2y = C_1$

$$6xy - 2x^3 - 3z^2 = C_2$$

1155 $\frac{dx}{xz} = \frac{dy}{yz} = \frac{dz}{xy\sqrt{z^2+1}}$

1) $\frac{dx}{xz} = \frac{dy}{yz} \Rightarrow x = C_1 y$

$\frac{dy}{yz} = \frac{dz}{C_1 y^2\sqrt{z^2+1}}$ $\quad C_1 y\,dy = \frac{z\,dz}{\sqrt{z^2+1}}$

$C_1 \cdot \frac{y^2}{2} = \frac{1}{2} \cdot \sqrt{z^2+1} \cdot 2 + C_2$

$\frac{C_1}{2} y^2 = \sqrt{z^2+1} + C_2$

$\begin{cases} xy = 2\sqrt{z^2+1} + 2C_2 \\ x = C_1 y \end{cases}$ $\quad \begin{cases} \frac{x}{y} = C_1 \\ \frac{xy}{2} - \sqrt{z^2+1} = C_2 \end{cases}$

1156 $\frac{dx}{x+y^2+z^2} = \frac{dy}{y} = \frac{dz}{z}$

$\frac{dy}{y} = \frac{dz}{z} \Rightarrow C_1 = \frac{y}{z}, \quad y = C_1 z$

$\frac{dx}{x+(C_1^2+1)z^2} = \frac{dz}{z}, \quad z\,dx = x + (C_1^2+1)z^2\,dz$

$\frac{dx}{dz} = \frac{x+(C_1^2+1)z^2}{z} = \frac{x}{z} + (C_1^2+1)z$

$x = (C_1^2+1)z^2 + C_2 z = \left(\frac{y^2}{z^2}+1\right)z^2 + C_2 z =$

$= y^2 + z^2 + C_2 z$

$$C_2 = \frac{x}{z} - \frac{y^2}{z} - z$$

(1158) $$-\frac{dx}{x^2} = \frac{dy}{xy - 2z^2} = \frac{dz}{xz}$$

$$\begin{cases} -\dfrac{dx}{x^2} = \dfrac{dz}{xz} & (1) \\ -\dfrac{dx}{x^2} = \dfrac{dy}{xy - 2z^2} & (2) \end{cases}$$

$$-\frac{dx}{x^2} = \frac{dy}{xy - 2z^2}$$

$$-xy\,dx + 2z^2\,dx = x^2\,dy$$

$$x^2 dy + xy\,dx = 2z^2\,dx$$

$$x(x\,dy + y\,dx) = 2z^2\,dx$$

$\{$ ф-ла $d(xy) = x\,dy + y\,dx$

$$x\,d(xy) = 2z^2\,dx$$

$$\begin{cases} x\,d(xy) = 2z^2\,dx & (2) \\ z\,dx = -x\,dz & (1) \end{cases}$$

$$\begin{cases} d(xy) = 2z^2\dfrac{dx}{x} \\ \dfrac{dx}{x} = -\dfrac{dz}{z} \end{cases}$$

$$\begin{cases} d(xy) = 2z^2\left(-\dfrac{dz}{z}\right) = -2z\,dz \\ dx/x = -dz/z \end{cases}$$

$$\begin{cases} \int dxy = -2\int z\,dz \\ \int dx/x + \int dz/z = 0 \end{cases}$$

$xy + z^2 = C_3$

$\ln|x| + \ln|z| = \ln C_2$

$$\begin{cases} xy + z^2 = C_3 \\ xz = C_2 \end{cases}$$

1162 $\dot{x} = xy, \quad \dot{y} = x^2 + y^2$;

$\varphi_1 = x \ln y - x^2 y, \quad \varphi_2 = \frac{y^2}{x^2} - 2\ln x$

Оскільки $\varphi_1 = C_1$, то $\frac{d}{dt}(\varphi_1) \equiv 0$

Отже, $\frac{d}{dt}(x \ln y - x^2 y) \equiv 0$, або

$$\dot{x} \ln y + \frac{x\dot{y}}{y} - 2x\dot{x}y - x^2\dot{y} \equiv 0$$

Підставивши, маємо:

$$xy \ln y + \frac{x}{y}(x^2 + y^2) - 3x^2y^2 - x^4 \equiv 0$$

Як видно, це неможливо, тому $\varphi_1 = C_1$ не є інтегралом цього р-ня

$$\frac{d}{dt}(\varphi_2) \equiv 0, \text{ або}$$

$$\frac{d}{dt}\left(\frac{y^2}{x^2} - 2\ln x\right) = \frac{2y\dot{y}x^2 - 2x\dot{x}y^2}{x^4} - \frac{2\dot{x}}{x} \equiv 0$$

$xy(x^2+y^2)-xy^3-x^3y\equiv 0,\quad x\neq 0$

Отже, $\psi_2=C_2$, а це означає, що наш вираз є першим інтегралом даної системи