

Off-policy learning & importance sampling

$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

what if we don't have samples from $p_{\theta}(\tau)$?

(we have samples from some $\bar{p}(\tau)$ instead)

$$J(\theta) = E_{\tau \sim \bar{p}(\tau)} \left[\frac{p_{\theta}(\tau)}{\bar{p}(\tau)} r(\tau) \right]$$

$$p_{\theta}(\tau) = p(\mathbf{s}_1) \prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)$$

$$\frac{p_{\theta}(\tau)}{\bar{p}(\tau)} = \frac{\cancel{p(\mathbf{s}_1)} \prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t) \cancel{p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)}}{\cancel{p(\mathbf{s}_1)} \prod_{t=1}^T \bar{\pi}(\mathbf{a}_t | \mathbf{s}_t) \cancel{p(\mathbf{s}_{t+1} | \mathbf{s}_t, \mathbf{a}_t)}} = \frac{\prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^T \bar{\pi}(\mathbf{a}_t | \mathbf{s}_t)}$$

importance sampling

$$\begin{aligned} E_{x \sim p(x)}[f(x)] &= \int p(x) f(x) dx \\ &= \int \frac{q(x)}{q(x)} p(x) f(x) dx \\ &= \int q(x) \frac{p(x)}{q(x)} f(x) dx \\ &= E_{x \sim q(x)} \left[\frac{p(x)}{q(x)} f(x) \right] \end{aligned}$$

Deriving the policy gradient with IS

$$\theta^* = \arg \max_{\theta} J(\theta)$$

$$J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

a convenient identity

$$p_{\theta}(\tau) \nabla_{\theta} \log p_{\theta}(\tau) = \nabla_{\theta} p_{\theta}(\tau)$$

can we estimate the value of some *new* parameters θ' ?

$$J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[\frac{p_{\theta'}(\tau)}{p_{\theta}(\tau)} r(\tau) \right]$$

the only bit that depends on θ'

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[\frac{\nabla_{\theta'} p_{\theta'}(\tau)}{p_{\theta}(\tau)} r(\tau) \right] = E_{\tau \sim p_{\theta}(\tau)} \left[\frac{\cancel{p_{\theta'}(\tau)}}{\cancel{p_{\theta}(\tau)}} \nabla_{\theta'} \log p_{\theta'}(\tau) r(\tau) \right]$$

now estimate locally, at $\theta = \theta'$: $\nabla_{\theta} J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[\nabla_{\theta} \log p_{\theta}(\tau) r(\tau)]$

The off-policy policy gradient

$$\theta^* = \arg \max_{\theta} J(\theta) \quad J(\theta) = E_{\tau \sim p_{\theta}(\tau)}[r(\tau)]$$

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[\frac{p_{\theta'}(\tau)}{p_{\theta}(\tau)} \nabla_{\theta'} \log \pi_{\theta'}(\tau) r(\tau) \right] \quad \text{when } \theta \neq \theta'$$

$$\frac{p_{\theta'}(\tau)}{p_{\theta}(\tau)} = \frac{\prod_{t=1}^T \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\prod_{t=1}^T \pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)}$$

$$= E_{\tau \sim p_{\theta}(\tau)} \left[\left(\prod_{t=1}^T \frac{\pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t)}{\pi_{\theta}(\mathbf{a}_t | \mathbf{s}_t)} \right) \left(\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \right) \left(\sum_{t=1}^T r(\mathbf{s}_t, \mathbf{a}_t) \right) \right] \text{ what about causality?}$$

$$= E_{\tau \sim p_{\theta}(\tau)} \left[\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \underbrace{\left(\prod_{t'=1}^t \frac{\pi_{\theta'}(\mathbf{a}_{t'} | \mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'} | \mathbf{s}_{t'})} \right)}_{\text{future actions don't affect current weight}} \left(\sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \left(\prod_{t''=t}^{t'} \frac{\pi_{\theta'}(\mathbf{a}_{t''} | \mathbf{s}_{t''})}{\pi_{\theta}(\mathbf{a}_{t''} | \mathbf{s}_{t''})} \right) \right) \right]$$

future actions don't affect current weight

if we ignore this, we get
a policy iteration algorithm
(more on this in a later lecture)

A first-order approximation for IS (preview)

$$\nabla_{\theta'} J(\theta') = E_{\tau \sim p_{\theta}(\tau)} \left[\sum_{t=1}^T \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_t | \mathbf{s}_t) \underbrace{\left(\prod_{t'=1}^t \frac{\pi_{\theta'}(\mathbf{a}_{t'} | \mathbf{s}_{t'})}{\pi_{\theta}(\mathbf{a}_{t'} | \mathbf{s}_{t'})} \right)}_{\text{exponential in } T} \left(\sum_{t'=t}^T r(\mathbf{s}_{t'}, \mathbf{a}_{t'}) \right) \right]$$

let's write the objective a bit differently...

on-policy policy gradient: $\nabla_{\theta} J(\theta) \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \nabla_{\theta} \log \pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$

$(\mathbf{s}_{i,t}, \mathbf{a}_{i,t}) \sim \pi_{\theta}(\mathbf{s}_t, \mathbf{a}_t)$

off-policy policy gradient: $\nabla_{\theta'} J(\theta') \approx \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \frac{\pi_{\theta'}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})}{\pi_{\theta}(\mathbf{s}_{i,t}, \mathbf{a}_{i,t})} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$

We'll see why this is reasonable later in the course!

$$= \frac{1}{N} \sum_{i=1}^N \sum_{t=1}^T \frac{\cancel{\pi_{\theta'}(\mathbf{s}_{i,t})}}{\cancel{\pi_{\theta}(\mathbf{s}_{i,t})}} \frac{\pi_{\theta'}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})}{\pi_{\theta}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t})} \nabla_{\theta'} \log \pi_{\theta'}(\mathbf{a}_{i,t} | \mathbf{s}_{i,t}) \hat{Q}_{i,t}$$

ignore this part