Precise 2D Point Set Registration Using Iterative Closest Algorithm and Correntropy

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***Abstract*—The iterative closest point (ICP) algorithm is fast and accurate for rigid point set registration, but it works badly when there are many outliers and noises in the point sets. This paper instead proposes a novel method based on the ICP algorithm to deal with this problem. Firstly, correntropy is introduced into the rigid registration problem and then a new energy function based on maximum correntropy criterion is proposed. After that, a new ICP algorithm based on correntropy is proposed, which performs well in dealing with rigid registration with noises and outliers. This new algorithm converges moronically from any given parameters, which is similar to the ICP algorithm. Experimental results demonstrate its accuracy and efficiency compared with the traditional ICP algorithm.**

Keywords—Iterative closest point (ICP); correntropy; outliers; noises; rigid registration.

I. Introduction

As a key technology of image registration, point set registration is one of the most important research issues in image analysis and computer vision, such as building correspondence in medical image [1]. The iteration closest point (ICP) algorithm [2-4] is known to be employed to solve this problem because of its accuracy and efficiency.

Until now, more and more related work has been done for the speed and robustness of the ICP algorithm. Fitzgibbon [5] employed the Levenberg-Marquardt algorithm to speed up ICP, and Sharp et al. [6] introduced invariant features to avoid the ICP algorithm form being trapped into local minimum. In addition, many researchers have studied point set registration with the outliers and noises. Chetverikov et al. [7] proposed the trimmed ICP (TrICP) algorithm for partially overlapping registration. Du et al. [8] proposed probability ICP for registration of point sets with noises.

However, the above mentioned work is based on Euclidean distance, which depends on the parameters greatly and cannot process the outliers and noises at the same time. It is well known that correntropy [9-10] can effectively deal with the outliers and noises, which is not sensitive to the parameters, so we propose a mathematical model based on correntropy. Therefore, we propose a new registration function based on maximum correntropy criterion (MCC), which is solved by a novel ICP algorithm. This algorithm is proved to be efficient and accurate in the experiments.

The rest of this paper is organized as follows. In section 2, a brief introduction about ICP algorithm is stated. In section 3, an optimization model based on correntropy is proposed and then the proposed method – the ICP algorithm based on MCC is given. In section 4, experimental results are shown to demonstrate the accuracy and efficiency of our algorithm. In the last section, a conclusion is drawn finally.

II. The ICP algorithm

In this part, we will take a brief review of the ICP algorithm. Given two point sets in $\mathbb{R}^n$, a data set $X \triangleq \{\vec{x}_i\}_{i=1}^{N_x} (N_x \in \mathbb{N})$ and a model set $Y \triangleq \{\vec{y}_i\}_{i=1}^{N_y} (N_y \in \mathbb{N})$. The goal of a rigid registration is to find a rotation matrix $\mathbf{R}$ and a translation vector $\vec{t}$, with which the data set X is in the best alignment with the model set Y. The least square (LS) problem based on Euclidean distance is formulated as follows:

$$\min_{\mathbf{R},\vec{t},j\in\{1,2,\ldots,N_y\}} \sum_{i=1}^{N_x} \| (\mathbf{R}\vec{x}_i + \vec{t}) - \vec{y}_j \|_2^2 \quad s.t. \quad \mathbf{R}^T\mathbf{R} = \mathbf{I}_m,\ \det(\mathbf{R}) = 1. \tag{1}$$

To solve this problem, the ICP algorithm is proposed, which iteratively calculates $\mathbf{R}$ and $\vec{t}$. In the kth iteration, there are mainly two steps.

Firstly, set up correspondence between two point sets:

$$c_k(i) = \arg\min_{j\in\{1,2,\ldots,N_y\}} (\|(\mathbf{R}_{k-1}\vec{x}_i + \vec{t}_{k-1}) - \vec{m}_j\|_2^2) \text{ for } i = 1,\ldots,N_x \tag{2}$$

Secondly, compute the new rotation matrix and translation vector by minimizing squared distance:

$$(\mathbf{R}_k, \vec{t}_k) = \arg\min_{\mathbf{R}^T\mathbf{R}=\mathbf{I}_m, \det(\mathbf{R})=1, \vec{t}} (\sum_{i=1}^{N_x} \| \mathbf{R}\vec{x}_i + \vec{t} - \vec{y}_{c_k(i)} \|_2^2) \tag{3}$$

The ICP algorithm runs iteratively until the registration error is small enough or k reaches the maximum number of iterations.

III. Our Algorithm

A. Problem Statement

Assume there are two point sets in $\mathbb{R}^n$, a data set $X \triangleq \{\vec{x}_i\}_{i=1}^{N_x} (N_x \in \mathbb{N})$ and a model set $Y \triangleq \{\vec{y}_i\}_{i=1}^{N_y} (N_y \in \mathbb{N})$. The goal of rigid registration is to find a rotation matrix $\mathbf{R}$ and a translation vector $\vec{t}$, with which the data set X can be in the best alignment with model set Y. Different from the problem stated above, there are lots of noises and outliers in the data set X, in which case the traditional ICP algorithm may suffer a large probability to fail.

It is known that the traditional ICP algorithm performs accurately and efficiently in handling rigid registration, but it could not deal with point set registration with the noises and outliers well. As the correntropy has good property of outlier rejection, which will be incorporated to the point set registration.

The correntropy between two data sets X and Y can be defined as follows:

$$v(\mathrm{X},\mathrm{Y}) = \sum_{i=1}^{N_x} e^{-\|\vec{x}_i - \vec{y}_{c(i)}\|_2^2/(2\sigma^2)}, \tag{4}$$

where $\vec{y}_{c(i)}$ matches $\vec{x}_i$, and σ is a free parameter.

Since the data X will undergo a rigid transformation to align with data set Y, which means there is a transformation function $T(\vec{x}_i) = \mathbf{R}\vec{x}_i + \vec{t}$ applied on X, so (4) can be rewritten as:

$$v(\mathrm{X},\mathrm{Y}) = \sum_{i=1}^{N_x} e^{-\|(\mathrm{R}\vec{x}_i + \vec{t}) - \vec{y}_{c(i)}\|_2^2/(2\sigma^2)} \tag{5}$$

Let $w(i) = \| \mathrm{R}\vec{x}_i + \vec{t} - \vec{y}_{(i)} \|_2^2 /(2\sigma^2)$, so (5) can be simplified as $v(\mathrm{X},\mathrm{Y}) = \sum_{i=1}^{N_x} e^{-w(i)}$.

In (5), if the data set X can align with the model set Y perfectly, which means there is a small Euclidean distance between $\vec{x}_i$ and $\vec{y}_{c(i)}$, the $w(i)$ will be very small with a limitation of 0 and $\mathrm{e}^{-w(i)}$ will be large but no more than 1. If the registration has not been completed, the $w(i)$ will be much larger and $\mathrm{e}^{-w(i)}$ will approximately equal to 0. Taking these properties into consideration, the correntropy is qualified to play a role as the energy function in rigid point registration. On the other hand, suppose there are some outliers or noises x_j in data set X, and there are no specific points in data set Y which can align with those noises, so the $w(j)$ will be larger than those that can be correctly registered, so the correntropy is smaller as well. This character seems that in the process of point registration, the noises and outliers will be assigned to lower weights while the points in order will be assigned to higher weights. It means that the impact caused by noises and outliers can be decreased or even be ignored and ideal points can lead the registration to a good result.

Therefore, the new energy function based on the correntropy between the correspondence is shown as follows:

$$E_\sigma(x_i, y_i) = e^{-\|(\mathbf{R}\vec{x}_i + \vec{t}) - \vec{y}_{c(i)}\|_2^2/(2\sigma^2)},$$

where $\mathbf{R}$ is the rotation matrix, $\vec{t}$ is the translation vector, $\vec{y}_{c(i)}$ denotes the point in the model set Y correspond with the point $\vec{x}_i$ in the data set X and σ is a free parameter.

Taking the properties of negative exponential function into consideration, to minimize the error of registration, the objective function should maximize the correntropy, which is shown as follows:

$$\begin{aligned} &\max_{\mathbf{R},\vec{t},j\in\{1,2,\ldots,N_y\}} \sum_{i=1}^{N_x} e^{-\|\mathbf{R}\vec{x}_i + \vec{t} - \vec{y}_{c(i)}\|_2^2/(2\sigma)} \\ &s.t.\quad \mathbf{R}^T\mathbf{R} = \mathbf{I}_m, \det(\mathbf{R}) = 1. \end{aligned} \tag{6}$$

B. The New ICP Algorithm Based on Correntropy

As the objective function (6) is similar to the function (1), we can use the similar algorithm to the ICP algorithm, which includes two steps in each iteration.

Step 1. Assume the (k-1)th rigid transformation is known, we can set up the kth correspondence between two point sets:

$$c_k(i) = \underset{j\in\{1,2,\ldots,N_y\}}{\arg\min} (\|(\mathbf{R}_{k-1}\vec{x}_i + \vec{t}_{k-1}) - \vec{m}_j\|_2^2) \quad \text{for } i = 1,\ldots,N_x$$

Step 2. Compute the kth rigid transformation according to the kth correspondence between two point sets.

The goal of our algorithm is to find the rotation matrix $\mathbf{R}$, translation vector $\vec{t}$ and the correspondence function $c(i)$, which is shown as follows:

$$(\mathbf{R},\vec{t}) = \underset{\substack{\mathbf{R}^T\mathbf{R}=\mathbf{I}_m \\ \det(\mathbf{R})=1,\vec{t}}}{\arg\max} \sum_{i=1}^{N_x} e^{-\|\mathbf{R}\vec{x}_i + \vec{t} - \vec{y}_{c(i)}\|_2^2/(2\sigma^2)} \tag{7}$$

The algorithm repeats until the registration error is small enough or k reaches the maximum number of iterations.

Step 1 of our algorithm can be solved by many methods such as k-d tree [11] or the nearest point search based on Delaunay tessellation [12]. Therefore, Step 2 is the key step for this new algorithm, which will be solved in the next section.

This new ICP algorithm based on correntropy works similar to the traditional ICP algorithm, which means that our

algorithm can converge monotonically to a local maximum from any given initial values.

C. Solve the Rigid Transformation

In this section, we will compute the rigid transformation $\mathbf{R}$ and $\vec{t}$ of Step 2 of our algorithm, and then give a closed-form solution.

Taking the derivative of (7) with respect to $\vec{t}$, we can get:

$$F(t)=\frac{\partial\sum_{i=1}^{N_x}e^{-w(i)}}{\partial\vec{t}}=\sum_{i=1}^{N_x}\left(-\frac{R\vec{x}_i+\vec{t}-\vec{y}_{c_k(i)}}{\sigma^2}e^{-\frac{\|\mathbf{R}\vec{x}_i+\vec{t}-\vec{y}_{c_k(i)}\|_2^2}{2\sigma^2}}\right) \quad (8)$$

To get an approximate result of $\vec{t}_k$, we fix the exponential term in (8). Hence, we substitute $\mathbf{R}_{k-1}$ into $\mathbf{R}$ and $\vec{t}_{k-1}$ into $\vec{t}$ in exponential term, which is expressed as follows:

$$p_k(i)=e^{-\frac{\|\mathbf{R}_{k-1}\vec{x}_i+\vec{t}_{k-1}-\vec{y}_{c_k(i)}\|_2^2}{2\sigma^2}}$$

Let $F(\vec{t})=0$, we can get:

$$\vec{t}_k=\left(\sum_{i=1}^{N_x}(\vec{y}_{c_k(i)}-\mathbf{R}\vec{x}_i)p_k(i)\right)/\sum_{i=1}^{N}p_k(i) \quad (9)$$

Therefore, to obtain the approximate results of $\vec{t}_k$, we need to compute the rotation $\mathbf{R}_k$.

Substituting $\vec{t}$ in the energy function (7) with (9), we can obtain the following objective function:

$$\mathbf{R}_k=\underset{\mathbf{R}^T\mathbf{R}=\mathbf{I}_m,\det(\mathbf{R})=1}{\arg\max}\sum_{i=1}^{N_x}e^{-\frac{\|\mathbf{R}\vec{q}_i-\vec{m}_i\|_2^2}{2\sigma^2}}, \quad (10)$$

where $\vec{q}_i=\vec{x}_i-\left(\sum_{i=1}^{N_x}p_k(i)\vec{x}_i\right)/\sum_{i=1}^{N_x}p_k(i)$ and $\vec{m}_i=\vec{y}_{c_k(i)}-\left(\sum_{i=1}^{N}p_k(i)\vec{y}_{c_k(i)}\right)/\sum_{i=1}^{N}p_k(i)$.

Consider $\mathbf{Q}$ and $\mathbf{M}$ as $N_x\times n$ matrices where each row represents a point $\vec{q}_i$ and $\vec{m}_i$. Then the energy function (10) can be rewritten as:

$$H(\mathbf{R})=\mathbf{1}_N^T G(\mathbf{M}-\mathbf{QR}^T), \quad (11)$$

where $G(\mathbf{M}-\mathbf{QR}^T)=\vec{g}$ is a column vector in which each row $G_i=\mathrm{e}^{-\|\mathbf{R}\vec{q}_i-\vec{m}_i\|_2^2/(2\sigma^2)}$.

Assume $D(\vec{g})=diag(\vec{g})$. According to the SVD algorithm, we can obtain $-\mathbf{Q}^T D(\vec{g})\mathbf{M}/\sigma^2=\mathbf{USV}^T$, where $\mathbf{U}$ and $\mathbf{V}$ are orthogonal matrices and $\mathbf{S}$ is a diagonal matrix. If we maximize (11), we can obtain the kth rotation matrix:

$$\mathbf{R}_k=\mathbf{VD}^{-1}\mathbf{U}^T, \quad (12)$$

where $\mathbf{D}=diag(d_i)$ and $d_i=1$ or -1.

IV. Experimental Results

To check the convergence and robustness of our method, experiments are done based on part B of CE-Shape-1 [13]. In addition, our algorithm is compared with the traditional ICP algorithm, and they are implemented by Matlab 2015a and run on PC with Intel Core 2.3GHz CPU and 8G RAM.

A. 2D Simulations

In the following part, we use 2D images for simulation experiments. The bat, bird and butterfly images are selected as the model data. To demonstrate that our algorithm can deal with outliers and noises well, we conduct three different experiments for various cases. Firstly, part of the wings of the bat image are cut. Secondly, an eye and the details on the tail are added in a bird image. Finally, part of the wings are cut out and 20% random noises are added to the butterfly image, which can be thought as salt and pepper noises. After that, a random rigid transformation is applied to the above different images with rotation matrix $\mathbf{R}_0$ and translation vector $\vec{t}_0$, so we can obtain the data sets. The compared results are shown as Fig. 1.

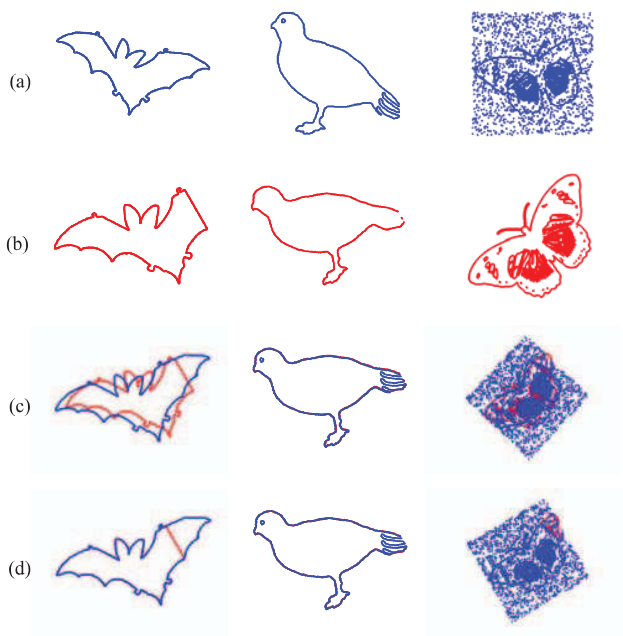

Fig. 1. Compared results of our algorithm and the ICP algorithm. (a) The data sets. (b) The model sets. (c) The traditional ICP algorithm. (d) Our algorithm.

In Fig. 1, it's easy to find that both our algorithm and traditional ICP algorithm can finish the rigid registration for the first two experiments, while our algorithm gets better results. In the third experiment, the new algorithm finishes the registration with a satisfying result while the traditional ICP algorithm fails.

To compare these two algorithms, we define $\varepsilon_{\mathbf{R}} = \|\mathbf{R}-\mathbf{R}_0\|_2$ and $\varepsilon_{\vec{t}} = \|\vec{t}-\vec{t}_0\|_2$. All of the results are displayed in the Table 1.

From Table 1, we can see that whatever condition changes, our algorithm gets smaller errors after the registration, which demonstrates the robustness of our algorithm in rigid registration with outliers and noises.

B. Experiments with 2D Real Images

In this section, three couples of real images which consist of the apple, heart and dog are used for experiments. There are outliers and noises in these three couples of images more or less. We compare our algorithm with the traditional ICP algorithm, and the compared results are shown in Fig. 2.

TABLE I. THE COMPARISON BETWEEN THE EXPERIMENTAL RESULTS AND THE GROUND TRUTH

Algorithm	The ICP Algorithm		Our Algorithm	
	$\varepsilon_{\mathbf{R}}$	$\varepsilon_{\vec{t}}$	$\varepsilon_{\mathbf{R}}$	$\varepsilon_{\vec{t}}$
bat	0.0154	53.7814	8.7872e-05	0.0596
bird	0.0067	1.2710	7.5771e-04	0.0351
butterfly	0.2100	115.9880	2.2361e-04	0.0719

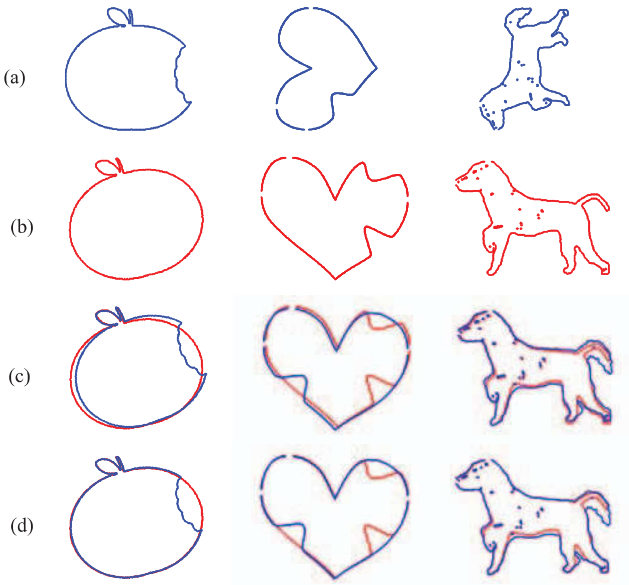

Fig. 2. Compared results of our algorithm and the ICP algorithm. (a) The data sets. (b) The model sets. (c) The traditional ICP algorithm. (d) Our algorithm.

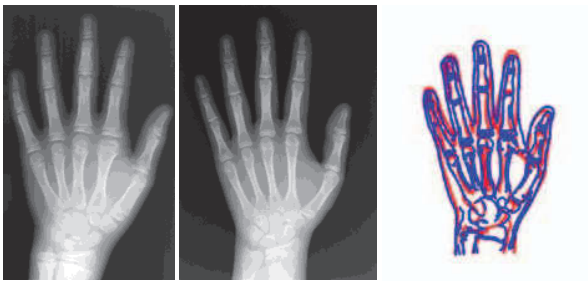

Fig. 3. The result of X-ray hand image registration

From Fig. 2, we can see that traditional ICP algorithm could not complete all point set registration, as it fails to register the apple and heart data sets . However, on the other hand, our algorithm works well in dealing with all these three data sets, which demonstrates the robustness of our algorithm again.

C. X-ray hand image registration

To demonstrate that our algorithm could deal with medical image registration, X-ray images are applied. In the following part, an X-ray hand image registration is carried out to prove the robustness of our algorithm. The result is shown as Fig. 3. The result shows that our algorithm works well on matching these two similar X-ray hand images.

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