Chapter 10

Frequency Response Techniques

Outline

1 Bode diagram

2 Nyquist diagram

3 Relation between Closed- and Open-Loop Frequency Responses

4 Stability in frequency

5 Example

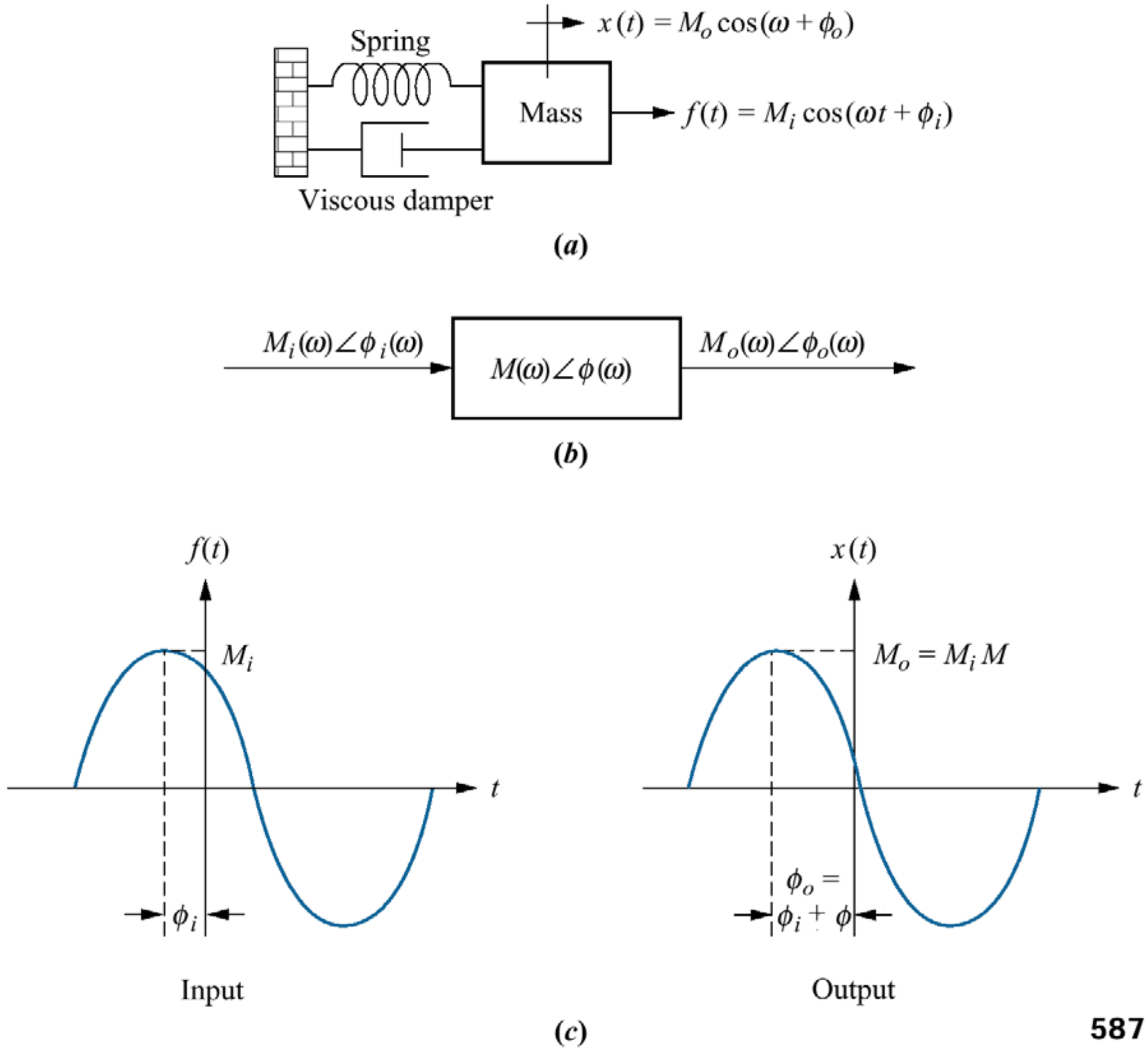
Concept of Frequency Response

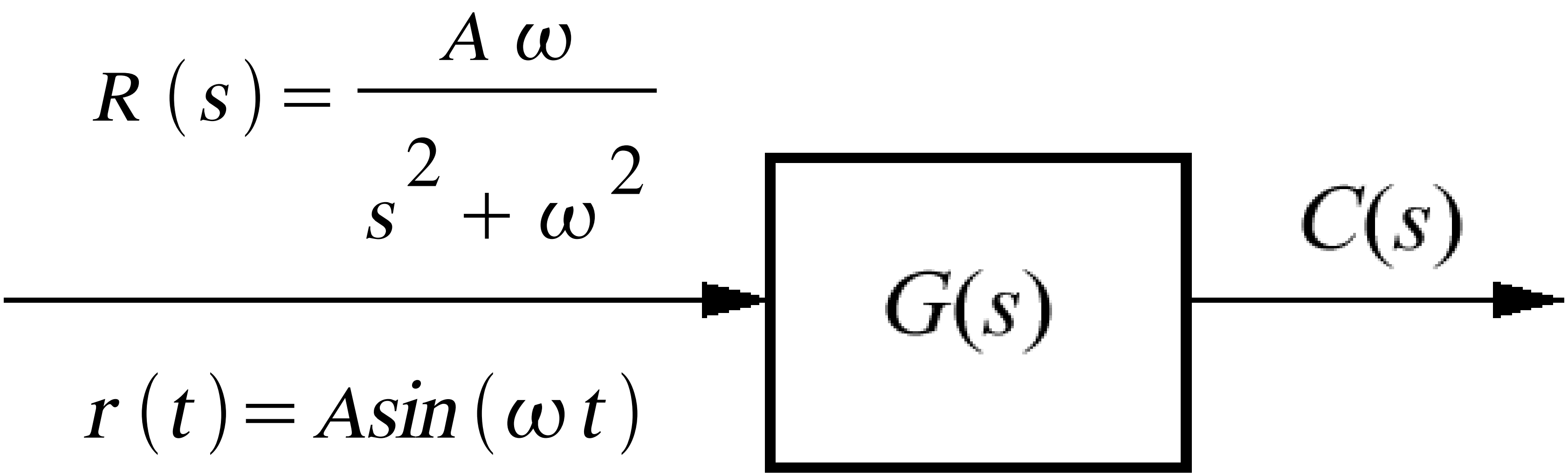
Sinusoidal frequency response:

a. system;

b. transfer function;

c. input and output waveforms





System with sinusoidal input

$$C(s) = G(s)\,A\,\frac{\omega}{s^2+\omega^2}$$

$$G(s) = \frac{\prod_{i=1}^{M}(s+z_i)}{\prod_{j=1}^{N}(s+p_j)}$$

$$G(j\omega) = \frac{\prod_{i=1}^{M}(z_i+j\omega)}{\prod_{j=1}^{N}(p_j+j\omega)} = |G(j\omega)|\,e^{j\phi}$$

$$\phi = \arctan\left(\frac{\Im[G(j\omega)]}{\Re[G(j\omega)]}\right)$$

$$\frac{\omega A}{s^2+\omega^2}G(s)=\frac{a_1}{s+j\omega}+\frac{a_2}{s-j\omega}+\ldots$$

$$a_1=\left[\frac{\omega A}{s-j\omega}\right]_{(s=-j\omega)} \qquad =\frac{-A}{2j}G(-j\omega)$$

$$a_2=\left[\frac{\omega A}{s+j\omega}\right]_{(s=j\omega)} \qquad =\frac{A}{2j}G(j\omega)$$

$$C(s) = -\frac{A}{2j}\frac{1}{s+j\omega}G(-j\omega) + \frac{A}{2j}\frac{1}{s-j\omega}G(j\omega)$$

$$G(j\omega) = |G(j\omega)|\,\mathrm{e}^{j\phi}$$

$$G(-j\omega) = |G(-j\omega)|\,\mathrm{e}^{-j\phi} = |G(j\omega)|\,\mathrm{e}^{-j\phi}$$

$$c(t) = \frac{A}{2j}|G(j\omega)|\,(-\mathrm{e}^{-j\omega t}\mathrm{e}^{-j\phi} + \mathrm{e}^{j\omega t}\mathrm{e}^{j\phi})$$

$$= A\,|G(j\omega)|\left(\frac{\mathrm{e}^{j(\omega t+\phi)} - \mathrm{e}^{-j(\omega t+\phi)}}{2j}\right)$$

$$c(t)=A|G(j\omega)|\sin(\omega t+\phi)$$

จากสมการจะเห็นได้ว่าเอาต์พุตของระบบเชิงเส้น (Linear system) จะมีขนาด(Gain) และ มุมเฟสที่เปลี่ยนไปจากอินพุต

Bode Diagram

Bode Diagram

1. Gain diagram – Plot between gain (dB) and frequency on semi-log diagram.
2. Phase diagram – Plot between phase (degree or radian) and frequency on semi-log diagram.

$$G(s) = \frac{K}{s^k} \frac{\prod_{i=1}^{M} (s + z_i)}{\prod_{j=1}^{N} (s + p_j)}$$

Gain diagram

$$dB = 20\log|G(j\omega)|$$

$$|G(j\omega)| = \frac{K\,|j\omega + z_1|\ldots|j\omega + z_M|}{|(j\omega)^k|\,|j\omega + p_1|\ldots|j\omega + p_N|}$$

$$\begin{aligned} dB = {} & 20\log(K) + 20\log\sqrt{z_1^2 + \omega^2} + \ldots + 20\log\sqrt{z_M^2 + \omega^2} \\ & - 20\log\omega^k - 20\log\sqrt{p_1^2 + \omega^2} - \ldots - 20\log\sqrt{p_N^2 + \omega^2} \end{aligned}$$

Phase diagram

$$\measuredangle G(j\omega) = \measuredangle(j\omega + z_1) + \ldots + \measuredangle(j\omega + z_M)$$
$$- \measuredangle(j\omega + p_1) - \ldots - \measuredangle(j\omega + p_N)$$

$$\phi_1 = \measuredangle(s + z_1) = \arctan\left(\frac{j\omega}{z_1}\right)$$
$$\vdots \qquad \vdots$$
$$\phi_M = \measuredangle(s + z_M) = \arctan\left(\frac{j\omega}{z_M}\right)$$

$$\theta_1 = \measuredangle(s+p_1) = \arctan\left(\frac{j\omega}{p_1}\right)$$

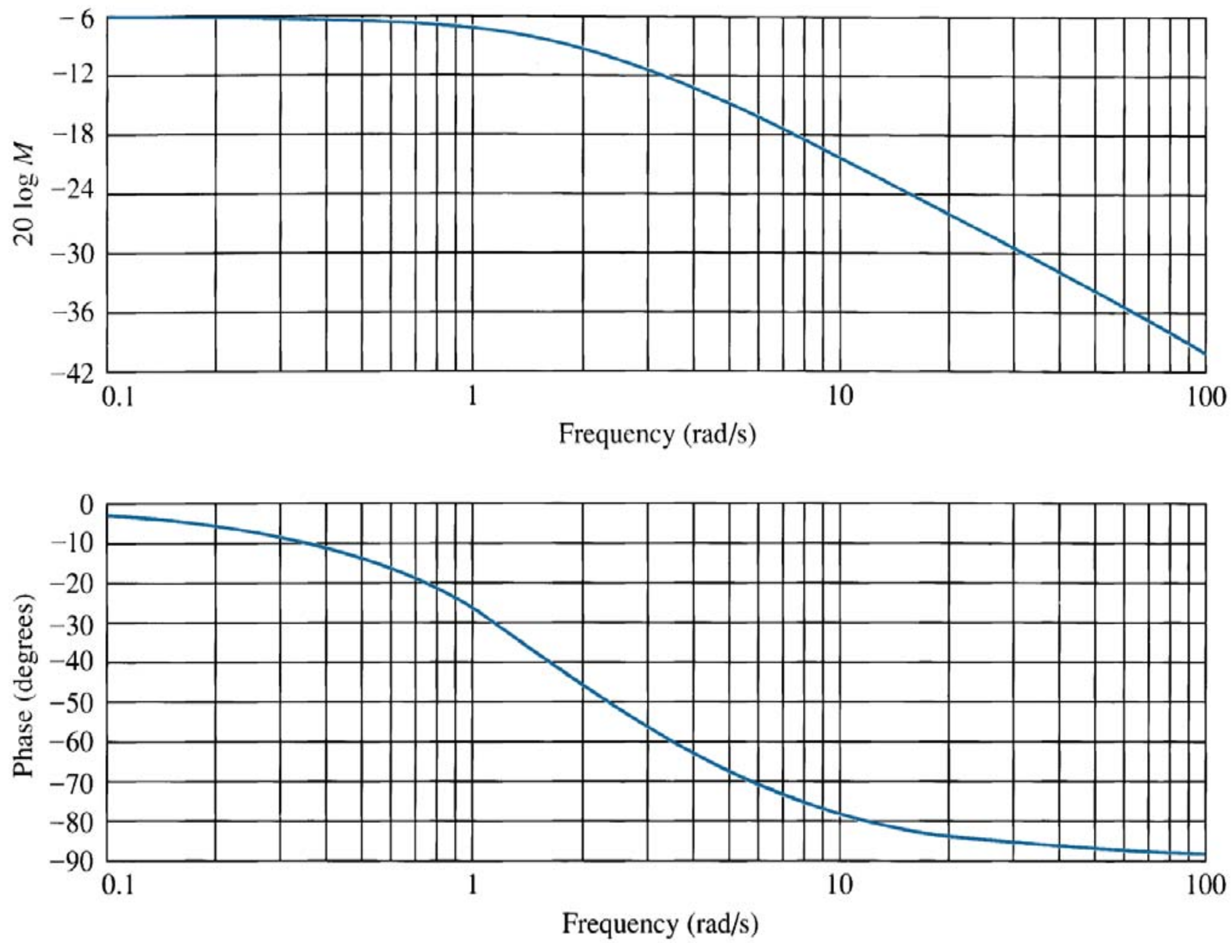
$$\vdots \qquad\qquad \vdots$$

$$\theta_N = \measuredangle(s+p_1) = \arctan\left(\frac{j\omega}{p_N}\right)$$

$$dB = 20\log(K) + 20\log\sqrt{z_1^2+\omega^2} + \ldots + 20\log\sqrt{z_M^2+\omega^2}$$
$$- 20\log\omega^k - 20\log\sqrt{p_1^2+\omega^2} - 20\log\sqrt{p_N^2+\omega^2}$$

$$\measuredangle G(j\omega) = \phi_1 + \ldots + \phi_M - \theta_1 - \ldots - \theta_N$$

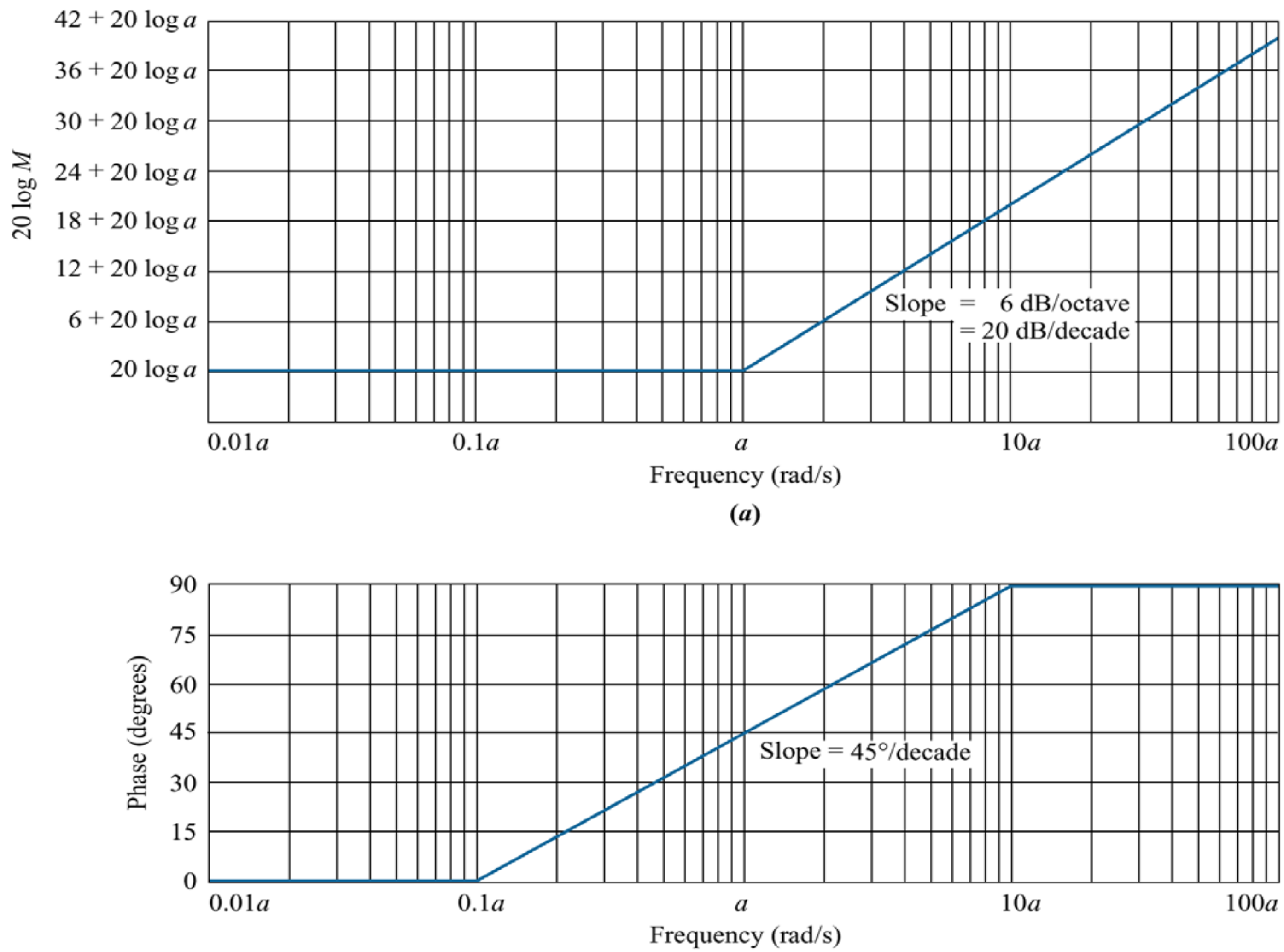
การพล๊อตนั้นจะทำอยู่ในช่วงความถี่(ω) ที่จะทำการศึกษา



Frequency response plots for $G(s) = 1/(s+2)$

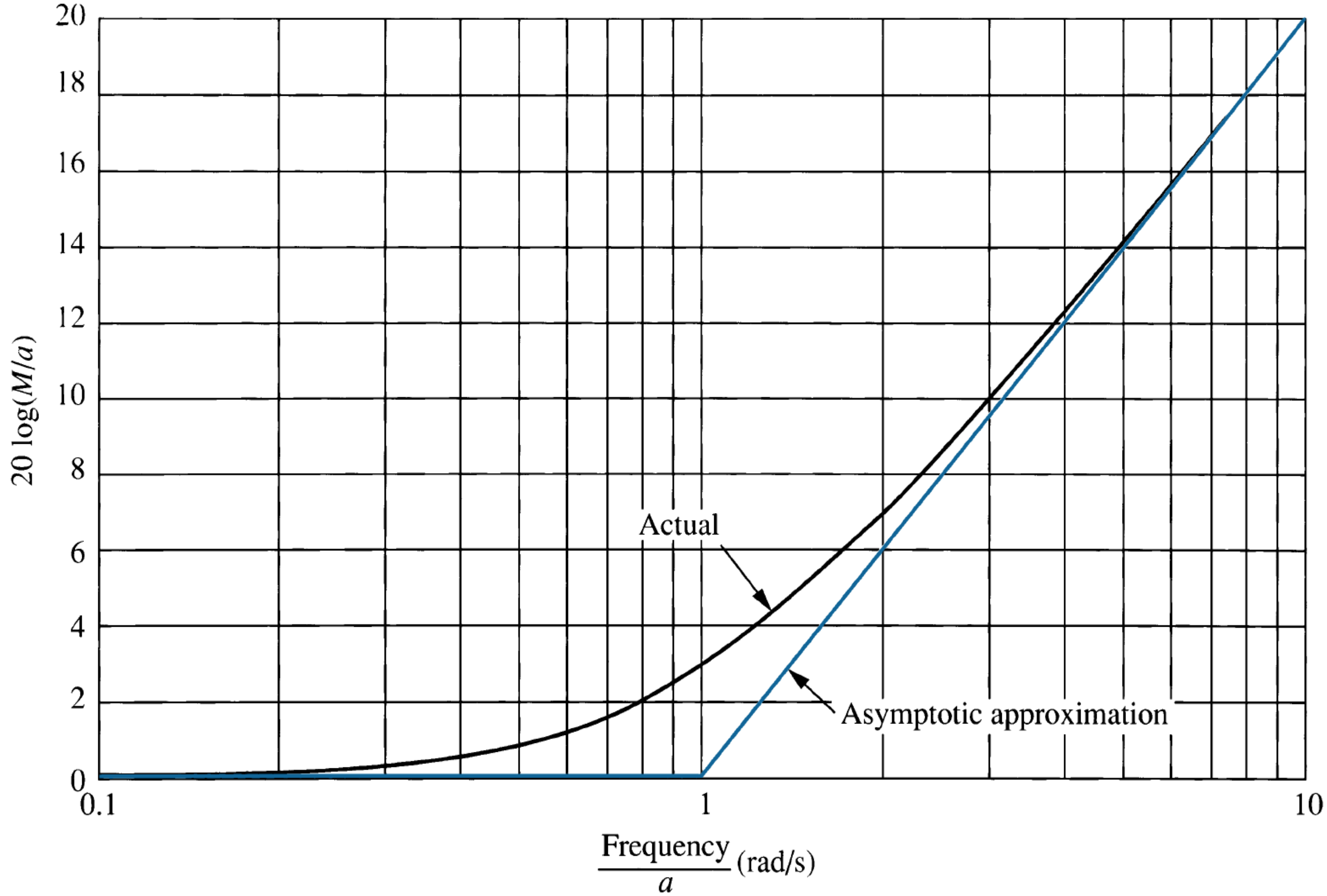
: separate magnitude and phase

Asymptotic Approximation

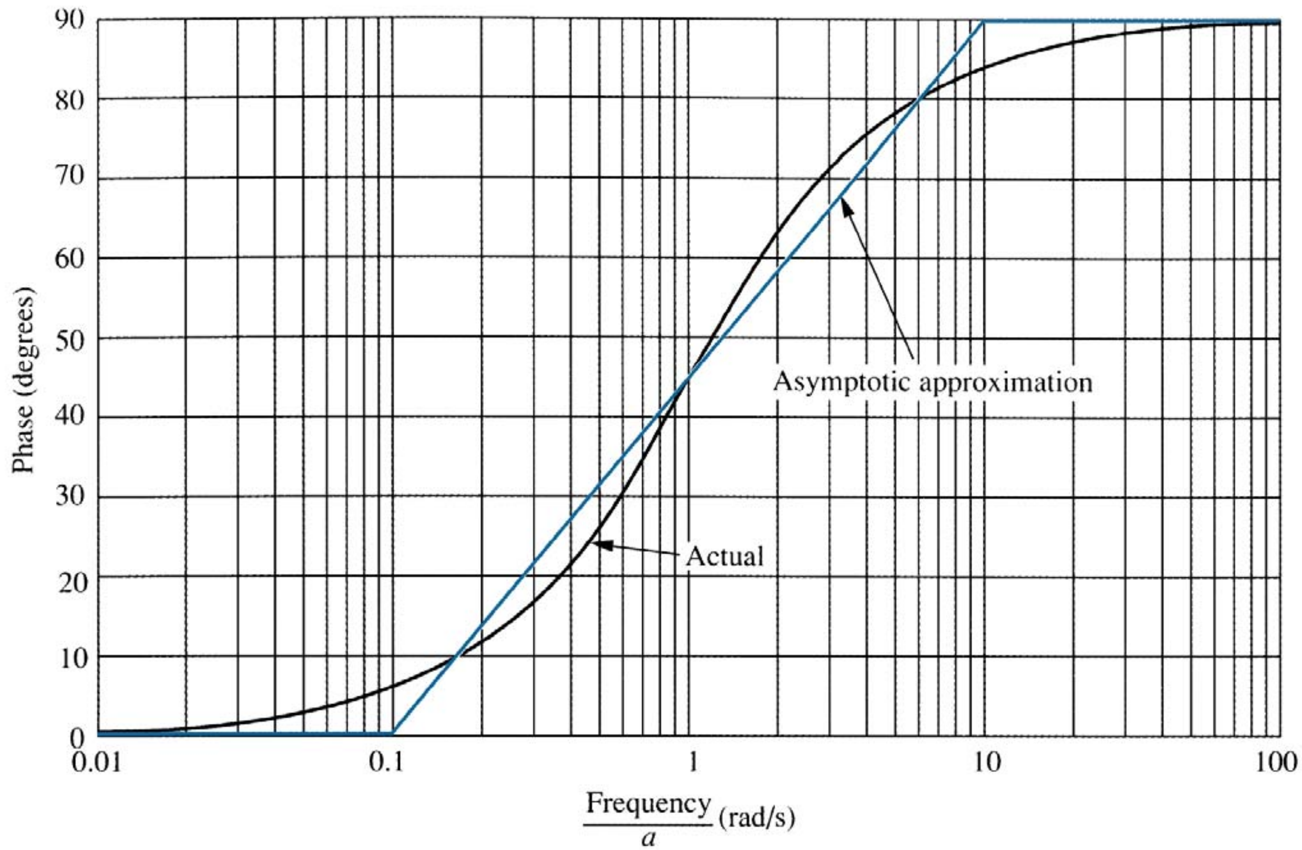


(*b*)

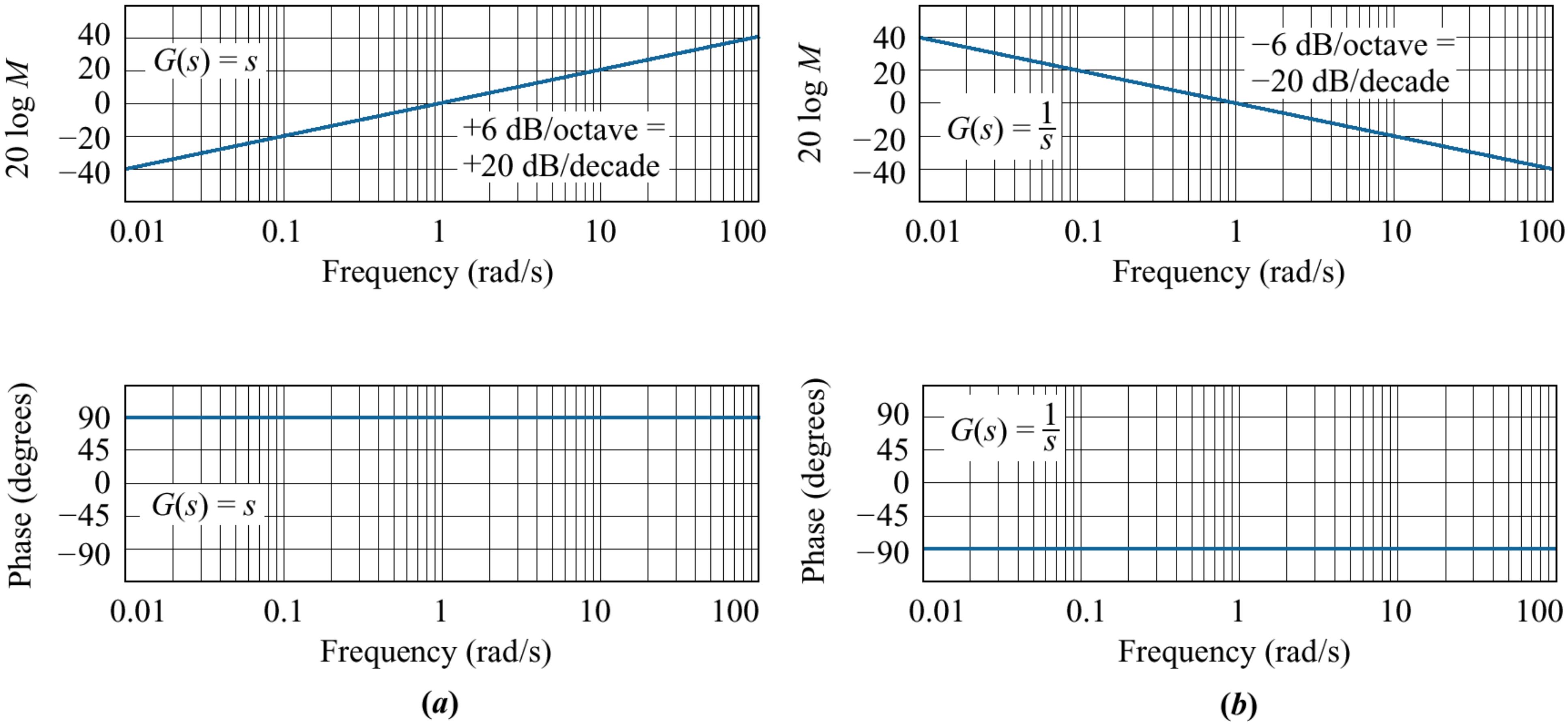
Bode plots of (s + a) : **a**. magnitude plot **b**. phase plot.



Asymptotic and actual normalized and scaled magnitude response of (s + a)

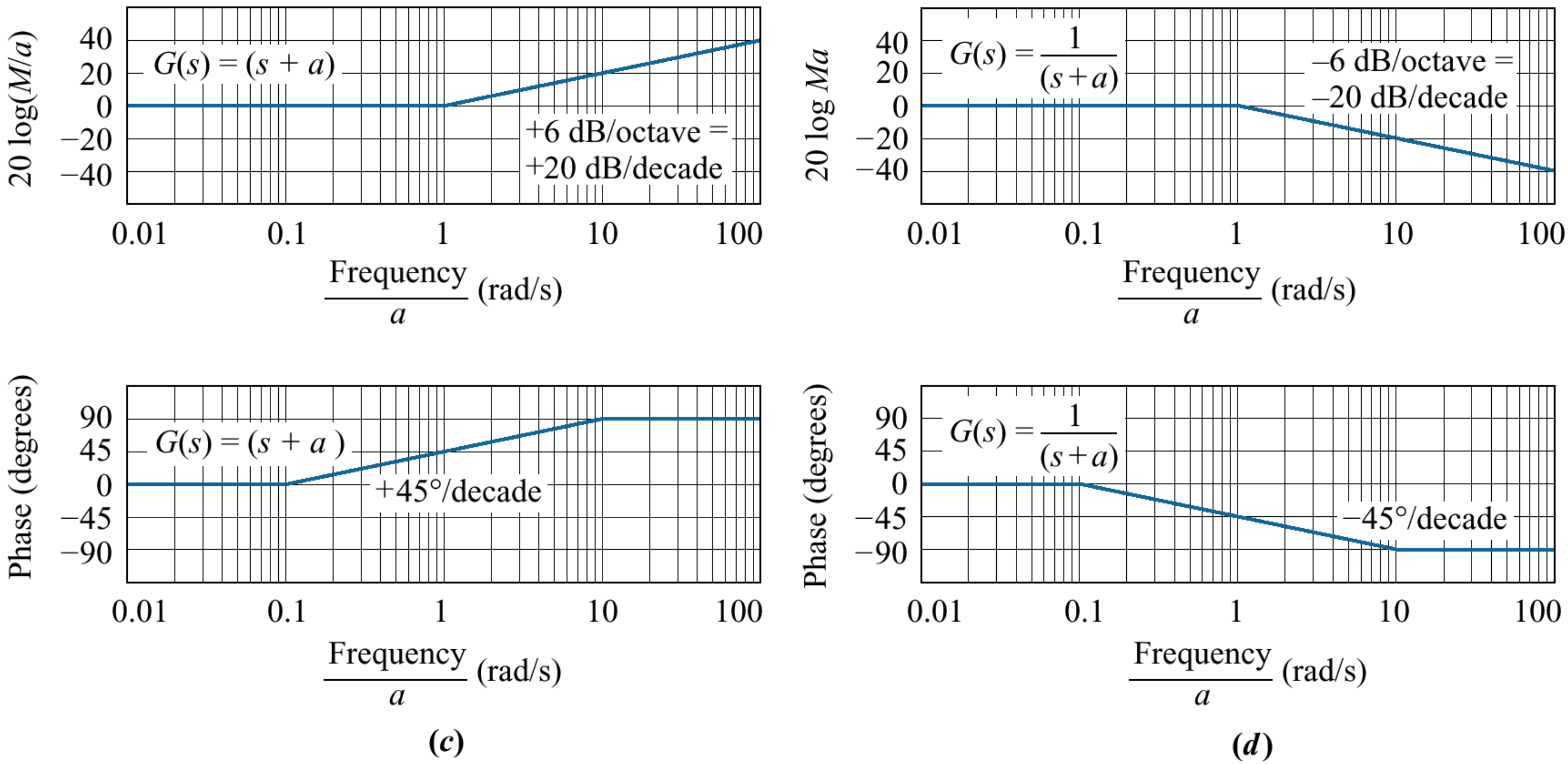


Asymptotic and actual normalized and scaled phase response of (s +a)



Normalized and scaled Bode plots for

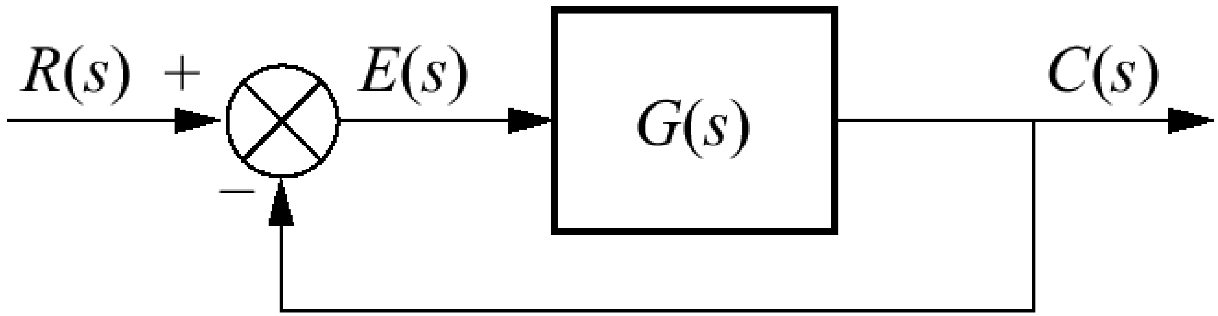
a. $G(s) = s$; **b.** $G(s) = 1/s$;



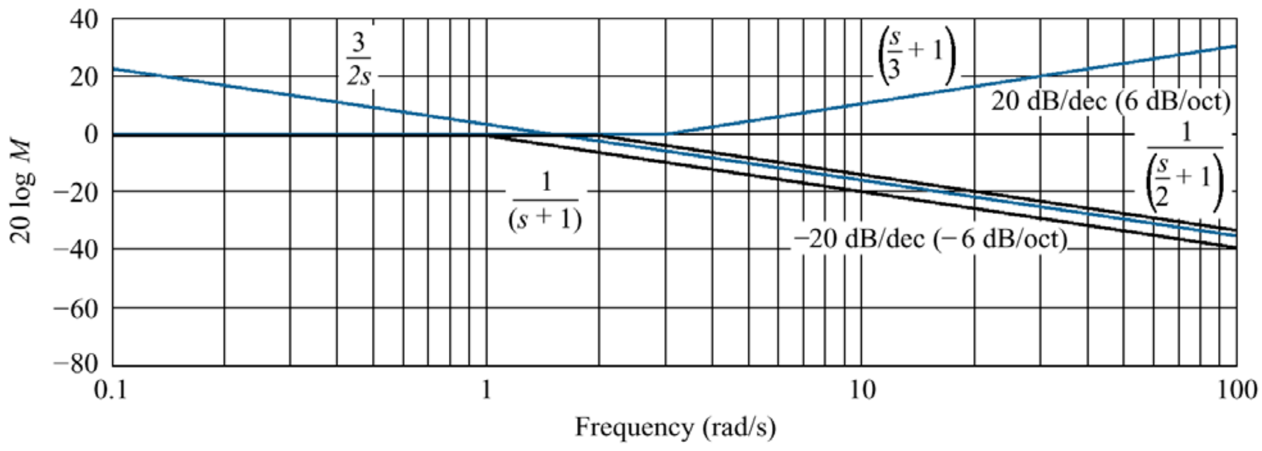
Normalized and scaled Bode plots for

c. $G(s) = (s + a)$; **d.** $G(s) = 1/(s + a)$

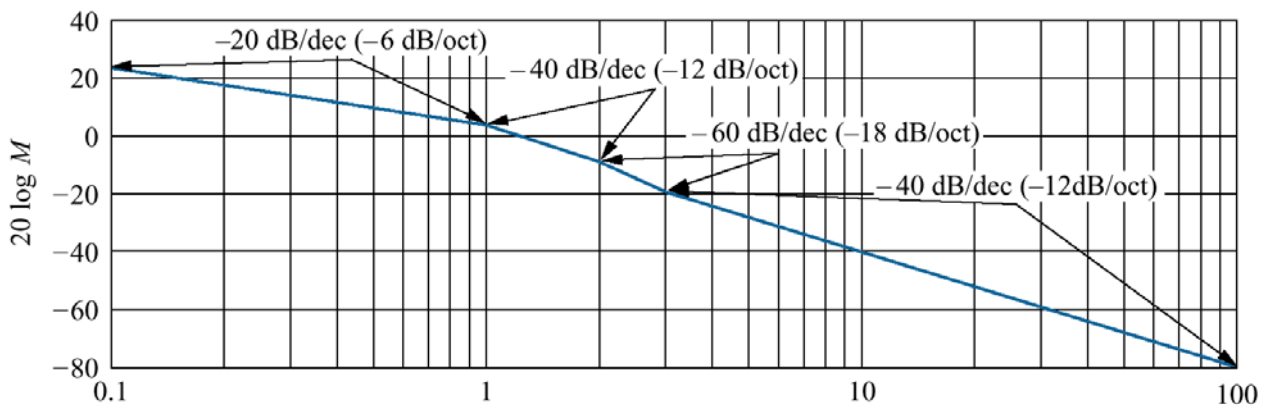
Example (Nise) Draw the Bode plot for the system shown below



$$G(s)=\frac{K(s+3)}{s(s+1)(s+2)}$$



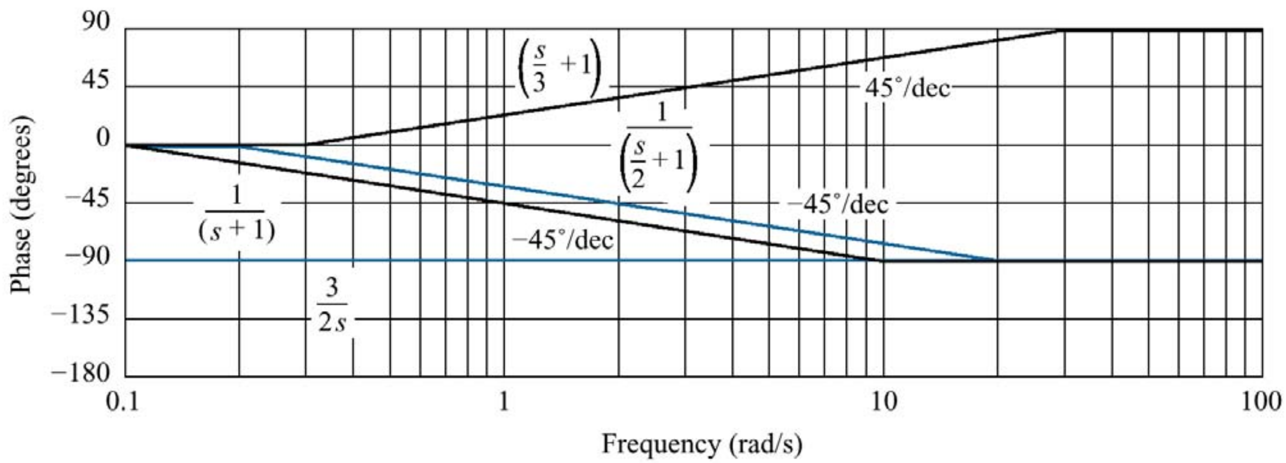
(*a*)



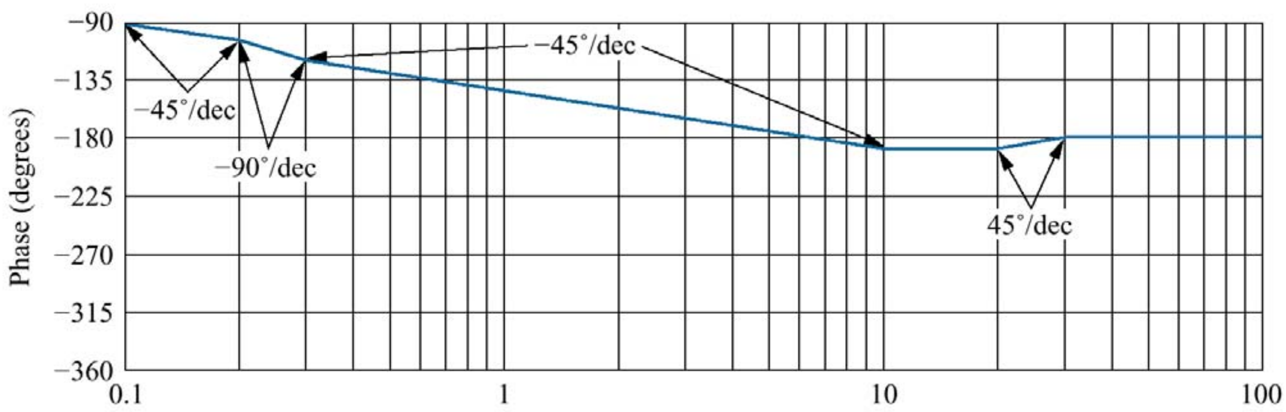
(*b*)

Bode log- magnitude plot :

a. components;   **b**. composite



(*a*)



(*b*)

Bode phase plot

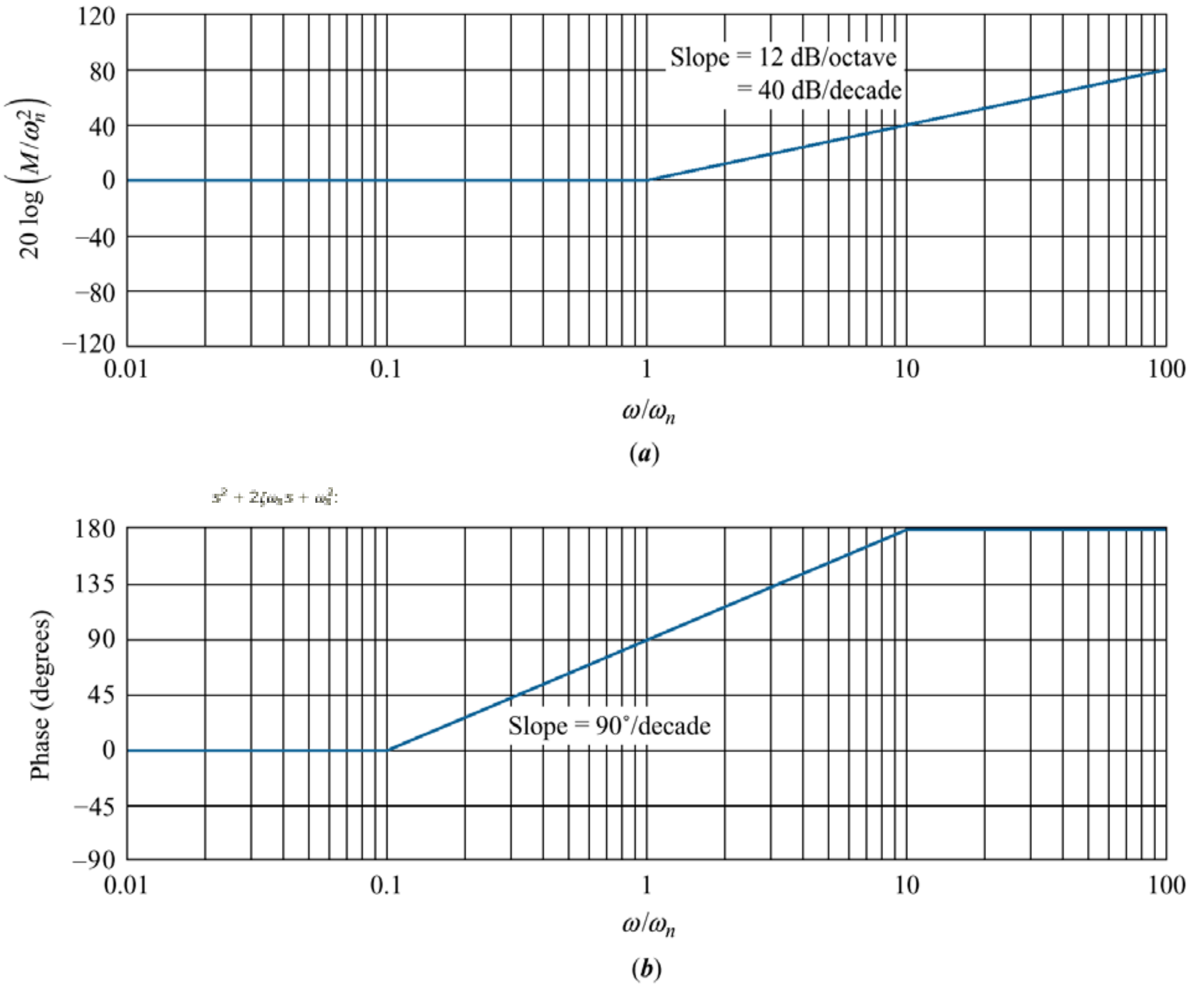
a. components; b. composite

Bode asymptotes for normalized and scaled $G(s) =$

$$s^2 + 2\zeta\omega_n s + \omega_n^2:$$

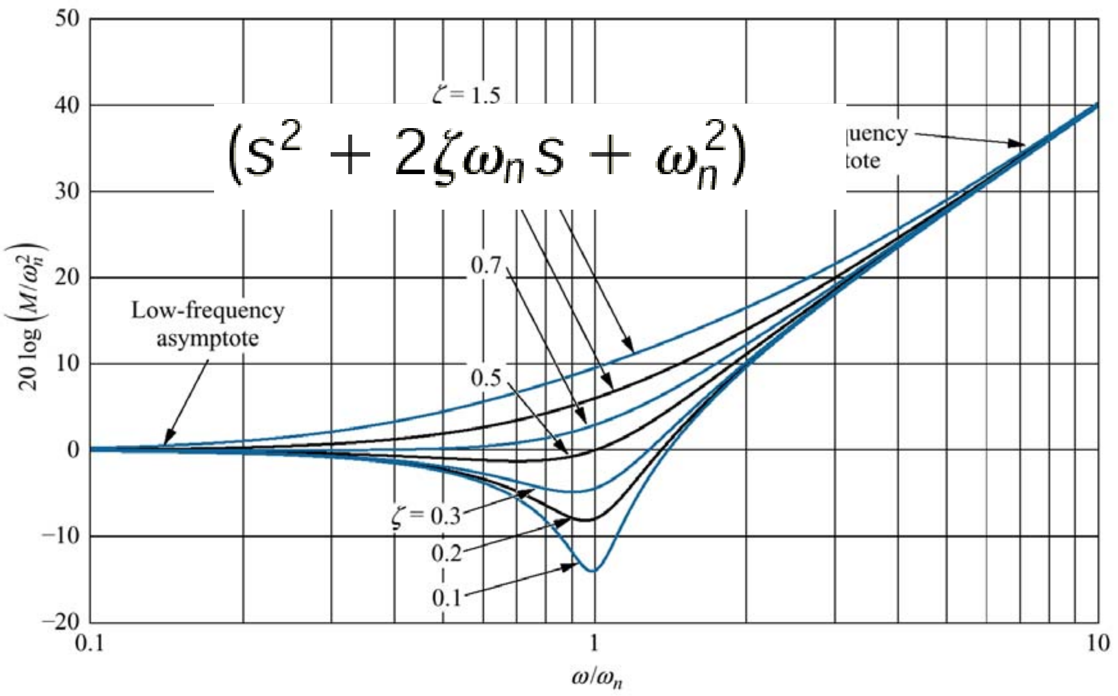
a. magnitude;

b. phase



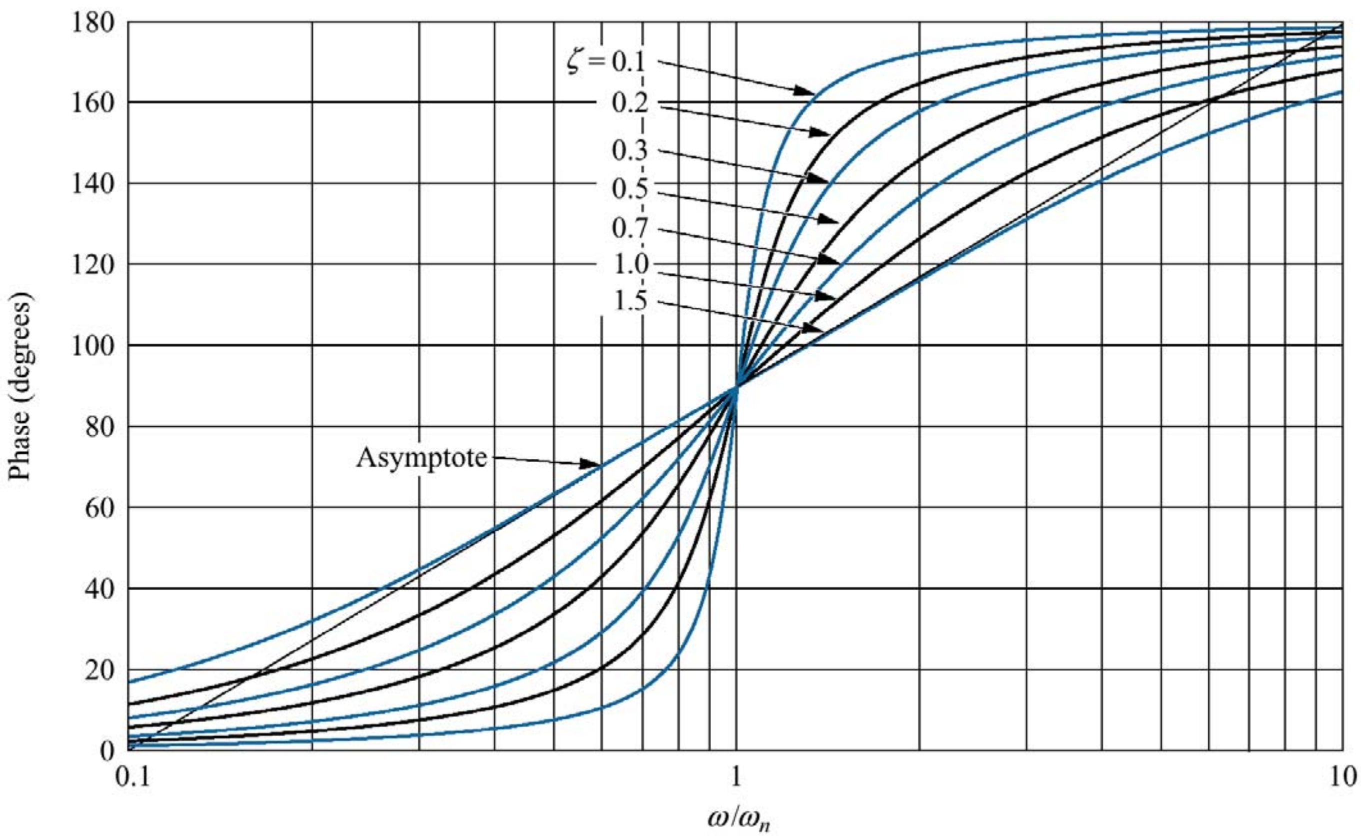
Normalized and scaled log-magnitude response for

$(s^2 + 2\zeta\omega_n s + \omega_n^2)$

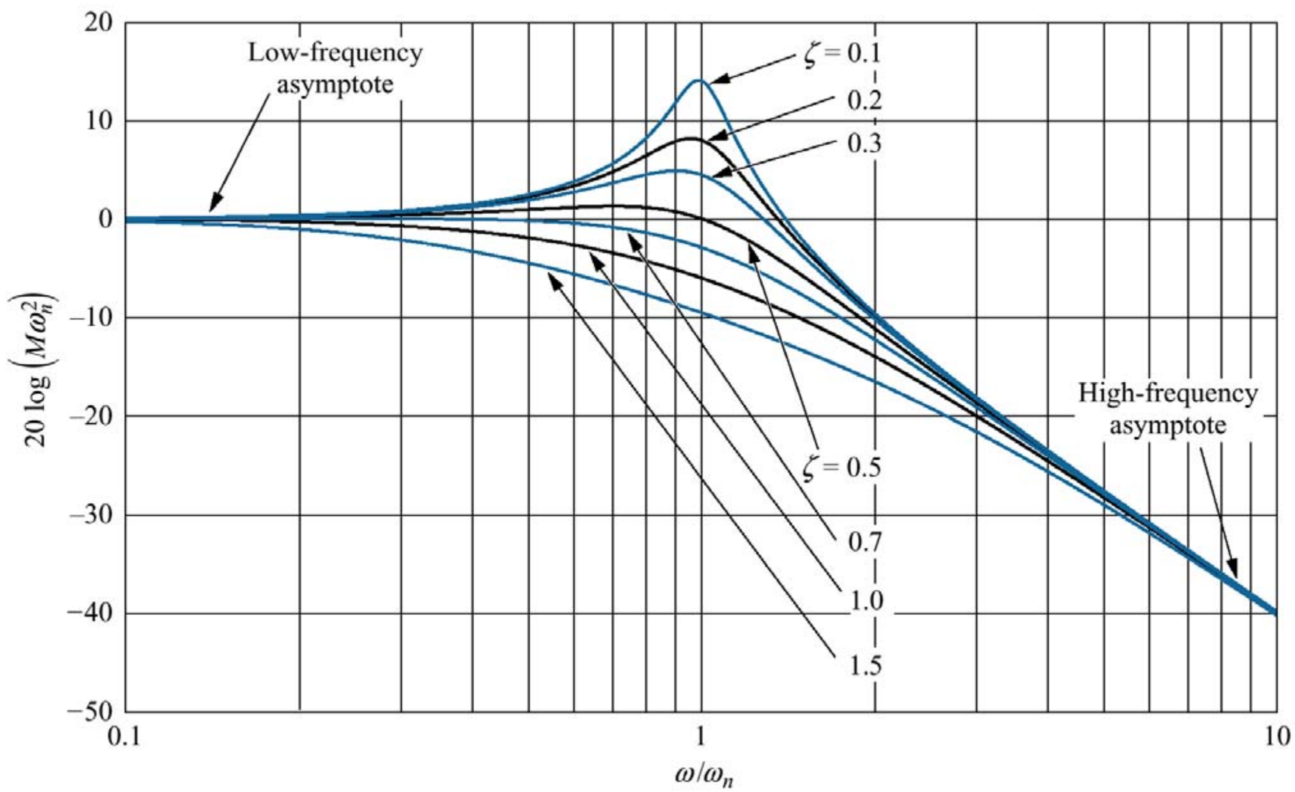


Scaled phase response for

$$(s^2 + 2\zeta\omega_n s + \omega_n^2)$$

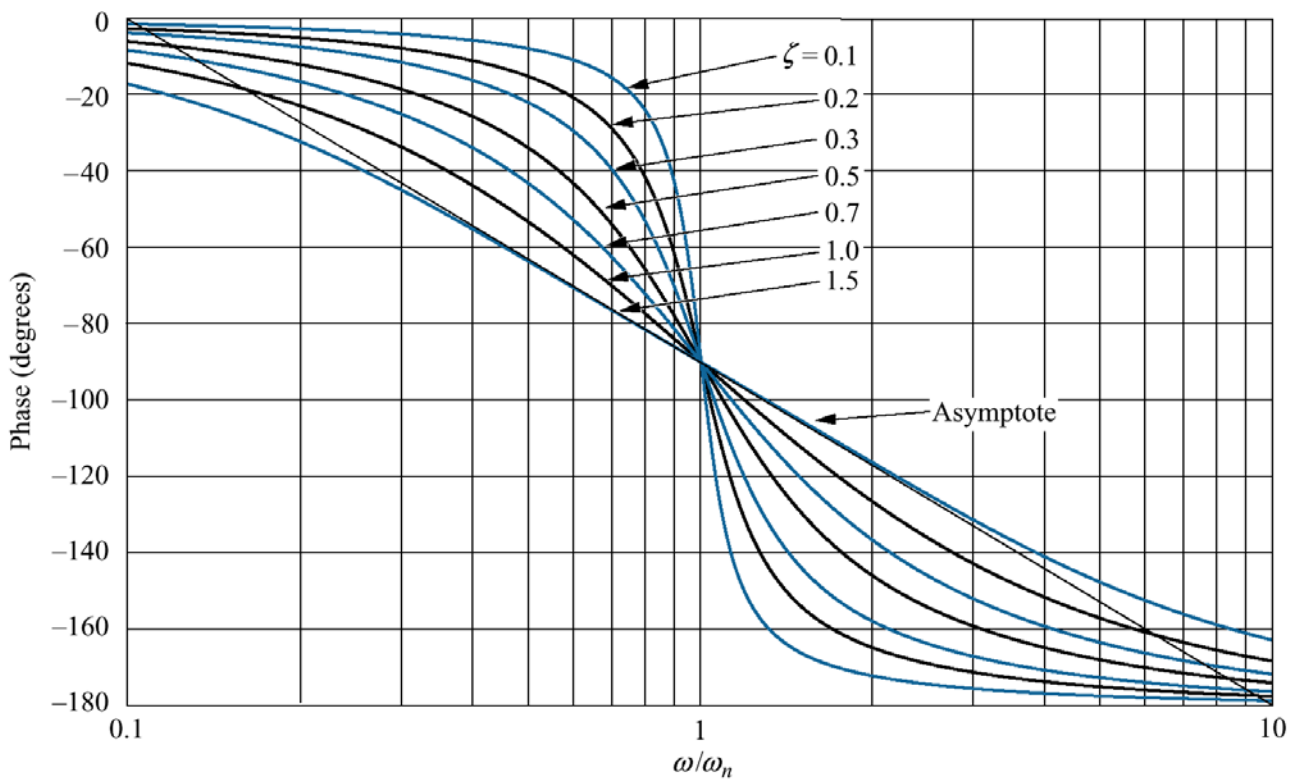


Normalized and scaled log magnitude response for $1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$



Scaled phase response for

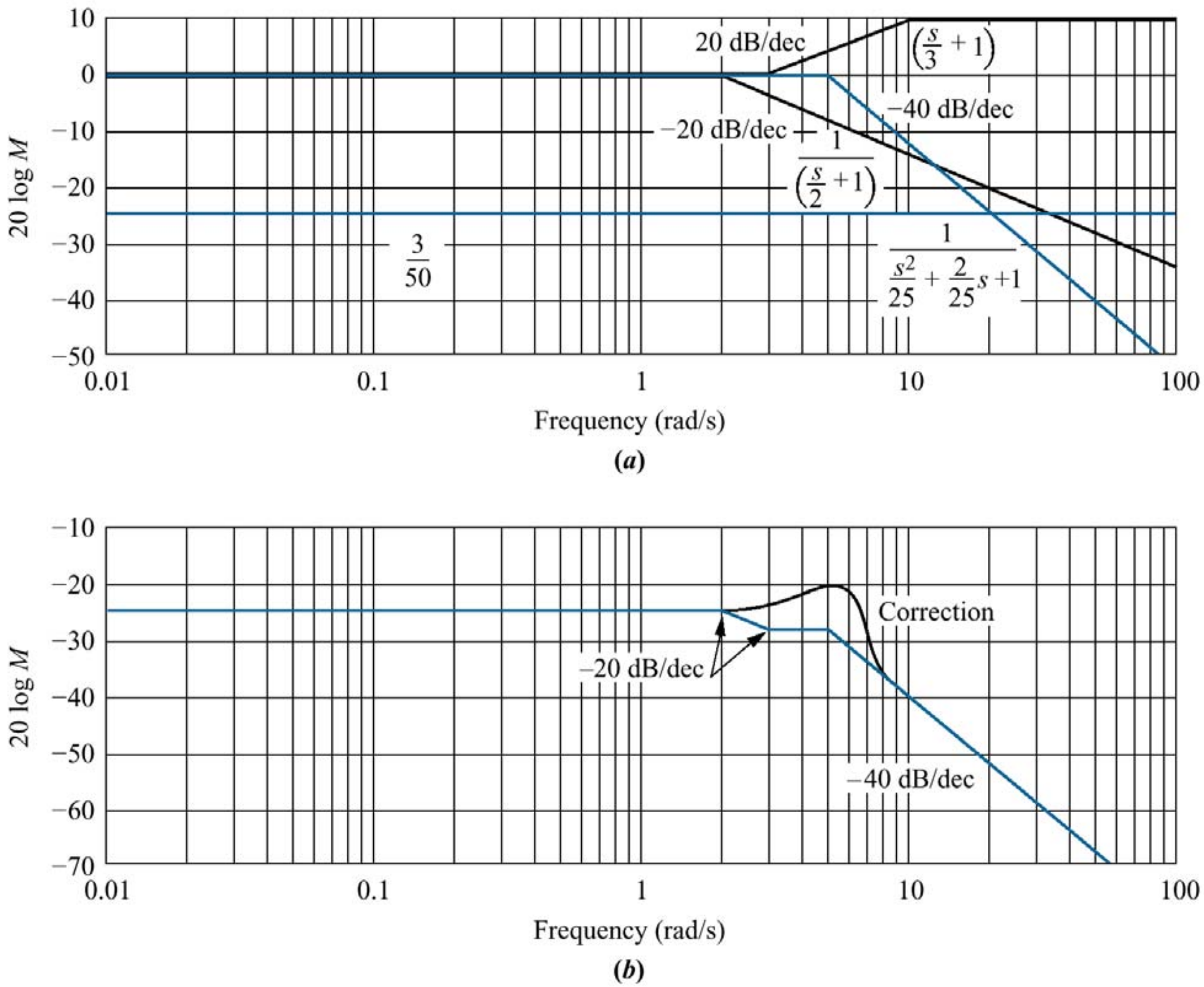
$1/(s^2 + 2\zeta\omega_n s + \omega_n^2)$



Bode magnitude plot for $G(s) = (s+3)/[(s+2)(s^2+2s+25)]$:

a. components;

b. composite



Polar plot

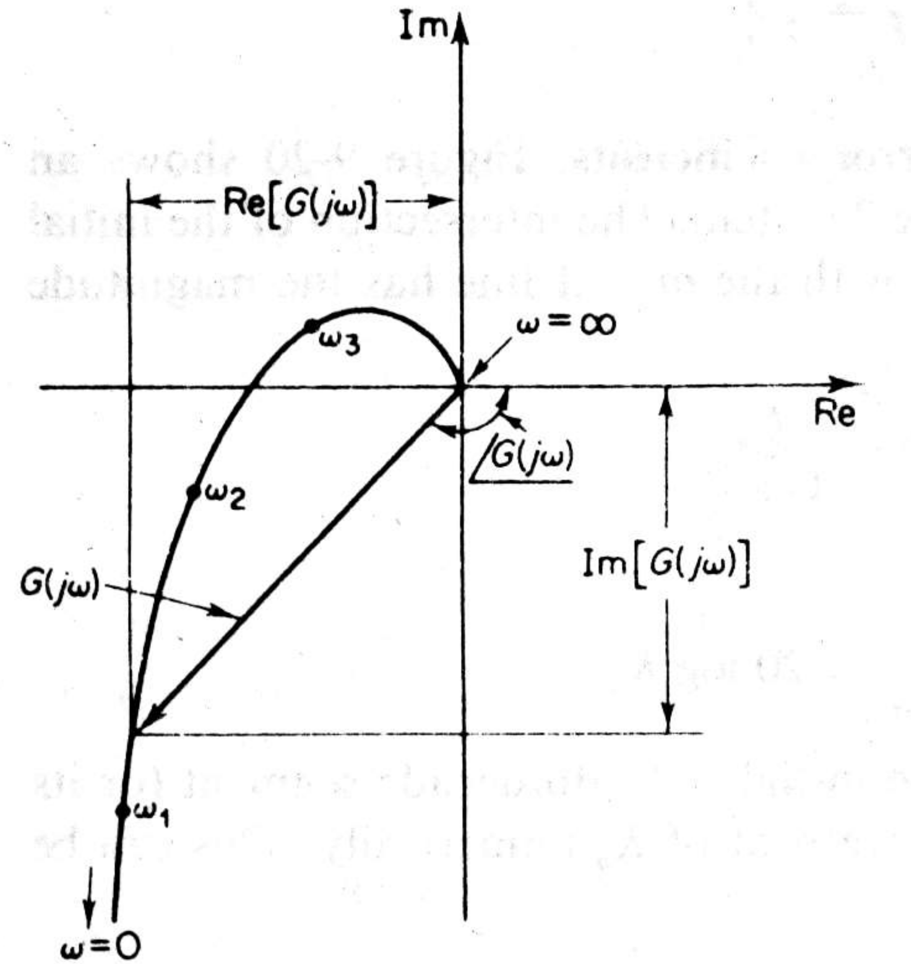


Fig. 9-21. Polar plot.

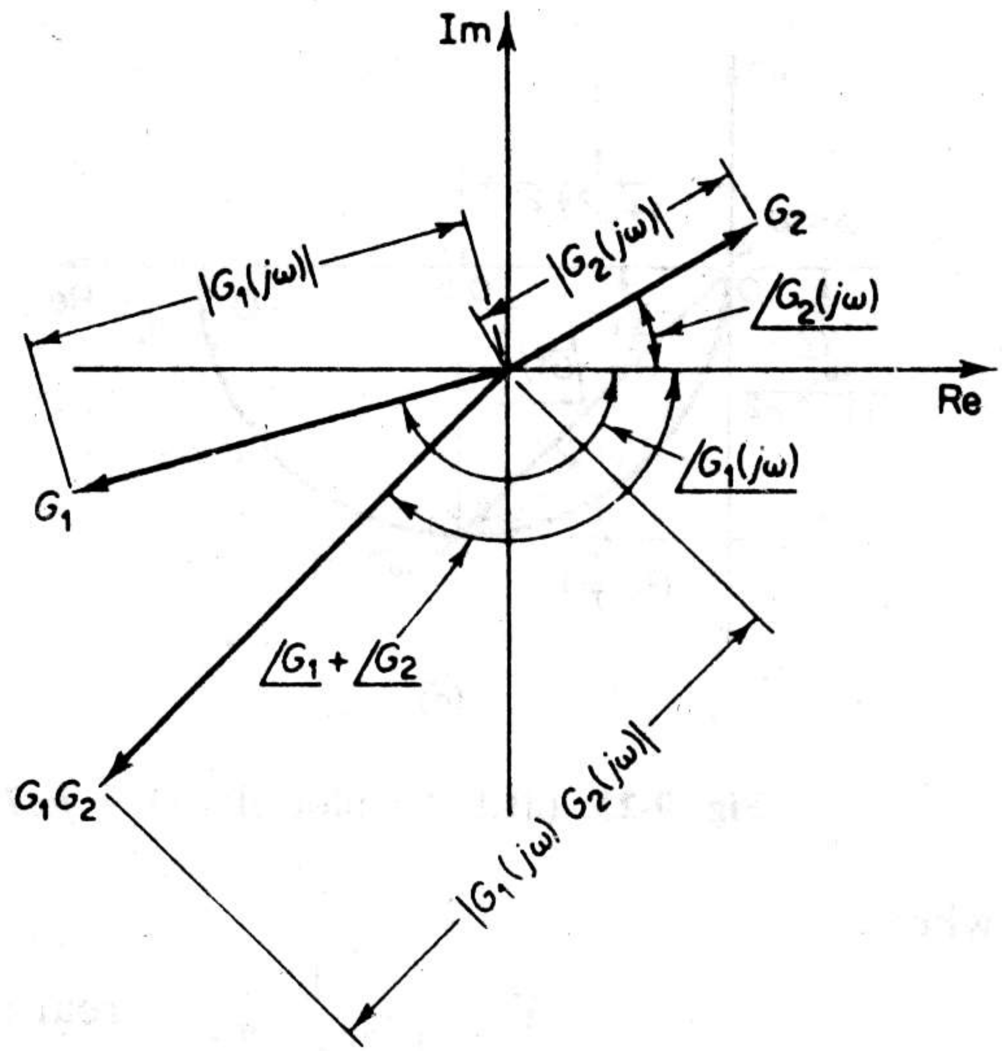


Fig. 9-22. Polar plots of $G_1(j\omega)$, $G_2(j\omega)$, and $G_1(j\omega)G_2(j\omega)$.

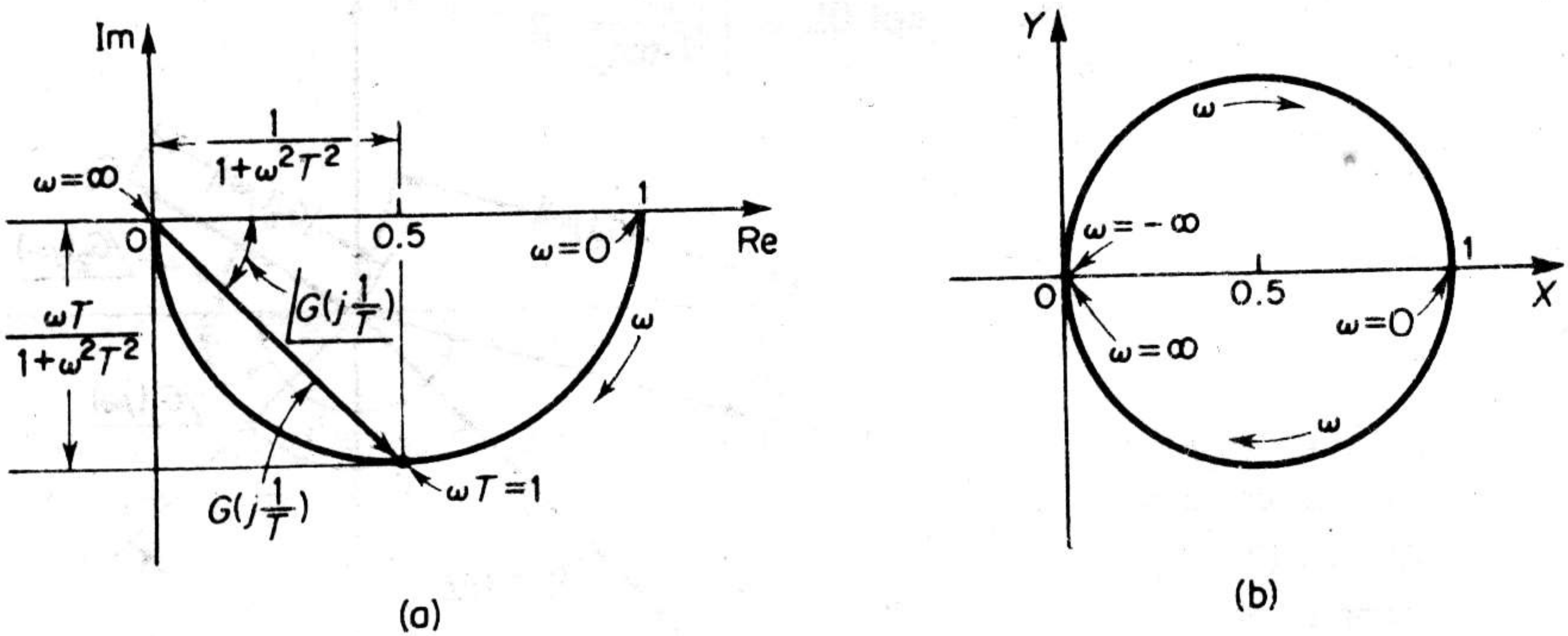


Fig. 9-23. (a) Polar plot of $1/(1 + j\omega T)$; (b) plot of $G(j\omega)$ in X-Y plane.

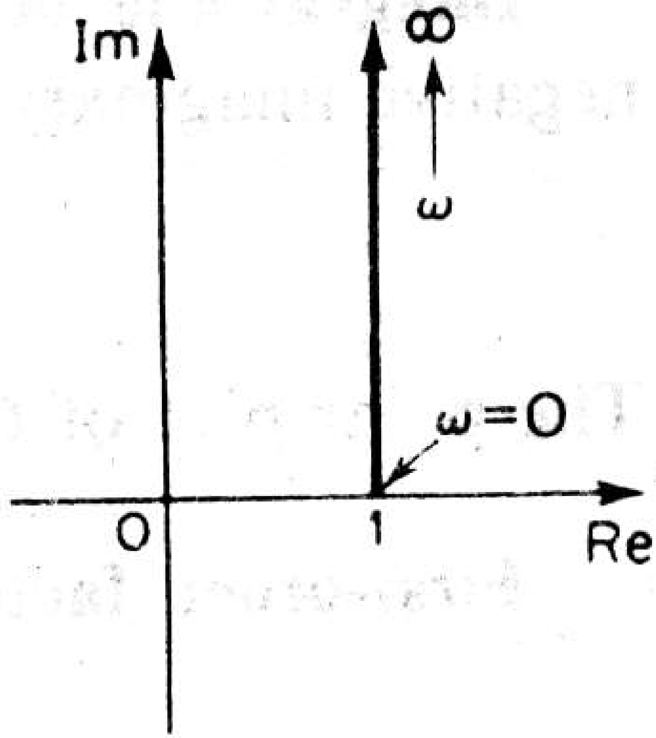


Fig. 9-24. Polar plot of $1 + j\omega T$.

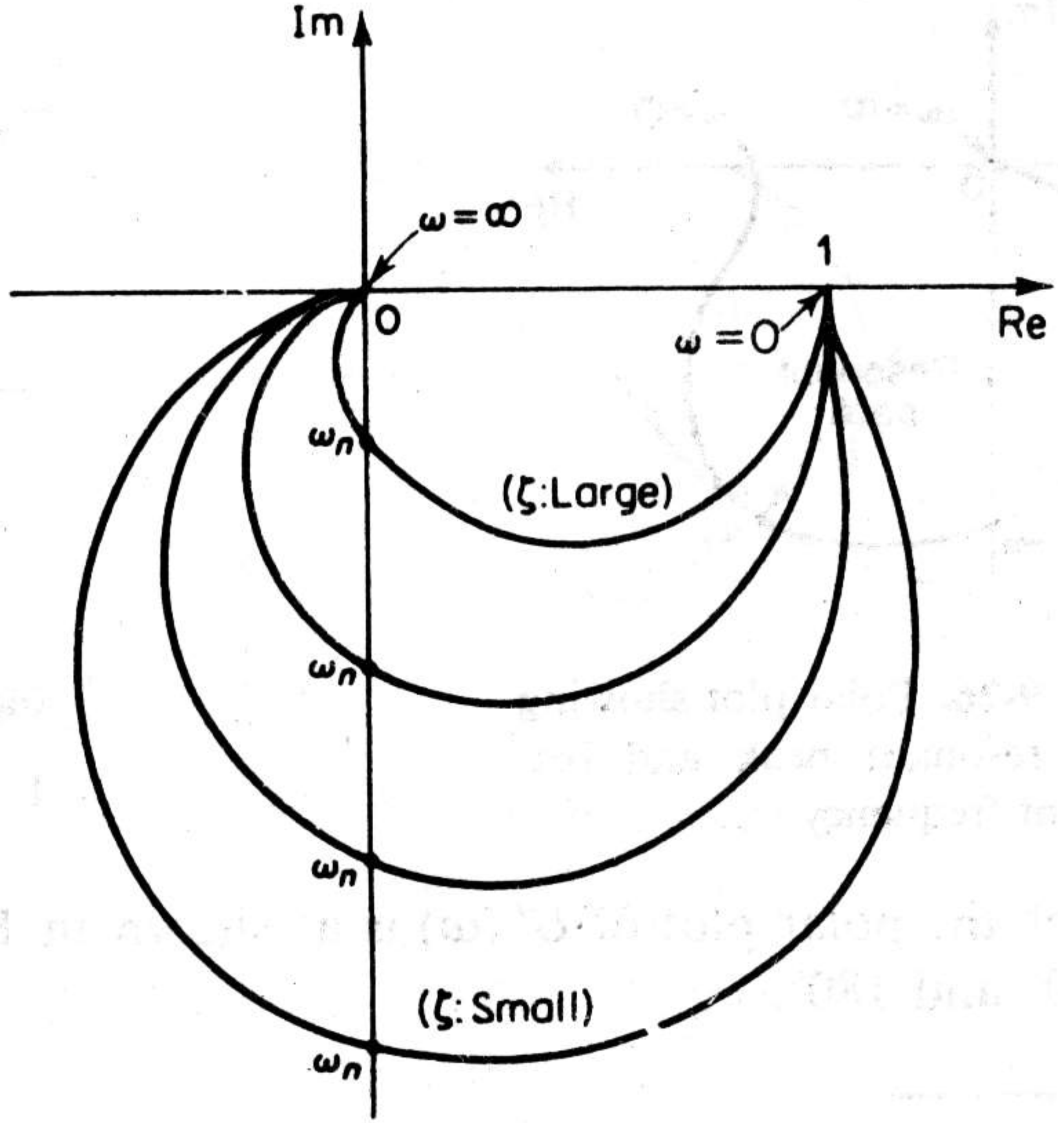


Fig. 9-25. Polar plots of

$$\frac{1}{1+2\zeta\left(j\frac{\omega}{\omega_n}\right)+\left(j\frac{\omega}{\omega_n}\right)^2}, \qquad (\zeta > 0).$$

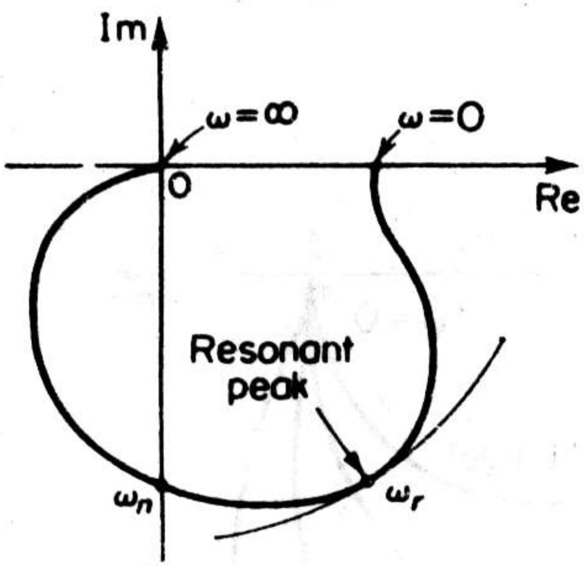


Fig. 9-26. Polar plot showing the resonant peak and resonant frequency ω_r.

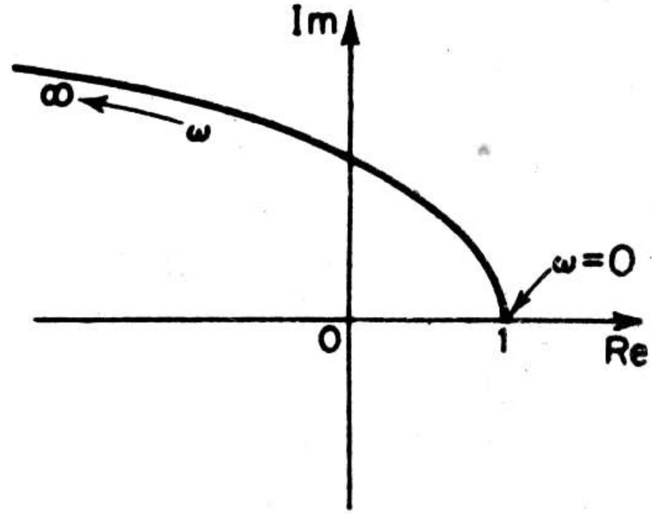


Fig. 9-27. Polar plot of $1 + 2\zeta\left(j\dfrac{\omega}{\omega_n}\right) + \left(j\dfrac{\omega}{\omega_n}\right)^2$, $\quad (\zeta > 0)$.

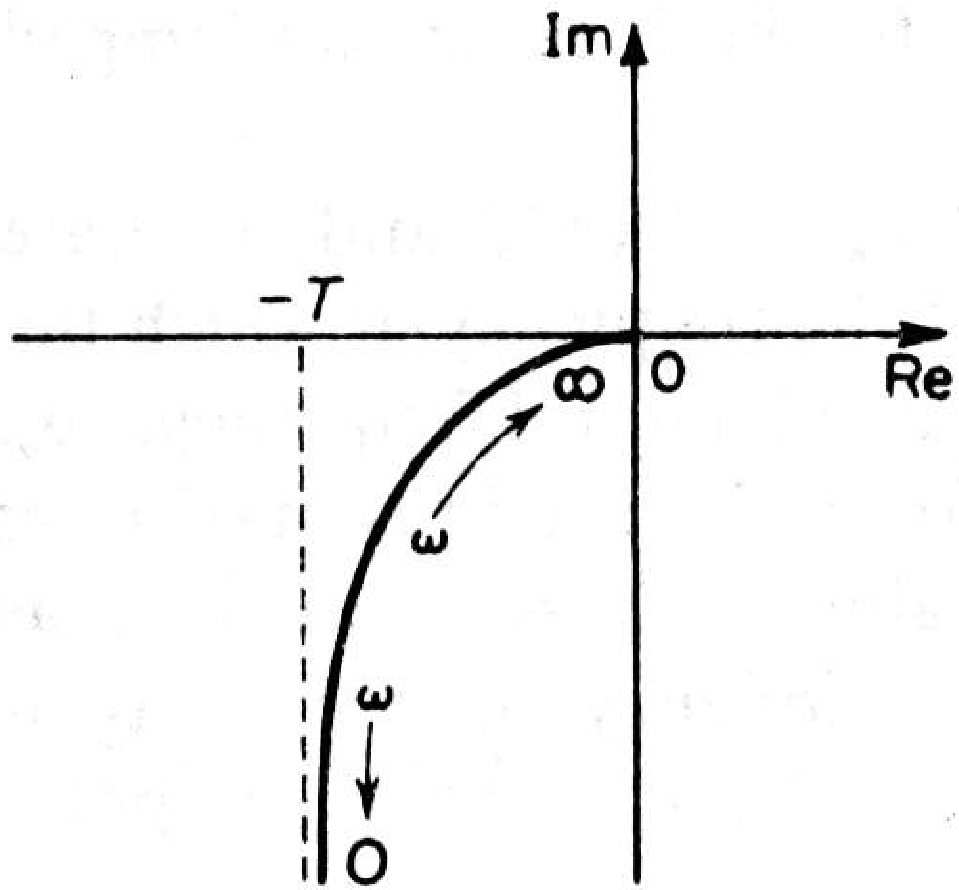


Fig. 9-28. Polar plot of $1/[j\omega(1+j\omega T)]$.

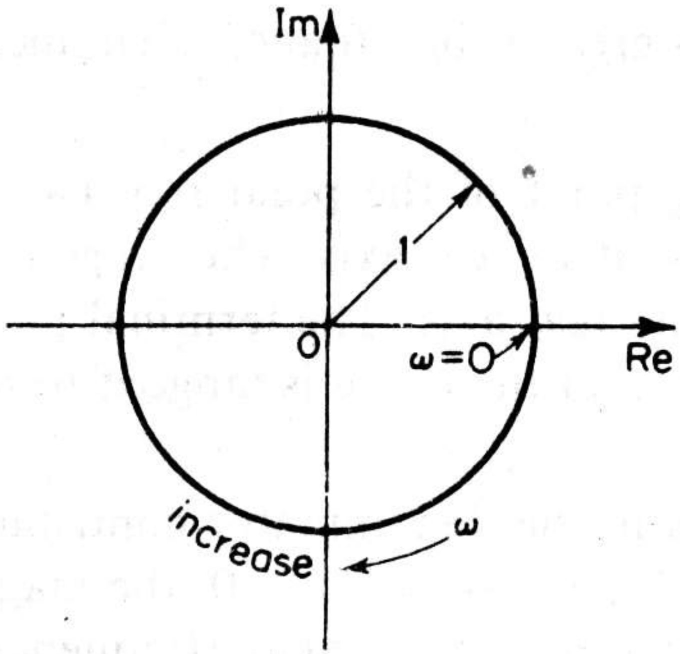


Fig. 9-29. Polar plot of transportation lag.

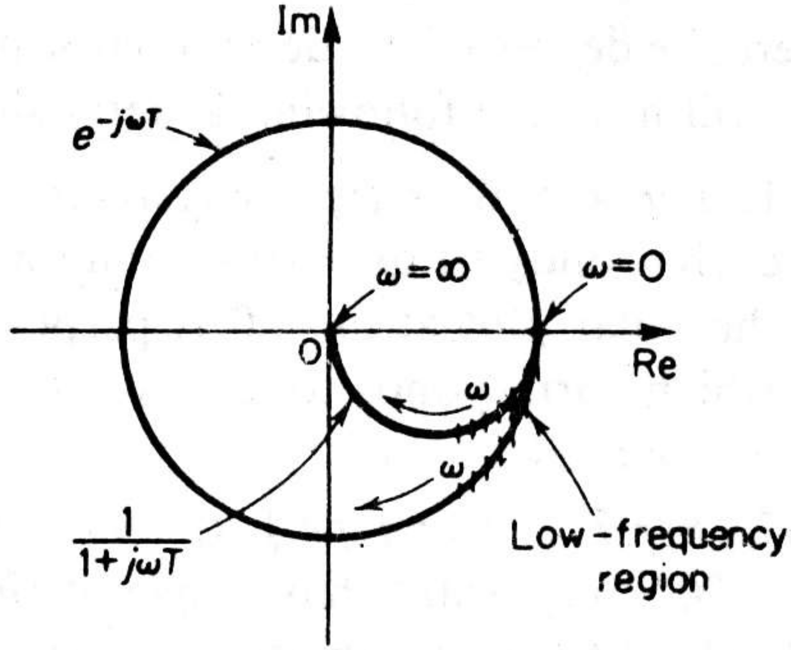


Fig. 9-30. Polar plots of $e^{-j\omega T}$ and $1/(1 + j\omega T)$.

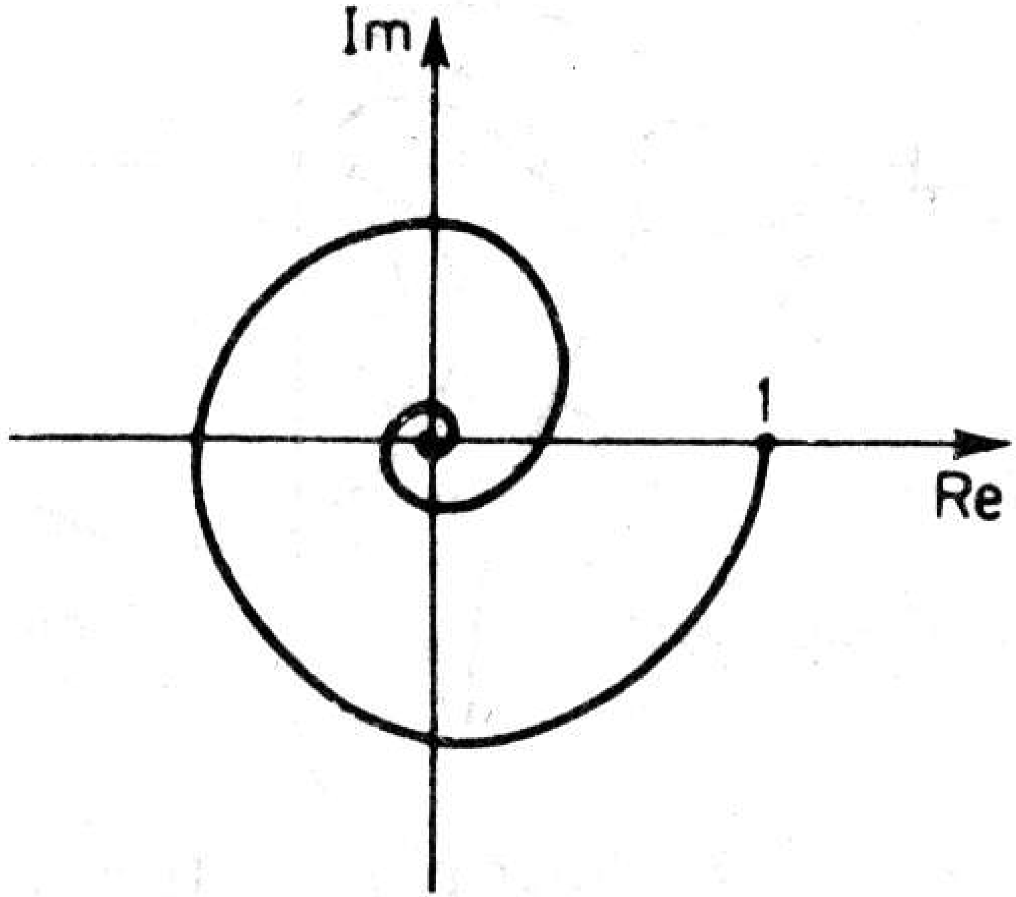


Fig. 9-31. Polar plot of $e^{-j\omega L}/(1 + j\omega T)$.

$$G(j\omega) = \frac{K(z_1 + j\omega)\ldots}{(j\omega)^k (p_1 + j\omega)\ldots}$$

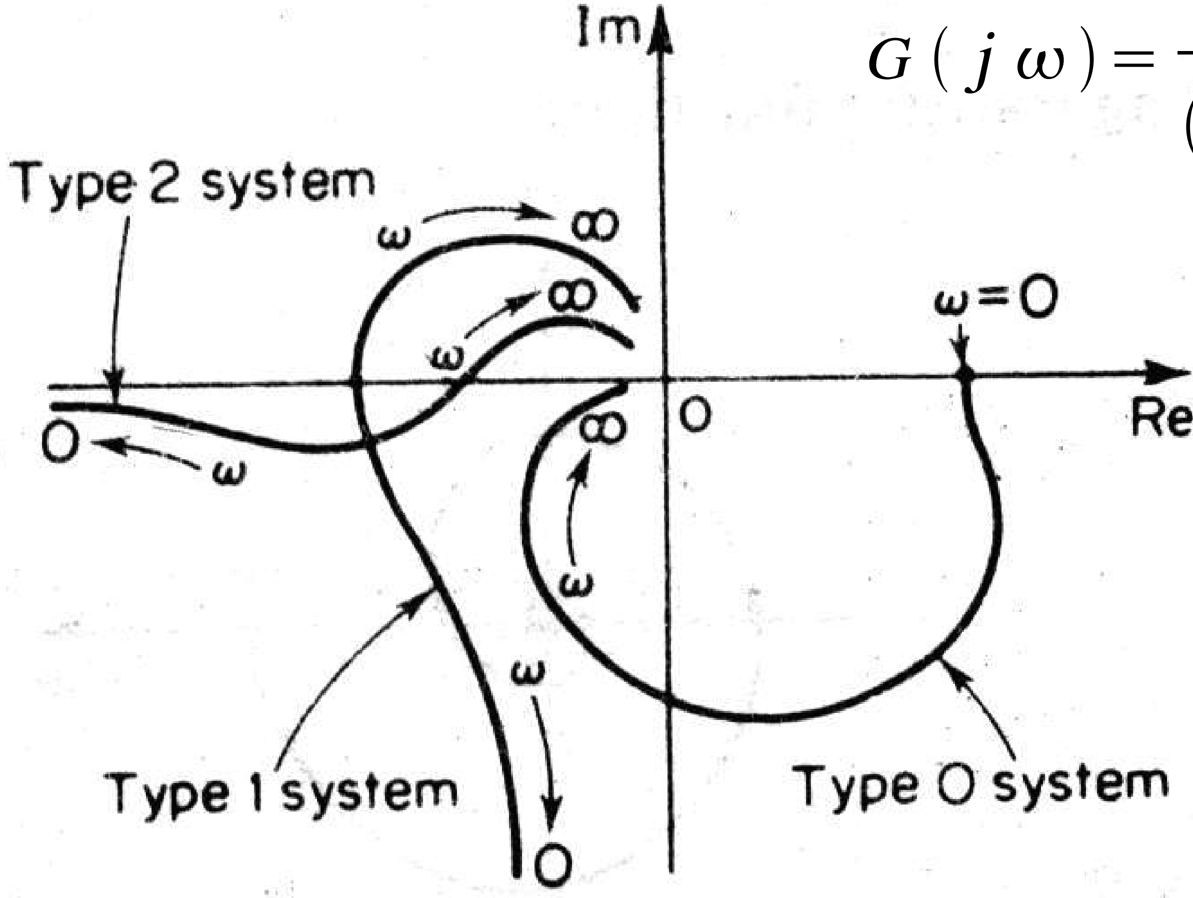
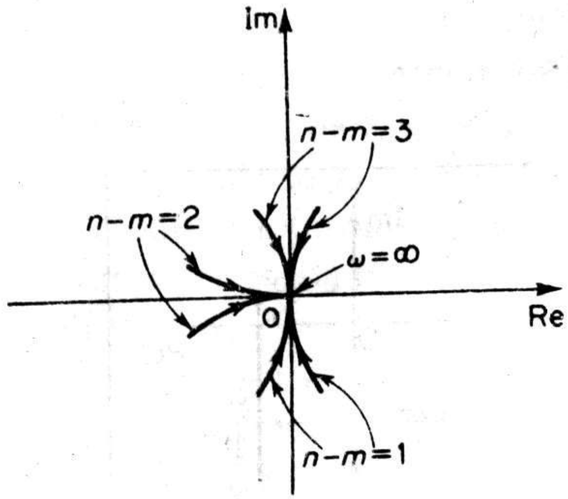


Fig. 9-32. Polar plots of type 0, type 1, and type 2 systems.



$$G(j\omega)=\frac{b_0(j\omega)^m+\cdots}{a_0(j\omega)^n+\cdots}$$

Fig. 9-33. Polar plots in the high-frequency range.

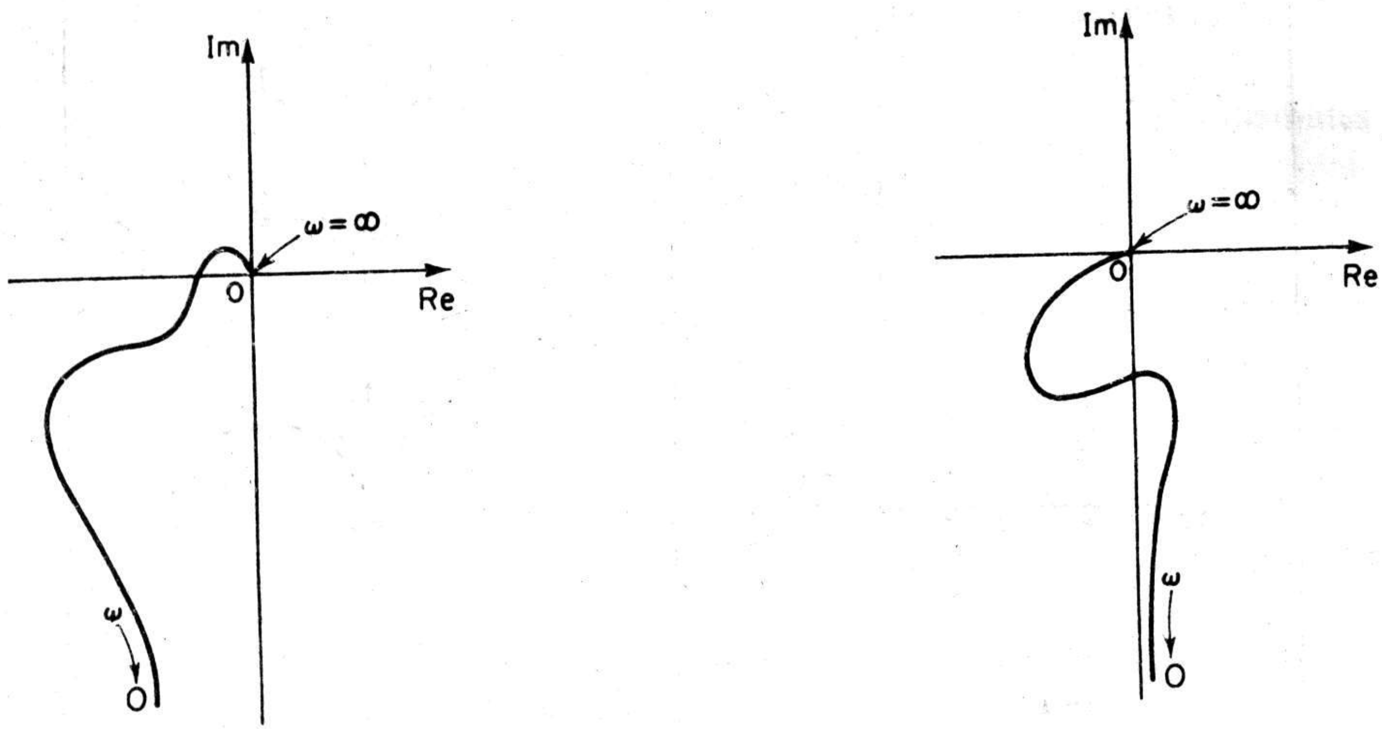
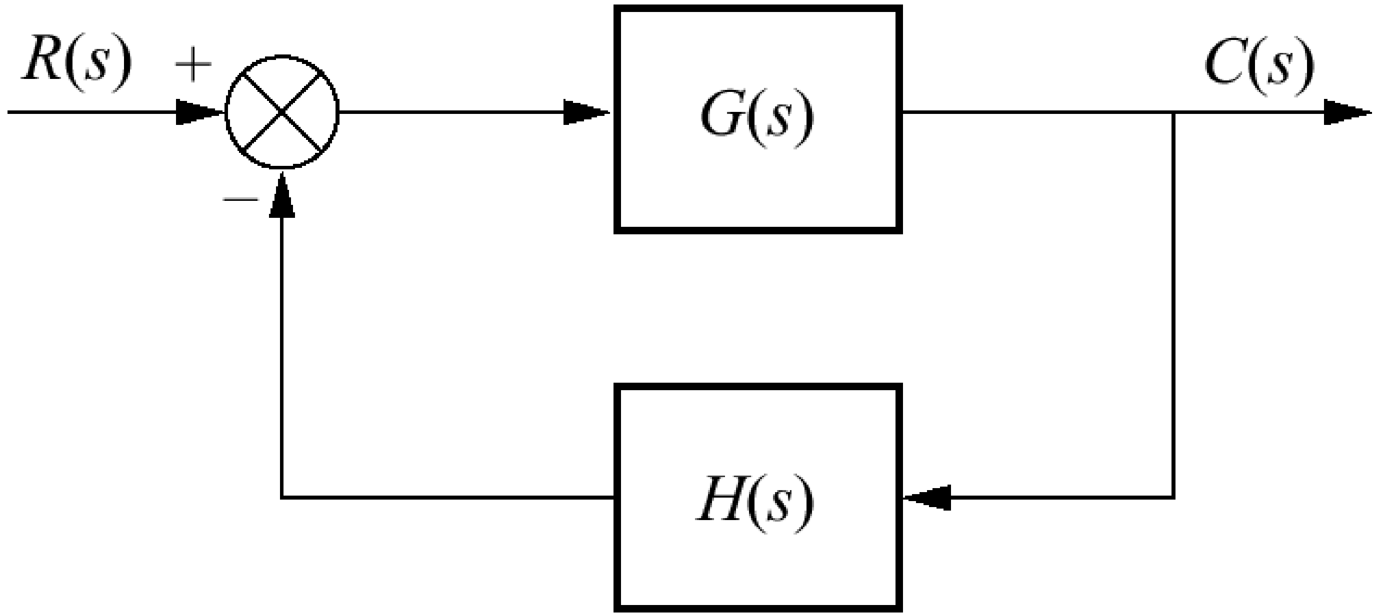


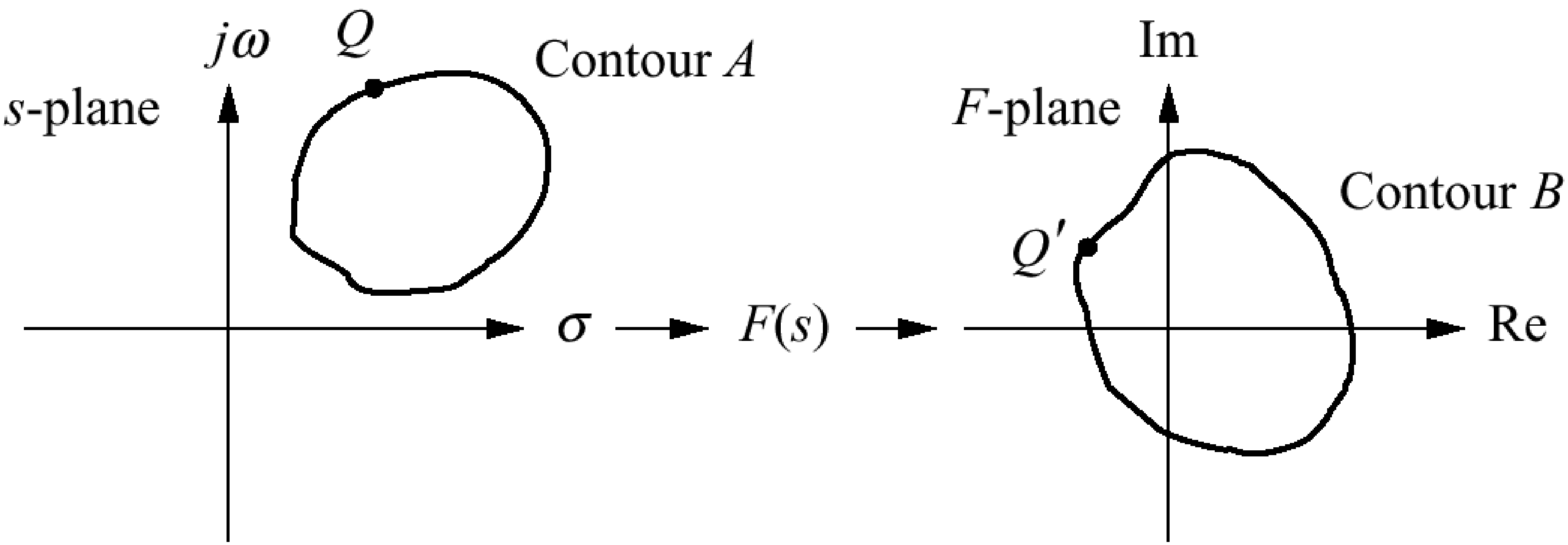
Fig. 9-34. Polar plots of transfer functions with numerator dynamics.

Introduction to the Nyquist Criterion

The NyC relates the stability of a closed-loop system to opened-loop frequency response and open loop pole location Thus knowledge of the opened-loop system's frequency response yields information about the stability of the closed-loop system.



Closed- loop control system



Mapping contour A through function $F(s)$ to contour B

Examples of contour mapping

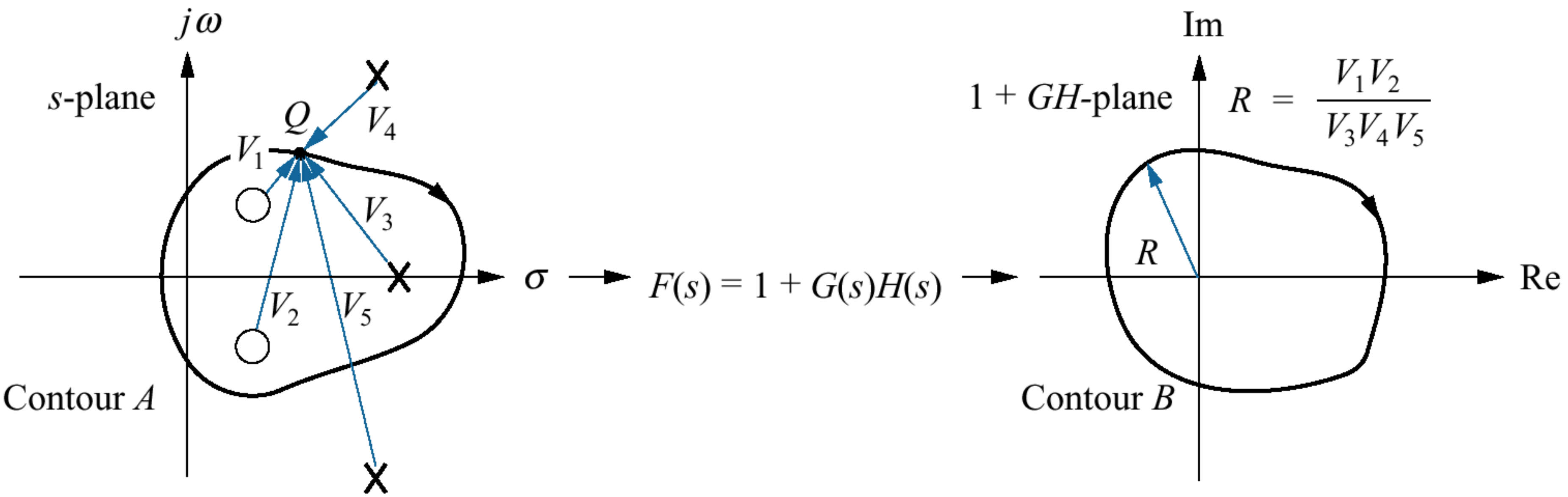
Zero outside the contour

Pole outside the contour

Zero inside the contour

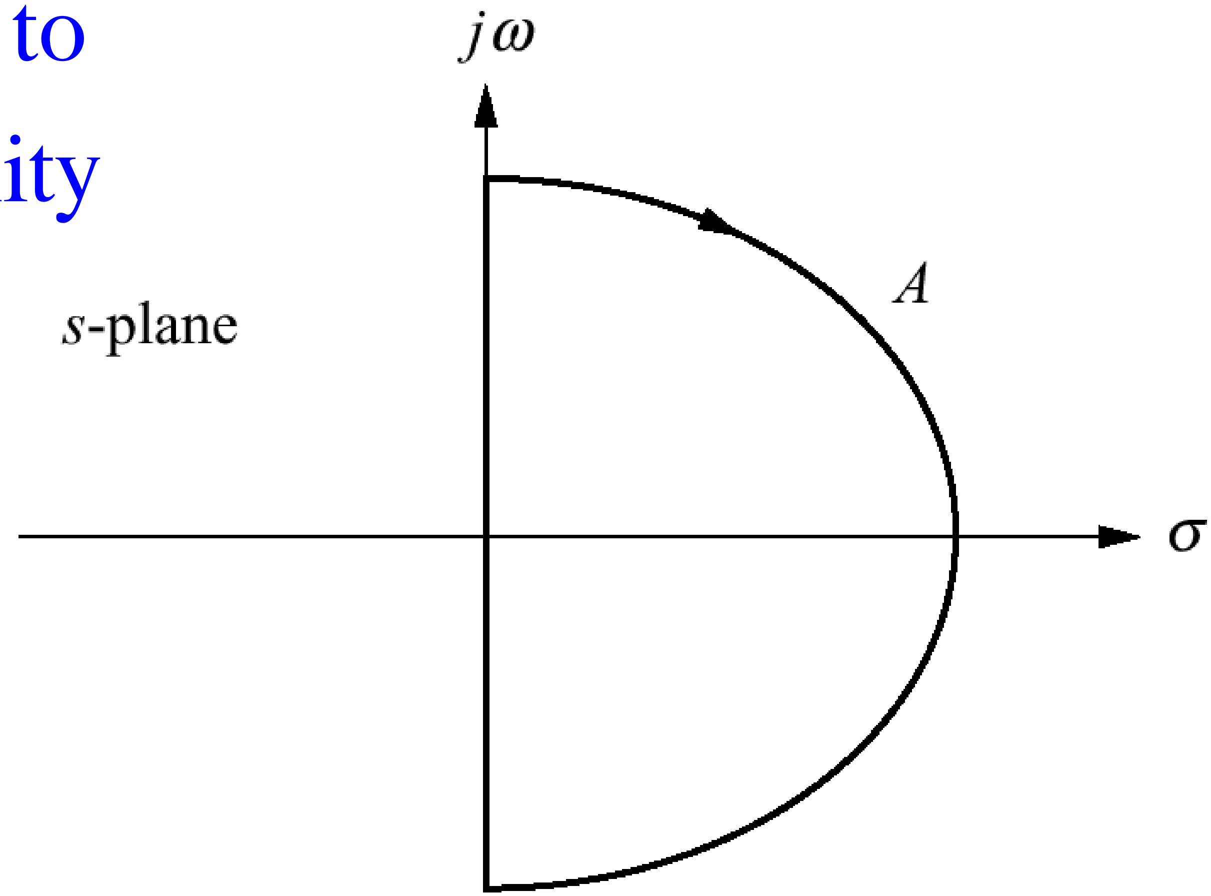
Pole inside the contour

Pole and zero inside the contour



Vector representation of mapping

Contour enclosing right half-plane to determine stability



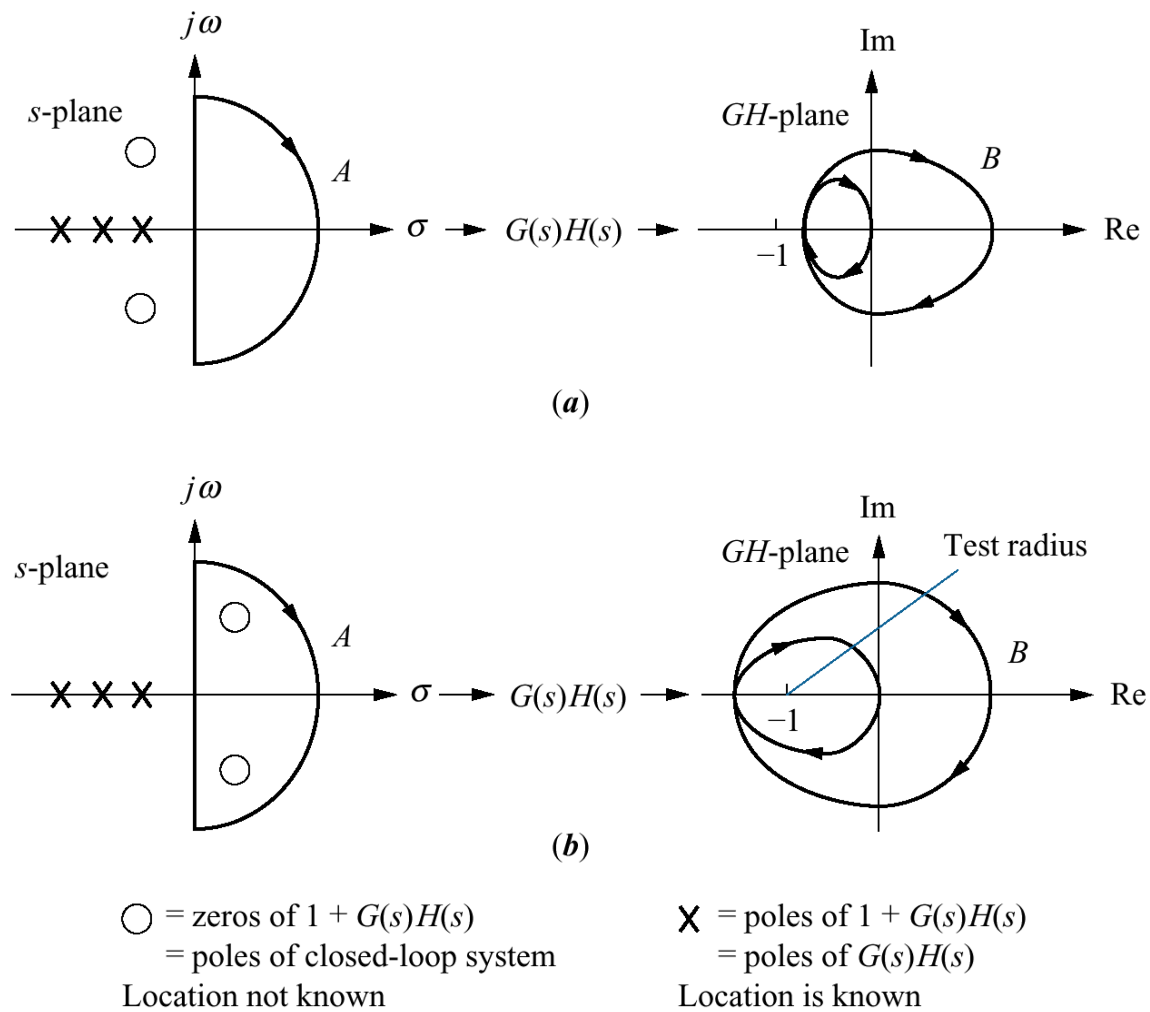
Nyquist Stability Criterion

If a contour A that encircles the entire right half-plane is mapped through G(s)H(s), then the number of closed-loop poles, Z, in the right half-plane equals the number of open-loop poles, P, that are in the right half plane minus the number of counterclockwise revolutions, N, around -1 of the mapping; that is, Z=P-N. The mapping is called the Nyquist diagram, or Nyquist plot of G(s)H(s).

Mapping examples:

a. contour does not enclose closed- loop poles;

b. contour does enclose closed- loop poles

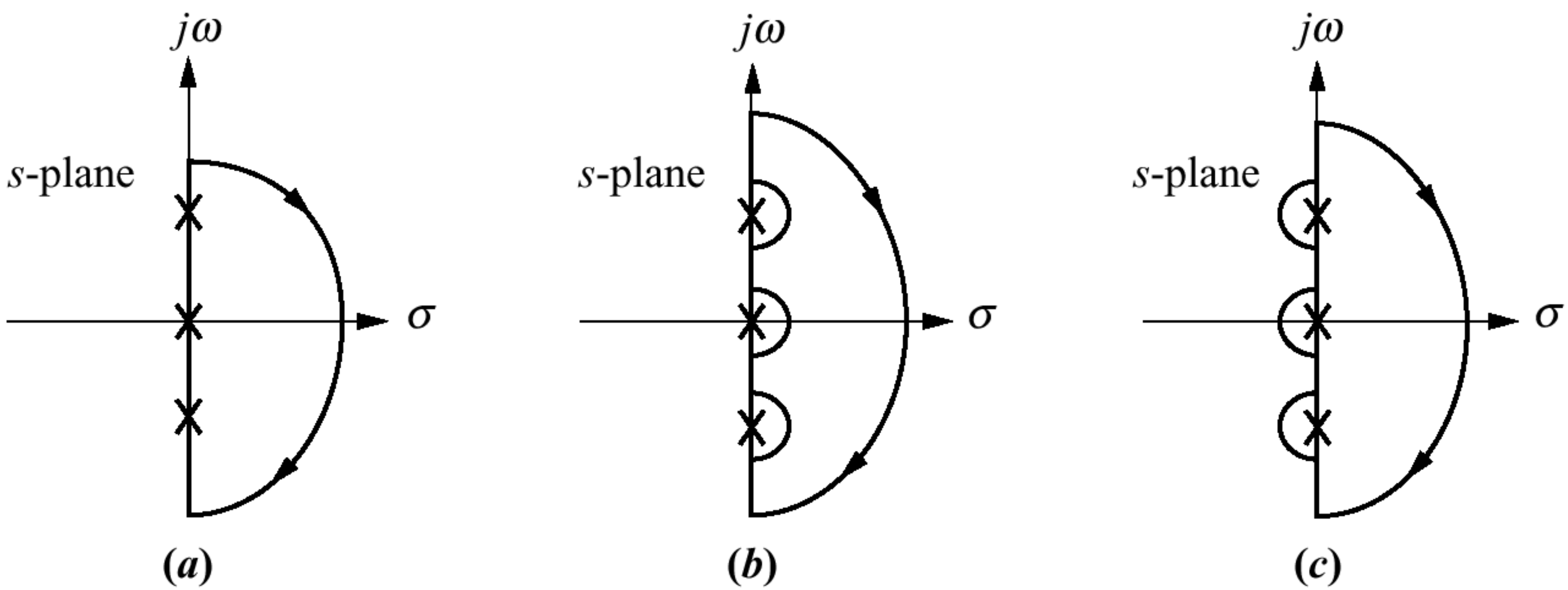


Detouring around open-loop poles:

a. poles on contour;

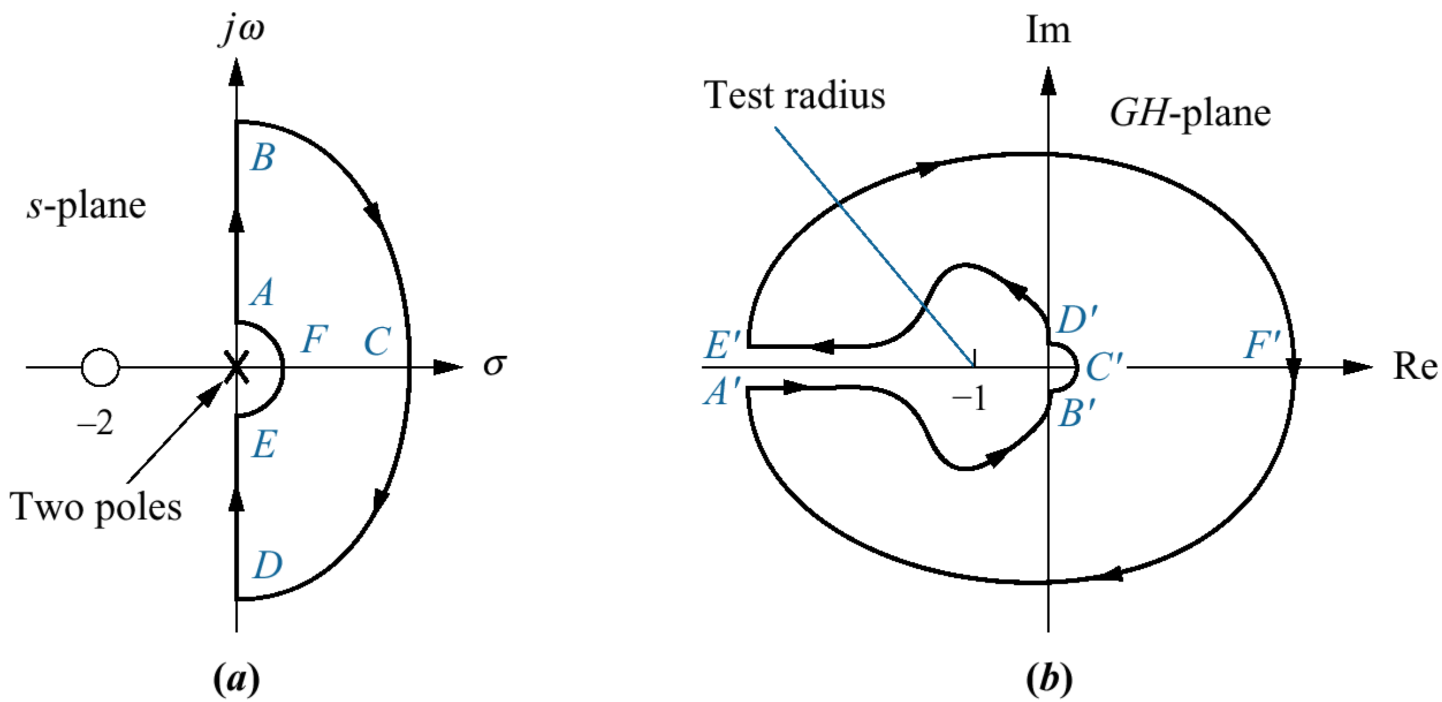
b. detour right;

c. detour left



a. Contour;

b. Nyquist diagram



Stability via the Nyquist Diagram

If the Nyquist path in s-plane encircles Z zeros and P poles of 1+G(s)H(s) and does not pass through any poles or zeros of 1+G(s)H(s) as a representative point s moves in the clockwise direction along the Nyquist path, then the corresponding contour in the G(s)H(s)-plane encircles the -1+j0 point N=Z-P time in clockwise direction (Negative values of N imply counter-clockwise encirclements)

1. There is no encirclement of the -1+j0 point. This impiles that the system is stable if there are no poles of G(s)H(s) in the right-half s-plane, otherwise the system is unstable.

2. There is a counterclockwise encirclement or encirclements of the -1+j0 point. In this case, the system is stable if the number of counterclockwise encirclements is the same as the nmber of polse of G(s)H(s) in the right-half s-plane, otherwise the system is unstable.

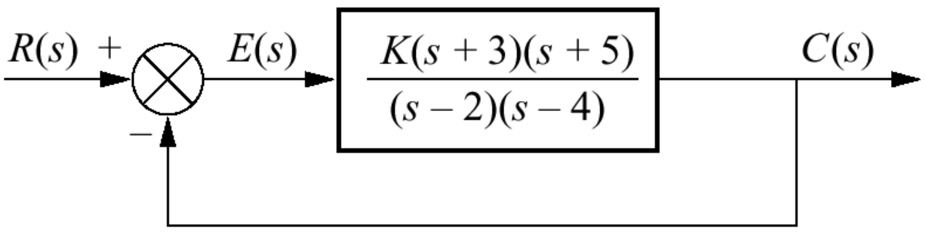
3. There is a clockwise encirclement or encirclements of the -1+j0 point. In this case, the system is unstable.

Demonstrating Nyquist stability:

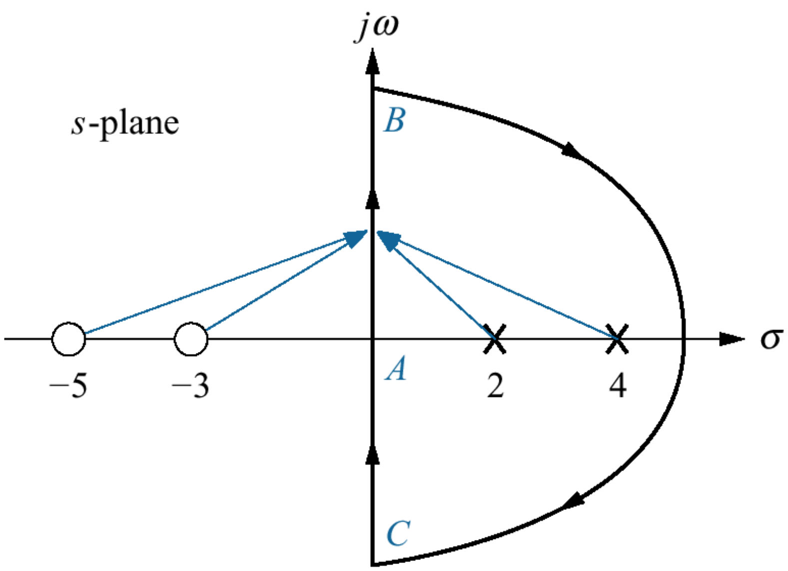
a. system;

b. contour;

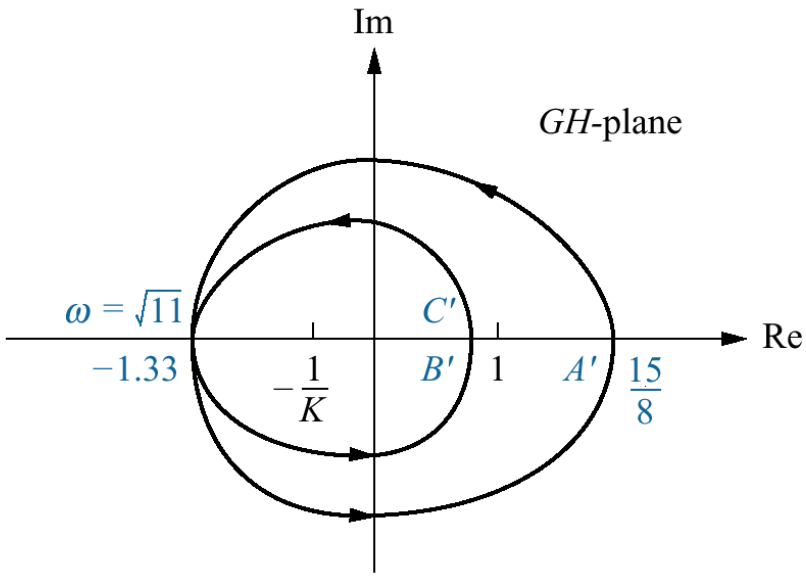
c. Nyquist diagram



(a)



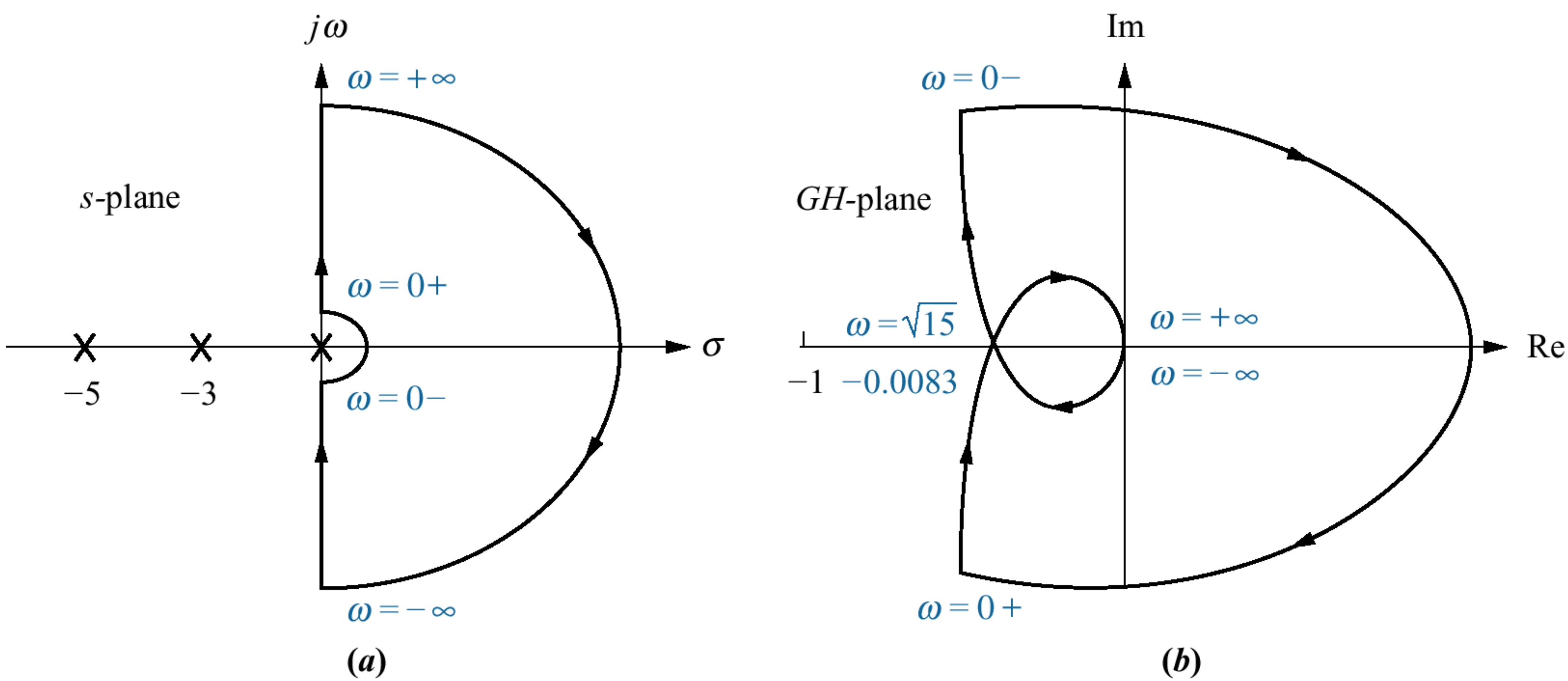
(b)



(c)

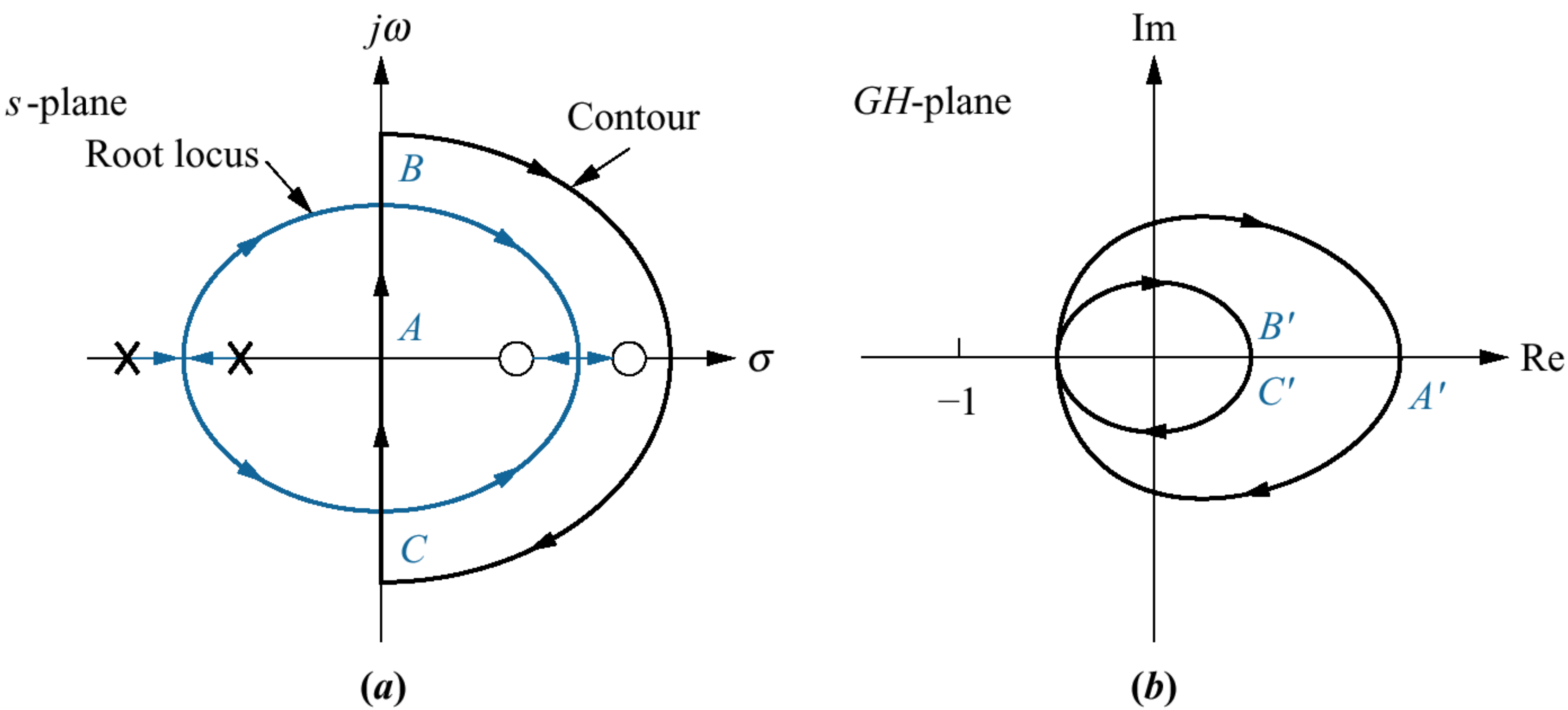
a. Contour ;

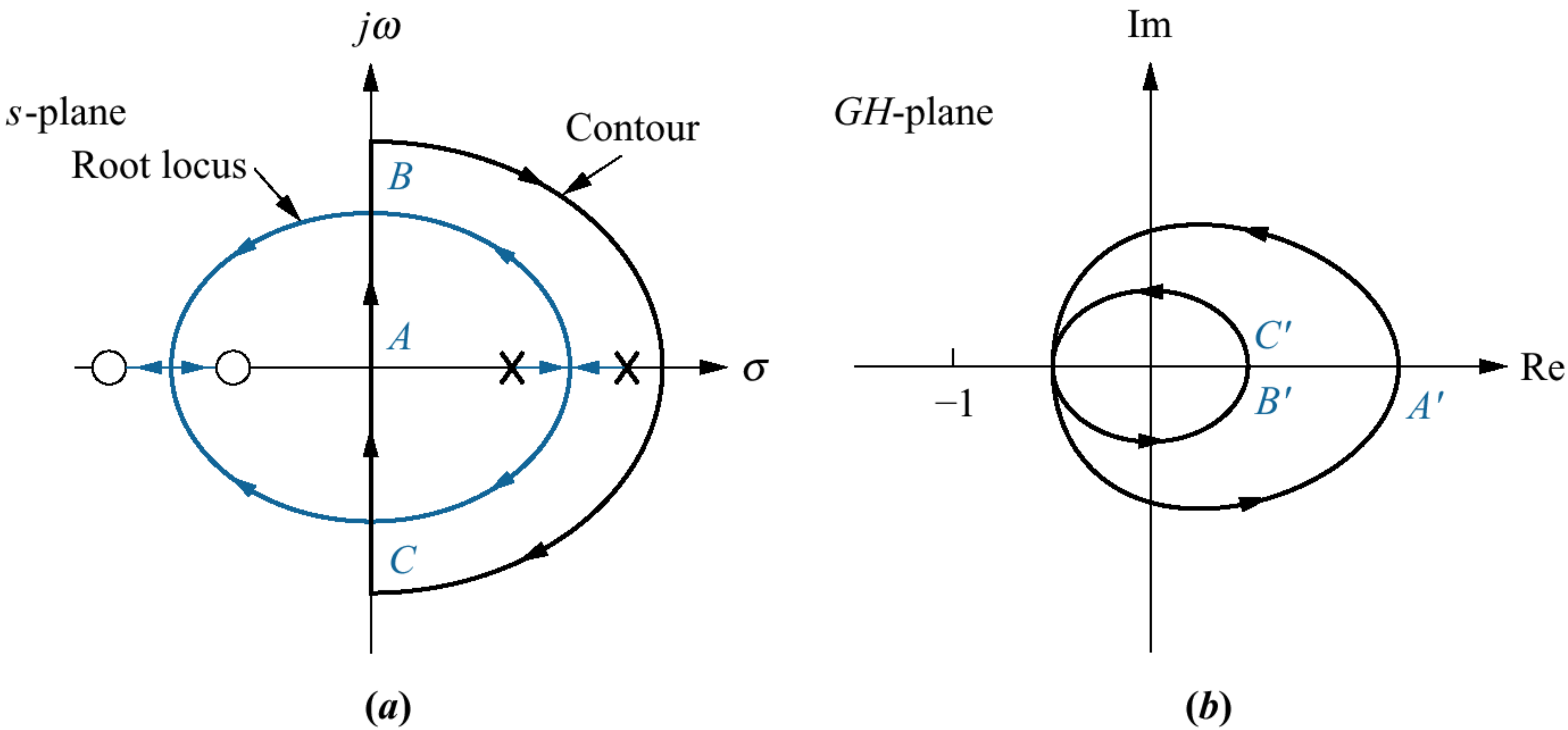
b. Nyquist diagram



a. Contour and root locus of system that is stable for small gain and unstable for large gain;

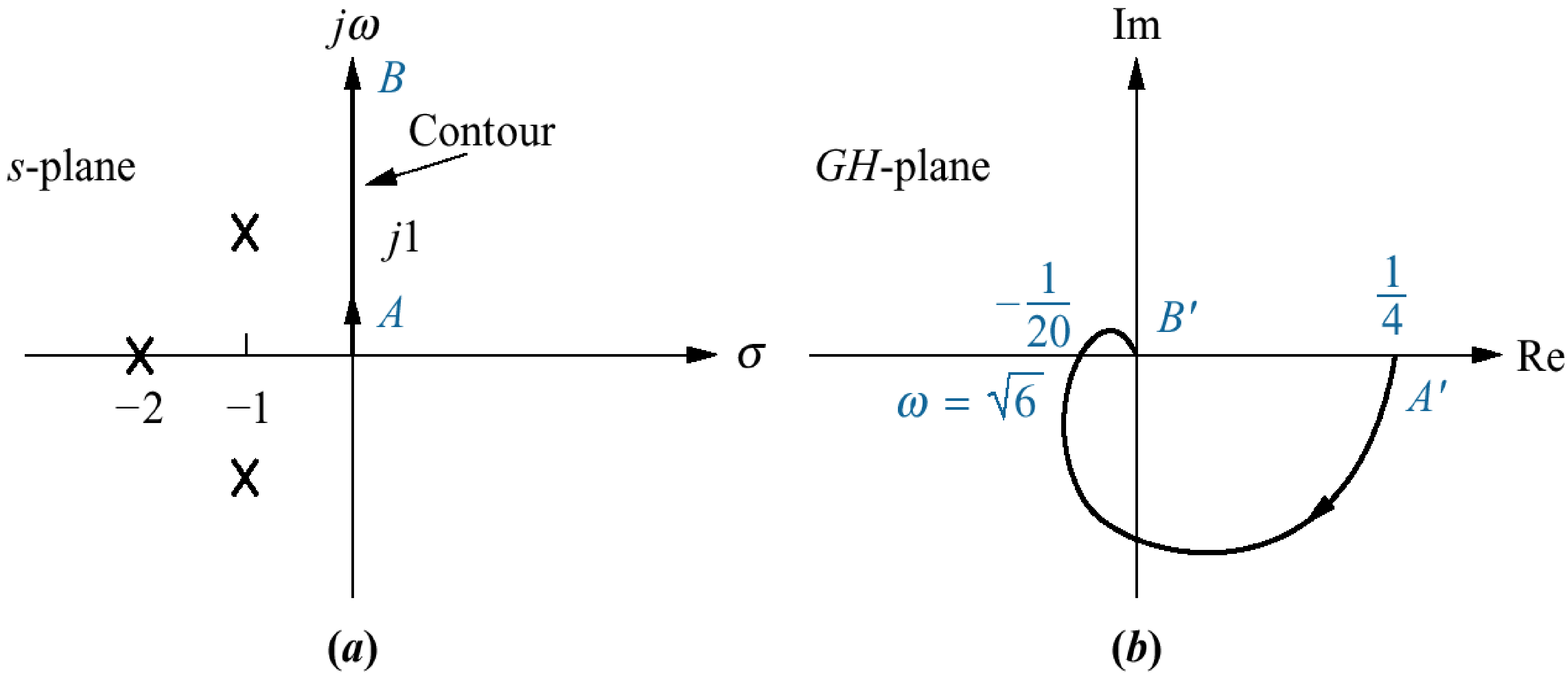
b. Nyquist diagram





a. Contour and root locus of system that is unstable for small gain and stable for large gain;

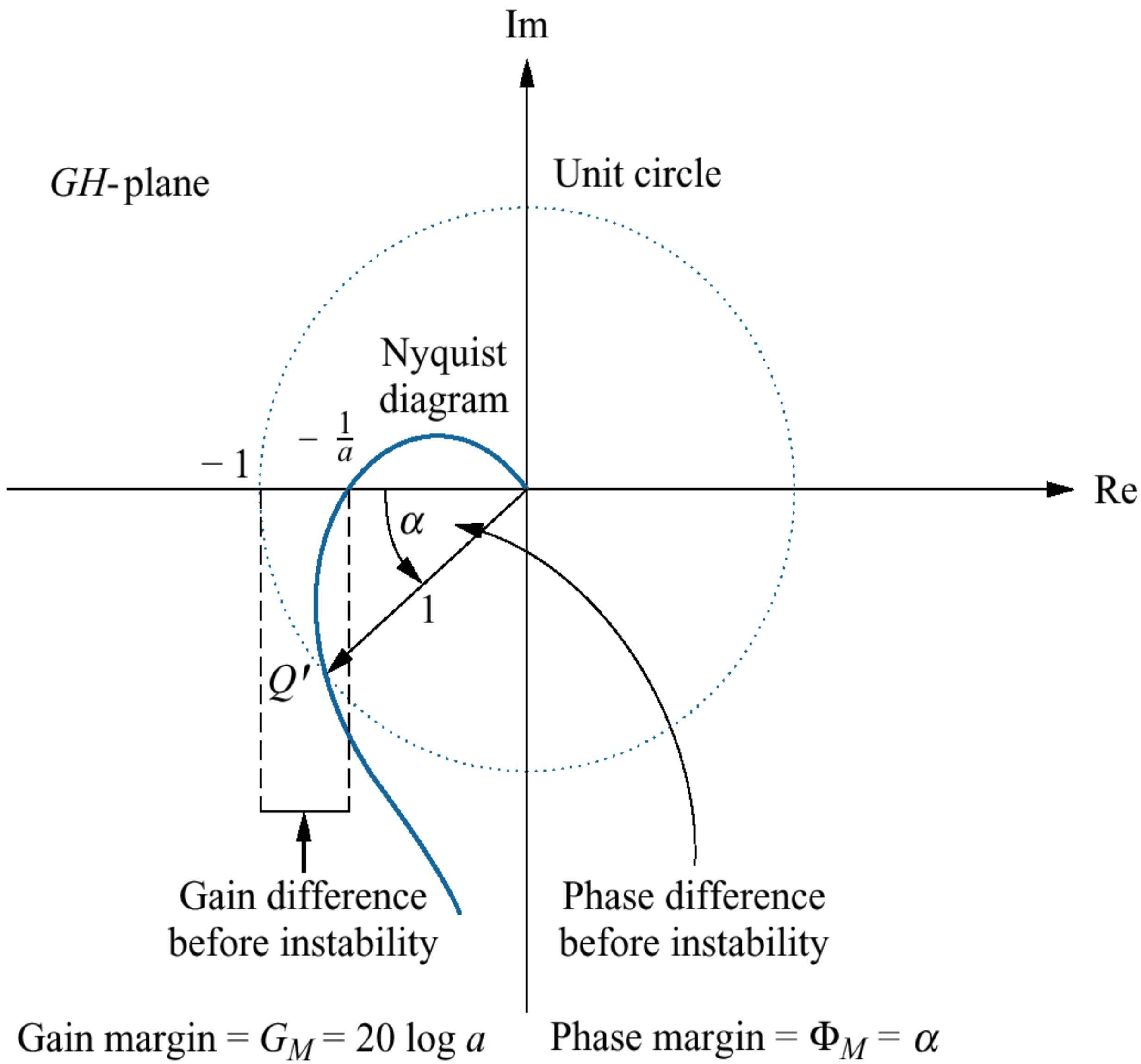
b. Nyquist diagram



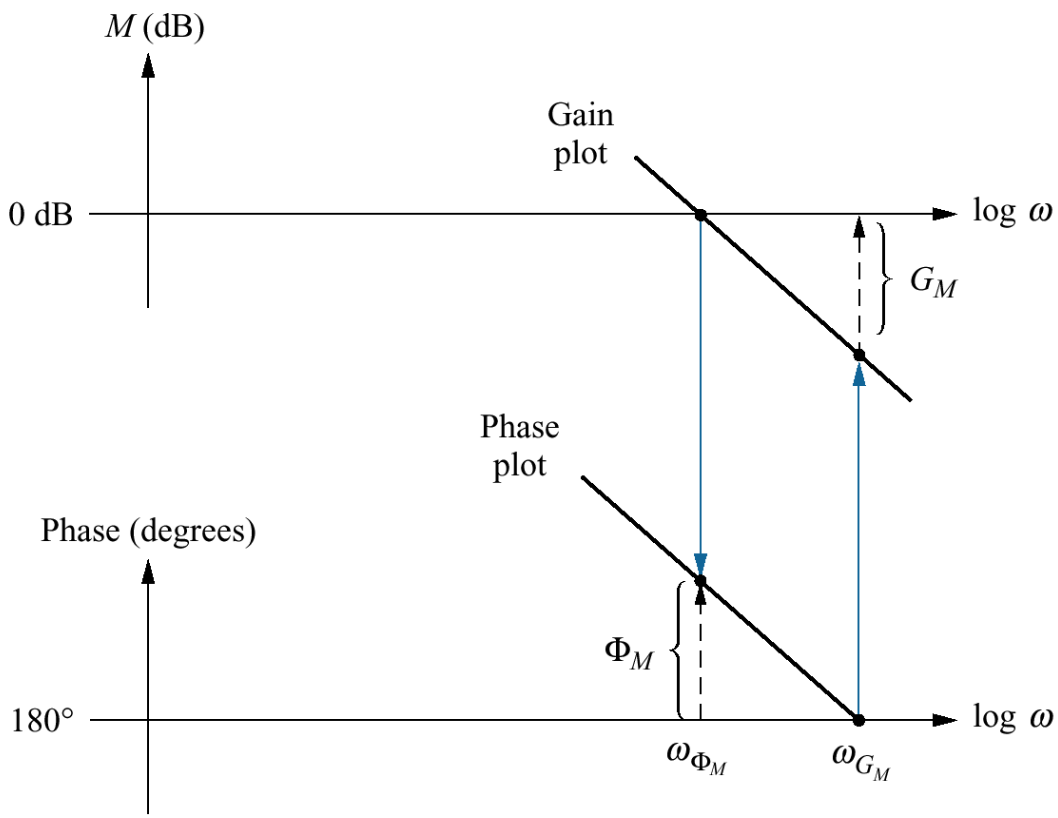
a. Portion of contour to be mapped for Example 10.7

b. Nyquist diagram of mapping of positive imaginary axis

Gain Margin and Phase Margin



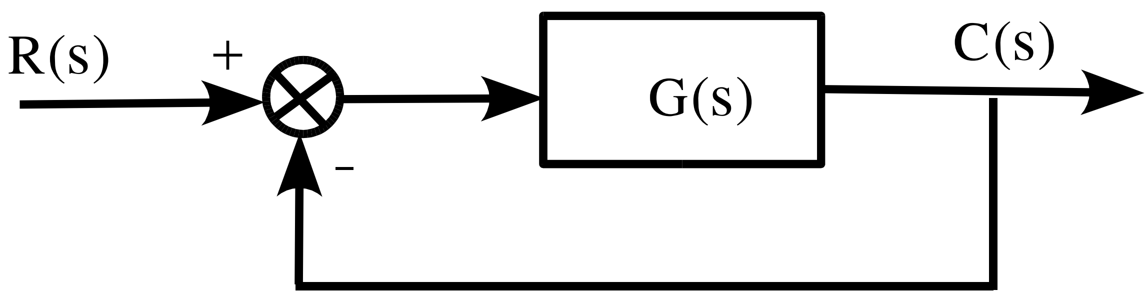
Nyquist diagram showing gain and phase margins



Gain and phase margins on the Bode diagrams

Relation between Closed-Loop and Open-Loop Frequency Response

Constant M cycles and Constant N cycles



Close-loop transfer function and frequency response

$$T(s) = \frac{G(s)}{1+G(s)} \qquad T(j\omega) = \frac{G(j\omega)}{1+G(j\omega)}$$

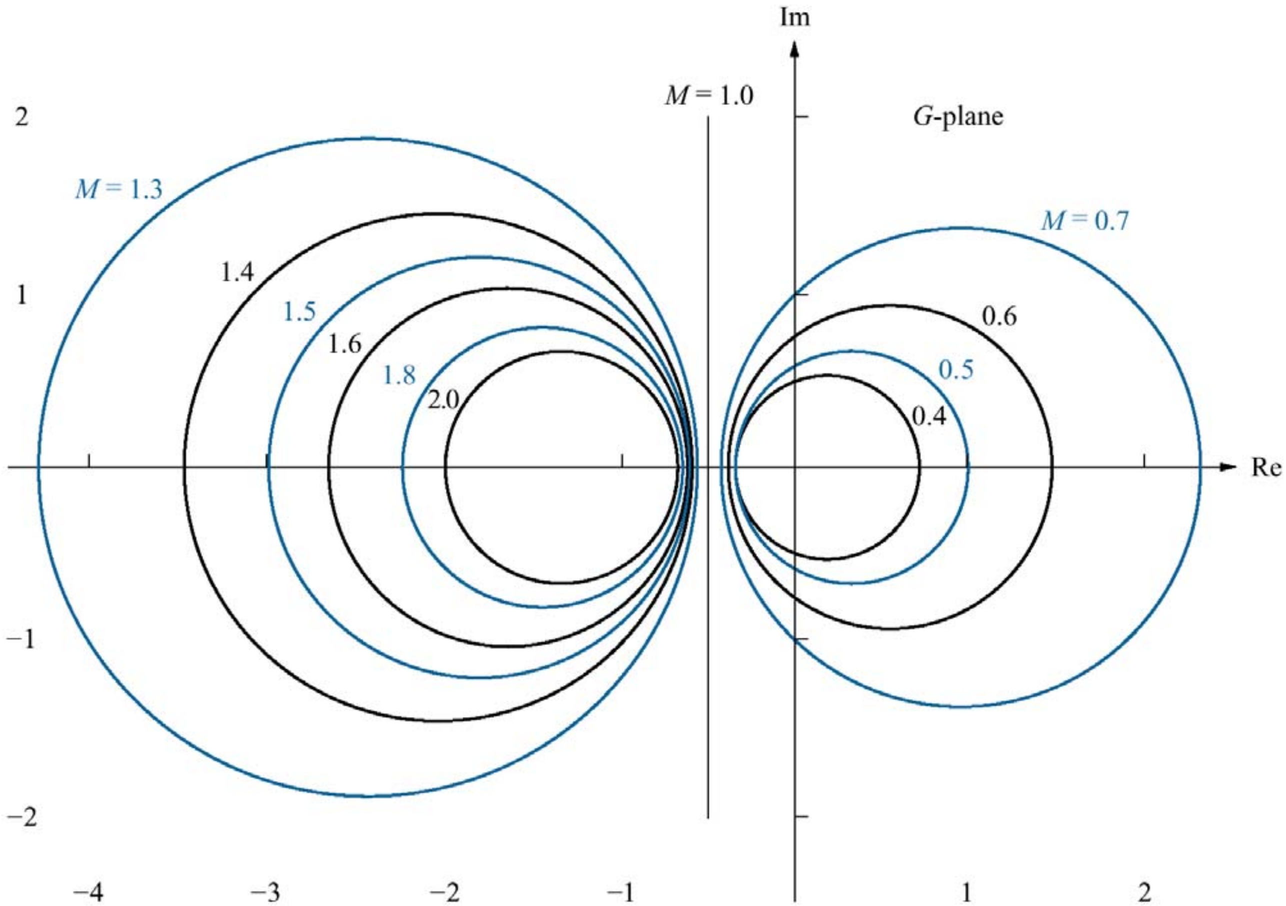
Let $$G(j\omega) = P(\omega) + jQ(\omega)$$

$$T(j\omega) = \frac{P(\omega) + jQ(\omega)}{(P(\omega)+1)^2 + Q^2(\omega)}$$

$$M^2 = |T^2(j\omega)| = \frac{P(\omega) + jQ(\omega)}{(P(\omega)+1)^2 + Q^2(\omega)}$$

$$\left(P + \frac{M^2}{M^2 - 1}\right)^2 + Q^2 = \frac{M^2}{(M^2-1)^2}$$

Constant *M* circles



$$\phi = \arctan \frac{Q(\omega)}{P(\omega)} - \arctan \frac{Q(\omega)}{P(\omega)+1}$$

$$\tan(A \pm B) = \frac{\tan A \pm \tan B}{1 \pm \tan A \tan B}$$

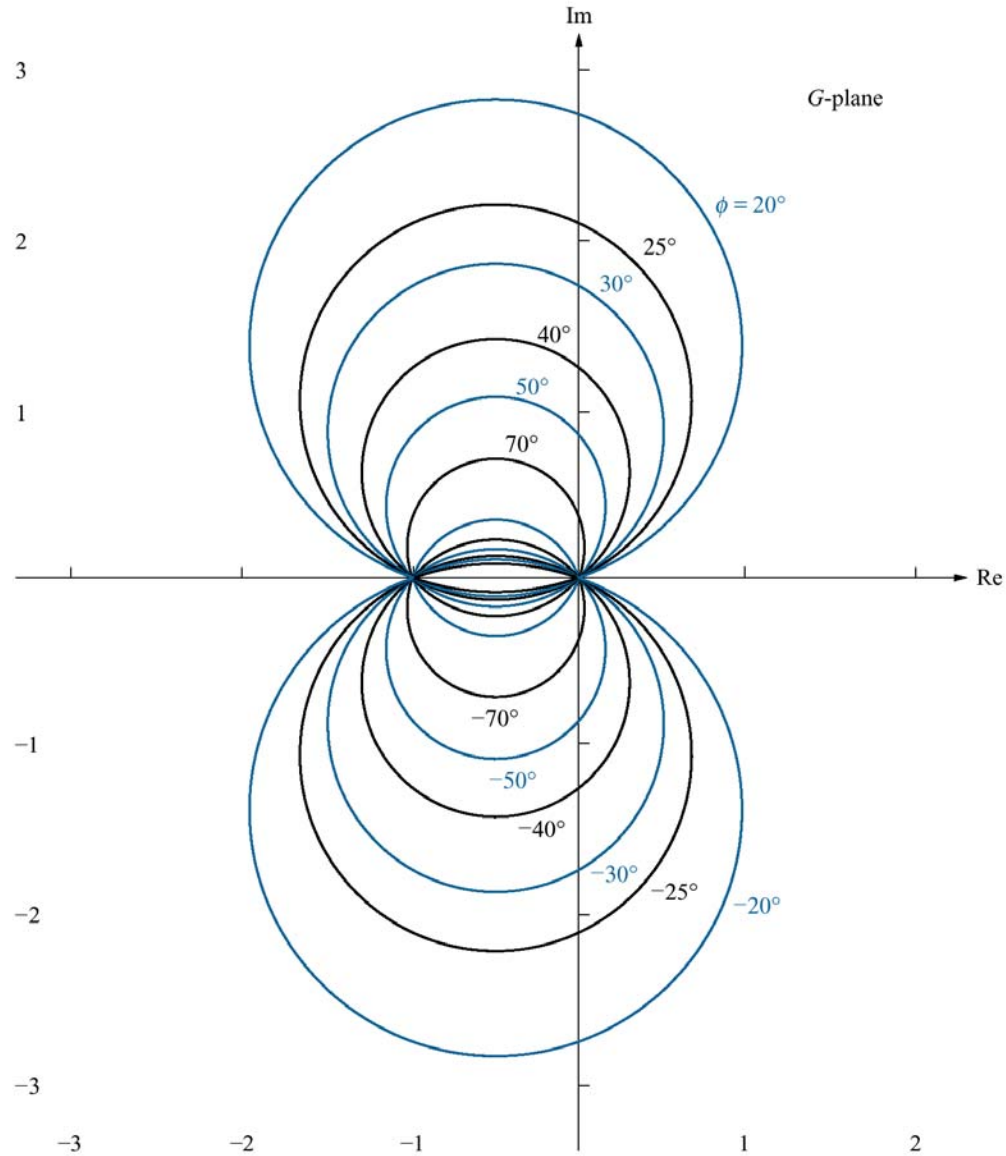
$$N = \tan(\phi) = \frac{\dfrac{Q(\omega)}{P(\omega)} - \dfrac{Q(\omega)}{P(\omega)+1}}{1 + \dfrac{Q(\omega)}{P(\omega)} \dfrac{Q(\omega)}{P(\omega)+1}}$$

$$N = \frac{Q}{P^2 + P + Q^2}$$

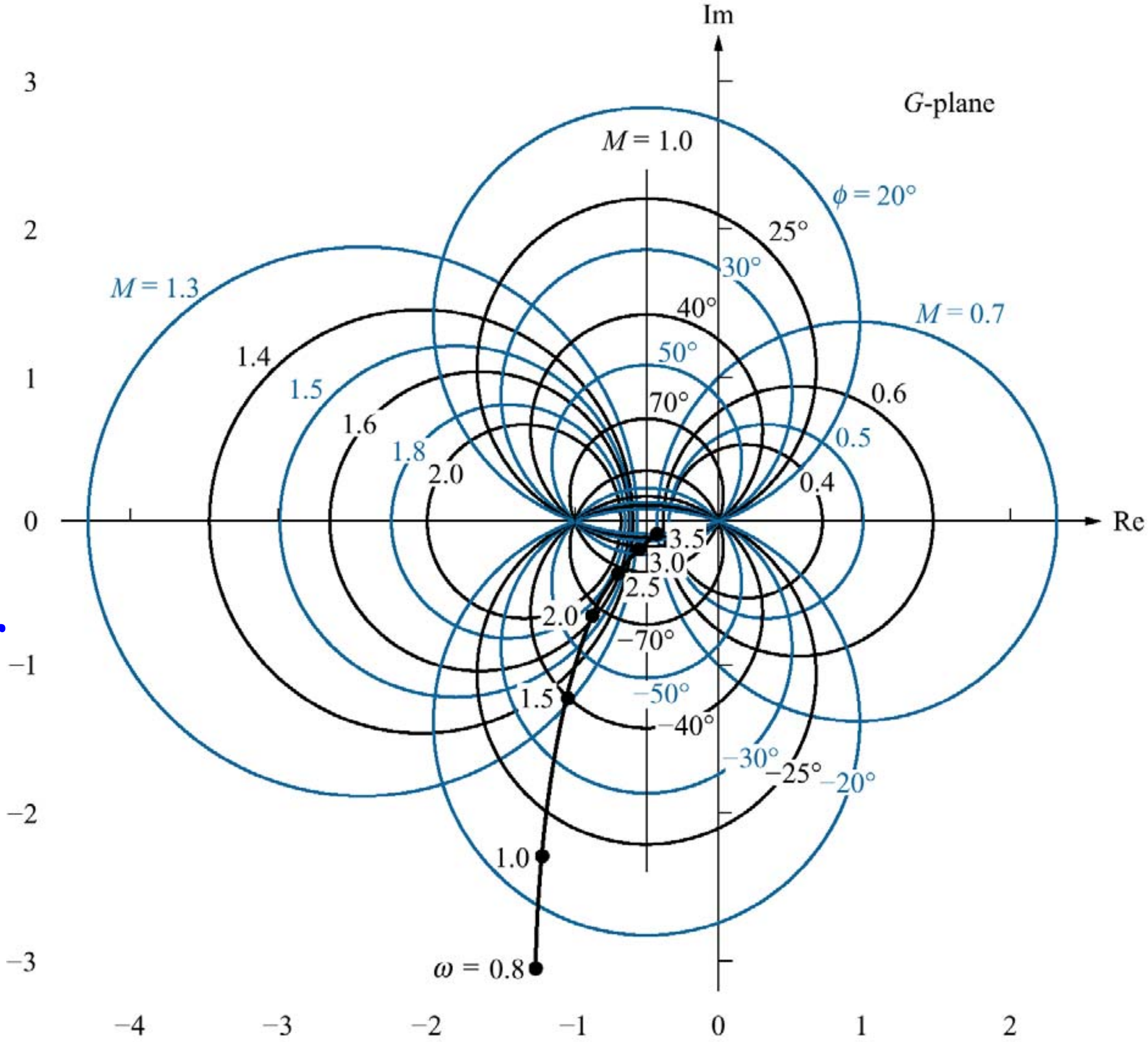
$$P^2 + P + Q^2 + \frac{Q}{N} = 0$$

$$\left(P + \frac{1}{2}\right)^2 + \left(Q - \frac{1}{2N}\right)^2 = \frac{N^2 + 1}{4N^2}$$

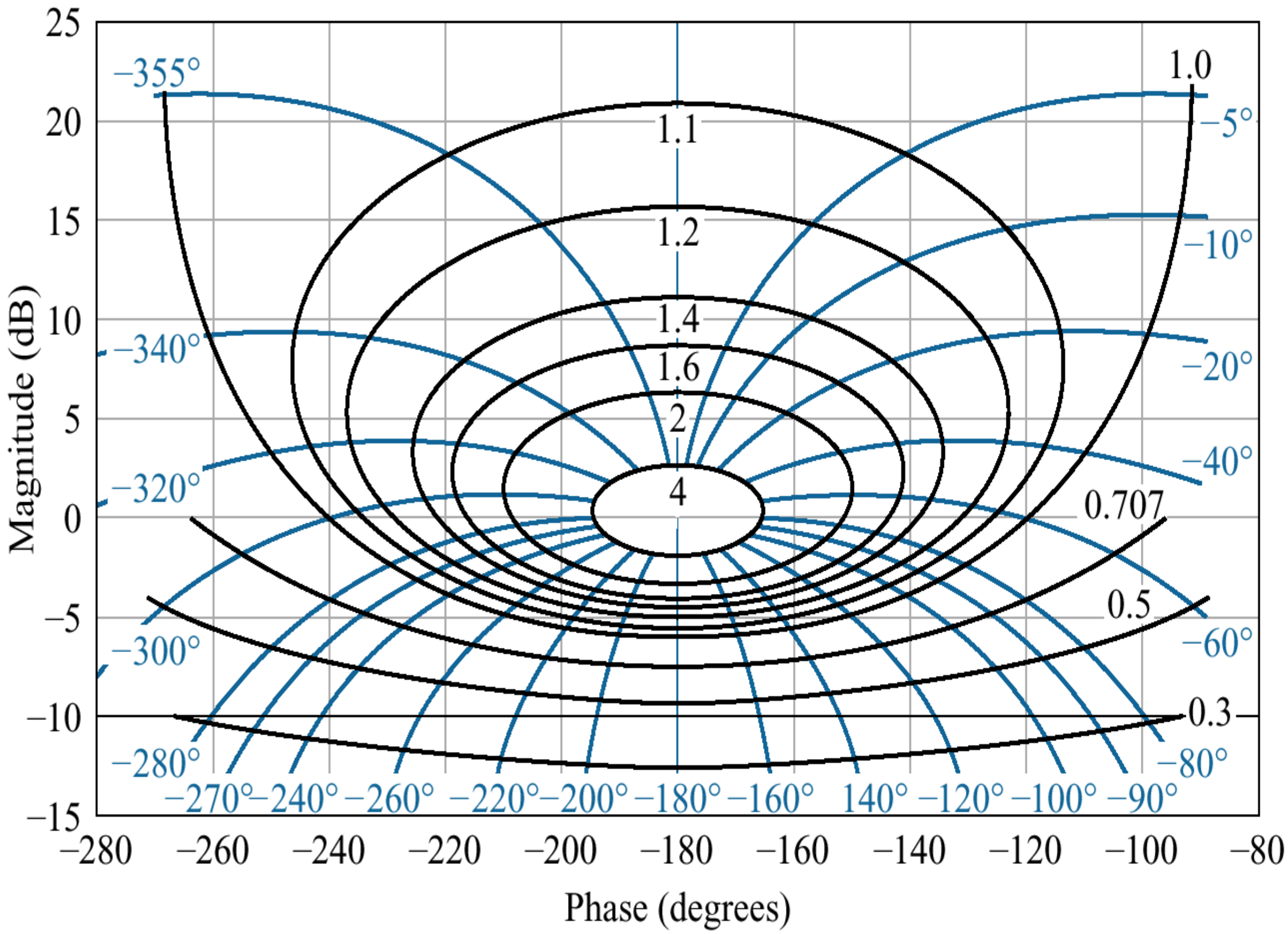
Constant N circles



Nyquist diagram for Example 10.11 and constant M and N circles



Nichols Chart



Steady-State Error Characteristics from Frequency Response

Typical unnormalized and unscaled Bode log-magnitude plots showing the value of static error constants:

a. Type 0;

b. Type 1;

c. Type 2