Chapter 4

KINEMATICS : MANIPULATOR POSITION

Outline

1 Joint
2 Link
3 General links
4 Assignment of coordinate frames
5 Trigonometric solution
6 A matrics
7 Homogeneous transformations

Principle of Robotics

Outline(cont.)

8 Direct kinematics
9 Vector solution
10 Solving general orientation transform
11 Inverse kinematics
12 Redundancy and degeneracies
13 Programming
14 Accuracy of the kinematic model

Principle of Robotics

Outline(cont.)

1 5 Efficiency of the kinematic

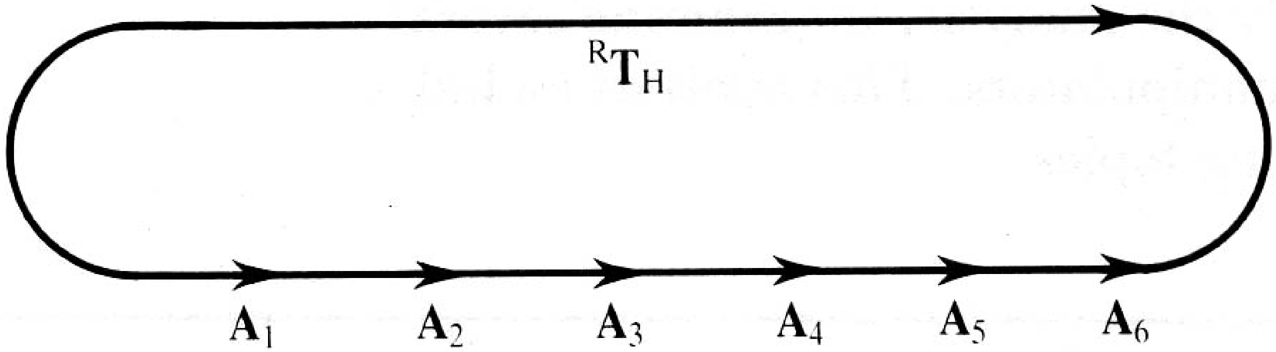
1 6 Example

Kinematic is the relationships between the position, velocities, and accelerations of the links of manipulator, where a manipulator is an arm, finger, or leg.

Serial link manipulator is a series of link, which connects the hand to the base, with each link connected to the next by an actuated joint.

A matrix is a homogeneous transformation matrix that describes the relationship between two links.

The first ***A*** matrix relates the first link to the base frame, and the last ***A*** matrix relates the hand frame to the last link.



Six-link manipulator transform graph.

$$ {}^{R}\mathbf{T}_{N} = {}^{R}\mathbf{T}_{\text{๑}}\,{}^{\text{๑}}\mathbf{T}_{\text{๒}}\cdots{}^{n-\text{๒}}\mathbf{T}_{n-\text{๑}}\,{}^{n-\text{๑}}\mathbf{T}_{H} = \boldsymbol{A}_{\text{๑}}\boldsymbol{A}_{\text{๒}}\cdots\boldsymbol{A}_{n-\text{๑}}\boldsymbol{A}_{n} $$

Direct kinematics involves solving the forward transformation equation to find location of the hand in terms of the angles and displacements between the links.

Inverse kinematics involves solving the inverse transformation equation to find the relationships between the links of the manipulator from the location of the hand in space.

Direct Kinematic Algorithm

๑. Move the manipulator to its zero position.
๒. Assign a coordinate frame to each link.
๓. Describe the rotations and translations between joints with link variables.
๔. Define the A matrices relating the links.
๕. Multiply the A matrices to calculate the manipulator transformation matrix ${}^{R}\mathbf{T}_{H}$.

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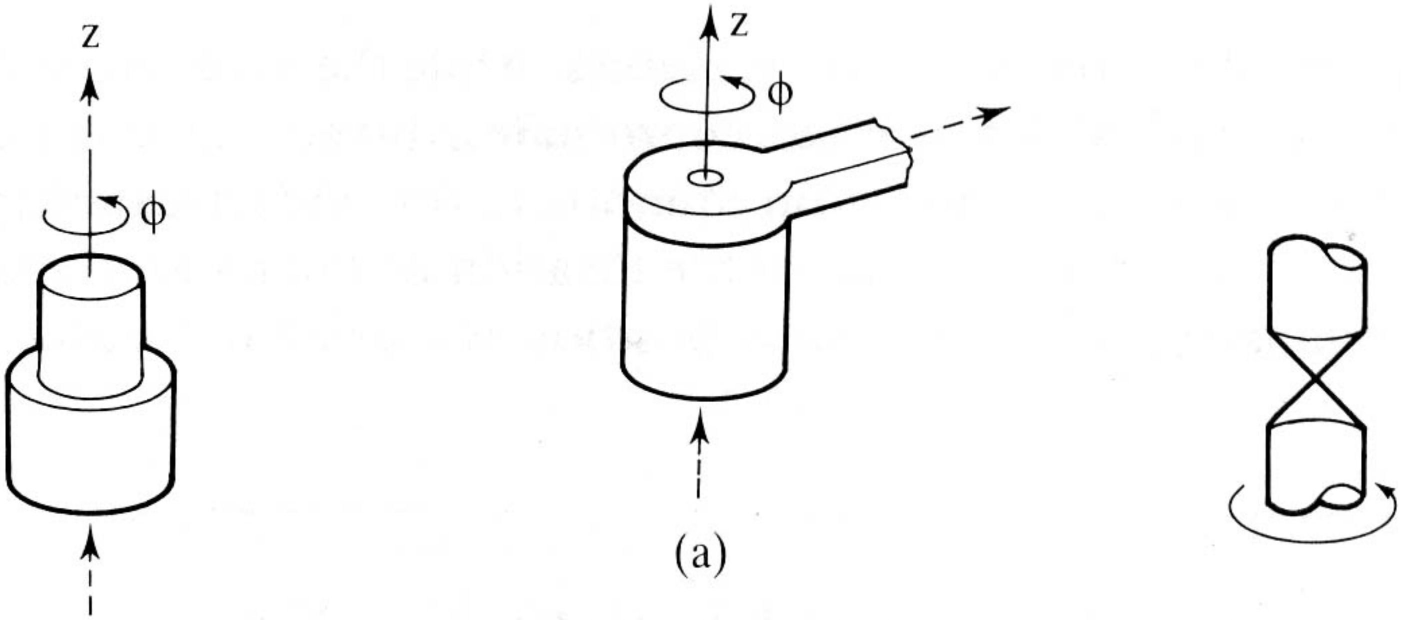
๖. Equate the manipulator transformation matrix and the general transformation matrix to obtain Cartesian coordinates in terms of joint coordinates.
๗. Equate the manipulator transformation matrix and the general orientation matrix to obtain the hand orientation angles.

๑ Joints

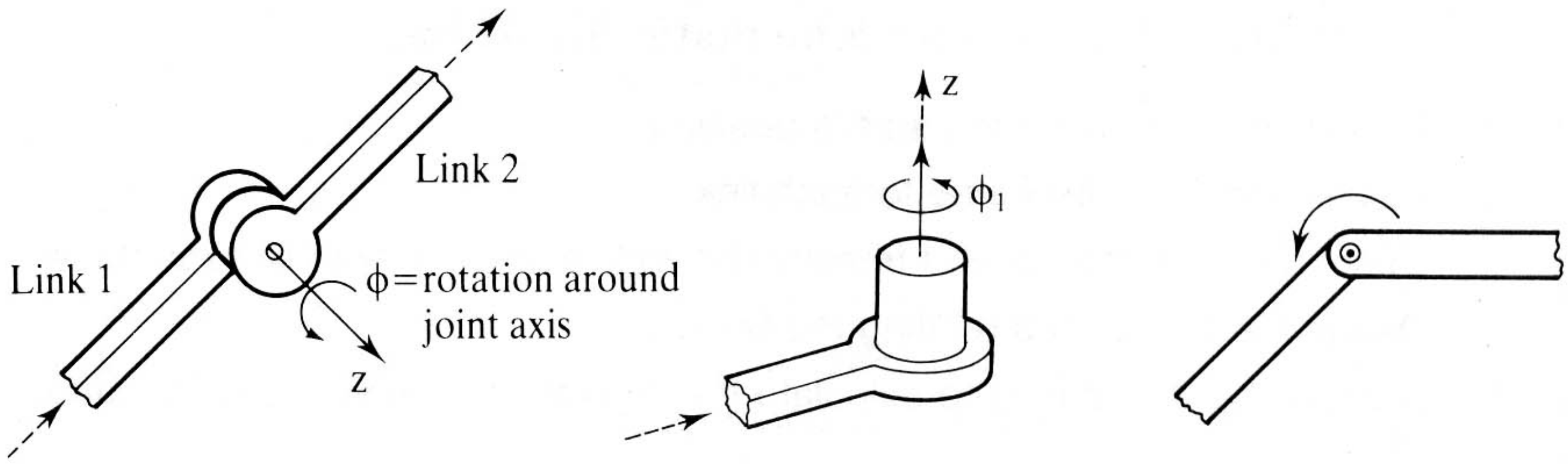
Two types of joints are commonly found in robots :

๑ Revolute or rotary joints provide one degree of rotation.

๒ Prismatic or Sliding joints provide one degree of translation.



Common joints found in robots : (a) Revolute joint with axis coincident with link



(b)

(b) Revolute joint with joint axis perpendicular to link

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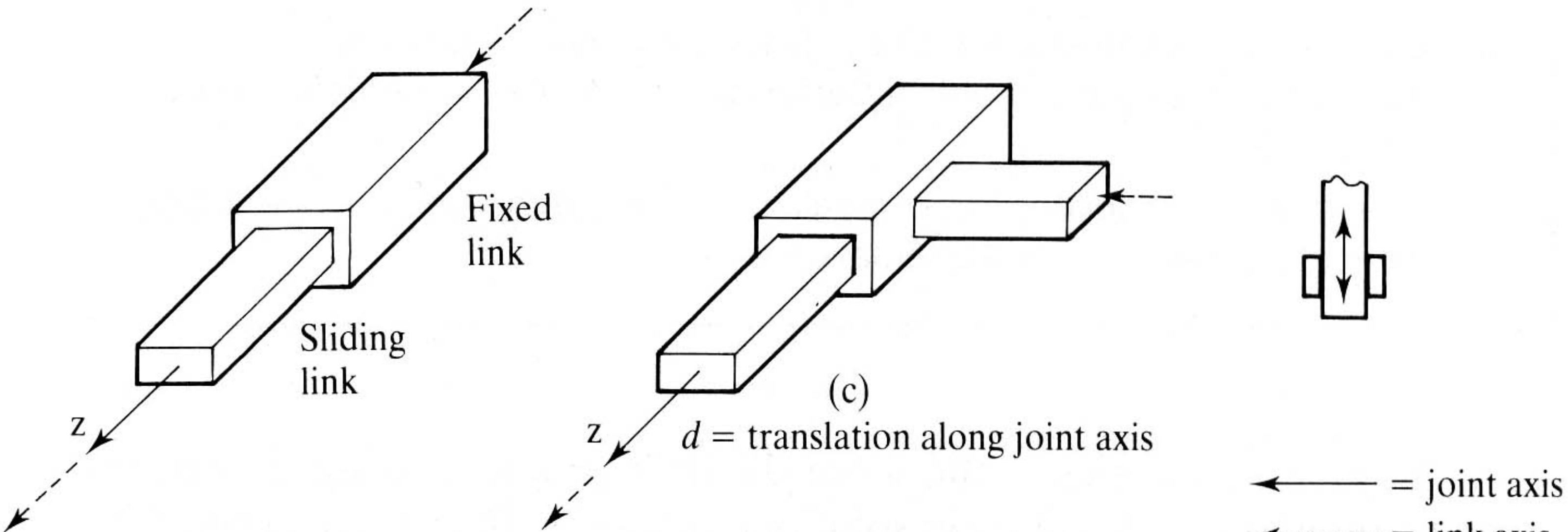


(c) Prismatic joint

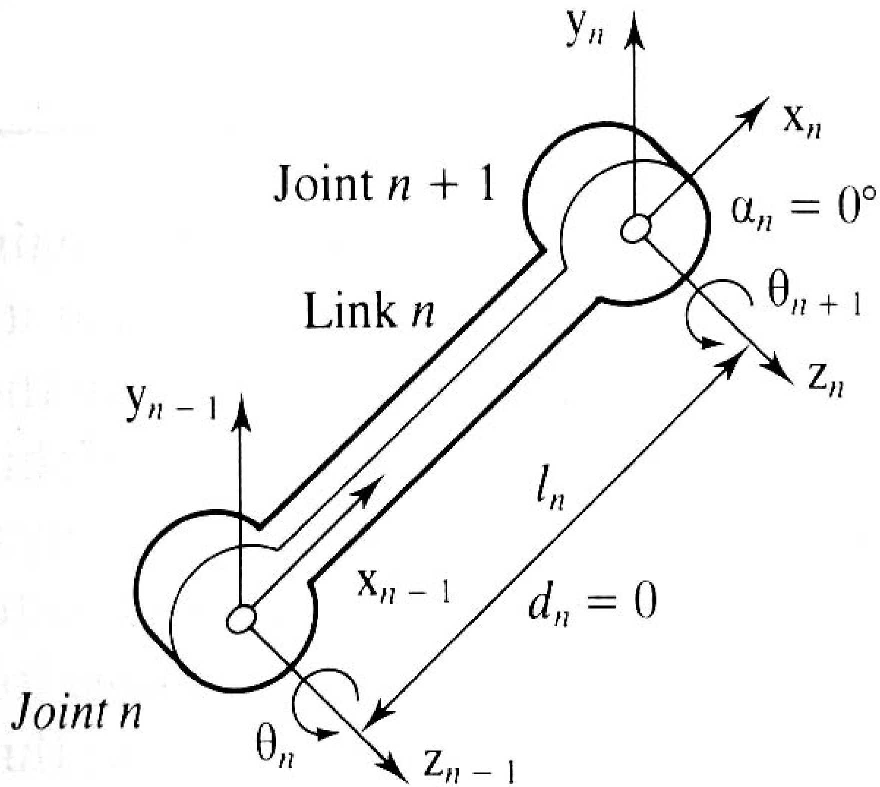
๒ Links

A ***link*** is a solid mechanical object which connect two joint.

The ***proximal end*** is the end closest to the base.

The ***distal end*** is the end furthest away from the base.

Type ๑ link has ๒ parallel revolute joint with no twist between axes; axes of the joints are parallel. These joint separated by a distance l_n, the length of link.



(a)

Typical link configulations connecting revolute joints. (a) Type ๑ link : parallel revolute joints with no twist between joint axes.

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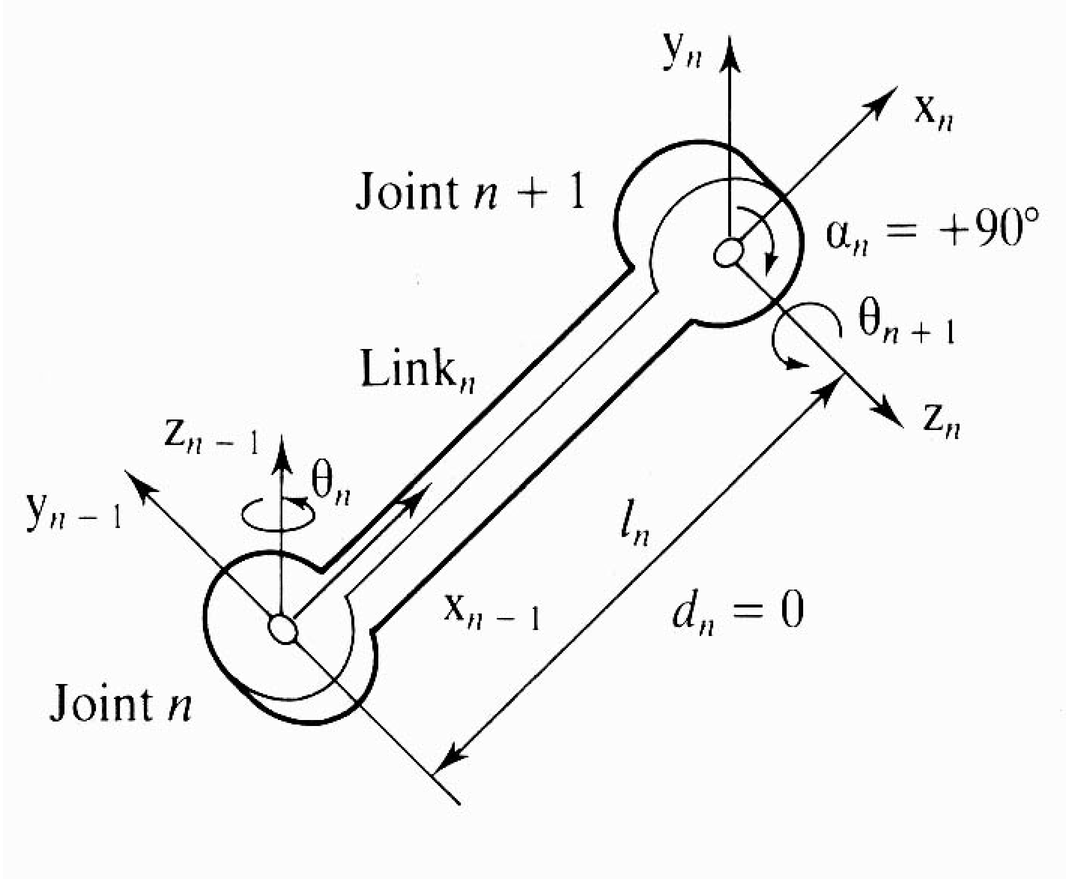
Type ๒ link If one of the joints in a type one link is twisted about the centre line of the link(axis $x_{n-๑}$), by the angle α_n



(b)

(b) Type ๒ link : parallel revolute joints with ๙๐ degree twist between joint axes.

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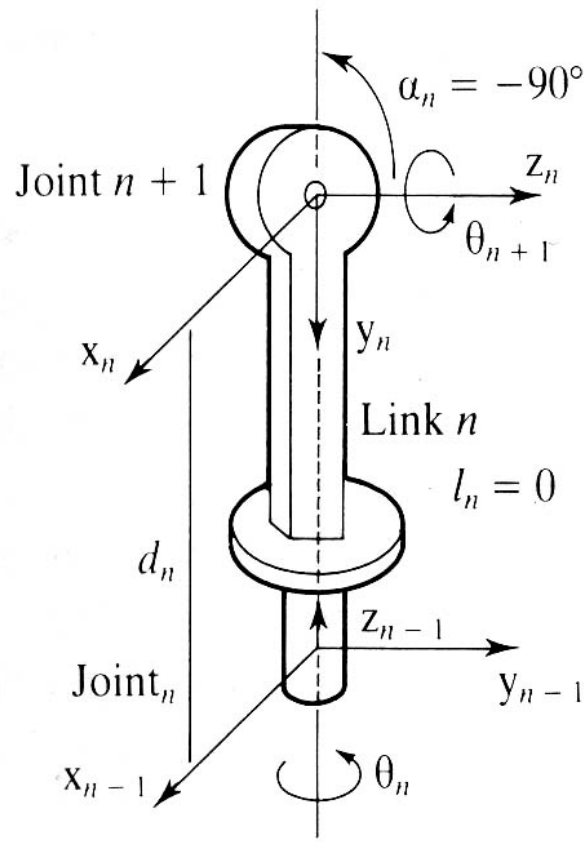
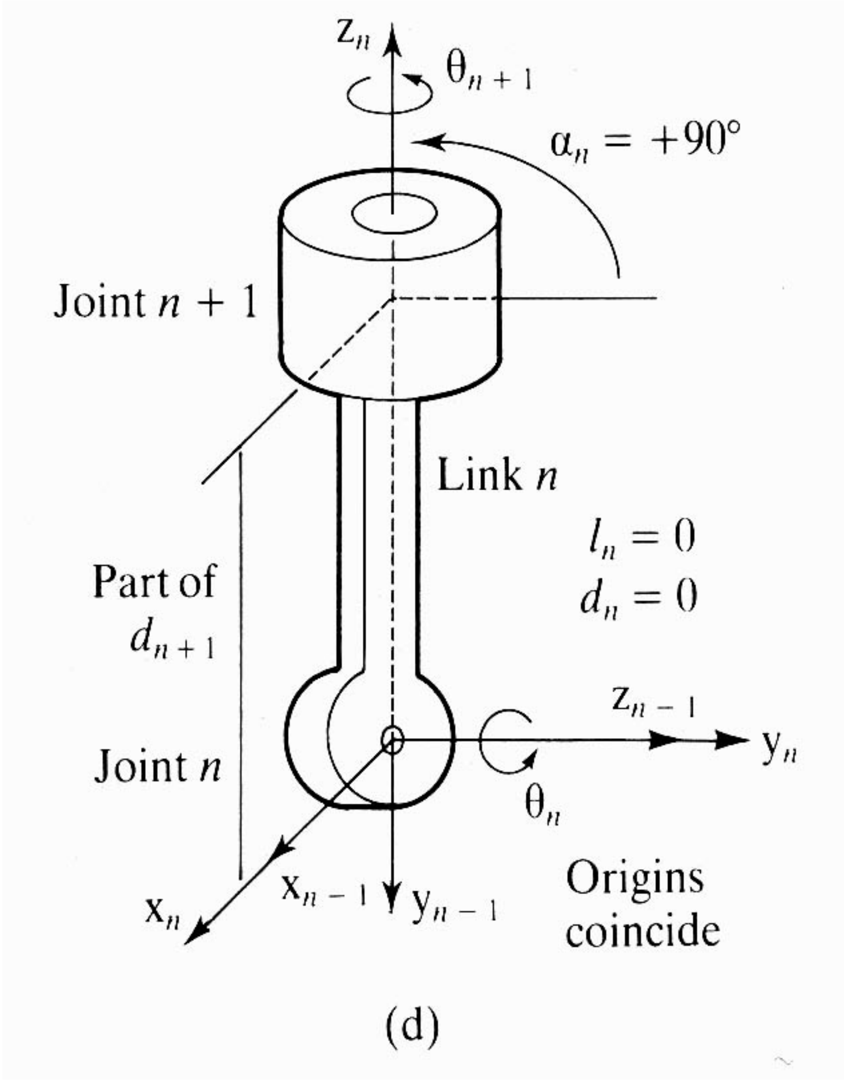
Type ๓ link If the joint n in the type ๑ link is rotated ๙๐ degree about the $y_{n-๑}$ axis so that the $z_{n-๑}$ axis is collinear with the centre line of the link. The significant difference between and the previous two link is that the joint axes intersect, whereas in the type ๑ and ๒ links they are parallel. The distance between joint axes is therefore zero, and consequently, the length of the link(l_n) is zero. However, there is a translation of distance (d_n) between the two joints.



(c)

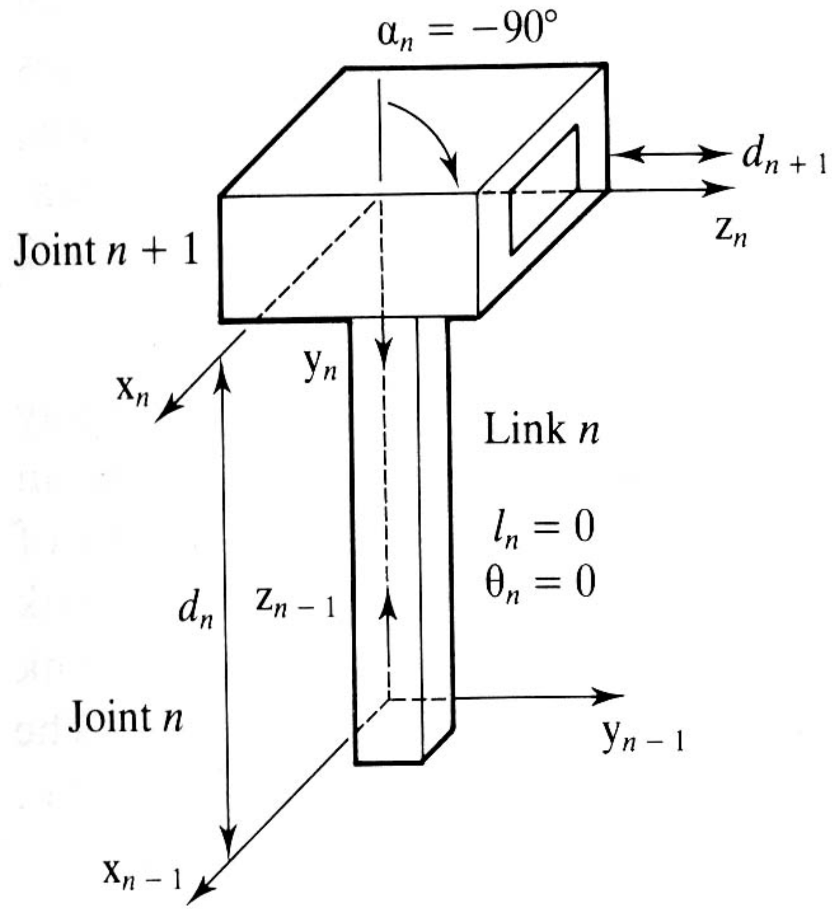
(c) Type ๓ link : revolute joints with intersecting axes-axes are perpendicular.

Type ๔ link the type ๔ link has the joints the other way to the type ๓ link. However, the assignment of axes results in significantly different values for link parameters. The origins of the axes for the two joints coincide, thus, both the length of the link and the distance between the links are zero.



(d) Type ๔ link : revolute joints with perpendicular axes-coordinate frame origins coincide.

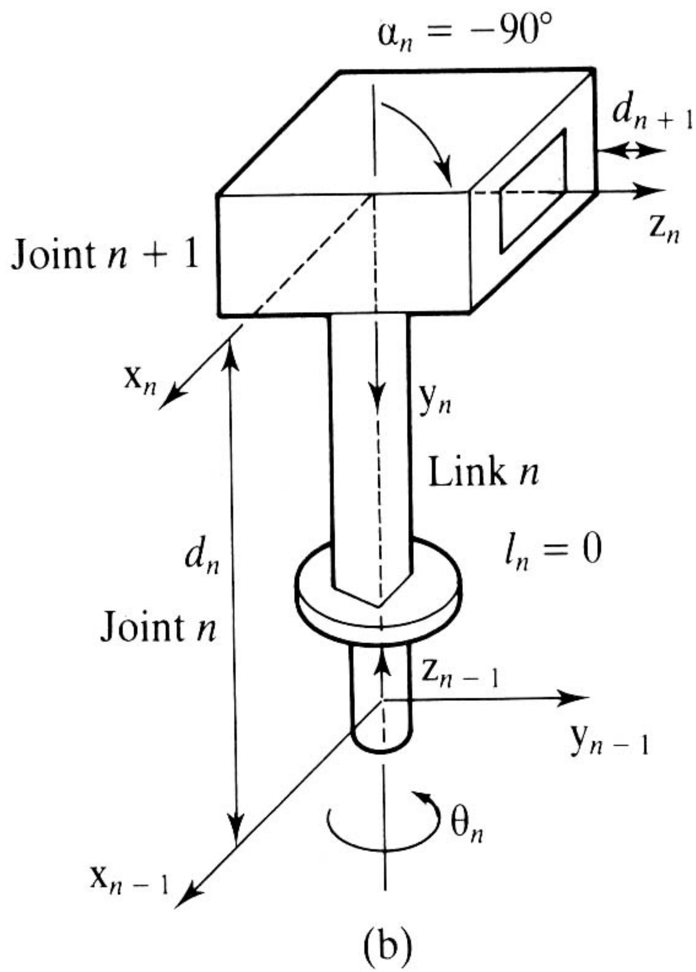
Type ๔ link The first of prismatic joints has ๒ prismatic joints, it cannot rotate, but there is a twist (angle α_n) between the z axes of the two joints. The link variable is a pure translation in the $z_{n-๑}$ direction by the distance d_n.



(a)

Typical link configulations connecting prismatic joints. (a) Type ๕ link : intersecting prismatic joints with ๙๐ degree twist angle.

Type ๖ link This link is the combination of revolute and prismatic joints. The joint variable has changed from the distance between the links (d_n) to the angle between the links(θ_n)



(b) Type ๖ link : intersecting revolute and prismatic joints with ๙๐ degree twist angle.

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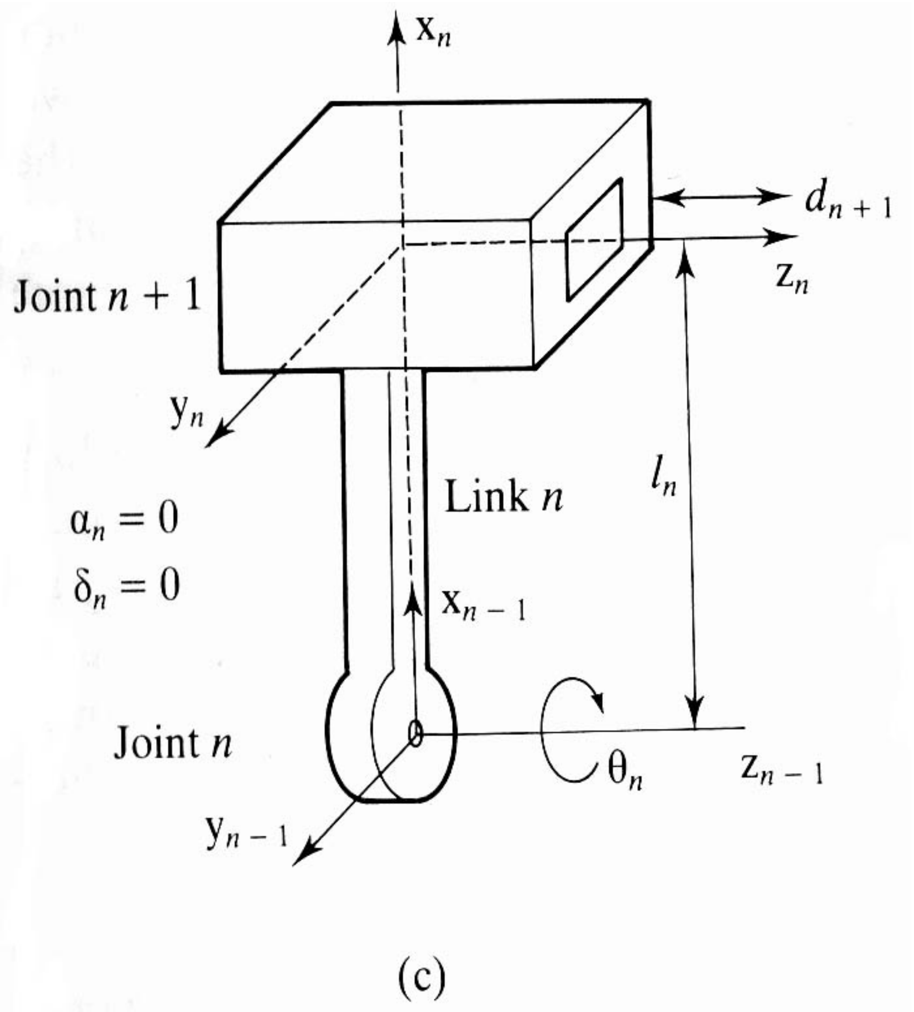
Type ๗ link The type ๗ link consist of a revolute joint whose axis is orthogonal to the link. The joint axes($z_{n-๑}$ and z_n) are parallel. The type ๗ link has the angle between the links as the joint variable, and has one degree of rotation and one degree of translation.



(c) Type ๗ link : parallel revolute and prismatic joints.

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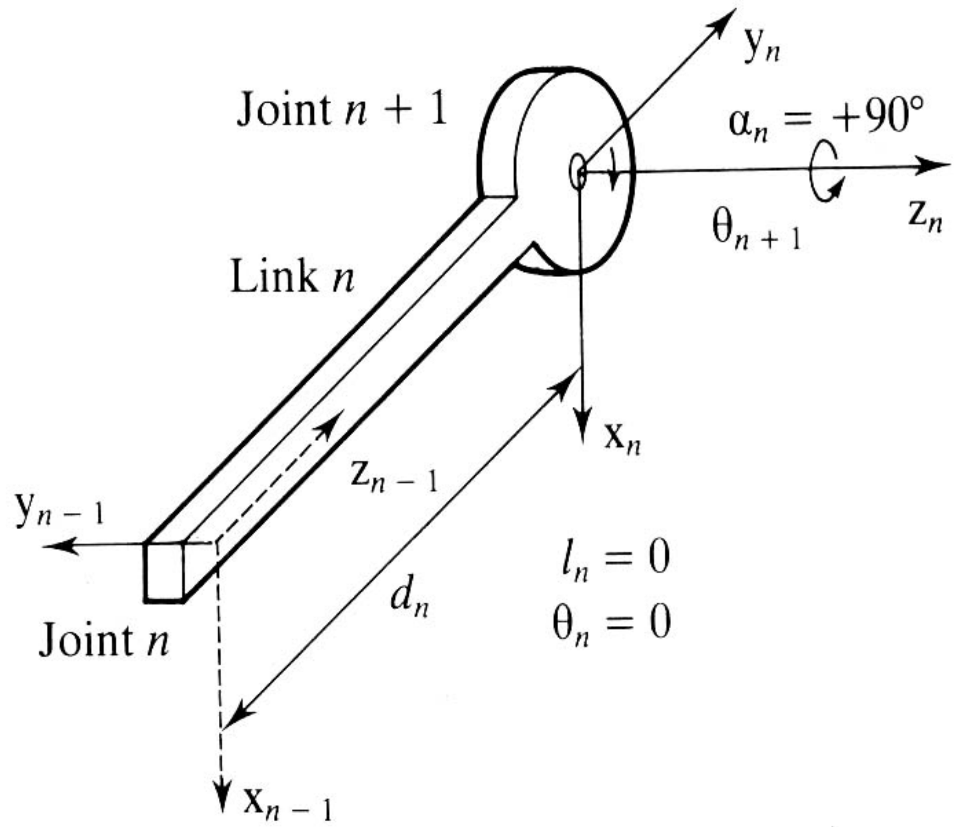
Type ๘ link The type ๗ link consist of a revolute joint whose axis is orthogonal to the link. The joint axes($z_{n-๑}$ and z_n) intersect. The type ๘ link has the distance between the links as the joint variable, and has one one degree of translation and degree of rotation.



(d)

(d) Type ๘ link : revolute and intersecting prismatic joints.

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๓ General links

The ***Common normal*** of link is the shortest line which is orthogonal to the axes of the joints at the ends of the link ($z_{n-๑}$ and z_n).

The ***length of the link*** is the length of the common normal.

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The ***twist angle*** is the angle that would exist between the joint axes if the origins of the joint frames were coincident.

The ***angle between the links*** is the angle between the common normals of successive links ($x_{n-๑}$ and x_n).

The ***distance between the links*** is the distance along the axis of the joint ($z_{n-๑}$), between the intersections of the common normals of the links with the joint axis.

The ***joint variable*** is the link parameter which can be varied.

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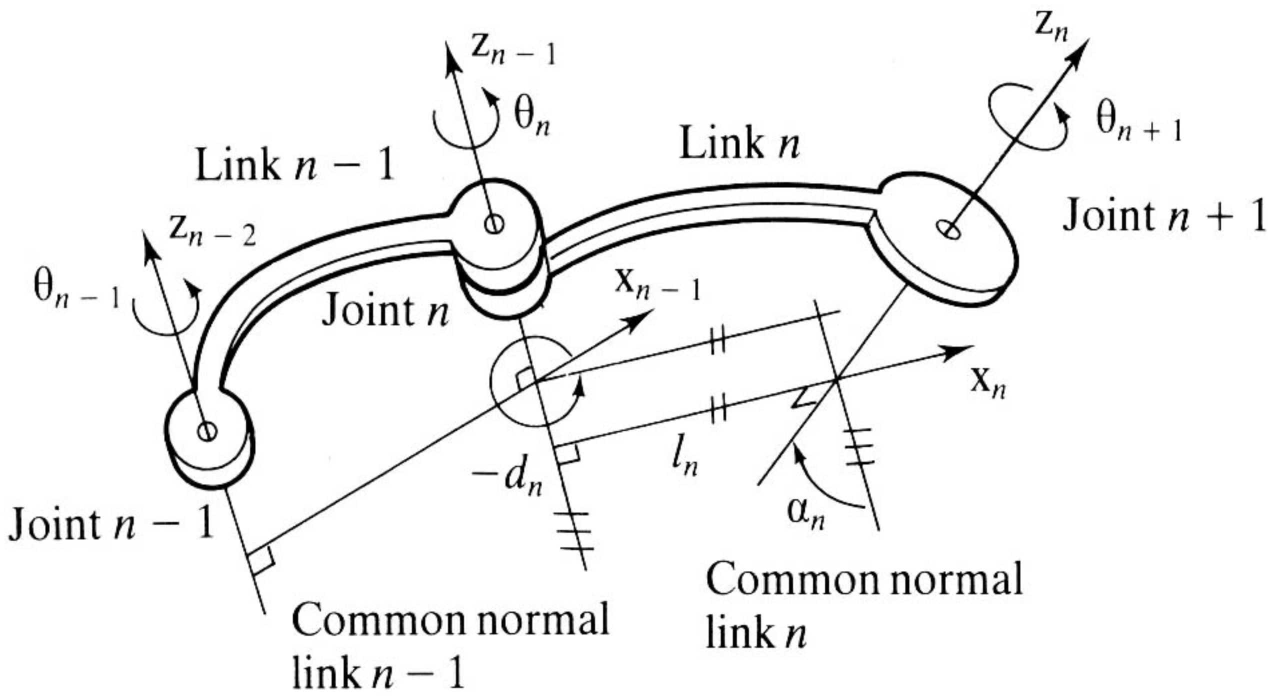
Link parameters :

๑ The length of the link (l_n).

๒ The twist angle between the joint axes (α_n).

๓ The angle between the link (θ_n).

๔ The distance between the links (d_n).



(a)

General links showing link parameters and coordinate frames (a) General link with revolute joints.

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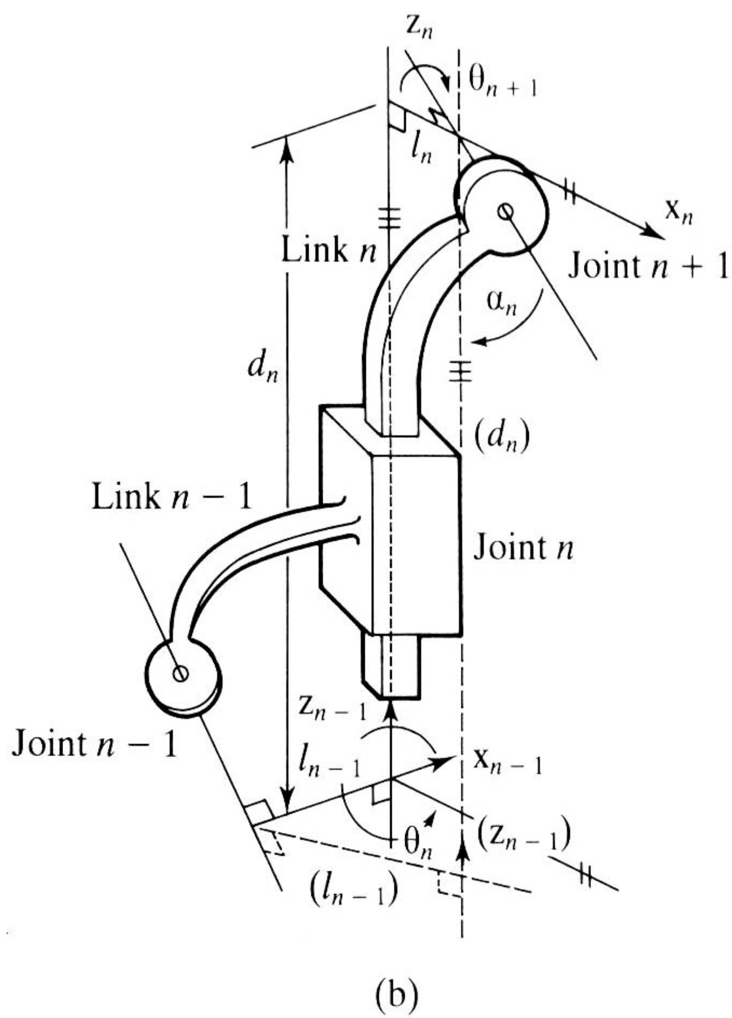

(b) General link with prismatic joints.

๔ Assignment of coordinate frames

The first step in the direct kinematic algorithm is to move the manipulator to its zero position. The zero position of the manipulator is the position where all the joint variables are zero.

The second step in the direct kinamatics algorithm is to assign a coordinate frame to each link in the manipulator.

Algorithm for assigning coordinate frames

๑ Starting at the base number of links from ๐ to n where n is the number of the links. The base is link ๐.

๒ One coordinate system is assigned to each link, each coordinate system is orthogonal, and the axes obey the right hand rule.

๓ The base coordinate frame (R or O) is assigned with axes parallel to the world coordinate frame. The origin of the base frame is coincident with the origin of joint ๑. This assumes that the axis of the first point is normal to the xy plane.

๔ Coordinate frames are attached to the link at the distal joint, (the joint farthest from the base). A frame is internal to the link it is attached to, and the succeeding link move relative to it. Thus, coordinate frame ๑ is at joint ๒ : the joint which connects link ๑ to link ๒.

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- The origin of the frame is located at the intersection of the common normal and the axis of the distal joint. If the axes of the joints are parallel, then the position of the origin is chosen to make the distance between the links(dn) zero, or minimum if there is an offset between the links. If the joint axes intersect, the origin is placed at the intersection of axes.

- ๖ The z axis is coindident with the joint axis. For prismatis joint, the direction of the z axis is in the direction of motion away from the joint. For a revolute joint, the direction of the z axis is determined from the positive direction of rotation around the z axis.

๗ The x axis is parallel to the common normal between the joint axes of the link. In the case of parallel axes, the x axis is coincident with the centre line of the link. If the axes intersect, there is no unique common normal and x axis is parallel, or anti-parallel, to the vector cross product of the z axes for the preceding link and this link($z_{n-๑} \times z_n$).

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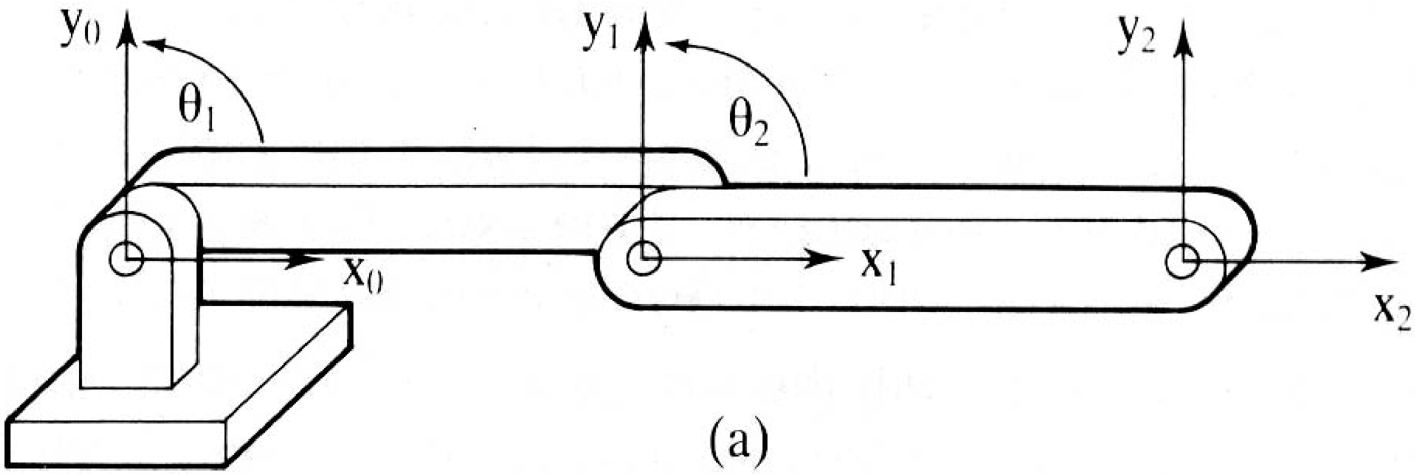
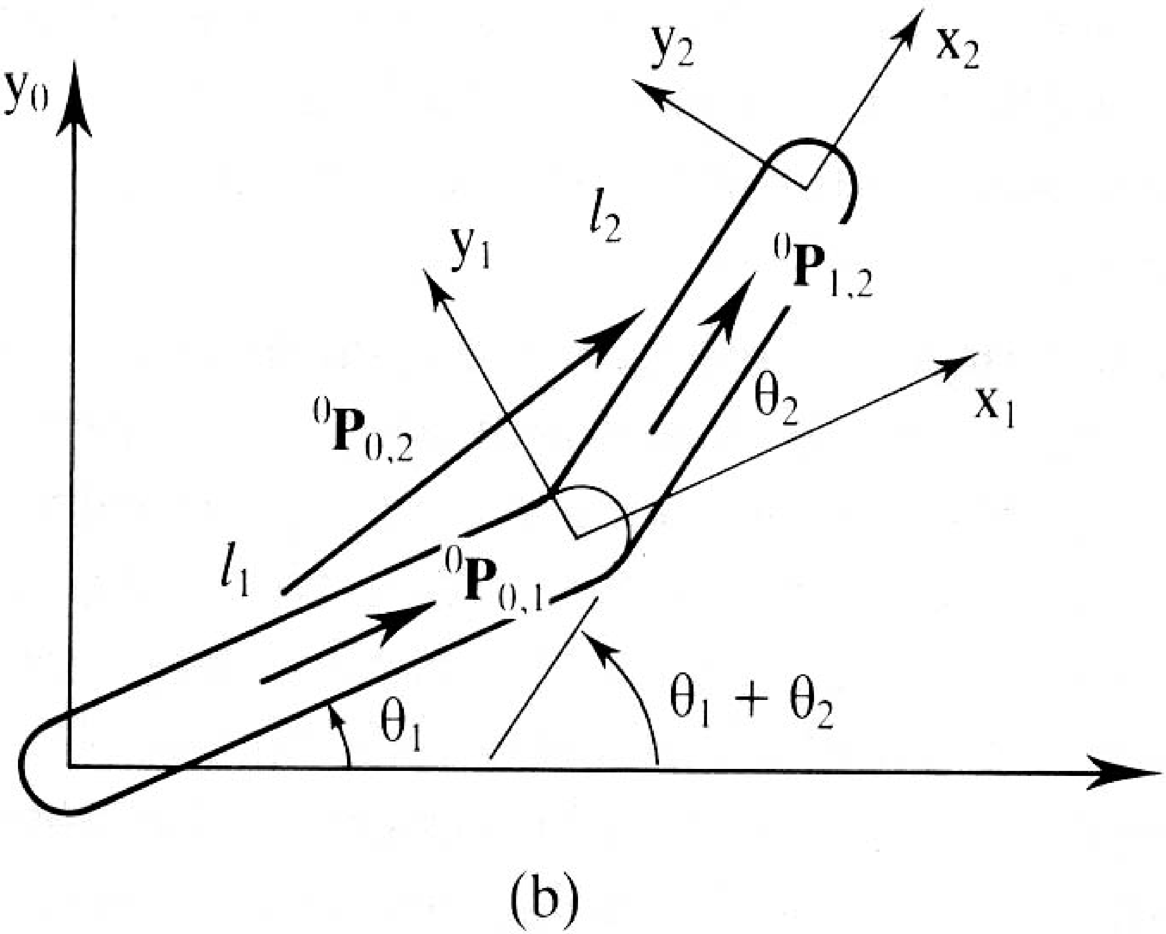
๘ The direction of the y axis can be found using the right hand rule.

๙ A coordinate frame is attached to the end of the final link(n), usually within the end effector or tool. If the robot has an articulated hand, or changes end effectors regularly, it may be neccessy to locate this coordinate frame at the tools plate, and have a separate hand transformation.

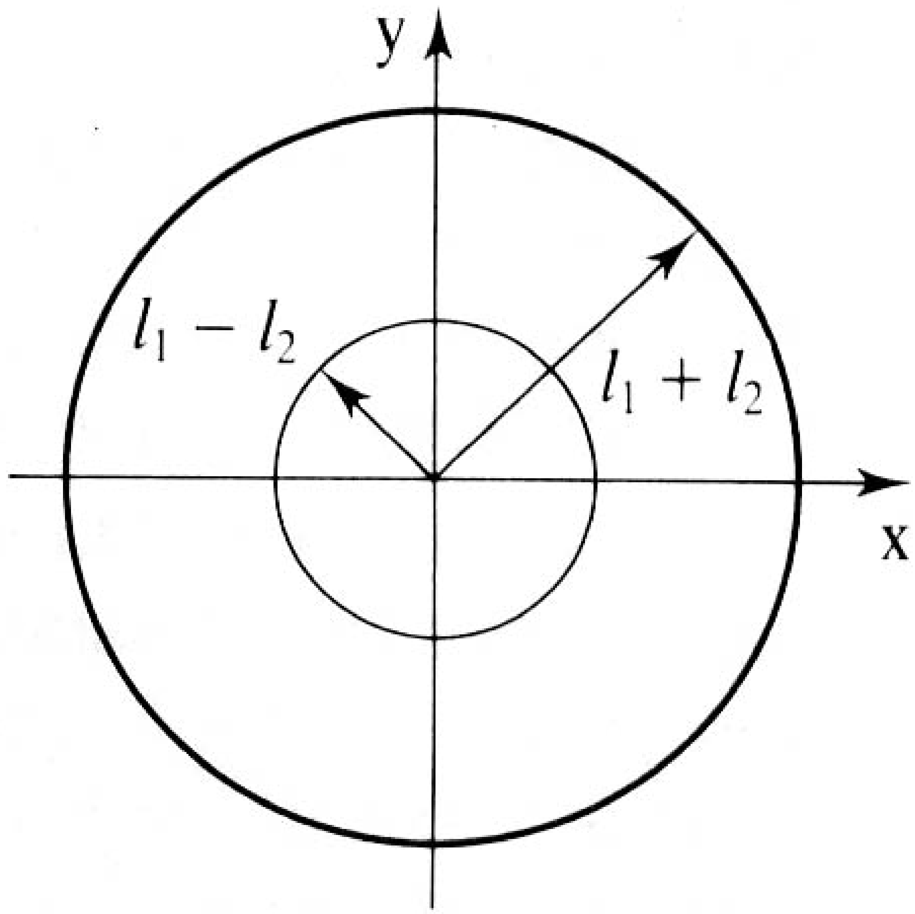
The z axis of this frame is in the same direction as the z axis of the frame assigned to the last joint (n-๑).



Type ๑ two-link : Manipulator (Horn, ๑๙๗๕) (a) Manipulator in zero position;



(b) Assignment of coordinate frames;

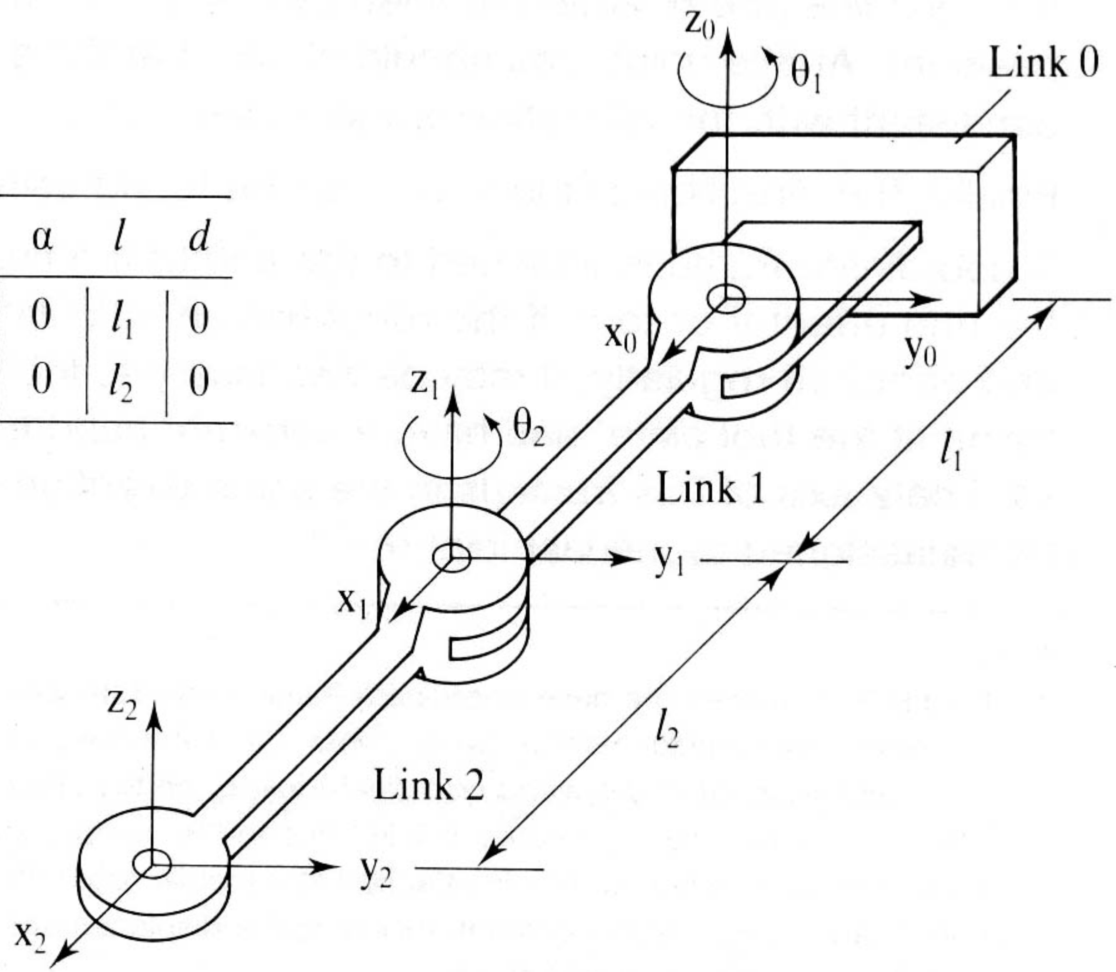


(c)

(c) Workspace;

Link	variable	θ	α	l	d
1	θ_1	θ_1	0	l_1	0
2	θ_2	θ_2	0	l_2	0

(d)



(e)

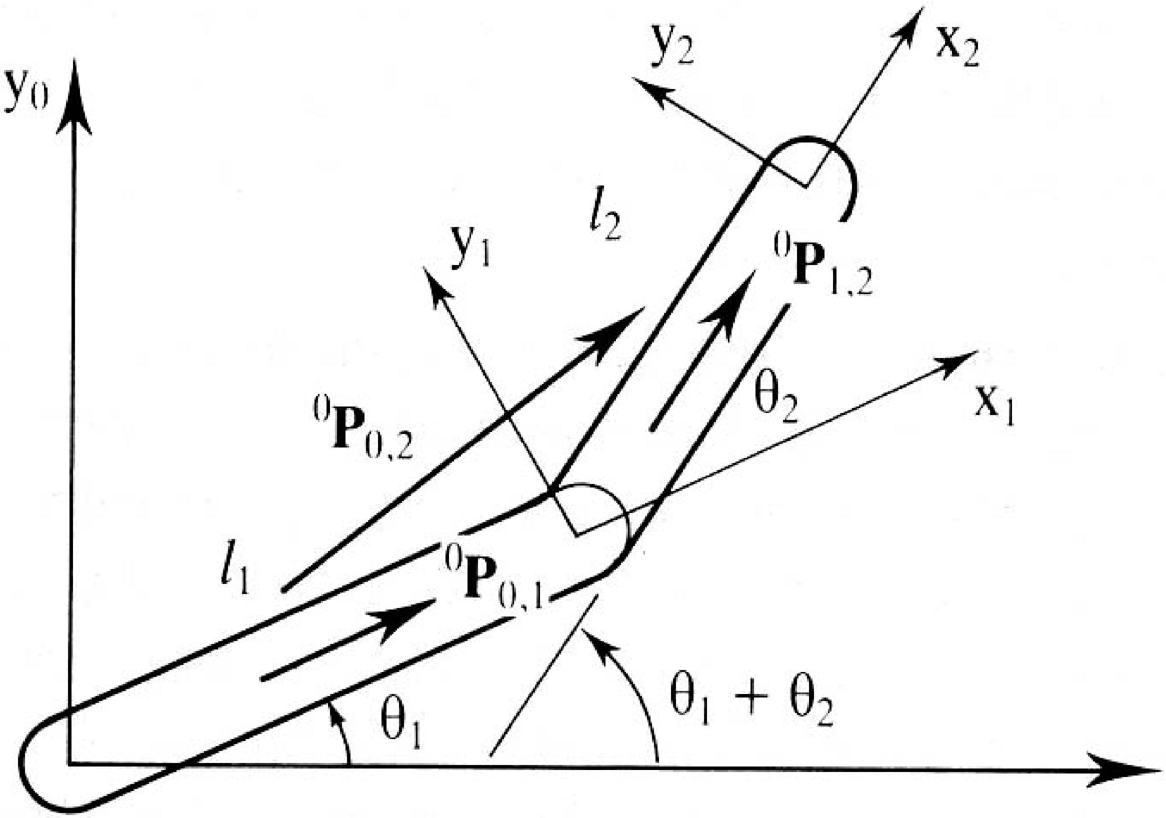
(d) Link Parameters; (e) ๓D representation e.g. SCARA robot.

& Trigonometric solution

A type ๑ two-link manipulator can be analysed using simple trigonometry, rather than homogeneous transformation equation.

$$^{๐}\boldsymbol{p}_{๐,๒} = {}^{๐}\boldsymbol{p}_{๐,๑} + {}^{๐}\boldsymbol{p}_{๑,๒}$$

$^{๐}\boldsymbol{p}_{i,j}$ is the vector from the origin of frame i to the origin of frame j as seen from the reference frame.



y_0
y_2
x_2
l_2
y_1
${}^0\mathbf{P}_{1,2}$
θ_2
x_1
${}^0\mathbf{P}_{0,2}$
l_1
${}^0\mathbf{P}_{0,1}$
θ_1
$\theta_1 + \theta_2$

(b)

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$$^{๐}\boldsymbol{p}_{๐,๑}=(l_{๑}\cos(\theta_{๑}),\, l_{๑}\sin(\theta_{๑}))$$

$$^{๐}\boldsymbol{p}_{๑,๒}=(l_{๒}\cos(\theta_{๑}+\theta_{๒}),\, l_{๒}\sin(\theta_{๑}+\theta_{๒}))$$

$$^{๐}\boldsymbol{p}_{๐,๒}=\begin{bmatrix}\boldsymbol{p}_{x}\\ \boldsymbol{p}_{y}\end{bmatrix}=\begin{bmatrix}(l_{๑}\cos(\theta_{๑})+l_{๒}\cos(\theta_{๑}+\theta_{๒}))\\ (l_{๑}\sin(\theta_{๑})+l_{๒}\sin(\theta_{๑}+\theta_{๒}))\end{bmatrix}$$

๖ A matrices

The third step in direct kinematic algorithm is to represent the homogeneous transformations between joints with A matrices. The transformation from one joint to the next can be specified by ๔ parameters.

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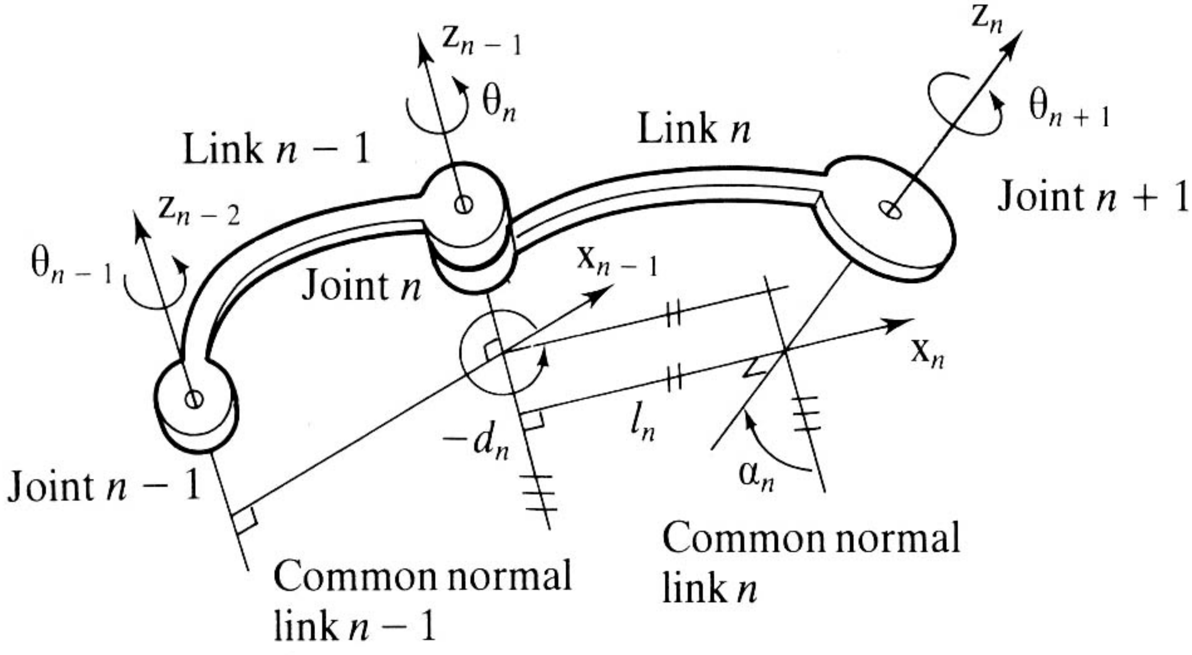


(a)

General links showing link parameters and coordinate frames (a) General link with revolute joints.

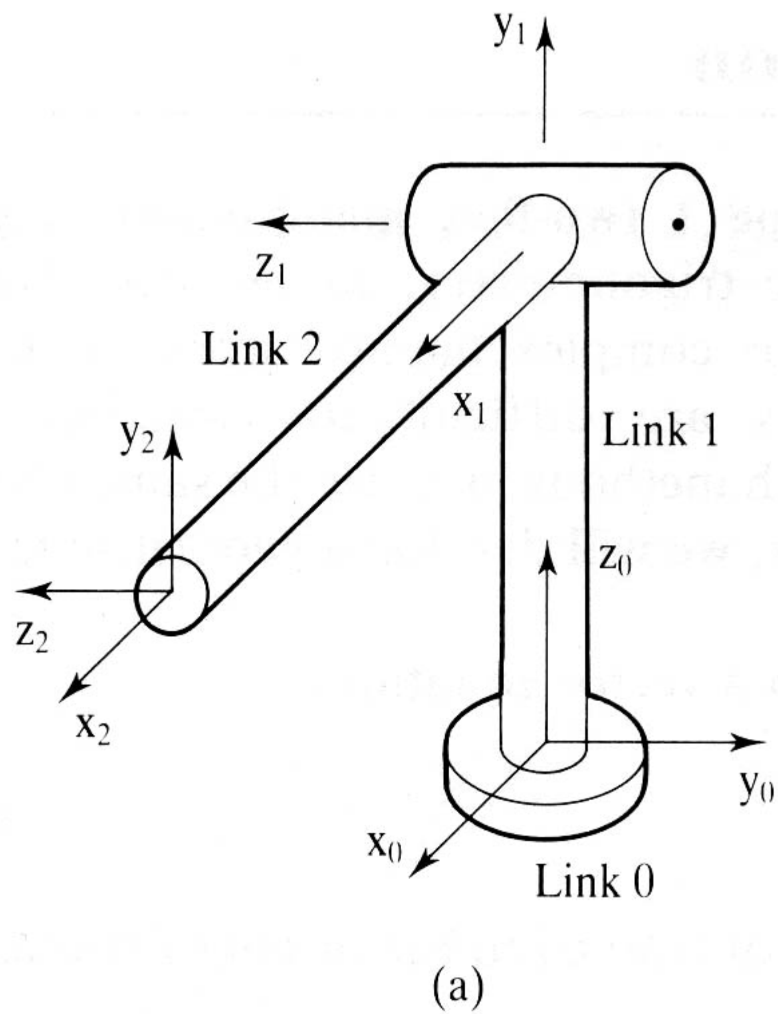
- a rotation about the $z_{n-๑}$ axis by the angle between the link(θ_n)
- a translation along the $z_{n-๑}$ axis of the distance between the links (d_n)
- a translation along the x_n axis(rotated $x_{n-๑}$ axis) of the length of the link (l_n)
- a rotation about the x_n axis of the twist angle (α_n)

$$A_n = \mathrm{Rot}(z,\theta)\mathrm{Trans}(๐,๐,d)\mathrm{Trans}(l,๐,๐)\mathrm{Rot}(x,\alpha)$$

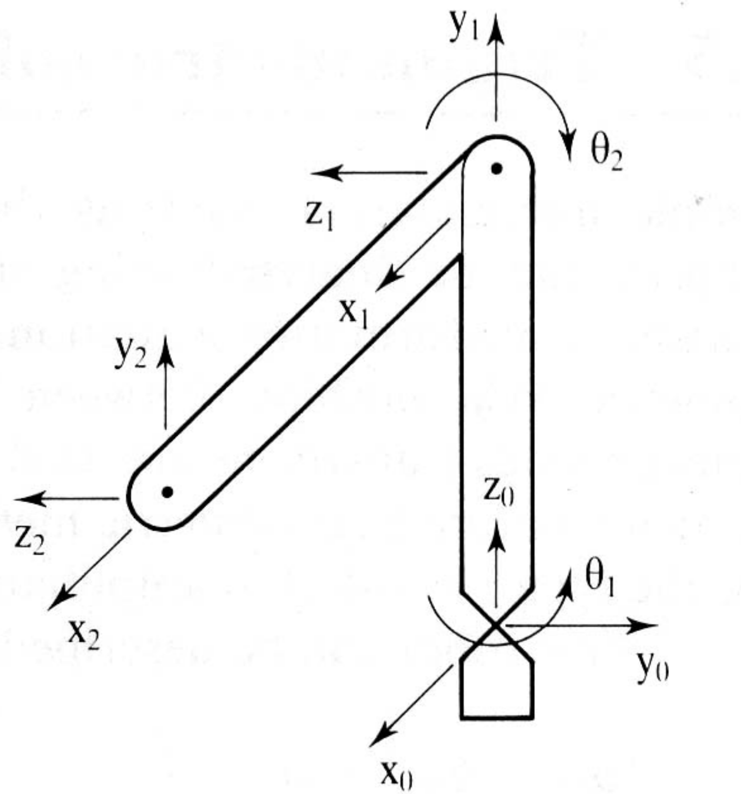
$$= \begin{bmatrix} \cos(\theta) & -\sin(\theta) & ๐ & ๐ \\ \sin(\theta) & \cos(\theta) & ๐ & ๐ \\ ๐ & ๐ & ๑ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix} \begin{bmatrix} ๑ & ๐ & ๐ & ๐ \\ ๐ & ๑ & ๐ & ๐ \\ ๐ & ๐ & ๑ & d \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$

$$\begin{bmatrix} ๑ & ๐ & ๐ & ๐ \\ ๐ & \cos(\alpha) & -\sin(\alpha) & ๐ \\ ๐ & \sin(\alpha) & \cos(\alpha) & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$

$$A_n = \begin{bmatrix} \cos(\theta) & -\sin(\theta)\cos(\alpha) & \sin(\theta)\sin(\alpha) & l\cos(\theta) \\ \sin(\theta) & \cos(\theta)\cos(\alpha) & -\cos(\theta)\sin(\alpha) & l\sin(\theta) \\ \text{๐} & \sin(\alpha) & \cos(\alpha) & d \\ \text{๐} & \text{๐} & \text{๐} & \text{๑} \end{bmatrix}$$

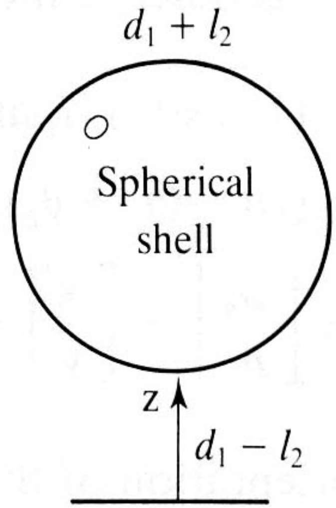


Type ๒ two-link manipulator. (a) Manipulator in zero position;



(b).

(b) Line diagram;



(c)

Link	variable	θ	α	l	d
1	θ_1	θ_1	90°	0	d_1
2	θ_2	θ_2	0	l_2	0

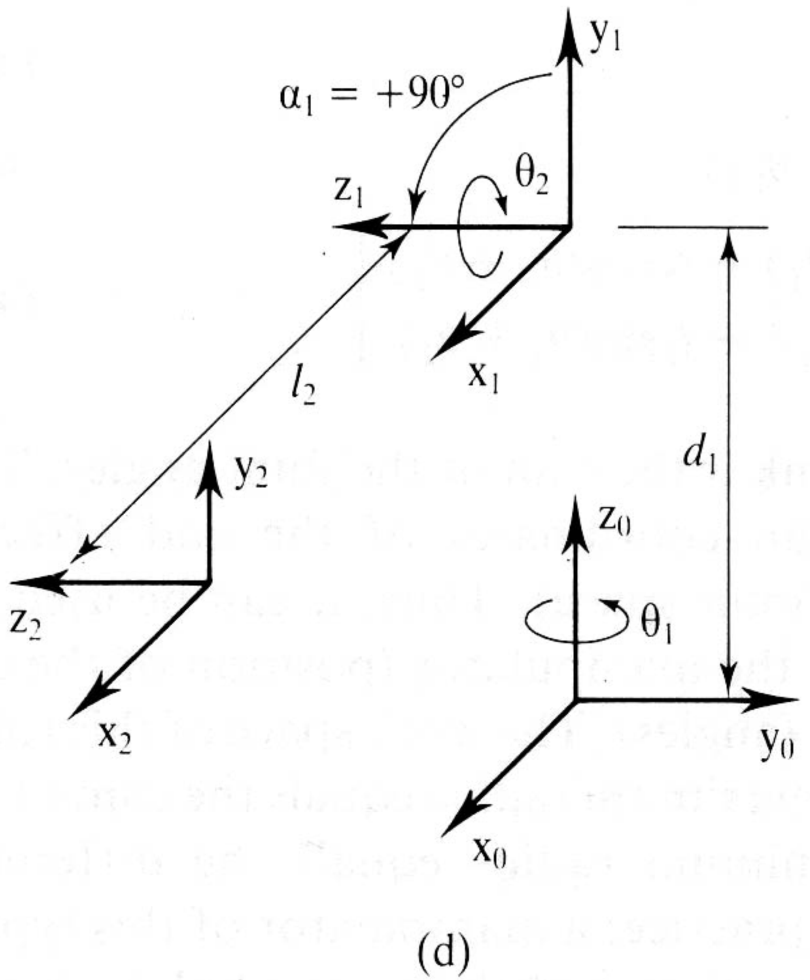
(e)

(c) Workspace; (e) Link parameters.

Example A matrix for the first link of the manipulator is :

$$A_n = \begin{bmatrix} \cos(\theta_๑) & ๐ & \sin(\theta_๑) & ๐ \\ \sin(\theta_๑) & ๐ & -\cos(\theta_๑) & ๐ \\ ๐ & ๑ & ๐ & d_๑ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$

$$A_n=\begin{bmatrix} C_๑ & ๐ & S_๑ & ๐ \\ S_๑ & ๐ & -C_๑ & ๐ \\ ๐ & ๑ & ๐ & d_๑ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$



(d) Assignment of coordinate frames;

๗ Homogeneous transformations

The fifth step in the direct kinematic algorithm is to multiply the A matrices to get the manipulator transform.

The homogeneous transform for the type ๑ two-link manipulator is :

$$
{}^{R}T_{H}=\begin{bmatrix} C_{๑} & -S_{๑} & ๐ & l_{๑}C_{๑} \\ S_{๑} & C_{๑} & ๐ & l_{๑}S_{๑} \\ ๐ & ๐ & ๑ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}\begin{bmatrix} C_{๒} & -S_{๒} & ๐ & l_{๒}C_{๒} \\ S_{๒} & C_{๒} & ๐ & l_{๒}S_{๒} \\ ๐ & ๐ & ๑ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

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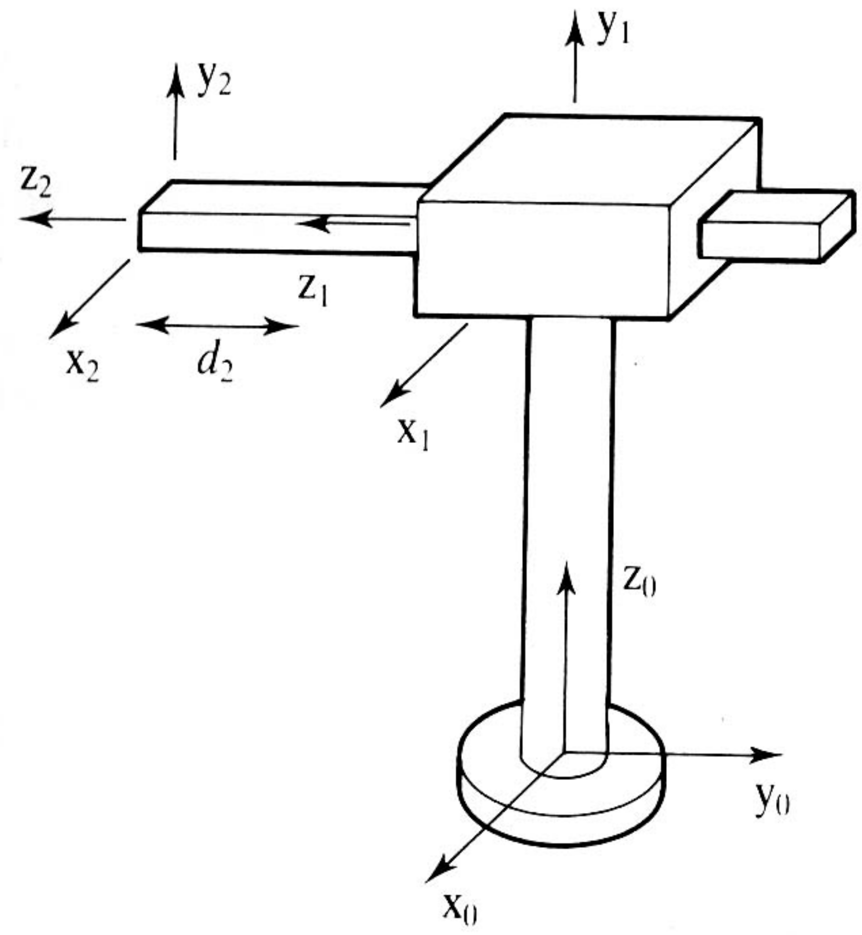
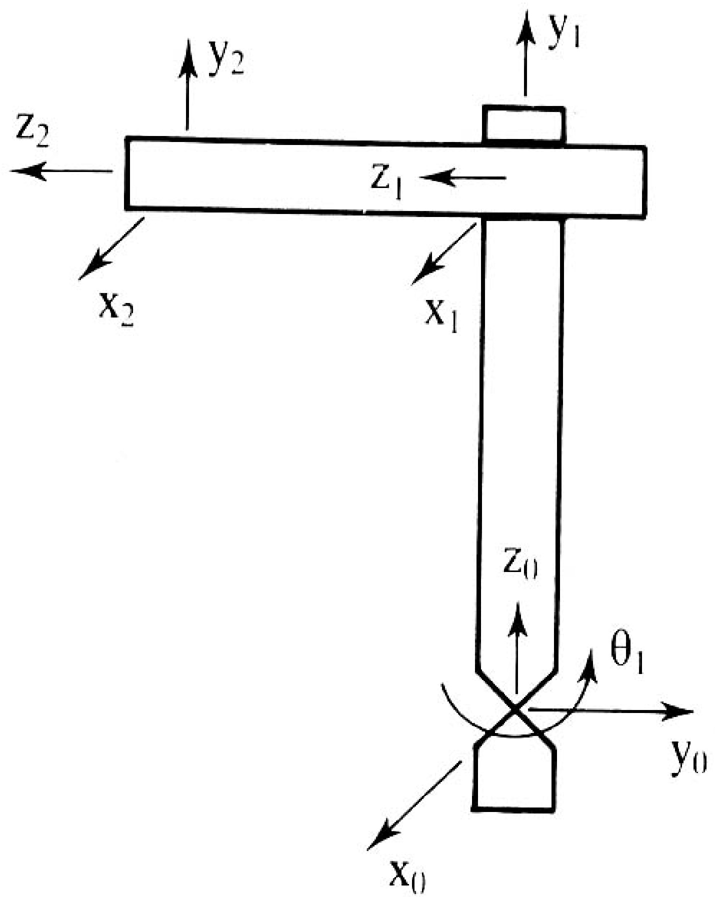
$$
{}^{R}T_{H} = \begin{bmatrix} C_{๑๒} & -S_{๑๒} & ๐ & l_{๑}C_{๑}+l_{๒}C_{๑๒} \\ S_{๑๒} & C_{๑๒} & ๐ & l_{๑}S_{๑}+l_{๒}S_{๑๒} \\ ๐ & ๐ & ๑ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

Where $S_{๑๒} = \sin(\theta_{๑}+\theta_{๒})$ $C_{๑๒} = \cos(\theta_{๑}+\theta_{๒})$



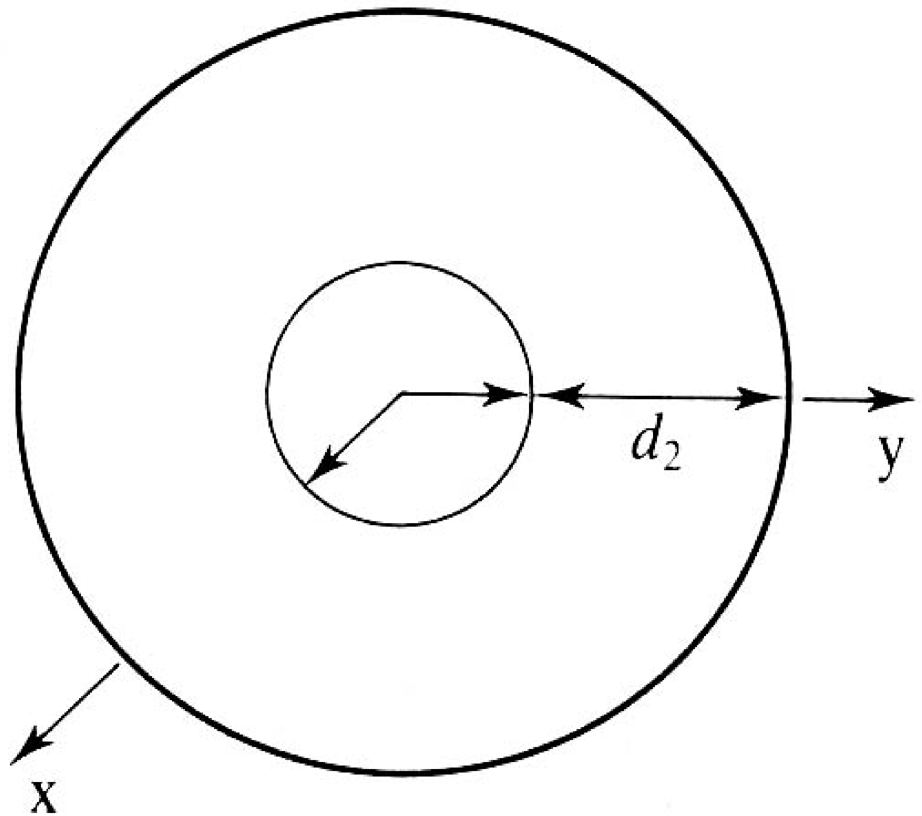
(a)

Type ๓ two-link manipulator. (a) Manipulator in zero position;



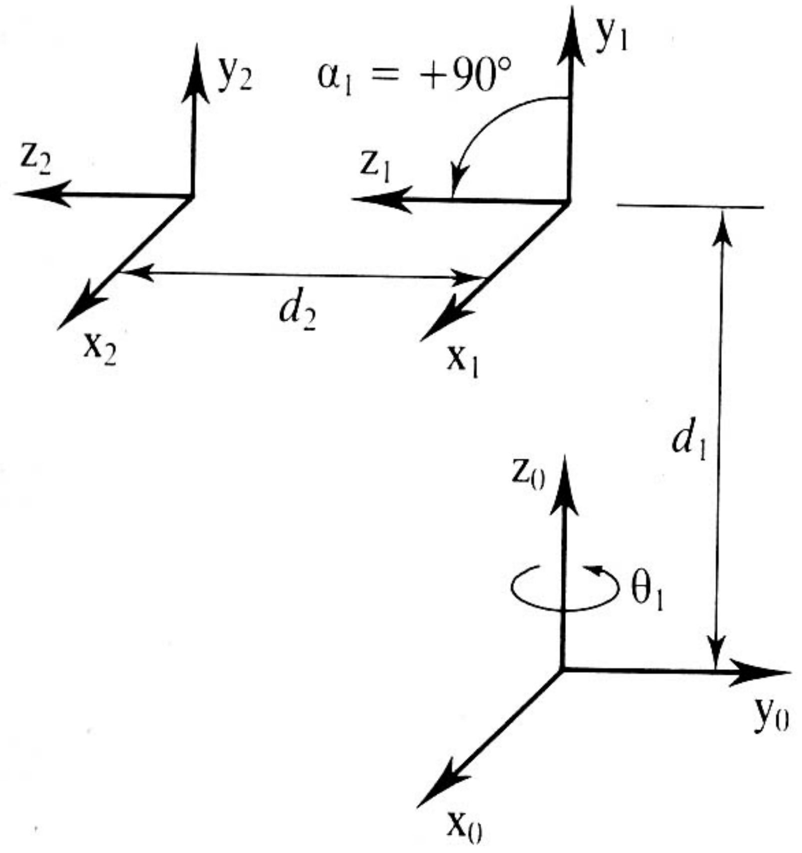
(b)

(b) Line diagram;



(c)

(c) Workspace;



(d)

(d) Assignment of coordinate frames;

Link	variable	θ	α	l	d
1	θ_1	θ_1	90°	0	d_1
2	d_2	0	0	0	d_2

(e)

(e) Link parameters.

Principle of Robotics

$$
{}^{R}T_{H}=\begin{bmatrix} C_{๑} & ๐ & S_{๑} & ๐ \\ S_{๑} & ๐ & -C_{๑} & ๐ \\ ๐ & ๑ & ๐ & d_{๑} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}\begin{bmatrix} ๑ & ๐ & ๐ & ๐ \\ ๐ & ๑ & ๐ & ๐ \\ ๐ & ๐ & ๑ & d_{๒} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

$$
{}^{R}T_{H}=\begin{bmatrix} C_{๑} & ๐ & S_{๑} & d_{๒}S_{๑} \\ S_{๑} & ๐ & -C_{๑} & -d_{๒}C_{๑} \\ ๐ & ๑ & ๐ & d_{๑} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

๘ Direct kinematics

The direct kinematic model describes the Cartesian coordinates and orientation angles of the tool plate in terms of the joint variables.

The location of the tool plate (or hand) in Cartesian space is described by the general transformation equation.

$$
{}^{\mathbf{R}}\mathbf{T}_{\mathbf{N}} = \begin{bmatrix} x_x & y_x & z_x & p_x \\ x_y & y_y & z_y & p_y \\ x_z & y_z & z_z & p_z \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix} = \begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{p} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

The position and orientation of the hand in joint space is described by the manipulator transformation equation.

$$^{R}\mathbf{T}_{N} = {}^{R}\mathbf{T}_{๑}{}^{๑}\mathbf{T}_{๒}\cdots{}^{n-๒}\mathbf{T}_{n-๑}{}^{n-๑}\mathbf{T}_{H} = \boldsymbol{A}_{๑}\boldsymbol{A}_{๒}\cdots\boldsymbol{A}_{n-๑}\boldsymbol{A}_{n}$$

The position and orientation of the manipulator in Cartesian space can be described with an orientation transform.

$$RPY(\phi, \theta, \psi) = Rot(z, \phi)\, Rot(y, \theta)\, Rot(x, \psi)$$

Principle of Robotics

The kinematic model of the manipulator is obtained by equating these three equation :

$$^{R}\mathbf{T}_{N}=\begin{bmatrix} \boldsymbol{x} & \boldsymbol{y} & \boldsymbol{z} & \boldsymbol{p} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}=\begin{bmatrix} x_x & y_x & z_x & p_x \\ x_y & y_y & z_y & p_y \\ x_z & y_z & z_z & p_z \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$

$$
= \begin{bmatrix} ๑ & ๐ & ๐ & p_x \\ ๐ & ๑ & ๐ & p_y \\ ๐ & ๐ & ๑ & p_z \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix} \begin{bmatrix} \textit{Orientation} \\ \textit{Transform} \\ \\ \end{bmatrix}
$$

Principle of Robotics

Example Find the orientation transform for the type ๑ two-link manipulator.

A rotation about the z axis of ϕ°, and the rotations about the x and y axes are fixed at ๐$^{\circ}$

$$RPY(\phi,\theta,\psi) = Rot(z,\phi)\,Rot(y,\theta)\,Rot(x,\psi)$$

$$= Rot(z,\phi)\,Rot(y,๐)\,Rot(x,๐)$$

$$= \begin{bmatrix} C(\phi) & -S(\phi) & ๐ & ๐ \\ S(\phi) & C(\phi) & ๐ & ๐ \\ ๐ & ๐ & ๑ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$

Principle of Robotics

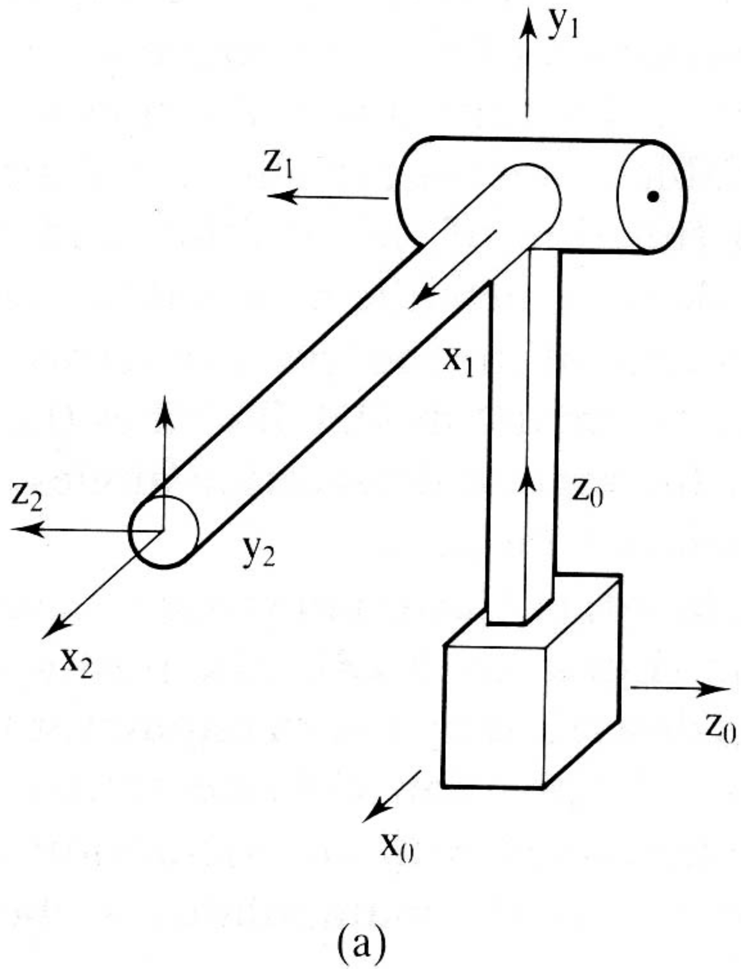
The type ๑ two-link manipulator, the kinematic model can make by assigning values to the ๑๒ components which make up the four vectors in the general transformation matrix.

$$
{}^{R}T_{H} = \begin{bmatrix} x_x & y_x & z_x & p_x \\ x_y & y_y & z_y & p_y \\ x_z & y_z & z_z & p_z \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix} = \begin{bmatrix} C_{๑๒} & -S_{๑๒} & ๐ & l_{๑}C_{๑} + l_{๒}C_{๑๒} \\ S_{๑๒} & C_{๑๒} & ๐ & l_{๑}S_{๑} + l_{๒}S_{๑๒} \\ ๐ & ๐ & ๑ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

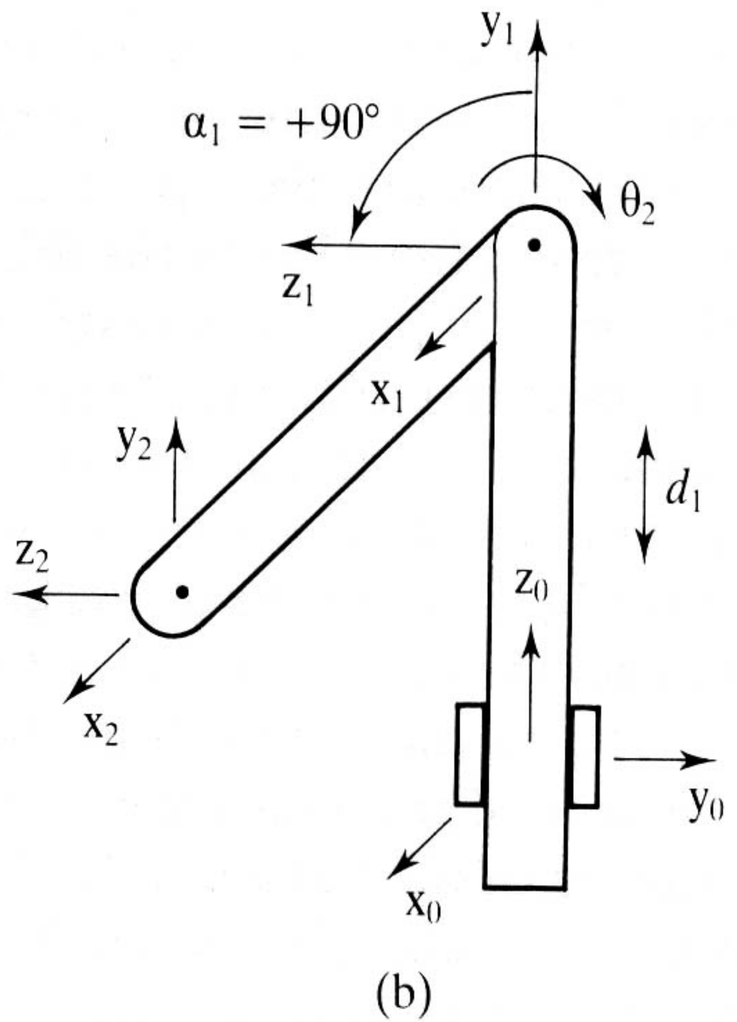
$$
= \begin{bmatrix} C(\phi) & -S(\phi) & ๐ & p_x \\ S(\phi) & C(\phi) & ๐ & p_y \\ ๐ & ๐ & ๑ & p_z \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

$$ {}^{๐}\boldsymbol{p}_{๐,๒} = \begin{bmatrix} \boldsymbol{p}_x \\ \boldsymbol{p}_y \end{bmatrix} = \begin{bmatrix} \mathrm{x} \\ \mathrm{y} \end{bmatrix} = \begin{bmatrix} (l_{๑}\cos(\theta_{๑}) + l_{๒}\cos(\theta_{๑} + \theta_{๒})) \\ (l_{๑}\sin(\theta_{๑}) + l_{๒}\sin(\theta_{๑} + \theta_{๒})) \end{bmatrix} $$

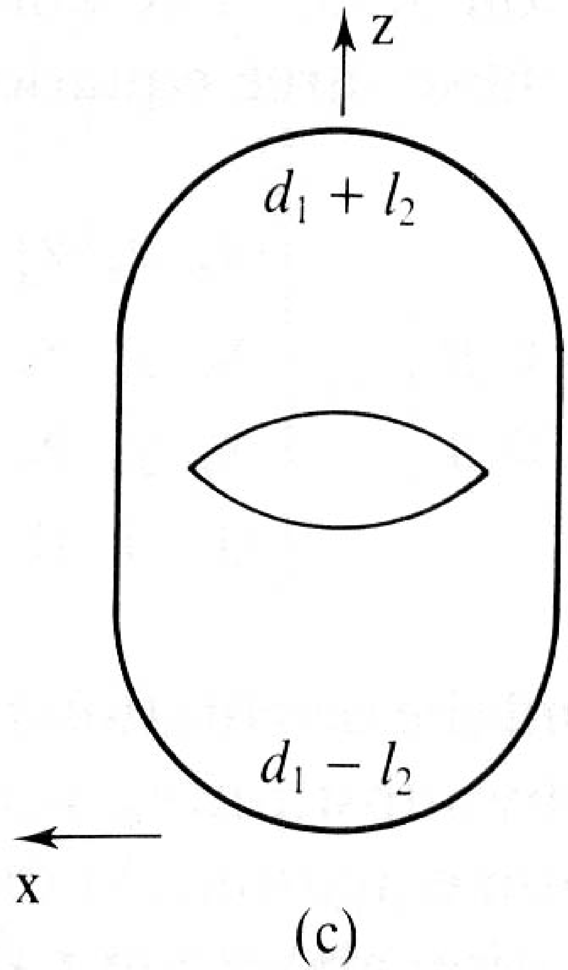
$$ C(\phi) = \cos(\phi) = C_{๑๒} = \cos(\theta_{๑} + \theta_{๒}) $$



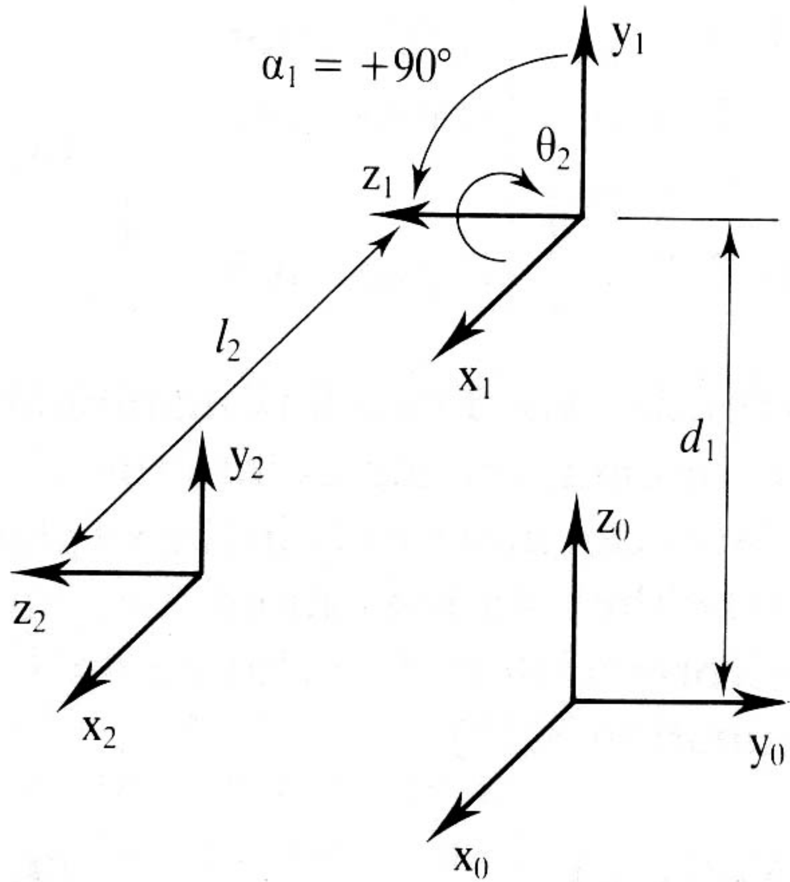
Type ๔ two-link manipulator. (a) Manipulator in zero position;



(b) Line diagram;



(c) Workspace;



(d)

(d) Assignment of coordinate frames;

Link	variable	θ	α	l	d
1	d_1	0	90°	0	d_1
2	θ_2	θ_2	0	l_2	0

(e)

(e) Link parameters;

The orientation transform includes a fixed rotation of ๐° about the $z_๐$ axis, a variable rotation of $-\theta$ about $y_๐$ axis($\theta_๒$) and a fix rotation of ๙๐° about the $x_๐$ axis ($\alpha_๑$)

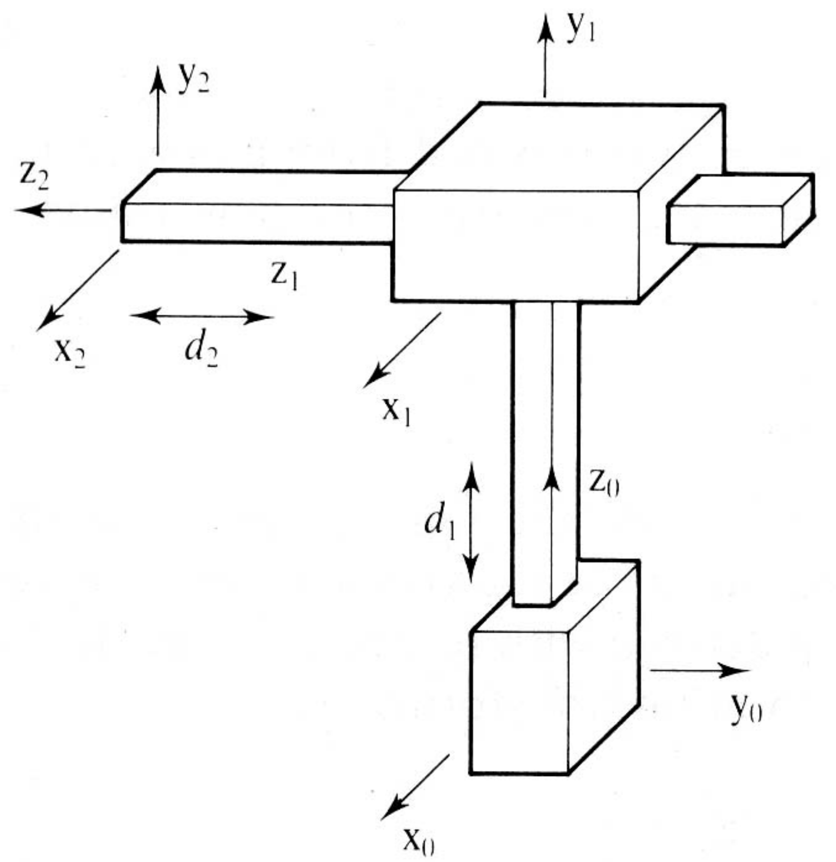
$$RPY(\phi,\theta,\psi)=Rot(z,๐)\,Rot(y,-\theta)\,Rot(x,๙๐)$$

$$=\begin{bmatrix} C(\theta) & -S(\theta) & ๐ & ๐ \\ ๐ & ๐ & -๑ & ๐ \\ S(\theta) & C(\theta) & ๐ & ๐ \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}$$

$$
{}^{R}T_{H}=\begin{bmatrix} C_{๒} & -S_{๒} & ๐ & l_{๒}C_{๒} \\ ๐ & ๐ & -๑ & ๐ \\ S_{๒} & C_{๒} & ๐ & d_{๑}+l_{๒}S_{๒} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}=\begin{bmatrix} x_{x} & y_{x} & z_{x} & p_{x} \\ x_{y} & y_{y} & z_{y} & p_{y} \\ x_{z} & y_{z} & z_{z} & p_{z} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

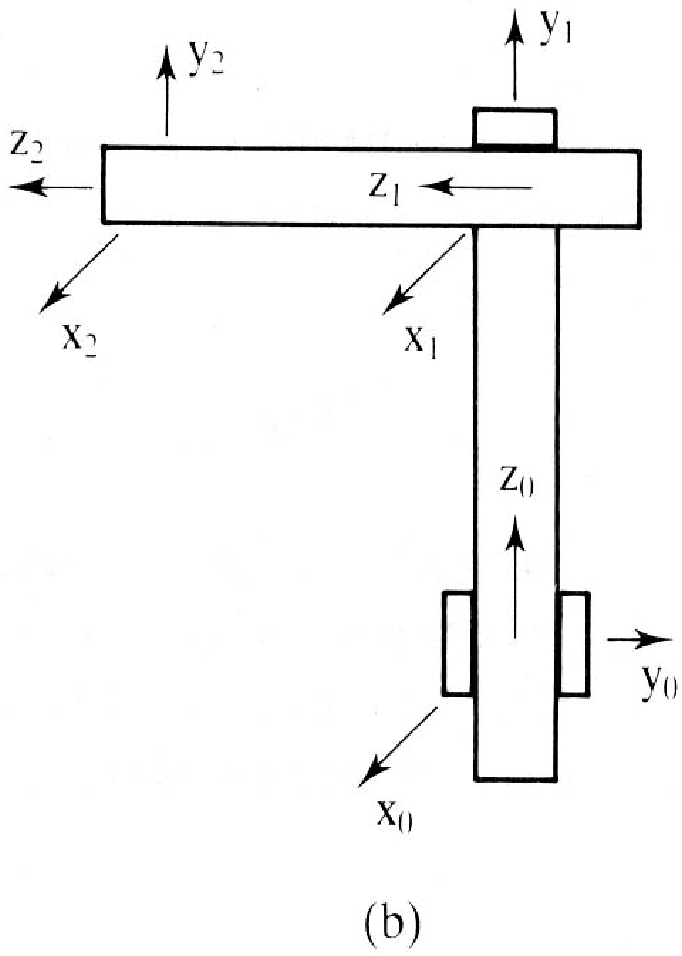
$$
=\begin{bmatrix} C(\theta) & -S(\theta) & ๐ & p_{x} \\ ๐ & ๐ & -๑ & p_{y} \\ S(\theta) & C(\theta) & ๐ & p_{z} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

$$p_x = l_๒ C_๒$$
$$p_z = d_๑ + l_๒ S_๒$$
$$\theta = -\theta_๒$$

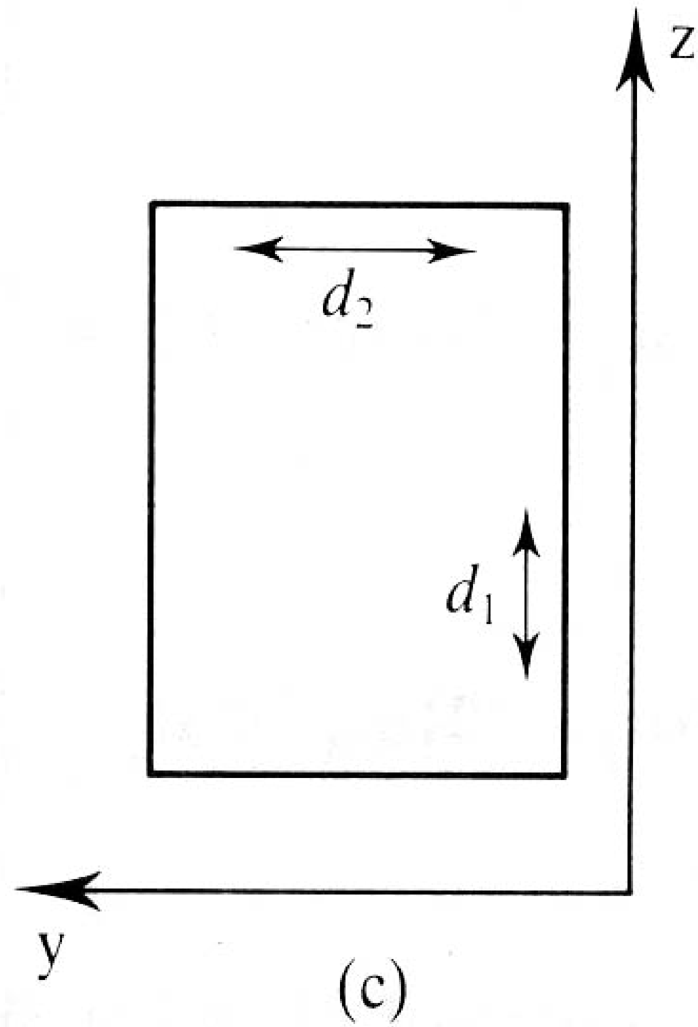


(a)

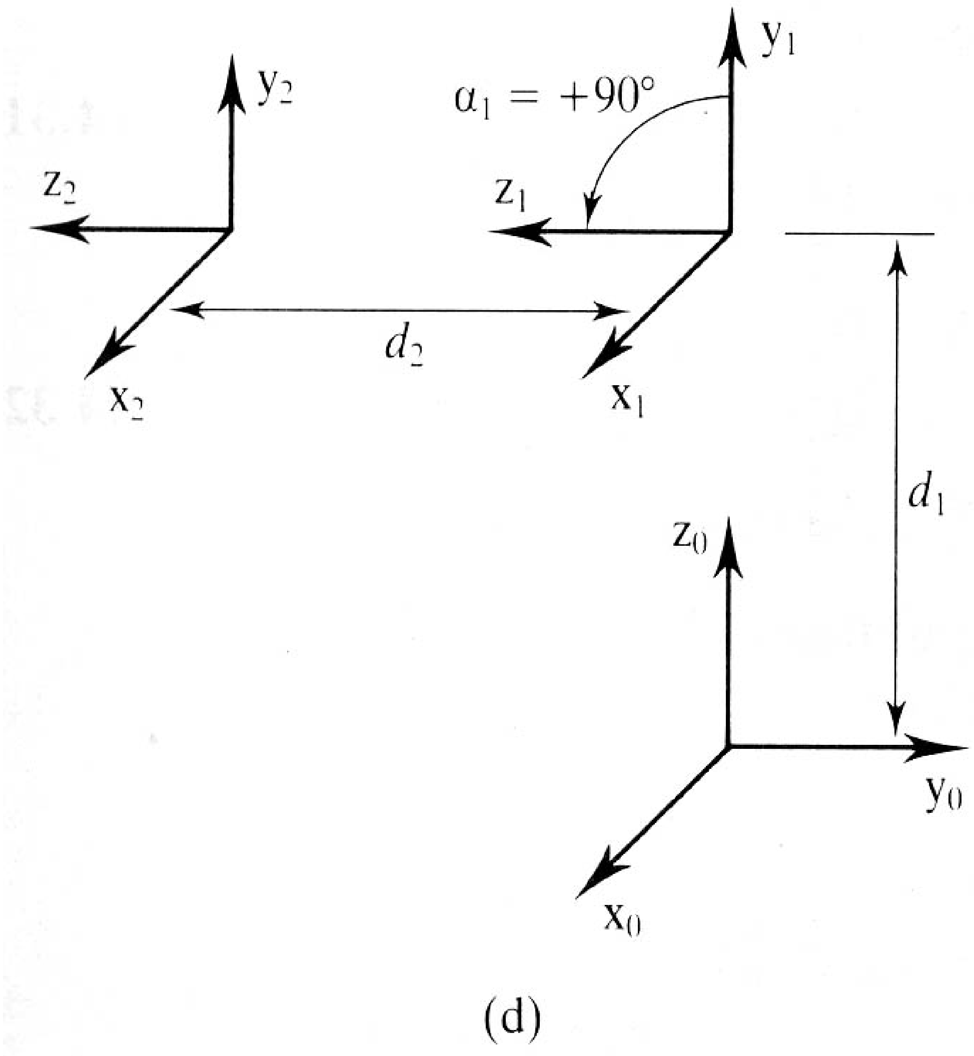
Type ๕ two-link manipulator. (a) Manipulator in zero position;



(b) Line diagram;



(c) Workspace;



(d) Assignment of coordinate frames;

Link	variable	θ	α	l	d
1	d_1	0	90°		d_1
2	d_2	0	0	0	d_2

(e)

(e) Link parameters;

Principle of Robotics

$$RPY(\phi,\theta,\psi)=Rot(z,๐)\,Rot(y,๐)\,Rot(x,๙๖)$$

$$
{}^{R}T_{H} = \begin{bmatrix} ๑ & ๐ & ๐ & ๐ \\ ๐ & ๐ & -๑ & -d_{๒} \\ ๐ & ๑ & ๐ & d_{๑} \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix} = \begin{bmatrix} x_x & y_x & z_x & p_x \\ x_y & y_y & z_y & p_y \\ x_z & y_z & z_z & p_z \\ ๐ & ๐ & ๐ & ๑ \end{bmatrix}
$$

$$
= \textbf{Trans}(y, -d_{๒})\,\textbf{Trans}(z, d_{๑})\,\textbf{Rot}(x, ๙๖)
$$

$$
p_y = -d_{๒}
$$

$$
p_z = d_{๑}
$$

$$
\psi = ๙๖
$$

Principle of Robotics

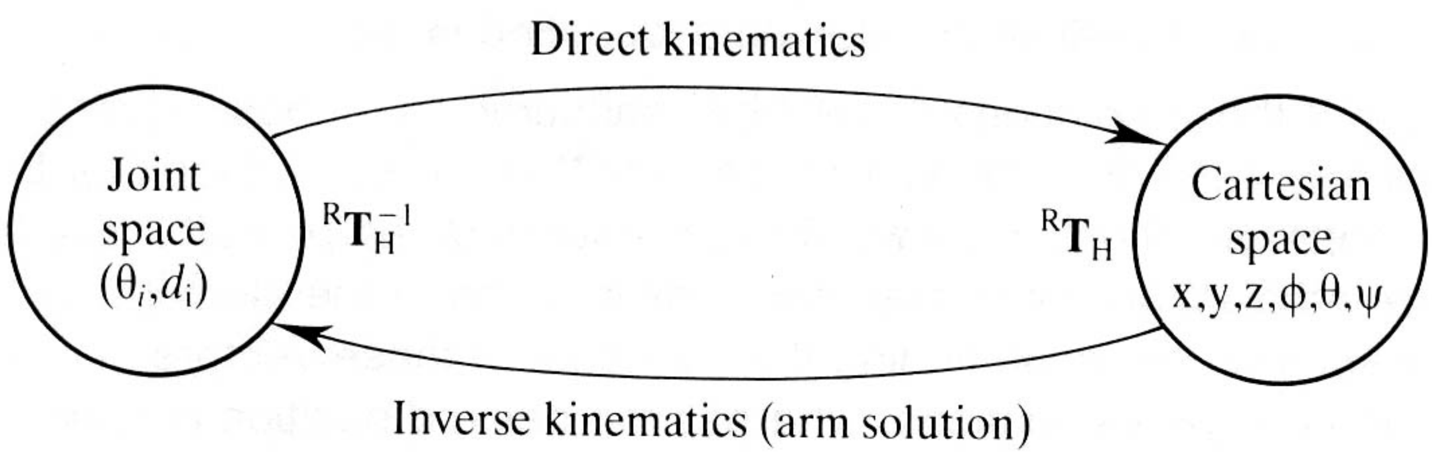
๙ Vector solution

We developed a trigonometric solution for the type ๑ two-link manipulator using vectors. Vector can only be added if they are defined with respect to the same coordinate frame. The vector describing the second link can be transformed from frame ๑ to frame ๐ with a rotation transform.

$${}^{\text{๐}}\boldsymbol{p}_{\text{๑,๒}} = {}^{\text{๐}}\mathbf{Rot}_{\text{๑}}\,{}^{\text{๑}}\boldsymbol{p}_{\text{๑,๒}}$$

๑๐ Solving the general orientation transform

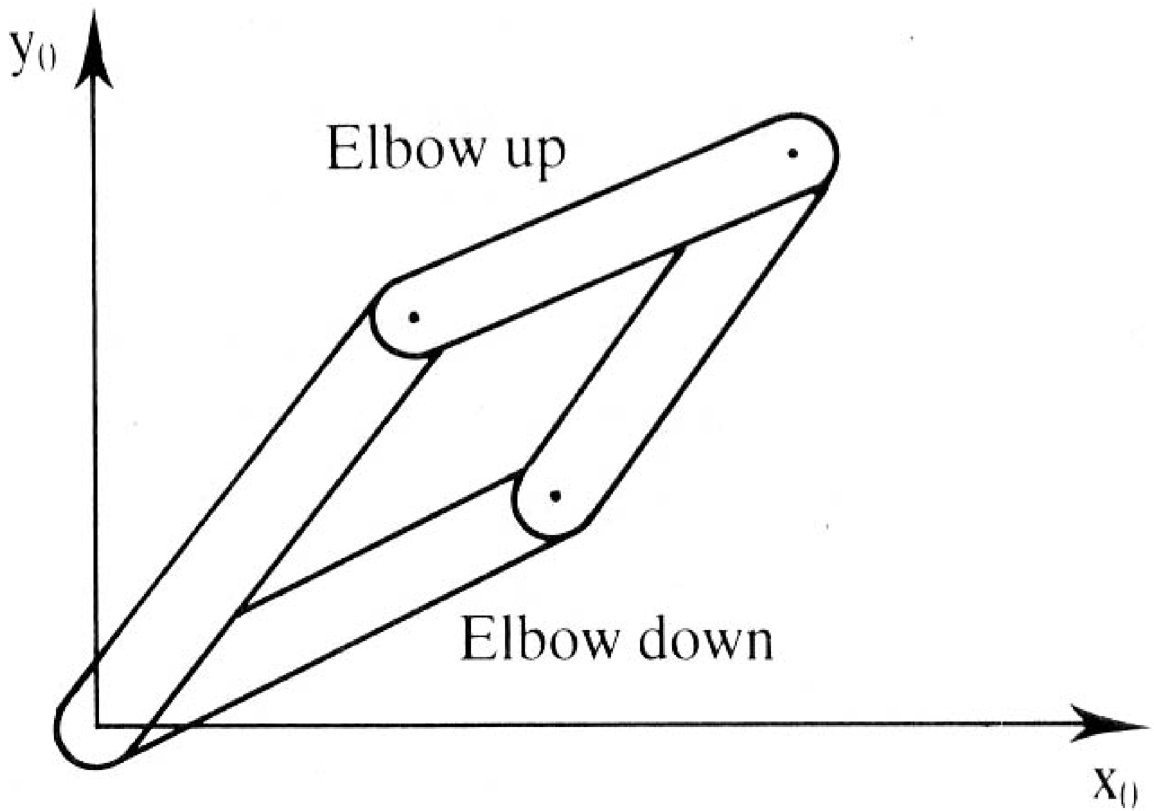
๑๑ Inverse kinematics



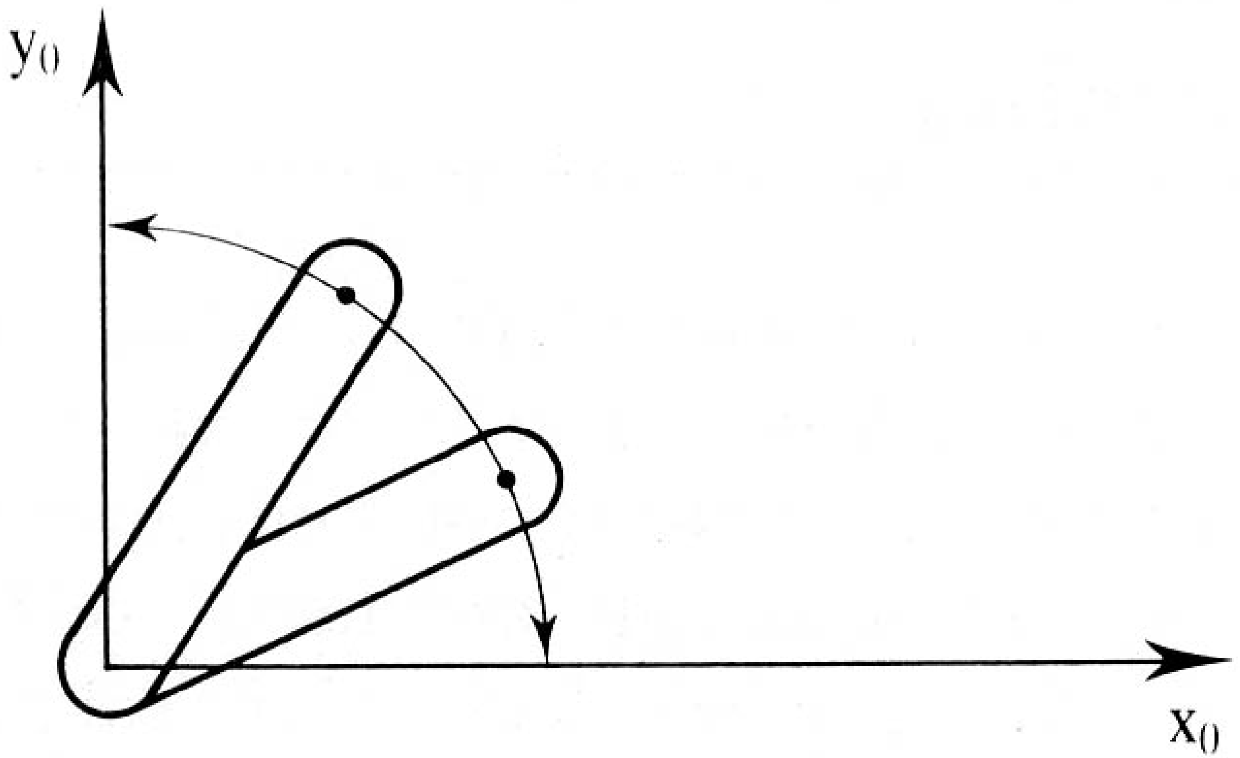
Mapping from joint to Cartesian space using a kinematic model.

๑๒ Redundancy and degeneracies

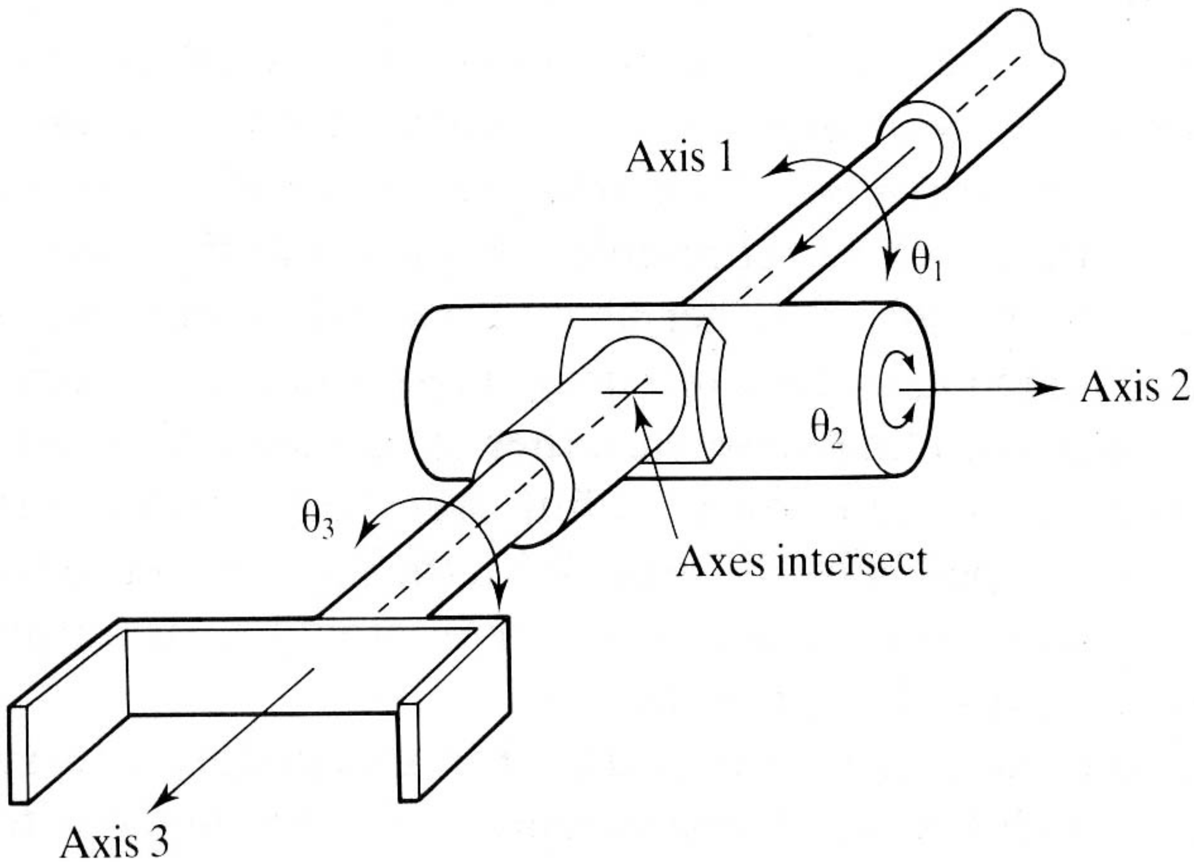
Principle of Robotics



Redundant solution for the Type ๑ two-link manipulator.



Degenerate solution for the Type ๑ two-link manipulator.



Degenerate wrist configuration.

๑๓ Programming

๑๔ Accuracy of the kinematic model

Mini-mover & arm with controller
and power supply.

๑๕ Efficiency of the kinematic solutions

๑๖ Example

Mini-mover & arm. (a) Manipulator in zero position;

(b) Kinematic model.

(c) Elbow triangle. $a_{๑}$=๑๙๕ mm

$a_{๒}$=$a_{๓}$=๑๗๗.๘ mm $a_{๔}$=๙๖.๕ mm

Six degree of freedom Puma Robot (simplified by neglecting l๓) (a) Puma manipulator;

(b) Manipulator in zero position;

(c) Assignment of coordinate frames.