Chapter 8

Steady-State Errors

Outline

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    2.2 Ramp function

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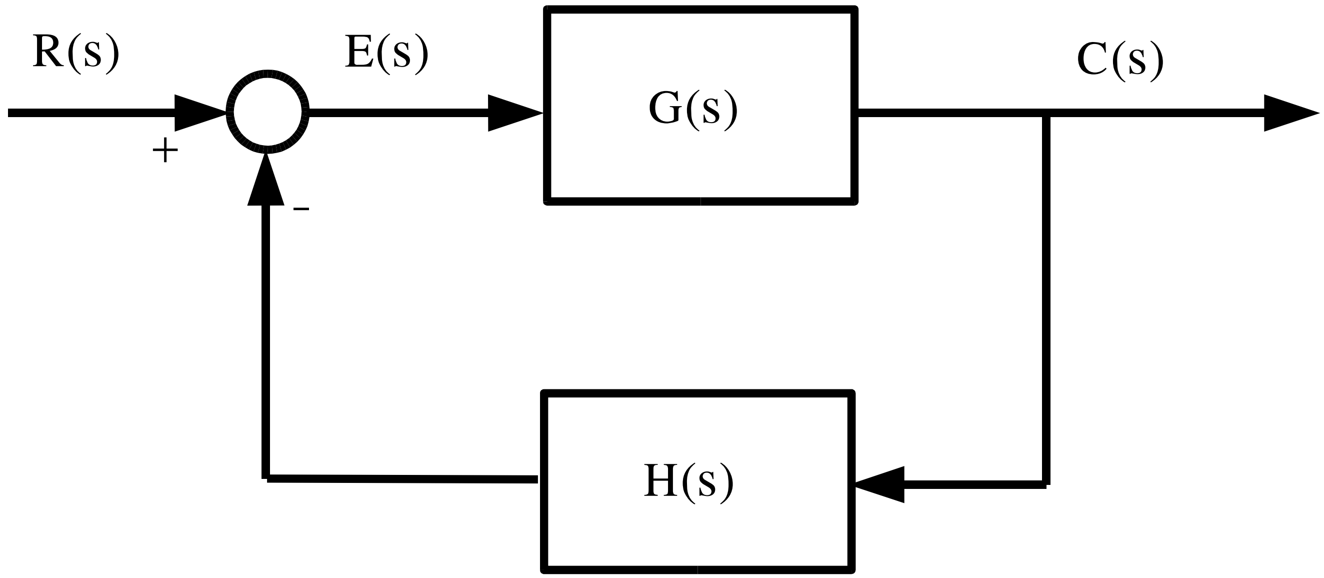
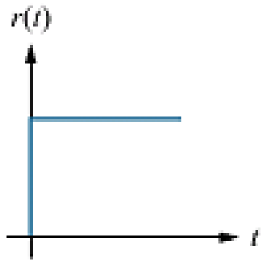
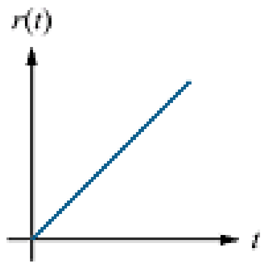
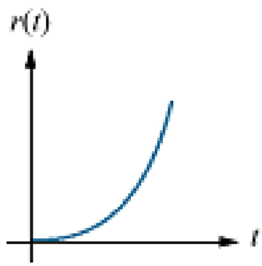
Definition and test input

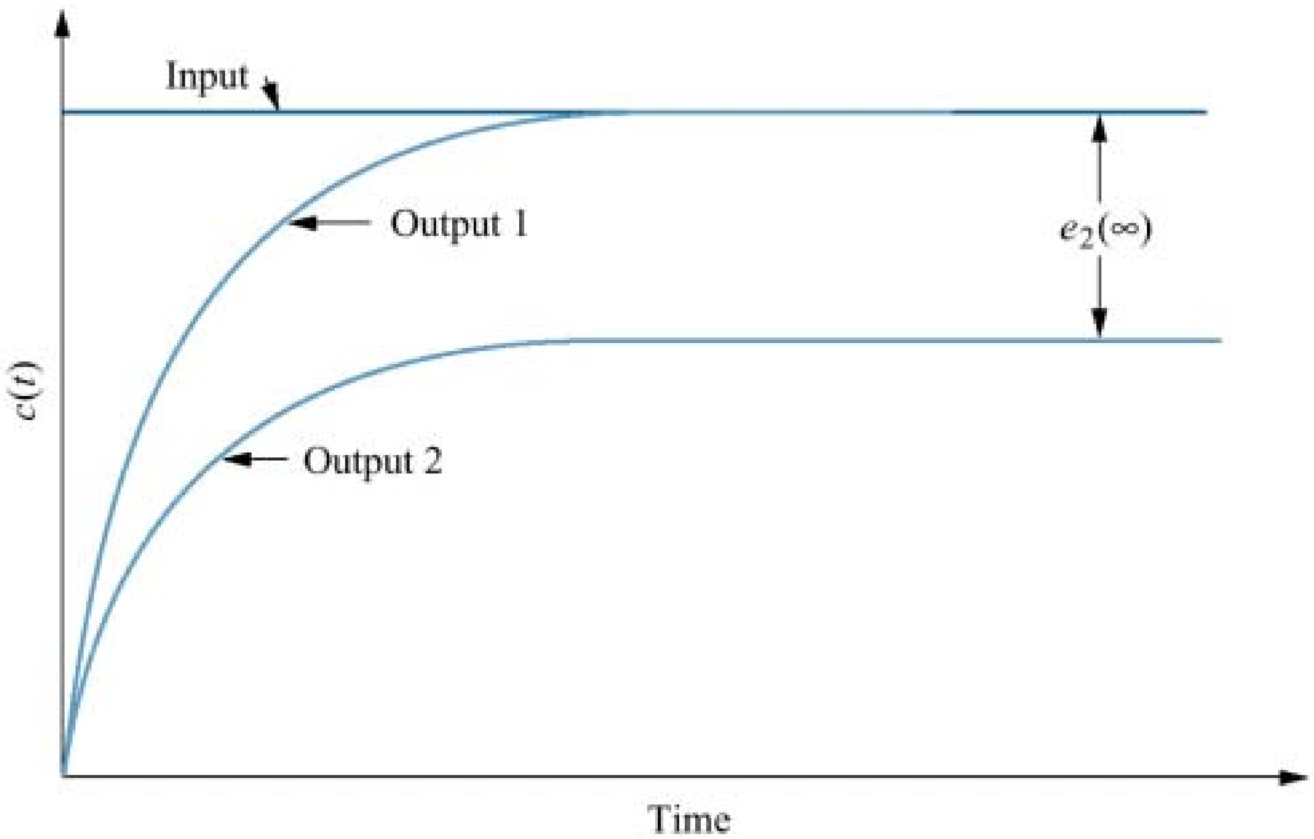


$$e(\infty) = \lim_{t \to \infty} e(t)$$

$$e(t) = L^{-1}[E(s)]$$

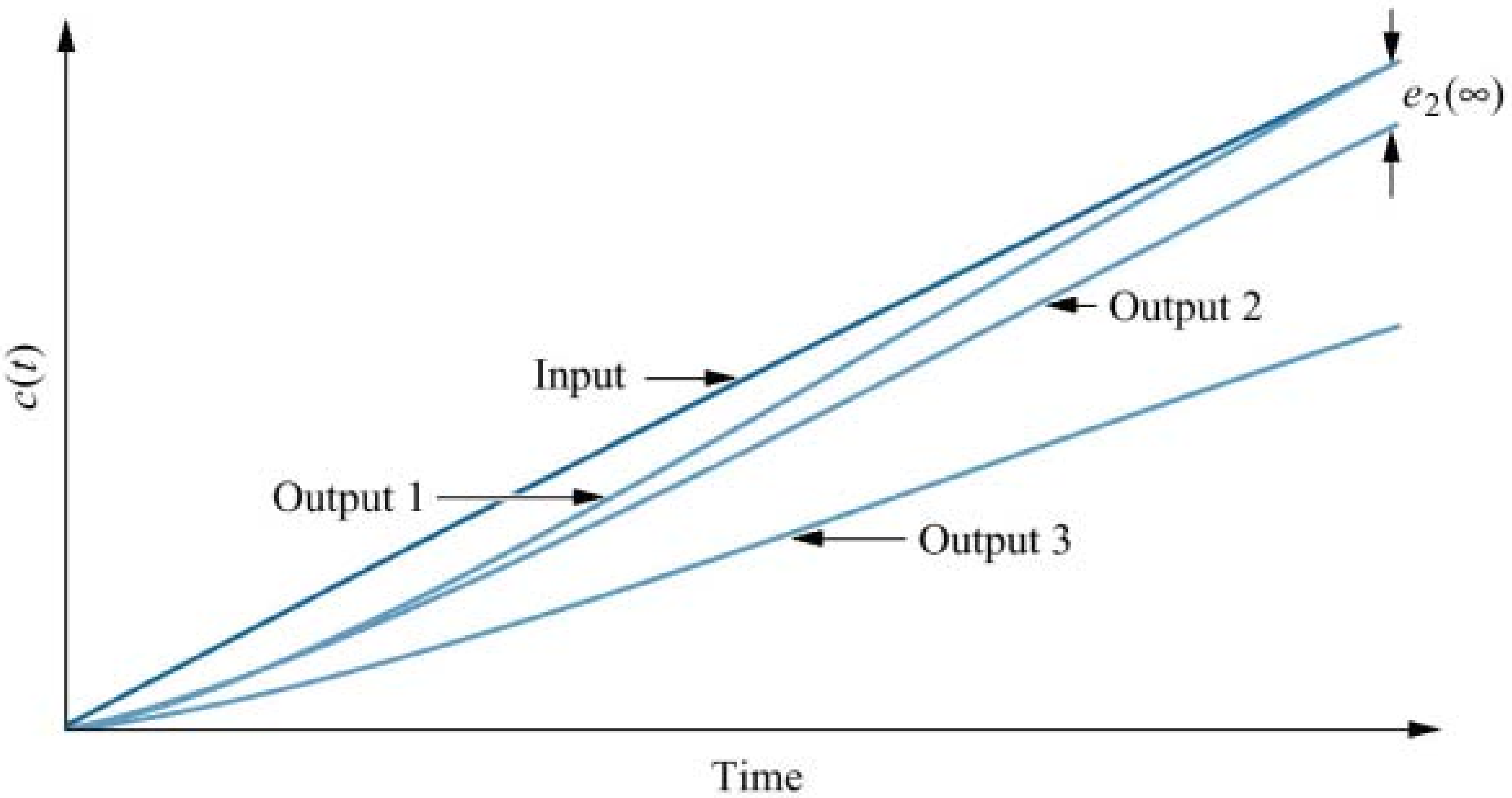
<table>
<tr><th>Waveform</th><th>Name</th><th>Physical interpretation</th><th>Time function</th><th>Laplace transform</th></tr>
<tr><td></td><td>Step</td><td>Constant position</td><td>1</td><td>$\frac{1}{s}$</td></tr>
<tr><td></td><td>Ramp</td><td>Constant velocity</td><td>t</td><td>$\frac{1}{s^2}$</td></tr>
<tr><td></td><td>Parabola</td><td>Constant acceleration</td><td>$\frac{1}{2}t^2$</td><td>$\frac{1}{s^3}$</td></tr>
</table>

Steady-state error



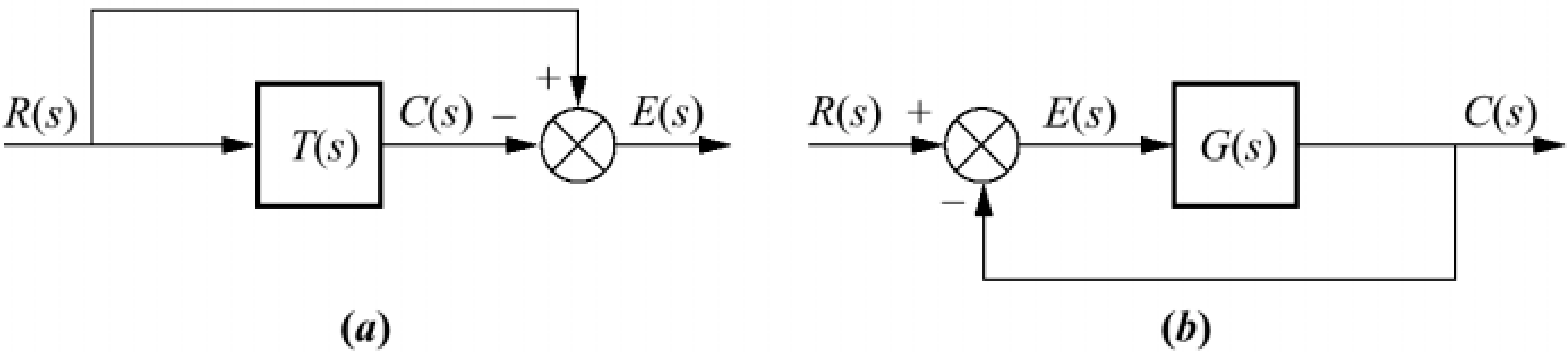
Step input

Steady-state error



Ramp input

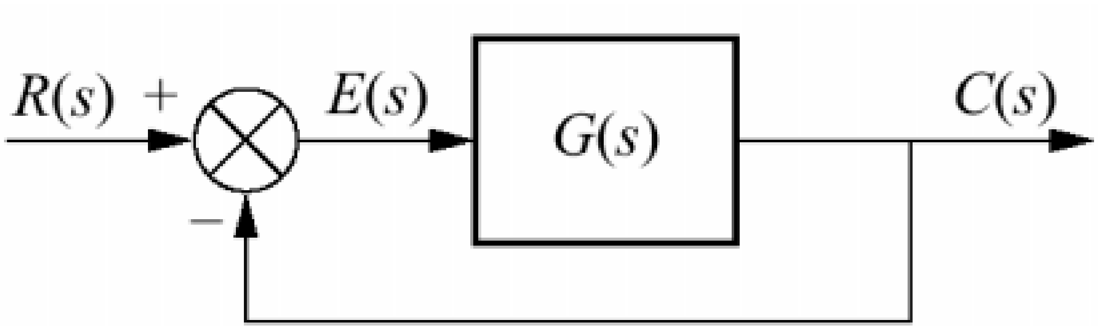
Closed-loop control system error



(a) general representation;

(b) representation for unity feedback systems

Steady-State Error for Unity Feedback Systems



$$E(s) = \frac{R(s)}{1+G(s)}$$

$$e(\infty) = \lim_{t \to \infty} e(t) = \lim_{s \to 0} sE(s)$$

Step input

$$e(\infty) = e_{step}(\infty) = \lim_{s \to 0} \frac{s(1/s)}{1+G(s)} = \frac{1}{1+\lim\limits_{s \to 0} G(s)}$$

$$e(\infty) \to 0 \qquad \text{When} \qquad \lim_{s \to 0} G(s) = \infty$$

Ramp input

$$e(\infty) = e_{ramp}(\infty) = \lim_{s \to 0} \frac{s(1/s^2)}{1 + G(s)} = \frac{1}{1 + \lim_{s \to 0} sG(s)}$$

$$e(\infty) \to 0 \qquad \text{When} \qquad \lim_{s \to 0} sG(s) = \infty$$

Parabolic input

$$e(\infty) = e_{para}(\infty) = \lim_{s \to 0} \frac{s(1/s^3)}{1+G(s)} = \frac{1}{1 + \lim_{s \to 0} s^2 G(s)}$$

$$e(\infty) \to 0 \qquad \text{When} \qquad \lim_{s \to 0} s^2 G(s) = \infty$$

Static Error Constants and System type

Position constant, K_p

$$K_p = \lim_{s \to 0} G(s)$$

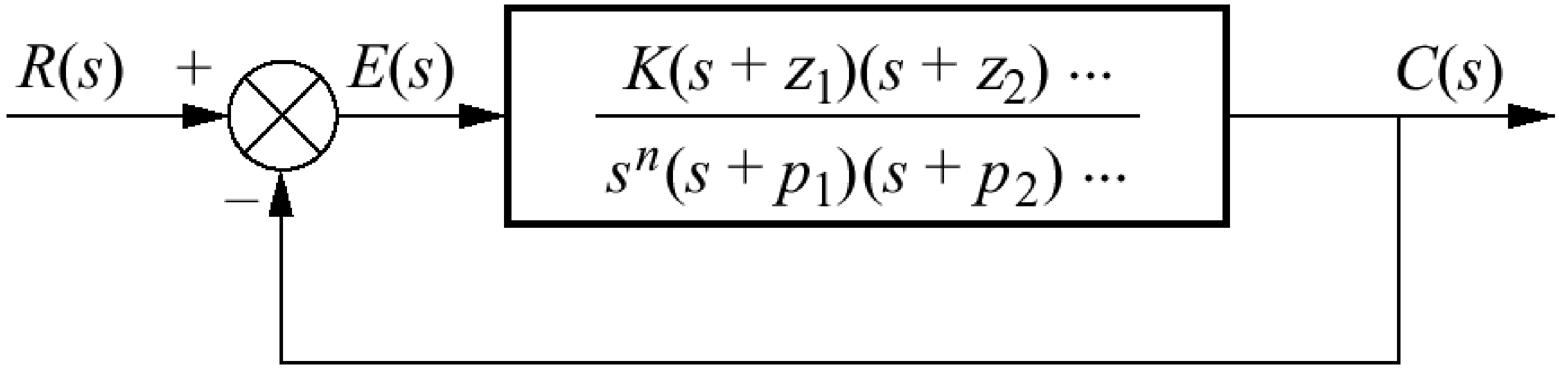
Velocity constant, K_v

$$K_v = \lim_{s \to 0} sG(s)$$

Acceleration constant, K_a

$$K_a = \lim_{s \to 0} s^2 G(s)$$

Feedback control system for defining system type



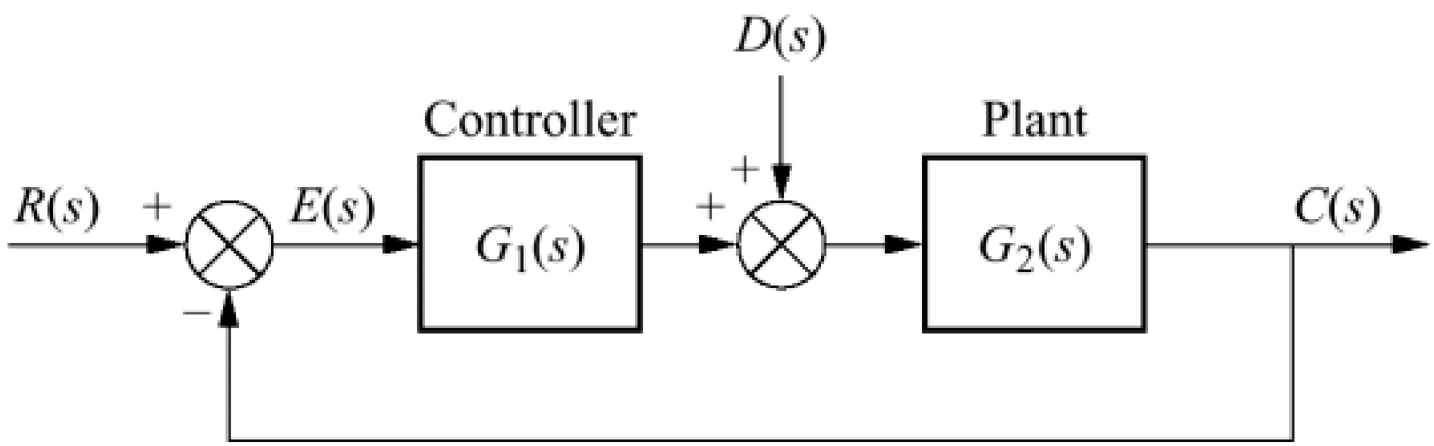
Input	Steady-state error formula	Type 0 Static error constant	Type 0 Error	Type 1 Static error constant	Type 1 Error	Type 2 Static error constant	Type 2 Error
Step, $u(t)$	$\frac{1}{1+K_p}$	$K_p =$ Constant	$\frac{1}{1+K_p}$	$K_p = \infty$	0	$K_p = \infty$	0
Ramp, $tu(t)$	$\frac{1}{K_v}$	$K_v = 0$	∞	$K_v =$ Constant	$\frac{1}{K_v}$	$K_v = \infty$	0
Parabola, $\frac{1}{2}t^2u(t)$	$\frac{1}{K_a}$	$K_a = 0$	∞	$K_a = 0$	∞	$K_a =$ Constant	$\frac{1}{K_a}$

Steady-State Error Specifications

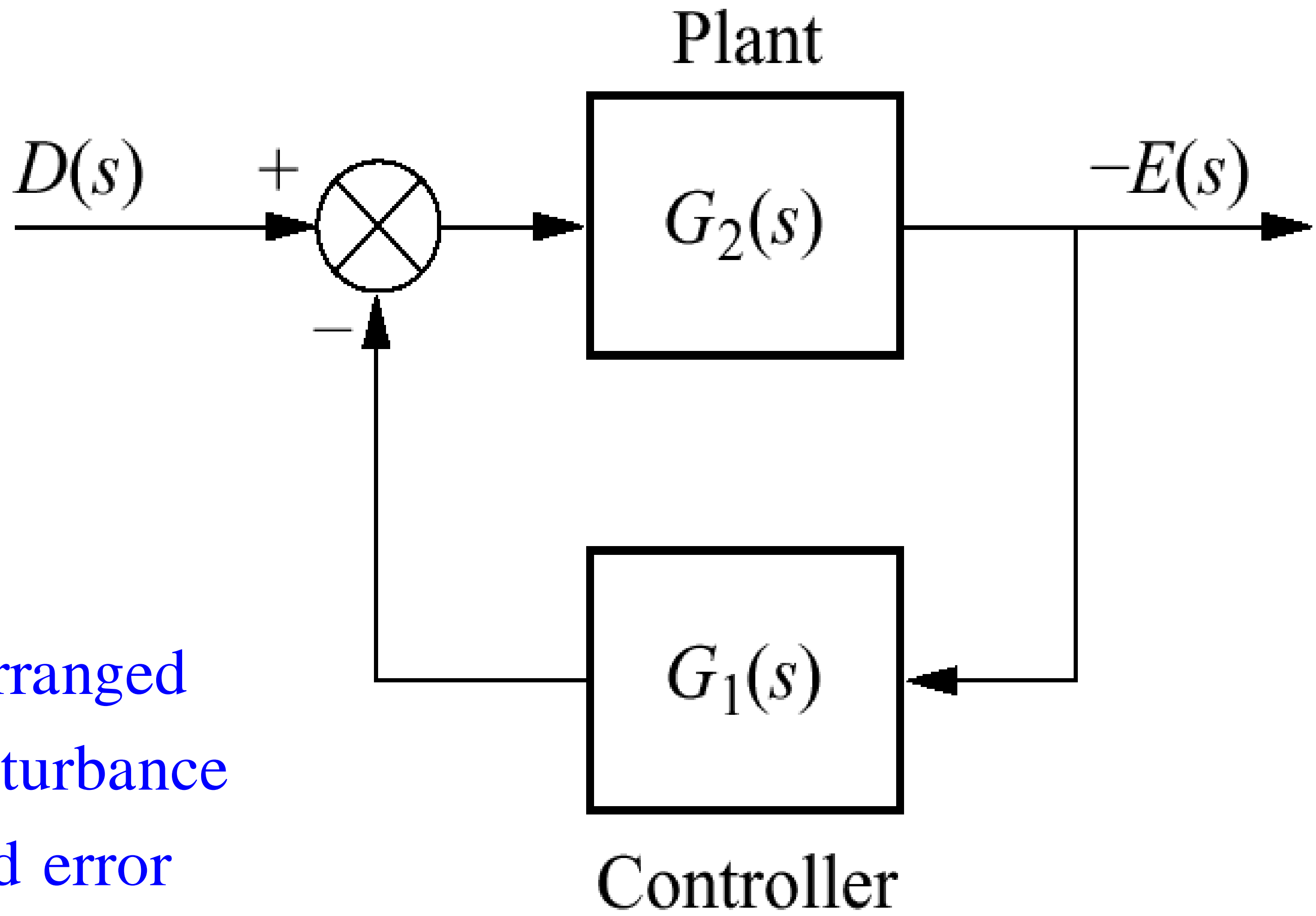
If a control system has the specification $K_v = 1000$, we can draw a conclusions :

1. The system is stable.
2. The system is of type 1.
3. A ramp input is the test signal.
4. The steady-state error between the input ramp and the output ramp is $1/K_v$ per unit of input slope.

Steady-state Error for Disturbances



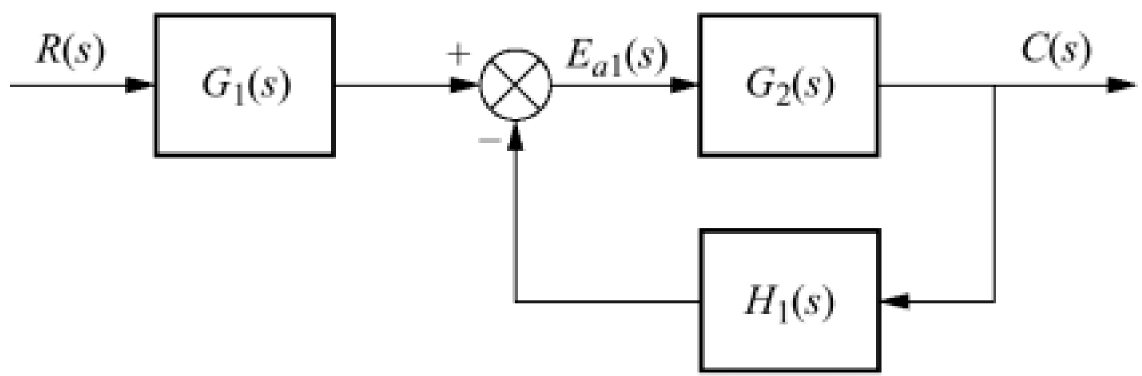
Feedback control system showing disturbance



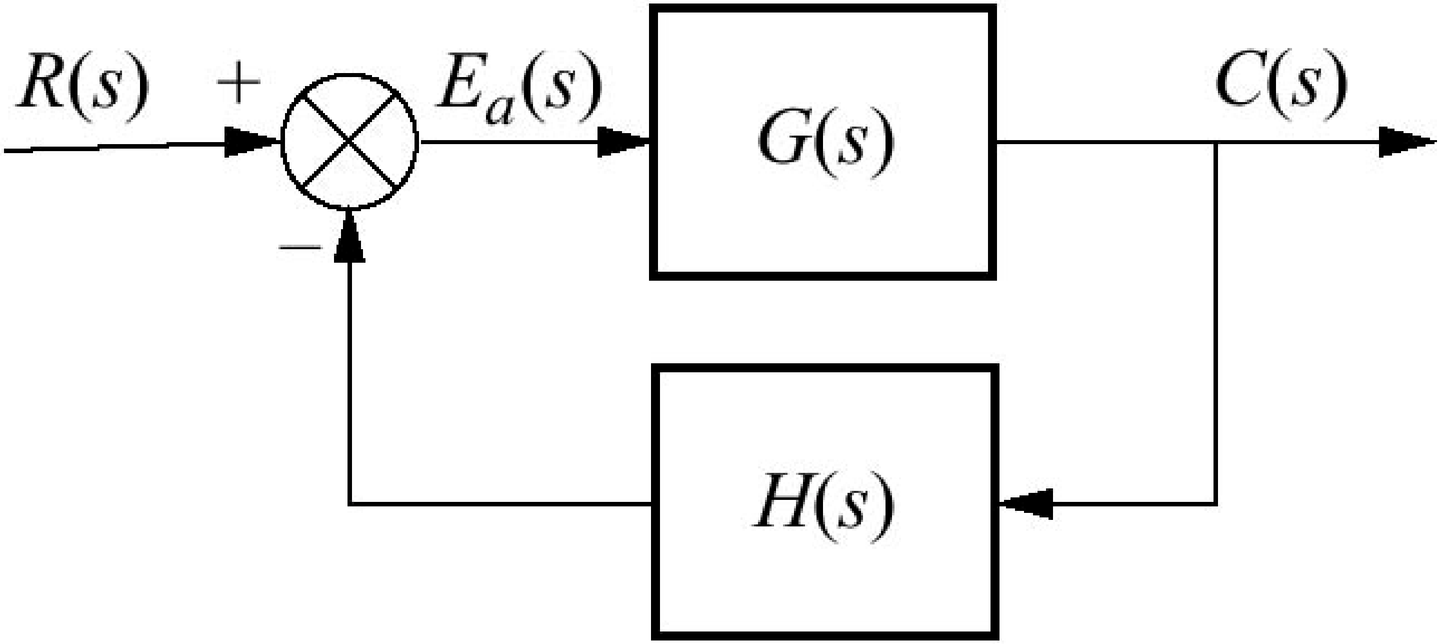
System rearranged to show disturbance as input and error as output, with R(s) = 0

Steady-state Error for Non-unity Feedback Systems

Forming an equivalent unity feedback system from a general nonunity feedback system

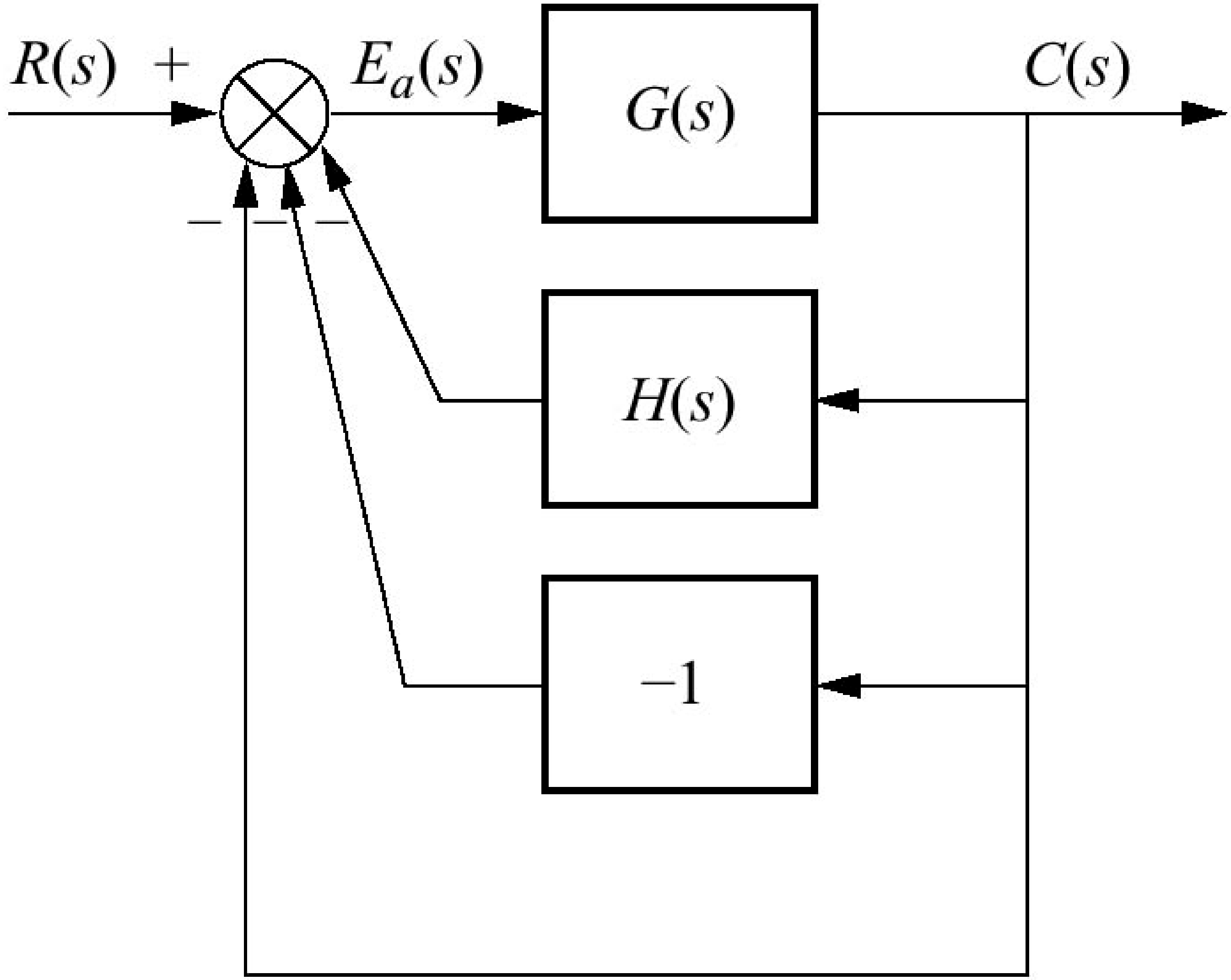


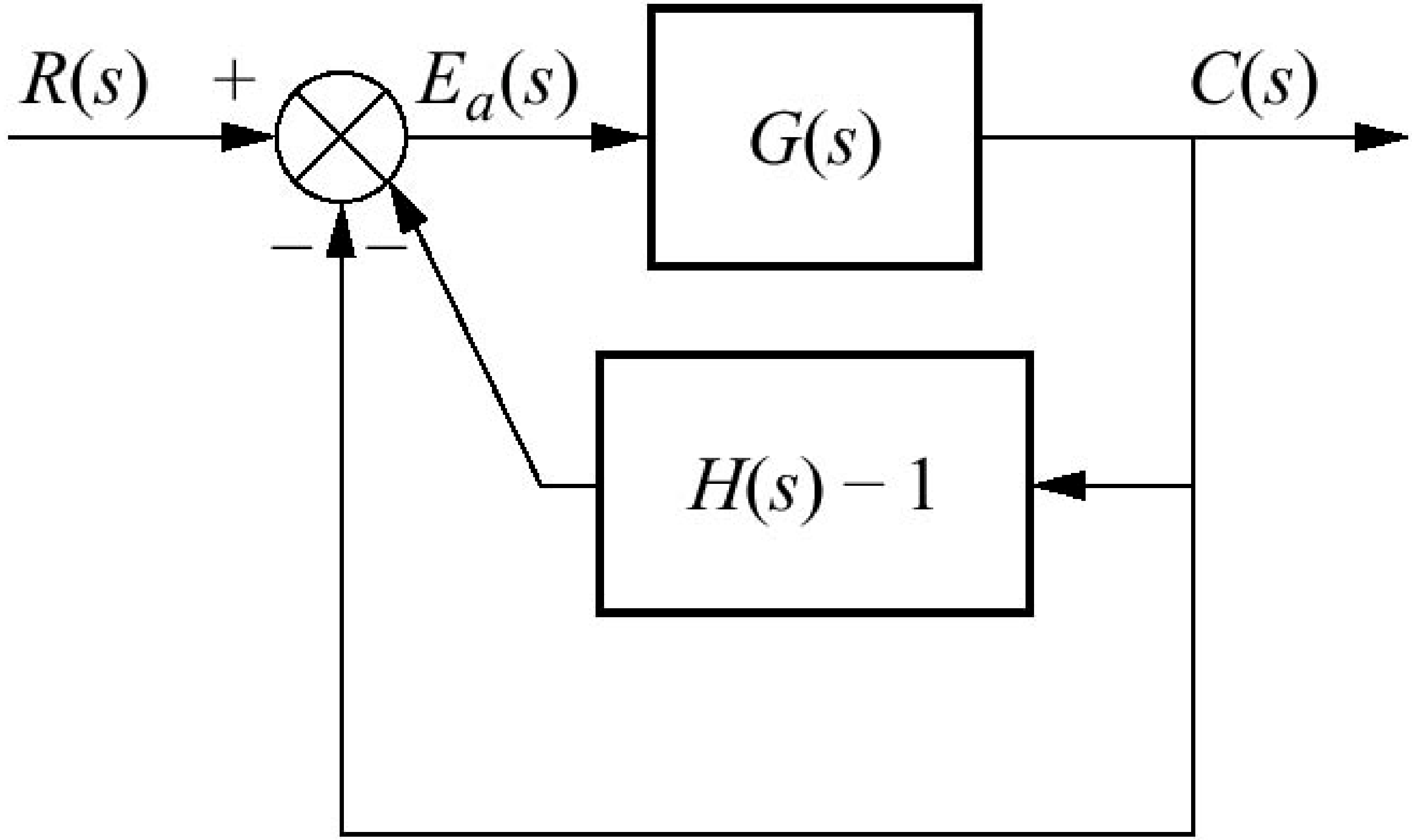
(a)



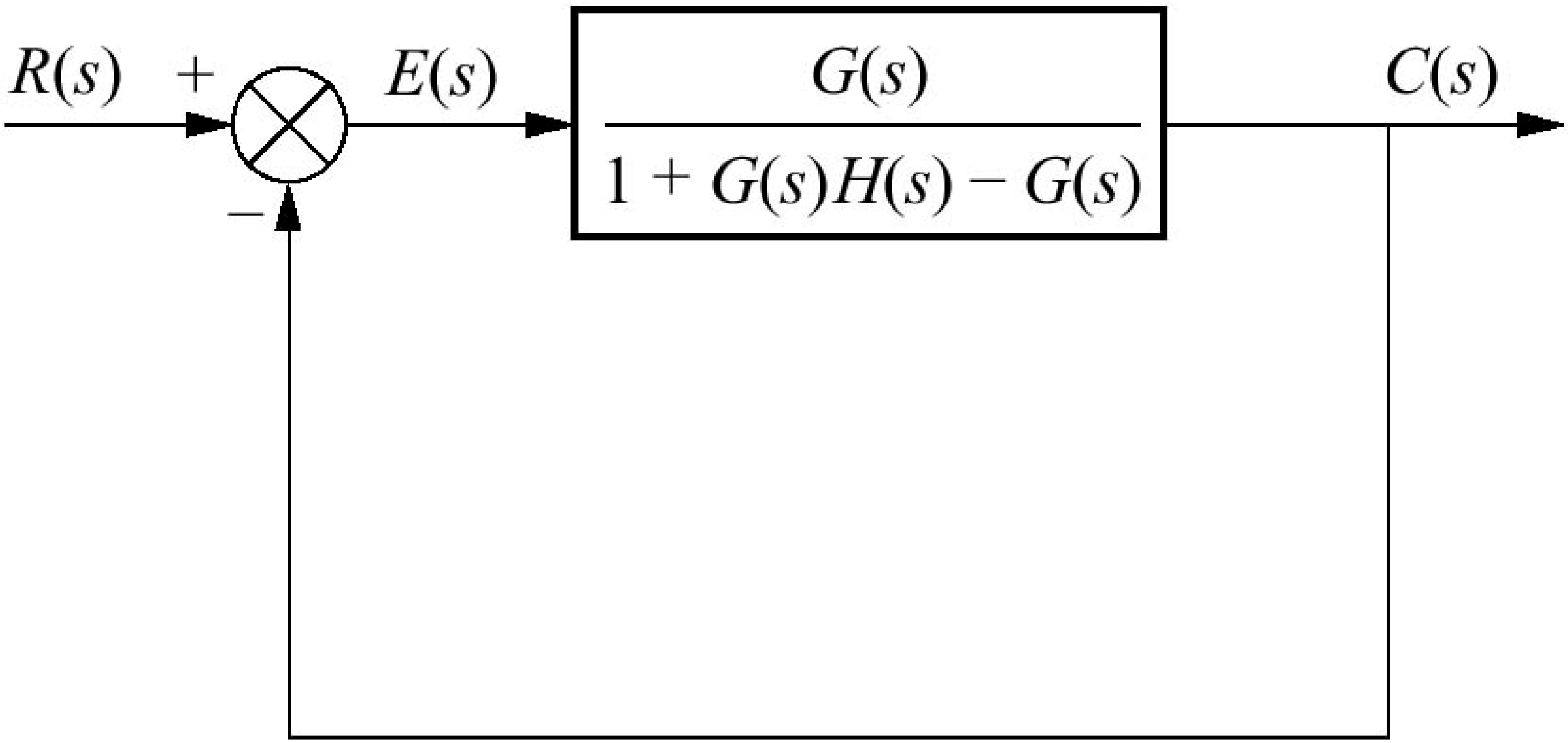
$$G(s) = G_1(s)G_2(s)$$

$$H(s) = H_1(s)/G_1(s)$$

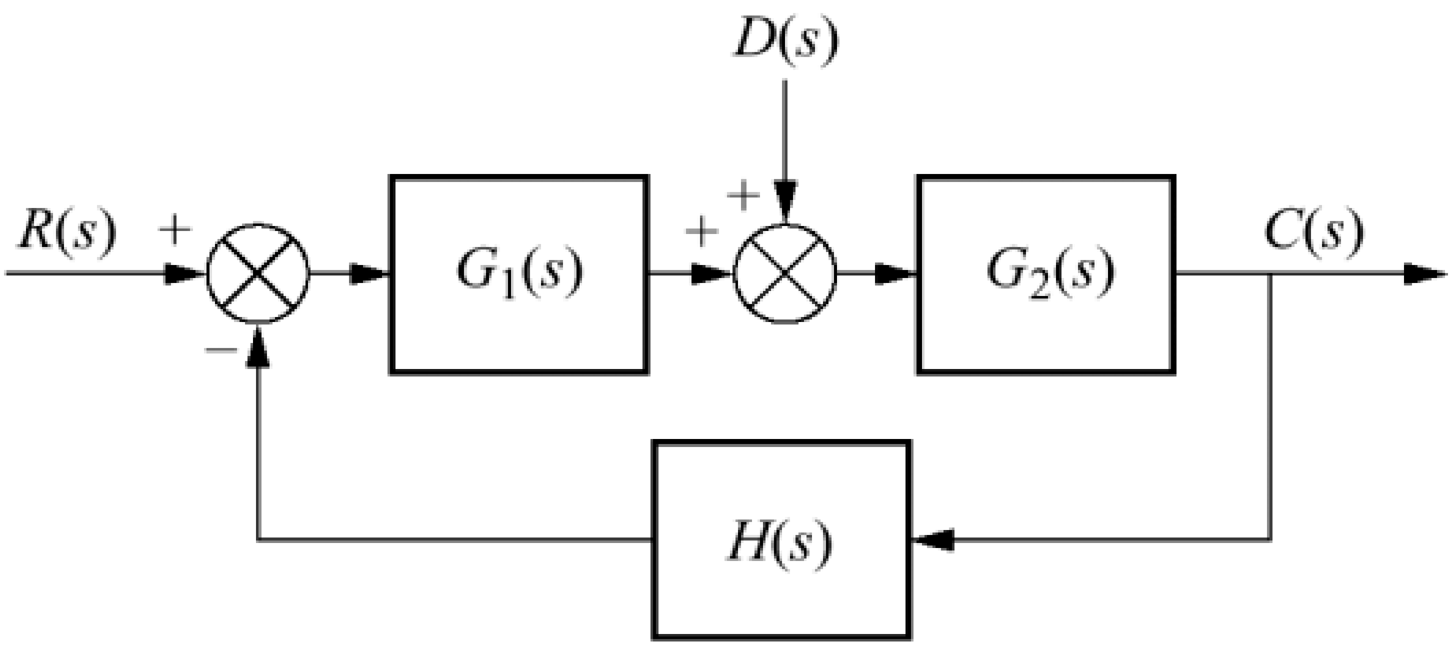

R(s)
+
Ea(s)
G(s)
C(s)
H(s)
−1
−
−
−



$R(s)$
+
$E_a(s)$
$G(s)$
$C(s)$
−
−
$H(s)-1$


R(s)
+
−
E(s)
G(s)
1 + G(s)H(s) − G(s)
C(s)

Nonunity feedback control system with disturbance



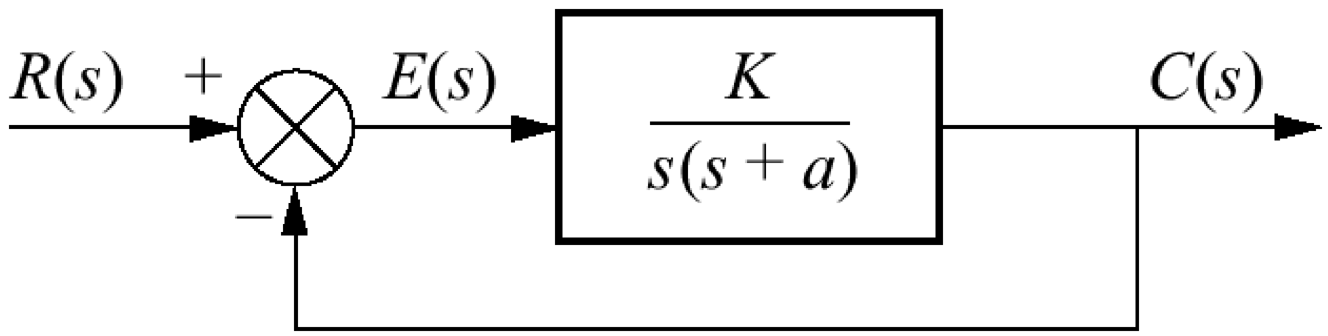
Sensitivity

$$S_{F:P} = \lim_{\Delta P \to 0} \frac{\text{Fractional change in the function}, F}{\text{Fractional change in the parameter}, P}$$

$$= \lim_{\Delta P \to 0} \frac{\Delta F / F}{\Delta P / P} = \lim_{\Delta P \to 0} \frac{P}{F} \frac{\Delta F}{\Delta P}$$

$$S_{F:P} = \frac{P}{F} \frac{\delta F}{\delta P}$$

Example (Nise) Calculate the sensitivity of the closed-loop transfer function to changes in parameter "a"



Steady-state Error for System in State space

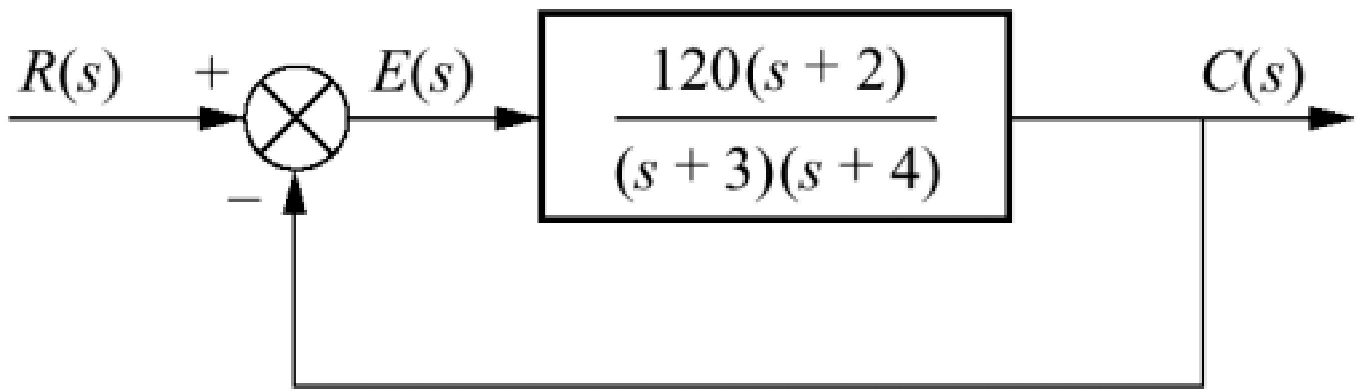
$$\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}r$$

$$y = \mathbf{C}\mathbf{x}$$

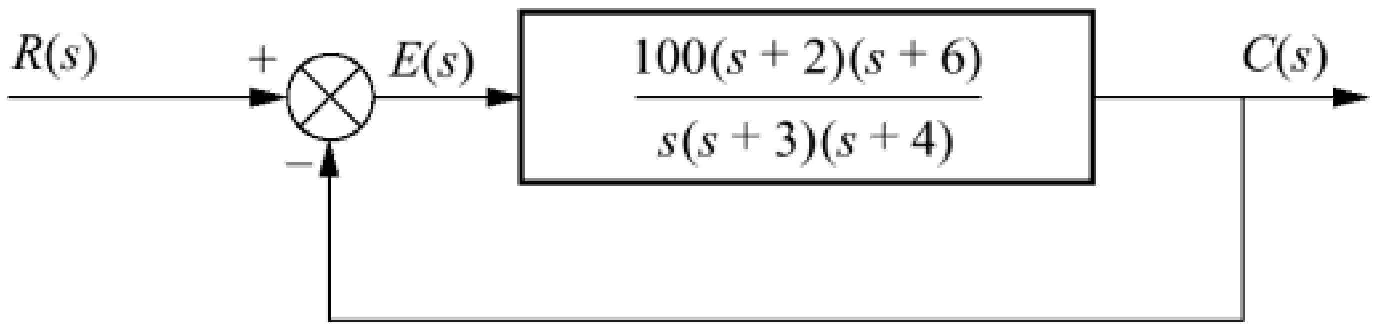
$$E(s) = R(s)\left[1 - C(sI - A)^{-1}B\right]$$

$$\lim_{s \to 0} sE(s) = \lim_{s \to 0} sR(s)\left[1 - \mathbf{C}(s\mathbf{I} - \mathbf{A})^{-1}\mathbf{B}\right]$$

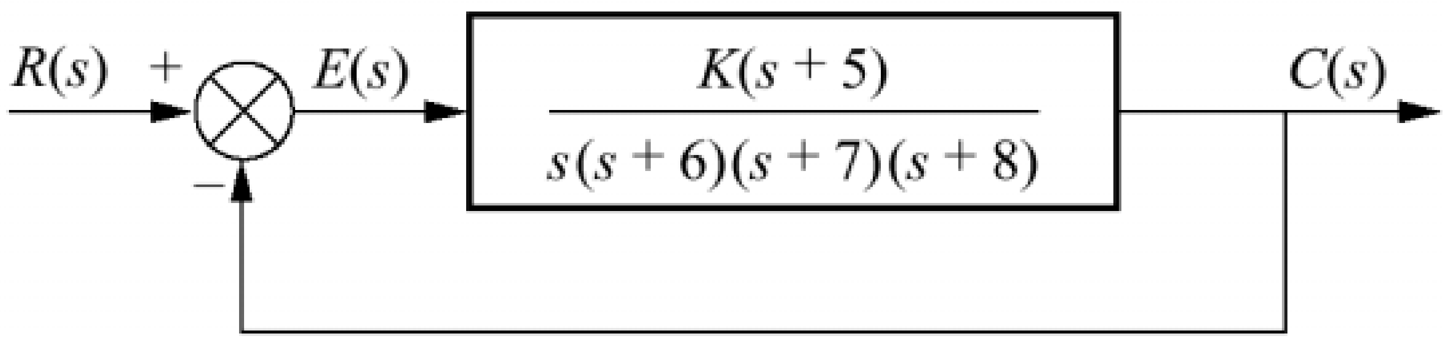
Example (Nise) Find the steady-state errors for inputs of $5u(t)$, $5tu(t)$, and $5t^2$ to the system shown below.



Example (Nise) Find the steady-state errors for inputs of $5u(t)$, $5tu(t)$, and $5t^2$ to the system shown below.



Example (Nise) Find the value of K so that there are 10% error in the steady state for the system shown below.



Example (Nise) Find the steady-state error for a unit step input of the system shown below.