A Critical Review of the Preetham Skylight Model

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ABSTRACT

The Preetham skylight model is currently one of the most widely used analytic models of skylight luminance in computer graphics. Despite its widespread use, very little work has been carried out to verify the results generated by the model, both in terms of the luminance patterns it generates, and in terms of numerical reliability and stability.

In this paper, we show where the model exhibits problematic behaviour, and compare the computed luminance values with the CIE 2003 Standard General Sky, and our own measurements of real, cloudless skies.

Keywords
Sky Luminance Models, Verification

1. INTRODUCTION

Over the past 20 years, the representation of the clear blue sky in computer graphics has been improved tremendously, from the then-common simple blue backgrounds to physically-based spectral models of skylight distribution.

A brute-force numerical simulation of radiation transport in the atmosphere still takes several hours on contemporary fast PCs, so for most practical applications, one still commonly chooses a fast approximative model of some sort.

This paper discusses the sky luminance values in the widely used skylight model published by Preetham [PSS99]. We contrast its luminance distributions to comparable distributions of the latest CIE skylight models. It appears that the range of atmospheric conditions Preetham's model can represent is more limited than previously assumed.

The rest of the paper is organised as follows: For brevity's sake, we do not present the entire state of the art in skylight models ourselves, but just give references which do, and concentrate on the Preetham model instead. We introduce the key components of the model, and in the largest section we show numerous colour-coded illustrations which help to identify where it is applicable, and also how it compares to the recent CIE models, as well as to our own measurements.

2. RELATED WORK

Skylight models have a long history in computer graphics. Sloup [Slo02] gives a good overview of all skylight models derived up to that point in time. Since then, the most ambitious overall atmospheric clear-sky simulation to have been presented is that of Haber *et al.* [HMS05], which is a brute-force numerical simulation of radiation transport in the atmosphere, and is even capable of simulating twilight phenomena. However, such a simulation takes several hours even on a contemporary PC. For most applications in computer graphics this is clearly impractical.

For illumination planning purposes, the CIE and ISO committees recently have released a joint standard series of analytic models for skylight luminance of clear and overcast skies [CIE04], which supersede previous standards. These models lack both spectral and colour information, so their immediate application to computer graphics is not possible; however, they still provide a valuable reference and are discussed in section 4.

3. PREETHAM'S MODEL

This analytic model [PSS99] was created by fitting simulation data from a skylight model of Nishita *et al.* [NDKY96] to the sky luminance approximation formula of Perez *et al.* [PSI93], which has also been adopted by the CIE in a modified form [CIE04]. The simulation and fitting only took turbidities in the range 2...6 into account. In the appendix of Preetham's paper, all necessary formulae are given for the implementation of the model. However, no comparison to measurements taken in nature seems to have been performed, and the paper is rather silent on limitations of the model.

Coordinate System

In the model, several angles are required:

θ_s	solar angle from zenith
θ	view angle from zenith
φ	view azimuth from solar azimuth
γ	angle between solar and view direction

For simplicity, the solar azimuth is always taken as $0°$, and the sky obviously is symmetric around the solar azimuth.

Turbidity

The key parameter to describe atmospheric conditions in Preetham's model is the atmospheric turbidity T, defined as the ratio of the optical thickness of the atmosphere including haze to that of a pure-air atmosphere: $T = (\tau_m + \tau_h)/\tau_m$. A (hypothetical) perfectly clean atmosphere has $T = 1$, and (rare, but observable) values of about 1.25 can be called "exceptionally clear", and 1.6 "very clear", as seen in figure 3 of Preetham's paper. Haze has $T \gtrsim 10$, and at this point at the latest we should also stop using a "clear-sky" model. Preetham states that turbidity is only a rough estimate for atmospheric conditions; it is quite a useful parameter nonetheless, since it offers an intuitive way of controlling the appearance of outdoor scenes.

A problem with zenith luminance Y_z

Preetham [PSS99, A.2] gives zenith luminance as

$$Y_z[\text{kcd/m}^2] = (4.0453T - 4.9710)\tan\chi$$
$$- 0.2155T + 2.4192 \qquad (1)$$

$$\text{where} \quad \chi = (\frac{4}{9} - \frac{T}{120})(\pi - 2\theta_s) \qquad (2)$$

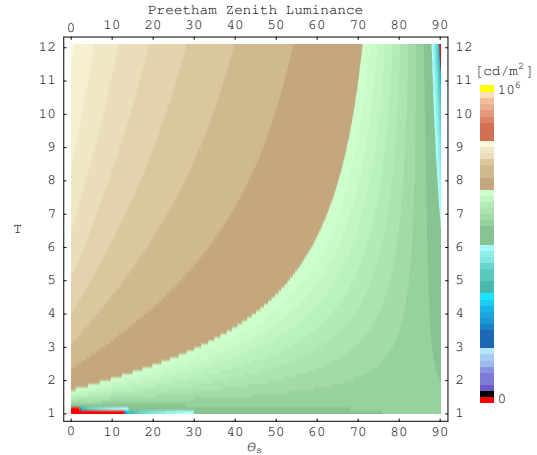Figure 1 shows values of Y_z dependent on solar zenith angle θ_s and turbidity T. As can be seen

Figure 1: Preetham zenith luminance Y_z (1) as function of solar zenith angle θ_s and turbidity T. Negative values are depicted in red.

in the plot, there are regions (marked red) near the border of valid input values where *the model yields negative zenith luminance!* So, from this plot alone, Preetham's model should not be used for exceptionally clear air ($T \lesssim 1.6$), and also values near sunrise/sunset ($\theta_s > 85°$) are apparently problematic, and fail for turbidities $T \gtrsim 10$.

Sky luminance Y

The sky luminance $Y(T, \theta, \theta_s, \varphi)$ is given as

$$Y = Y_z \frac{\mathcal{F}(\theta, \gamma)}{\mathcal{F}(0, \theta_s)}, \quad \text{where} \qquad (3)$$

$$\mathcal{F}(\theta, \gamma) = (1 + Ae^{\frac{B}{\cos\theta}})(1 + Ce^{D\gamma} + E\cos^2\gamma) \quad (4)$$

and A, B, C, D, E are given as functions of T. Figures 6 and 7 show skylight distributions relative to the zenith brightness and absolute luminance values for different values of T and θ_s.

4. CIE STANDARD SKY

The CIE, together with the ISO, has recently published fifteen standard sky luminance distributions [CIE04], which are described in Table 1. These distributions were created and refined from a long series of skylight measurements and previous standards. A sixteenth type describes the "traditional" overcast sky.

The typical application of the CIE models is illumination estimation for architectural design [KD06]. Unfortunately the model only provides luminance data, so that direct application to (colorised) computer graphics is not possible.

1 **CIE Standard Overcast Sky**: Steep luminance
  gradation towards zenith, azimuthal uniformity
2 Overcast, with steep luminance gradation and
  slight brightening towards the sun
3 Overcast, moderately graded with azimuthal uniformity
4 Overcast, moderately graded and slight brightening towards the sun
5 **Sky of uniform luminance**
6 Partly cloudy sky, no gradation towards zenith,
  slight brightening towards the sun
7 Partly cloudy sky, no gradation towards zenith,
  brighter circumsolar region
8 Partly cloudy sky, no gradation towards zenith, distinct solar corona
9 Partly cloudy, with the obscured sun
10 Partly cloudy, with brighter circumsolar region
11 White-blue sky with distinct solar corona
12 **CIE Standard Clear Sky, low luminance turbidity**
13 **CIE Standard Clear Sky, polluted atmosphere**
14 Cloudless turbid sky with broad solar corona
15 White-blue turbid sky with broad solar corona
16 CIE Traditional Overcast Sky

Table 1: The CIE 2003 Standard General Sky models [CIE04]

From the descriptions it appears that the only CIE skylight distributions comparable to the Preetham model are CIE Types 11–15. Note that the CIE skylight distribution type designations are in no way related to Preetham's turbidity! The CIE models do not have a single intuitive parameter, however, they also use eq. (3) and a modified form of the Perez equation (4), which, using the symbols from above, is:

$$\mathcal{F}_{CIE2003}(\theta, \gamma) =$$
$$(1 + Ae^{\frac{B}{\cos\theta}})(1 + C(e^{D\gamma} - e^{D\frac{\pi}{2}}) + E\cos^2\gamma) \quad (5)$$

with values A, B, C, D, E tabulated for the 15 standardized distributions [CIE04, DK02, KD06]. The sixteenth distribution is the classic formula of Moon and Spencer (1942):

$$Y(\theta) = Y_z \frac{1 + 2\cos\theta}{3} \quad (6)$$

Figure 4 shows skylight luminances of the 16 CIE 2003 Standard General Skylight distributions relative to the zenith luminance.
The standard does not provide absolute zenith luminances Y_z, but Kittler (one of the authors of the CIE standard) and Darula [KD06, DK02] provide another five-parameter equation and a table of typical values for sky types 1–15, which are applicable for solar zenith angles $10° \lesssim \theta_s < 90°$ [DK02]. Figure 5 shows all 15 "regular" CIE 2003 Standard General Skylight distributions with their given maximum and minimum recommended values.

5. MEASUREMENTS
For comparison with the Preetham model, we made several measurement cycles of skylight dis-
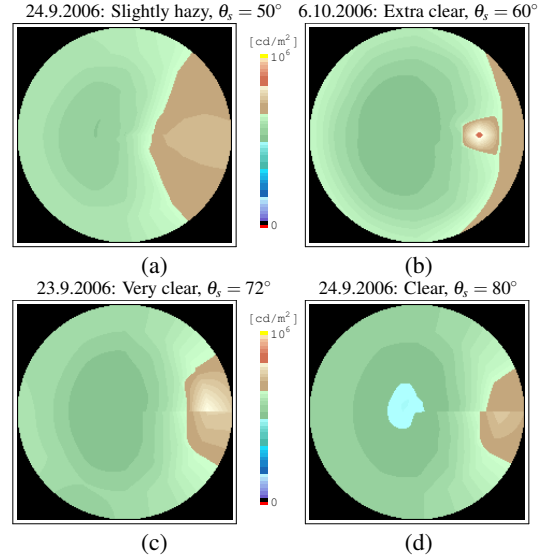

Figure 2: Measured sky luminances $[cd/m^2]$ for solar zenith angles $\theta_s = \{50°, 60°, 72°, 80°\}$ with low turbidities. The discontinuity in the solar vertical (towards right) shows the development during the 1/2-hour measurement process.

tributions using a Minolta LS-110 Luminance Meter on a tripod with coordinate indications. Figure 2 shows 4 typical results for bright, sunny days. What can be clearly seen is a distinct dark zone on the hemisphere opposite the sun and a slight overall horizon brightening.

6. COMPARISON
Figure 6 shows skylight distributions of the original Preetham model and an (obviously fruitless) attempt to improve the Preetham model by using equation (5) for various values of turbidity T and solar zenith angle θ_s, relative to the zenith luminance. Each small circle represents a full 180° hemisphere in stereographic projection. With increasing θ_s, the sun sinks towards the right horizon.
Figure 7 shows skylight distributions for the same data, again with both variants of the function $\mathcal{F}$, but providing absolute luminance values from the Preetham model. Compared to the recommended values from the CIE standard and [KD06] (Figures 4 and 5) and real-world measurements (Fig.2) we note the following shortcomings:

- The results for low turbidities $T < 2$ are definitely wrong: A very clear atmosphere only has a moderate brightening along the horizon (Fig. 2(b)), and not the wide, extremely

bright zone visible in the plots in Figure 7 (*cf.* also Figure 3), leading to a totally unrealistic "horizon glow". Even for $T \lesssim 2.4$ the horizon is shown unnaturally bright when the sun is low ($\theta_s \gtrsim 60°$), and the brightening around the sun is too weak.

- The Preetham model does not properly reproduce the noticeable darkening of the sky in the antisolar hemisphere when the sun is low, with luminance values about 2–5 times too high.

- Also, the brightness peak towards the sun is not as steep as it can be measured or is modelled by the CIE Clear-Sky models.

The CIE Clear Sky (Type 12) is best approximated with $T \approx 2.3 \dots 2.5$, however, differences remain, esp. the mentioned darkening is too weak when $\theta_s \gtrsim 60°$. Comparing Preetham and CIE models for higher turbidities, we found some potentially usable similarities, but with different absolute luminances (with values from [KD06]):

Preetham T	10	5	2.9
CIE Type	6	7	10
$Y_{Pr}/Y_{CIE}\|_{15°<\theta_s<70°}$	> 2	$\gtrsim 1$	$\lesssim 1.25$

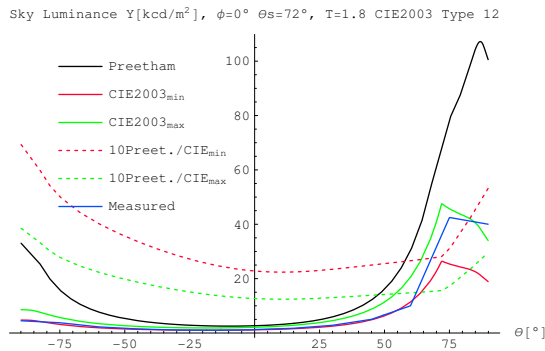Sky Luminance Y[kcd/m²], ϕ=0° θs=72°, T=1.8 CIE2003 Type 12

Figure 3: A cut through the solar vertical for a very clear sky ($T = 1.8$). The black line represents luminance values from the Preetham model, the red and green full lines are maximum and minimum, resp., in the CIE model #12 with values recommended by [KD06], and the blue line shows a result from our measurements (*cf.* Fig. 2(c)). The dashed lines show the relation between Preetham and CIE models (scaled 10 times for clarity and to fit the scale). Clearly, the horizon area below the sun (right end) and also on the opposite side (and all around the sky, see Fig. 7) is far too bright (brighter than the immediate solar vicinity)!

7. CONCLUSION AND FURTHER WORK

We have provided comparisons between the sky luminance values of the popular Preetham skylight model, the ISO/CIE 2003 Standard General Sky luminance distributions and a few measurements taken by ourselves.

Our measurements of typical mid-European early autumn clear sky luminance distributions yielded luminance patterns which are not reproducible by the Preetham model, but which are reasonably close to the CIE Clear Sky models.

From the way it was created, the Preetham model can at most be as good as the Nishita model [NDKY96] with simulation of (at most) second order scattering. The data fit was done for turbidities $2 \dots 6$, so outside this range, it should not be used. We showed it breaks down numerically for $T \lesssim 1.9$ and $T \gtrsim 10$.

An analytic skylight model certainly is required for outdoor scenes, when the skylight should be used as light source, and the sky as visible background. A better simulation of atmospheric scattering such as [HMS05] should be used as base to improve or replace Preetham's model, so that a data fit will hopefully provide more natural results.

8. ACKNOWLEDGMENTS

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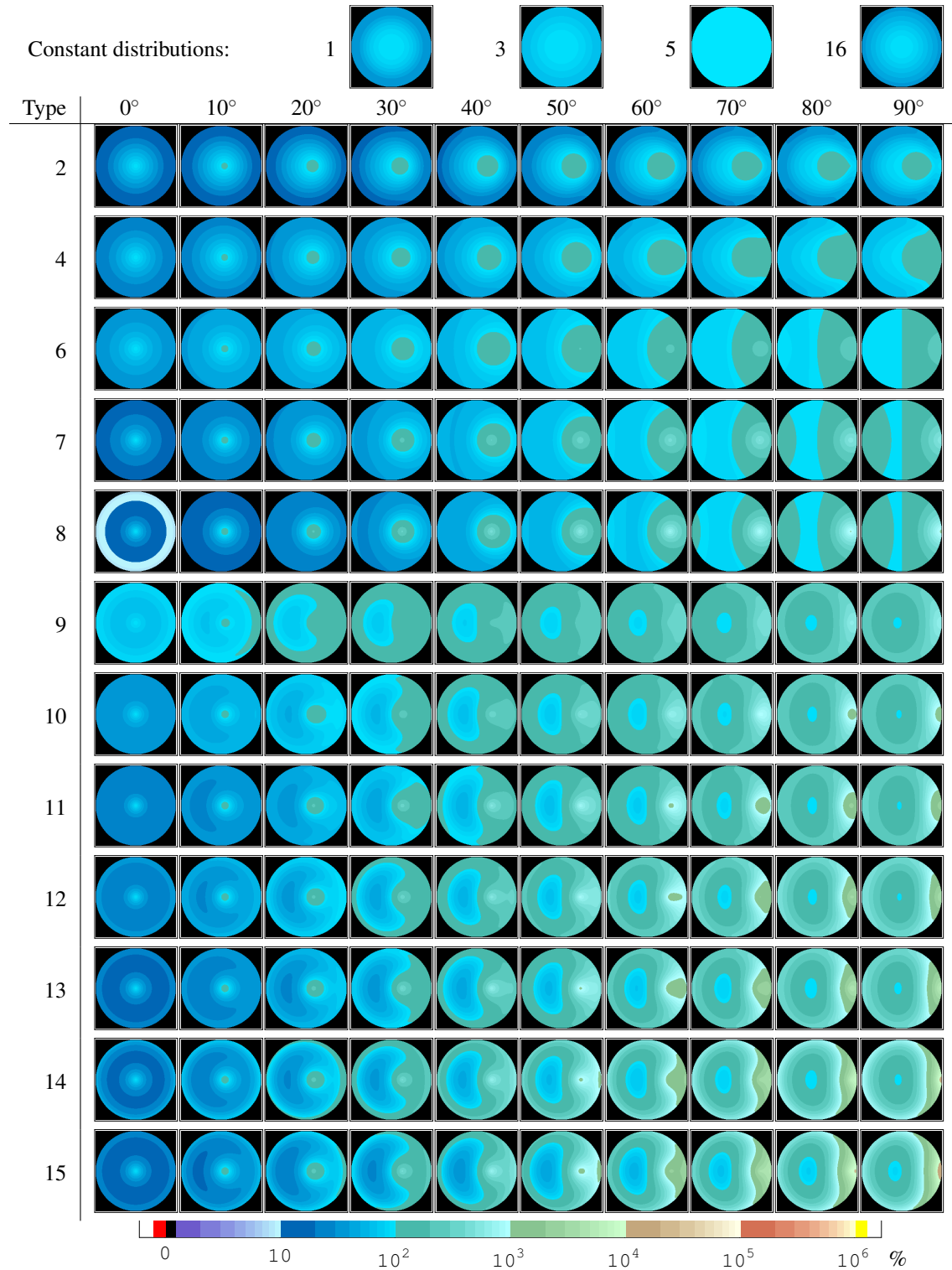
Figure 4: Relative sky luminance distributions of the 16 CIE 2003 Standard General Sky models [CIE04], for various solar zenith distances θ_s. The scale is logarithmic in percent of zenith luminance, and colour hues are used to separate the decades of magnitude.
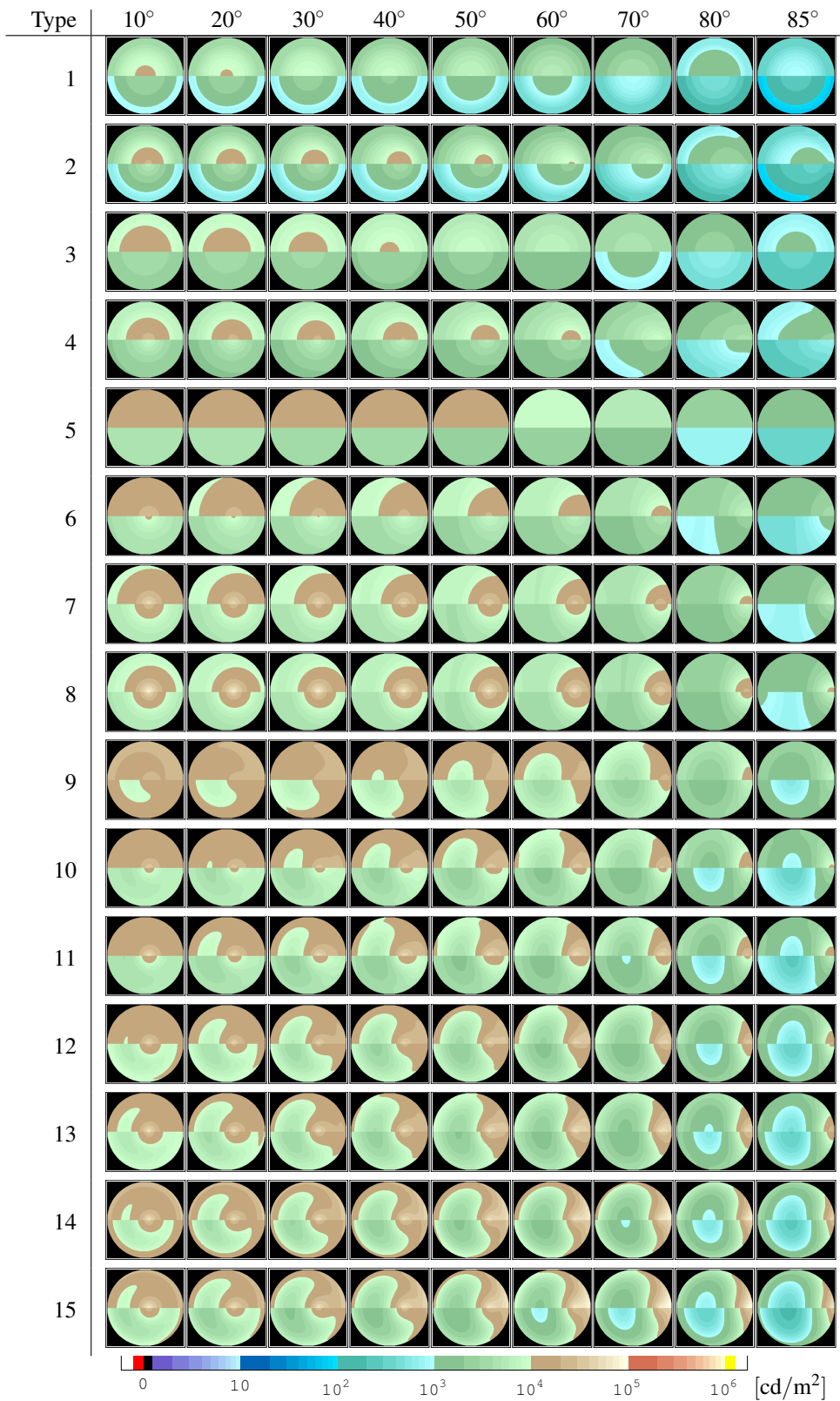
Figure 5: Absolute sky luminance distributions based on the CIE 2003 Standard General Sky models, for various solar zenith distances θ_s. Each upper half-image shows the approximate maximum, the lower half the minimum recommended values after [KD06]. The scale is logarithmic in $[\text{cd/m}^2]$.
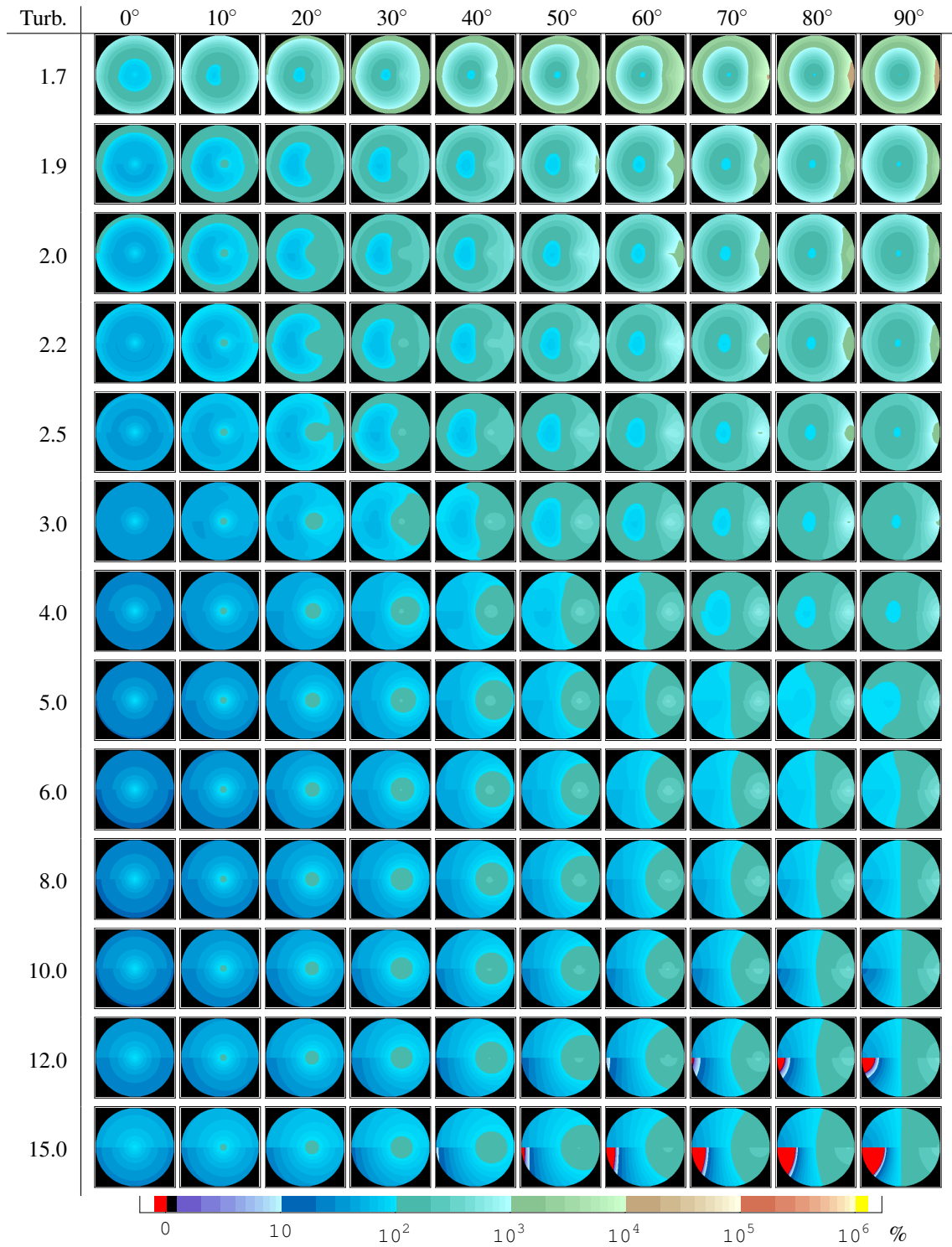
Figure 6: Relative sky luminance distributions of the Preetham model, for various solar zenith distances θ_s and turbidities. The upper half-images show values from the original model, the lower show values achieved by replacing equation (4) by (5), which however does not significantly improve the results and was just done for trial purposes. The scale is logarithmic in percent of zenith luminance.
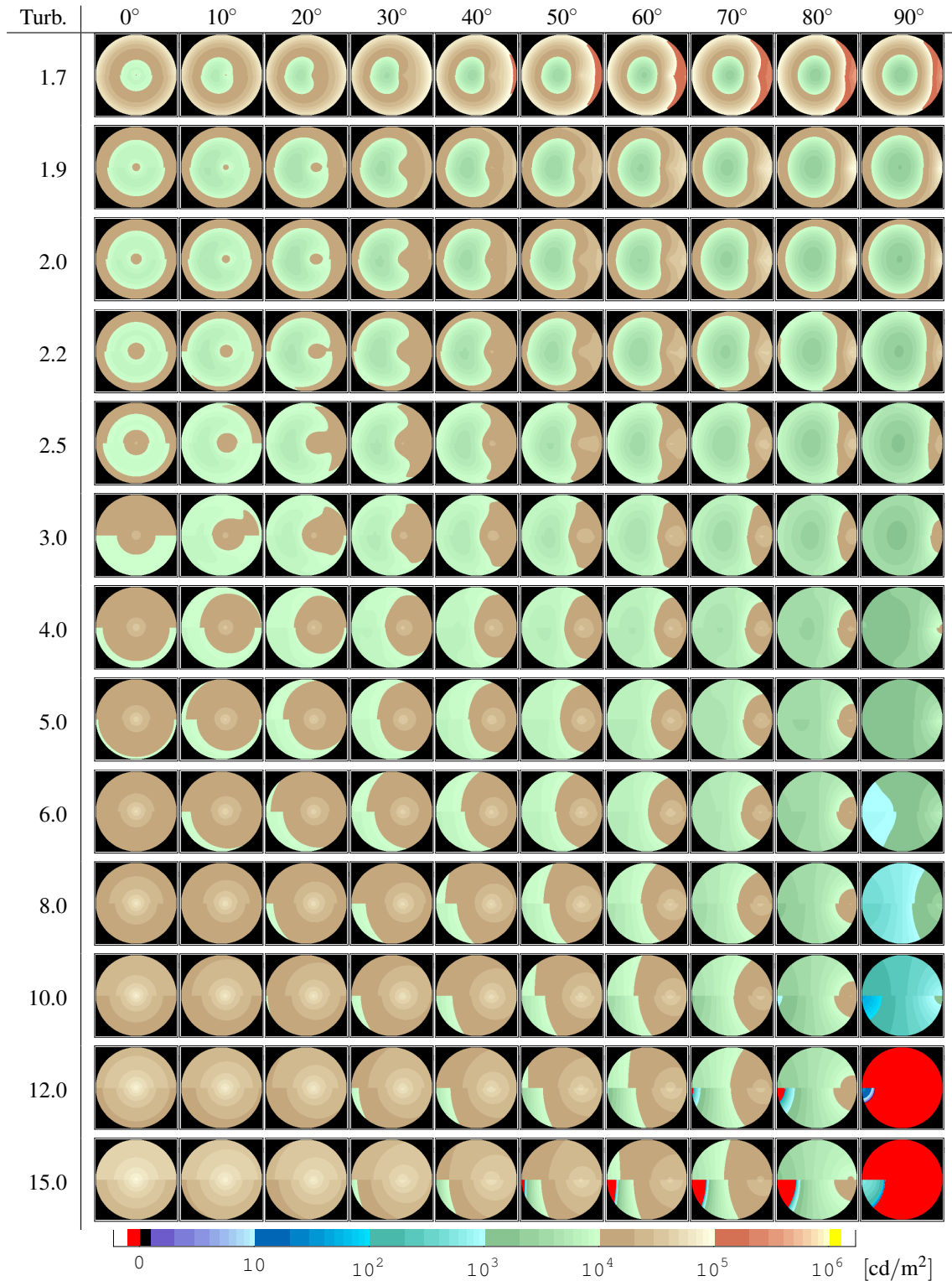
Figure 7: Absolute sky luminance distributions of the Preetham model, for various solar zenith distances θ_s and turbidities. The upper half-images show values from the original model, the lower show values achieved by replacing equation (4) by (5), which however does not significantly improve the results and was just done for trial purposes. The scale is logarithmic in $[\mathrm{cd/m^2}]$.