

Example: Attribute Selection with Information Gain

Class P: buys_computer = "yes"

Class N: buys_computer = "no"

$$Info(D) = I(9,5) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) = 0.940$$

age	p _i	n _i	I(p _i , n _i)
<=30	2	3	0.971
31...40	4	0	0
>40	3	2	0.971

age	income	student	credit_rating	buys_computer
<=30	high	no	fair	no
<=30	high	no	excellent	no
31...40	high	no	fair	yes
>40	medium	no	fair	yes
>40	low	yes	fair	yes
>40	low	yes	excellent	no
31...40	low	yes	excellent	yes
<=30	medium	no	fair	no
<=30	low	yes	fair	yes
>40	medium	yes	fair	yes
<=30	medium	yes	excellent	yes
31...40	medium	no	excellent	yes
31...40	high	yes	fair	yes
>40	medium	no	excellent	no

age	p _i	n _i	I(p _i , n _i)
<= 30	2	2	1
31...40	3	0	0
> 40	3	2	0.971

income	p _i	n _i	I(p _i , n _i)
h	2	2	1
me	4	1	0.722
l	2	1	0.918

student	p _i	n _i	i(p _i , n _i)
y	3	3	1
n	5	1	

credit	p _i	n _i	i(p _i , n _i)
f	6	1	0.594
e	2	3	0.911

$$Info_{age}(D) = \frac{5}{14} I(2,3) + \frac{4}{14} I(4,0) + \frac{5}{14} I(3,2) = 0.694$$

$\frac{5}{14} I(2,3)$ means "age <=30" has 5 out of 14 samples, with 2 yes'es and 3 no's.

Hence

$$Gain(age) = Info(D) - Info_{age}(D) = 0.246$$

Similarly, we can get

$$Gain(income) = 0.029$$

$$Gain(student) = 0.151$$

$$Gain(credit_rating) = 0.048$$

$$Info(D) = I(9,5) = -\frac{9}{14} \log_2\left(\frac{9}{14}\right) - \frac{5}{14} \log_2\left(\frac{5}{14}\right) = 0.919$$

$$Info_{age}(D) = \frac{5}{14} i(2,3) + \frac{3}{14} i(3,0) + \frac{5}{14} i(3,2)$$

$$= \frac{5}{14} \left[-\frac{2}{4} \log_2\left(\frac{2}{4}\right) - \frac{3}{4} \log_2\left(\frac{3}{4}\right) \right] + \frac{3}{14} \left[-\frac{3}{3} \log_2\left(\frac{3}{3}\right) - \frac{0}{3} \log_2\left(\frac{0}{3}\right) \right] + \frac{5}{14} \left[-\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) \right]$$

$$= \frac{5}{14} (1) + \frac{3}{14} (0) + \frac{5}{14} (0.918)$$

$$= 0.754$$

$$Info_{income}(D) = \frac{4}{14} i(2,2) + \frac{5}{14} i(4,1) + \frac{3}{14} i(2,1)$$

$$= \frac{4}{14} (1) + \frac{5}{14} \left[-\frac{4}{5} \log_2\left(\frac{4}{5}\right) - \frac{1}{5} \log_2\left(\frac{1}{5}\right) \right] + \frac{3}{14} \left[-\frac{2}{3} \log_2\left(\frac{2}{3}\right) - \frac{1}{3} \log_2\left(\frac{1}{3}\right) \right]$$

$$= \frac{4}{14} + \frac{5}{14} (0.722) + \frac{3}{14} (0.918)$$

$$= 0.864$$

$$Info_{student}(D) = \frac{6}{14} i(3,3) + \frac{5}{14} i(5,1)$$

$$= \frac{6}{14} (1) + \frac{5}{14} \left[-\frac{3}{5} \log_2\left(\frac{3}{5}\right) - \frac{2}{5} \log_2\left(\frac{2}{5}\right) \right]$$

$$= \frac{6}{14} + \frac{5}{14} (0.850)$$

$$= 0.915$$

$$Info_{credit}(D) = \frac{7}{14} i(6,1) + \frac{5}{14} i(2,3)$$

$$= \frac{7}{14} \left[-\frac{1}{7} \log_2\left(\frac{1}{7}\right) - \frac{6}{7} \log_2\left(\frac{6}{7}\right) \right] + \frac{5}{14} \left[-\frac{2}{5} \log_2\left(\frac{2}{5}\right) - \frac{3}{5} \log_2\left(\frac{3}{5}\right) \right]$$

$$= \frac{7}{14} (0.591) + \frac{5}{14} (0.918)$$

$$= 0.750$$

$$\text{Gain}(\text{age}) = 0.918 - 0.738 = 0.18$$

$$\theta_{\text{gain}(\text{income})} = 0.918 - 0.864 = 0.054$$

$$\text{Gain}(\text{student}) = 0.918 - 0.895, 0.093$$

$$\text{Gain (credit)} = 0.918 - 0.750 = 0.168$$

Gain (Age) \rightarrow $\frac{1}{\text{Age}}$ is the root node