Exercise 3.1

1. Which of the following expressions are meaningful? Which are meaningless? Explain.
   (a) $(\mathbf{a} \cdot \mathbf{b}) \cdot \mathbf{c}$ (b) $(\mathbf{a} \cdot \mathbf{b})\mathbf{c}$
   (c) $|\mathbf{a}|(\mathbf{b} \cdot \mathbf{c})$ (d) $\mathbf{a} \cdot (\mathbf{b} + \mathbf{c})$
   (e) $\mathbf{a} \cdot \mathbf{b} + \mathbf{c}$ (f) $|\mathbf{a}| \cdot (\mathbf{b} + \mathbf{c})$
2. Find the dot product of two vectors if their lengths are 6 and $\frac{1}{3}$ and the angle between them is $\pi/4$.

3–8 ■ Find $\mathbf{a} \cdot \mathbf{b}$.

3. $|\mathbf{a}| = 6, \quad |\mathbf{b}| = 5, \quad$ the angle between $\mathbf{a}$ and $\mathbf{b}$ is $2\pi/3$
4. $\mathbf{a} = \langle -2, 3 \rangle, \quad \mathbf{b} = \langle 0.7, 1.2 \rangle$
5. $\mathbf{a} = \langle 4, 1, \frac{1}{4} \rangle, \quad \mathbf{b} = \langle 6, -3, -8 \rangle$
6. $\mathbf{a} = \langle s, 2s, 3s \rangle, \quad \mathbf{b} = \langle t, -t, 5t \rangle$
7. $\mathbf{a} = \mathbf{i} - 2\mathbf{j} + 3\mathbf{k}, \quad \mathbf{b} = 5\mathbf{i} + 9\mathbf{k}$
8. $\mathbf{a} = 4\mathbf{j} - 3\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + 4\mathbf{j} + 6\mathbf{k}$

Exercise 3.2

1–7 ■ Find the cross product $\mathbf{a} \times \mathbf{b}$ and verify that it is orthogonal to both $\mathbf{a}$ and $\mathbf{b}$.

1. $\mathbf{a} = \langle 1, 2, 0 \rangle, \quad \mathbf{b} = \langle 0, 3, 1 \rangle$
2. $\mathbf{a} = \langle 5, 1, 4 \rangle, \quad \mathbf{b} = \langle -1, 0, 2 \rangle$
3. $\mathbf{a} = 2\mathbf{i} + \mathbf{j} - \mathbf{k}, \quad \mathbf{b} = \mathbf{j} + 2\mathbf{k}$
4. $\mathbf{a} = \mathbf{i} - \mathbf{j} + \mathbf{k}, \quad \mathbf{b} = \mathbf{i} + \mathbf{j} + \mathbf{k}$
5. $\mathbf{a} = 3\mathbf{i} + 2\mathbf{j} + 4\mathbf{k}, \quad \mathbf{b} = \mathbf{i} - 2\mathbf{j} - 3\mathbf{k}$
6. $\mathbf{a} = \mathbf{i} + e^t\mathbf{j} + e^{-t}\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} + e^t\mathbf{j} - e^{-t}\mathbf{k}$
7. $\mathbf{a} = \langle t, t^2, t^3 \rangle, \quad \mathbf{b} = \langle 1, 2t, 3t^2 \rangle$

Exercise 3.3

1. Determine whether each statement is true or false.

(a) Two lines parallel to a third line are parallel.
(b) Two lines perpendicular to a third line are parallel.
(c) Two planes parallel to a third plane are parallel.
(d) Two planes perpendicular to a third plane are parallel.
(e) Two lines parallel to a plane are parallel.
(f) Two lines perpendicular to a plane are parallel.
(g) Two planes parallel to a line are parallel.
(h) Two planes perpendicular to a line are parallel.
(i) Two planes either intersect or are parallel.
(j) Two lines either intersect or are parallel.
(k) A plane and a line either intersect or are parallel.

2–5 ▪ Find a vector equation and parametric equations for the line.

2. The line through the point $(1, 0, -3)$ and parallel to the vector $2\mathbf{i} - 4\mathbf{j} + 5\mathbf{k}$

3. The line through the point $(-2, 4, 10)$ and parallel to the vector $\langle 3, 1, -8 \rangle$

4. The line through the origin and parallel to the line $x = 2t$, $y = 1 - t$, $z = 4 + 3t$

5. The line through the point $(1, 0, 6)$ and perpendicular to the plane $x + 3y + z = 5$

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6–10 ▪ Find parametric equations and symmetric equations for the line.

6. The line through the points $(6, 1, -3)$ and $(2, 4, 5)$

7. The line through the points $\left(0, \frac{1}{2}, 1\right)$ and $(2, 1, -3)$

8. The line through $(2, 1, 0)$ and perpendicular to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{j} + \mathbf{k}$

9. The line through $(1, -1, 1)$ and parallel to the line $x + 2 = \frac{1}{2}y = z - 3$

10. The line of intersection of the planes $x + y + z = 1$ and $x + z = 0$

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11. Is the line through $(-4, -6, 1)$ and $(-2, 0 -3)$ parallel to the line through $(10, 18, 4)$ and $(5, 3, 14)$?

12. Is the line through $(4, 1, -1)$ and $(2, 5, 3)$ perpendicular to the line through $(-3, 2, 0)$ and $(5, 1, 4)$?

13. (a) Find symmetric equations for the line that passes through the point $(0, 2, -1)$ and is parallel to the line with parametric equations $x = 1 + 2t$, $y = 3t$, $z = 5 - 7t$.

(b) Find the points in which the required line in part (a) intersects the coordinate planes.

14. (a) Find parametric equations for the line through $(5, 1, 0)$ that is perpendicular to the plane $2x - y + z = 1$.

(b) In what points does this line intersect the coordinate planes?

15. Find a vector equation for the line segment from $(2, -1, 4)$ to $(4, 6, 1)$.

16. Find parametric equations for the line segment from $(10, 3, 1)$ to $(5, 6, -3)$.

17–20 ■ Determine whether the lines L_1 and L_2 are parallel, skew, or intersecting. If they intersect, find the point of intersection.

17. L_1: $x = -6t, \quad y = 1 + 9t, \quad z = -3t$

L_2: $x = 1 + 2s, \quad y = 4 - 3s, \quad z = s$

18. L_1: $x = 1 + 2t, \quad y = 3t, \quad z = 2 - t$

L_2: $x = -1 + s, \quad y = 4 + s, \quad z = 1 + 3s$

19. L_1: $\dfrac{x}{1} = \dfrac{y - 1}{2} = \dfrac{z - 2}{3}$

L_2: $\dfrac{x - 3}{-4} = \dfrac{y - 2}{-3} = \dfrac{z - 1}{2}$

20. L_1: $\dfrac{x - 1}{2} = \dfrac{y - 3}{2} = \dfrac{z - 2}{-1}$

L_2: $\dfrac{x - 2}{1} = \dfrac{y - 6}{-1} = \dfrac{z + 2}{3}$

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21–30 ■ Find an equation of the plane.

21. The plane through the point (6, 3, 2) and perpendicular to the vector $\langle -2, 1, 5 \rangle$

22. The plane through the point (4, 0, −3) and with normal vector $\mathbf{j} + 2\mathbf{k}$

23. The plane through the origin and parallel to the plane $2x - y + 3z = 1$

24. The plane that contains the line $x = 3 + 2t$, $y = t$, $z = 8 - t$ and is parallel to the plane $2x + 4y + 8z = 17$

25. The plane through the points (0, 1, 1), (1, 0, 1), and (1, 1, 0)

26. The plane through the origin and the points (2, −4, 6) and (5, 1, 3)

27. The plane that passes through the point (6, 0, −2) and contains the line $x = 4 - 2t$, $y = 3 + 5t$, $z = 7 + 4t$

28. The plane that passes through the point (1, −1, 1) and contains the line with symmetric equations $x = 2y = 3z$

29. The plane that passes through the point (−1, 2, 1) and contains the line of intersection of the planes $x + y - z = 2$ and $2x - y + 3z = 1$

30. The plane that passes through the line of intersection of the planes $x - z = 1$ and $y + 2z = 3$ and is perpendicular to the plane $x + y - 2z = 1$

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31. Find the point at which the line $x = 3 - t$, $y = 2 + t$, $z = 5t$ intersects the plane $x - y + 2z = 9$.

32. Where does the line through (1, 0, 1) and (4, −2, 2) intersect the plane $x + y + z = 6$?