

## FlowNet 3D

### 1. Problem definition

$$P = \{x_i \mid i = 1, \dots, n_1\}, Q = \{y_j \mid j = 1, \dots, n_2\}, x_i, y_j \in \mathbb{R}^3$$

$$d_i = x_i' - x_i$$

$$\text{goal: } D = \{d_i \mid i = 1, \dots, n_1\}$$

### 2. Summary:

FlowNet 3D: estimate scene flow from a pair of consecutive point clouds  
end-to-end  $t$  and  $t+1$

two new layers:

① flow embedding layer: correlate two point clouds

② set up conv layer: propagate features from one set to the other

### 3. FlowNet 3D Architecture

① set conv layer: from PointNet++

hierarchical feature learning, translation-invariant

$$n \text{ points } p_i = \{x_i, f_i\}, x_i \in \mathbb{R}^3, f_i \in \mathbb{R}^c, i = 1, \dots, n$$

↓ set conv layer

$$\begin{array}{l} \text{sub-sampled } n' \text{ points } p_j' = \{x_j', f_j'\}, x_j' \in \mathbb{R}^3, f_j' \in \mathbb{R}^{c'}, j = 1, \dots, n' \\ \text{farthest point sampling} \qquad \qquad \text{region center} \end{array}$$

For each region centered at  $x_j'$

$$f_j' = \max_{\{i \mid \|x_i - x_j'\| \leq r\}} \{ \underbrace{h(f_i, x_i - x_j')}_{\text{concatenate}} \}$$

$$h: \mathbb{R}^{c+3} \longrightarrow \mathbb{R}^{c'}, \text{ a MLP}$$

max: element-wise max pooling

### ② flow embedding layer

mix two point clouds

$$\text{input: } \{p_i = (x_i, f_i)\}_{i=1}^{n_1}, \{q_j = (y_j, g_j)\}_{j=1}^{n_2}$$

$$x_i, y_j \in \mathbb{R}^3, f_i, g_j \in \mathbb{R}^c$$

$$\text{output: } \{o_i = (x_i, e_i)\}_{i=1}^{n_1}$$

For each  $x_i$ :

$$e_i = \max_{\{j | \|y_j - x_i\| \leq r\}} \{h(f_i, g_j, y_j - x_i)\}$$

\* For each  $x_i$ , we consider multiple softly corresponding points  $y_j$  and make a "weighted" decision

\*  $\|y_j - x_i\|$  alternative:  $\text{dist}(f_i, g_j)$   
but the previous is better.

\*  $\{o_i\}$  further go through several set conv layers.

③ Set upconv layer: flow refinement

input:  $\{p_i = \{x_i, f_i\} \mid i = 1, \dots, n\}$  降采样后的点数  
 $\{x'_j \mid j = 1, \dots, n'\}$  P中的点数  
output:  $\{x'_j, f'_j\}_{j=1}^{n'}$

For each region centered at  $x'_j$

$$f'_j = \max_{\{i | \|x_i - x'_j\| \leq r\}} \{h(f_i, x_i - x'_j)\}$$

concatenate

\* alternative way to upsample: 3D interpolation

$$f'_j = \sum_{\{i | \|x_i - x'_j\| \leq r\}} \frac{w(x_i, x'_j)}{\text{normalized inverse-distance weight function}} f_i$$

but the previous is better.

\* a final regression layer to output  $R^3$  predicted scene flow

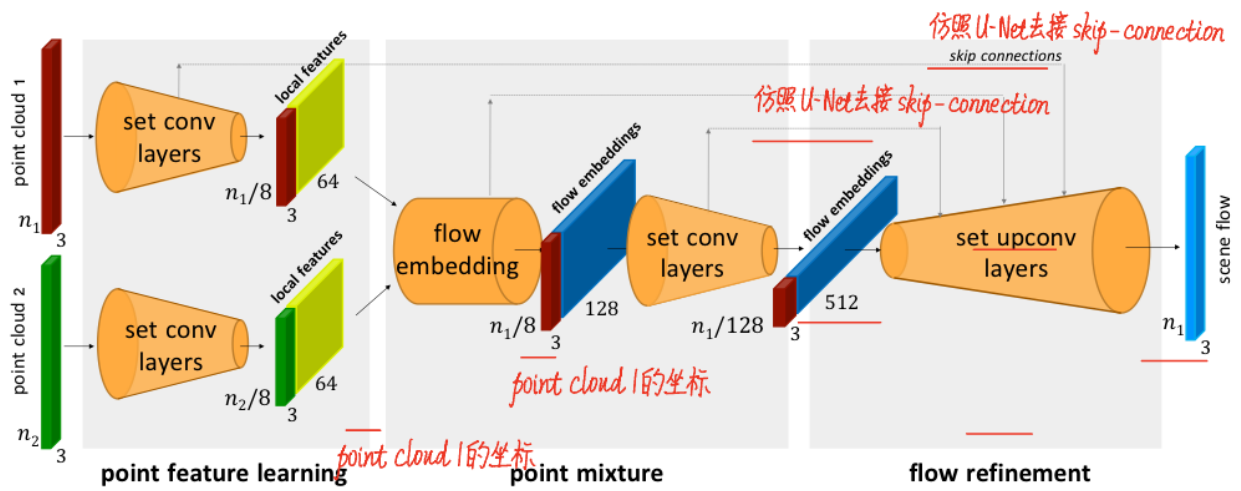


Figure 3: **FlowNet3D architecture.** Given two frames of point clouds, the network learns to predict the scene flow as translational motion vectors for each point of the first frame. See Fig. 2 for illustrations of the layers and Sec. 4.4 for more details on the network architecture.

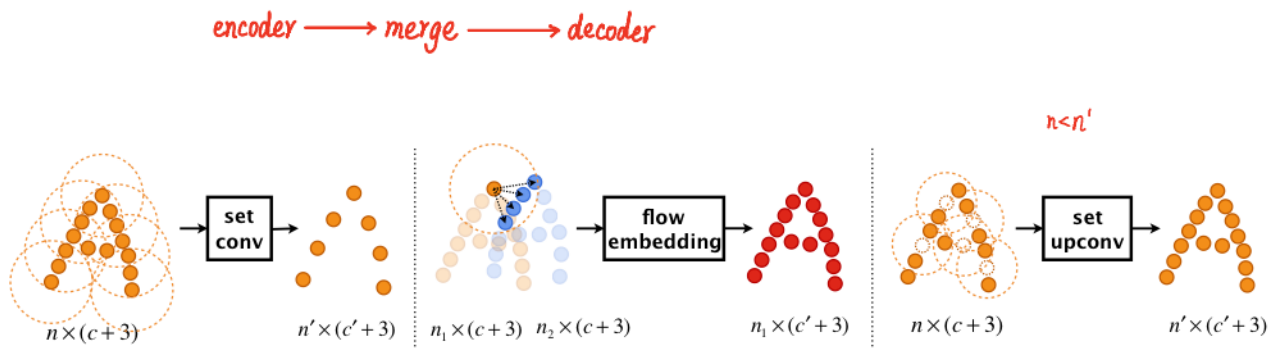


Figure 2: **Three trainable layers for point cloud processing.** *Left:* the *set conv* layer to learn deep point cloud features. *Middle:* the *flow embedding* layer to learn geometric relations between two point clouds to infer motions. *Right:* the *set upconv* layer to up-sample and propagate point features in a learnable way.

#### 4. Other notes

① training loss:

$$P = \{\mathbf{x}_i\}_{i=1}^{n_1}, Q = \{\mathbf{y}_j\}_{j=1}^{n_2}$$

$$D = F(P, Q; \Theta) = \{d_i\}_{i=1}^{n_1}$$

FlowNet3D parameters

$$\text{ground truth: } D^* = \{d_i^*\}_{i=1}^{n_1}$$

$$\text{backward flow: } \{d_i'\}_{i=1}^{n_1} = F(P', P; \Theta)$$

$$\text{where } P' = \{\mathbf{x}_i + d_i\}_{i=1}^{n_1}$$

$$L(P, Q, D^*, \Theta) = \frac{1}{n_1} \sum_{i=1}^{n_1} \left\{ \|d_i - d_i^*\| + \lambda \underbrace{\|d_i' + d_i\|}_{\text{cycle-consistency term}} \right\}$$

② down-sample introduces noise  $\longrightarrow$  inference with random re-sampling

#### 5. Application

<1> 3D scan registration

<2> motion segmentation