1 Project Objective & Business Context

Model the impact of different reinsurance structures on an insurer's loss distribution, volatility, and capital requirements. The project is framed from the perspective of a general insurance company purchasing reinsurance to manage tail risk and improve solvency.

2 Portfolio & Claim Modelling

The portfolio is assumed to consist of property of insurance risk and the annual aggregate losses are modelled using a frequency-severity frame work, which is often adapted in general insurance and re-insurance applications.

For Claim frequency, Negative Binomial Distribution was assumed to be followed such that over-dispersion was allowed. It was parameterised with a mean of 25 claims per year and a variance of 50 such that greater variability would be implied compare to Poisson Distribution.

For claim severity, Pareto Type I was assumed which reflect the heavy-tailed nature of property insurance losses. A minimum amount of 25,000 and shape parameter $\alpha = 1.8$ was assumed. It was also assumed that claim frequency followed a Negative Binomial Distribution with mean 25 and variance 50.

To face volatile and heavy-tailed claim outcomes, reinsurance is bought to reduce overall variance (or volatility) of the (net) annual aggregate claim amount and improve solvency metrics.

3 Base Line Loss Analysis (Without Reinsurance)

Annual aggregate losses were simulated using Monte Carlo methods under the frequency-severity model described in Section 2. For each simulated year, claims frequency were generated from a Negative Binomial distribution and individual severity were generated from a Pareto Distribution, then summing to obtain the annual aggregate loss. 100,000 years of simulation was considered. Key summary statistics (mean and standard deviation) were calculated from the 100,000 aggregate losses alongside with tail risk measure including VaR and TVaR at the selected confidence levels (99% and 99.5%). These baseline results will be used to compare the with portfolio with reinsurance structure.

4 Loss Analysis (With Reinsurance)

To manage reduce volatility and manage tail risk observed in the baseline loss distribution, three reinsurance structures were considered: proportional (quota share) reinsurance, per-risk excess-of-loss (XL) reinsurance, and aggregate excess-of-loss reinsurance. These structures represent commonly used mechanisms in general insurance and reinsurance practice, each targeting different aspects of portfolio risk.

Per-risk Excess-of-Loss (XL) reinsurance applies at the individual claim level. Losses more than a predefined retention level are ceded to the reinsurer up to a specified limit. This structure is particularly effective in transferring tail risk driven by large individual claims, which is a key feature of heavy-tailed property loss distributions.

Quota Share reinsurance, a fixed proportion of every claim is ceded to the reinsurer. This structure provides broad risk sharing across all loss sizes and is effective in smoothing volatility, though it does not specifically target extreme tail events.

Aggregate XL reinsurance applies at the annual portfolio level. The reinsurer covers total annual losses in excess of an aggregate retention. This structure is designed to protect against unusually adverse underwriting years, including those driven by claim clustering or multiple large losses.

Table 1 – Key Statistic of Gross-to-Net Losses

METRIC	NO REINSURANCE	PER-RISK XL	PROPORTIONAL (QUOTE SHARE)	AGGREGATE XL
MEAN ANNUAL LOSS	1,406,385	1,304,500	1,125,108	£1,392,773
STD DEV ANNUAL LOSS	994,315	863,304	795,452	£905,426
VAR99%	3,557,886	2,520,131	2,846,309	£3,500,000
TVAR99%	6,222,349	4,548,373	4,977,879	£4,787,080
VAR99.5%	4,510,436	2,828,581	3,608,348	3,500,000
TVAR99.5%	8,493,571	6,451,452	6,794,857	£4,787,080
CAPITAL AT VAR 99.5%	3,104,051	1,524,081	2,483,241	2,107,227
CAPITAL RELIEF	0	1,579,970	620,810	996,824
RELIEF %	0	50.9%	20.0%	32.1%

Each reinsurance structure was applied to the simulated annual aggregate losses, and the resulting net loss distributions were analysed using the same summary statistics as in the baseline case. Table 1 summarises the mean, standard deviation, VaR, TVaR and solvency capital metrics across all scenarios. The results highlight clear differences in how each reinsurance structure reshapes the loss distribution. Quota share reinsurance reduces the mean and standard deviation, reflecting the proportional sharing of all losses. However, its impact on extreme tail risk is relatively limited when compared to non-proportional structures.

Per-risk XL reinsurance achieves a pronounced reduction in tail risk, as evidenced by the substantial decrease in both VaR and TVaR at the 99% and 99.5% confidence levels. This reflects the effectiveness of XL reinsurance in removing large individual claims that dominate the tail of the distribution.

Aggregate XL reinsurance produces a different pattern. While its impact on the mean and standard deviation is modest, the VaR at both the 99% and 99.5% confidence levels is capped at the aggregate retention. This behaviour arises because losses exceeding the retention within the covered layer are fully ceded, resulting in a flat segment of the quantile function over this confidence range.

5 Capital Modelling and Capital Relief

Solvency capital was approximated using a one-year risk measure framework. Capital was defined as the excess of the 99.5th percentile of the annual aggregate loss distribution over its mean, representing the buffer required to absorb unexpected losses beyond those covered by pricing and reserving.

Capital requirements were calculated on both a gross and net basis to quantify the effect of reinsurance. Capital relief was measured as the reduction in required capital relative to the no-reinsurance case.

The results indicate that **per-risk XL reinsurance provides the greatest capital relief**, reducing required capital by approximately 51%. This reflects its strong effectiveness in removing large individual losses that drive solvency capital. **Aggregate XL reinsurance** also delivers meaningful capital relief by imposing a hard cap on annual losses, though its impact on overall volatility is limited. **Quota share reinsurance** provides the smallest capital relief, as it reduces losses proportionally rather than targeting extreme tail events.

These results demonstrate that different reinsurance structures serve distinct risk-management objectives and that capital efficiency varies significantly across treaty types.

6 Conclusion

This project demonstrates how different reinsurance structures influence the distribution of insurance losses and associated solvency capital requirements. Using a Monte Carlo frequency-severity framework with heavy-tailed claim severity, it was shown that proportional and non-proportional reinsurance have different impacts on volatility, tail risk and capital.

Per-risk excess-of-loss reinsurance was found to be the most capital-efficient structure, providing substantial relief by removing large individual claims from the tail of the loss distribution. Aggregate excess-of-loss reinsurance imposed a hard cap on annual losses at relevant confidence levels, offering strong protection against adverse underwriting years. Quota share reinsurance primarily reduced volatility and expected losses but delivered comparatively lower capital efficiency.

Overall, the results highlight the critical role of reinsurance in managing tail risk and optimising capital requirements, and they illustrate how actuarial modelling can be used to support reinsurance design and strategic risk management decisions.

7 Limitations and Further Extensions

The analysis relies on simplifying assumptions, including independence between claims, static exposure, and the absence of inflation, expenses, and dependency structures. Future extensions could incorporate catastrophe event modelling, dependency between frequency and severity, dynamic exposure growth, and explicit reinsurance pricing using cost-of-capital frameworks.