ASSIGNMENT 6

(1) Describe all 2×2 orthogonal matrices. Prove that the action of an orthogonal matrix on $\mathbb{R}^2$ is composition of rotation and a reflection about a line.

(2) Let $v, w \in \mathbb{R}^n$ with $n \geq 2$, with $||v|| = ||w||$. Prove that there exists an orthogonal matrix A of order n such that $A(v) = w$ and $det(A) = 1$.

(3) Find the eigen values and eigen vectors of the matrix

$$\begin{pmatrix} -1 & 2 & 2 \\ 2 & 2 & 2 \\ -3 & -6 & -6 \end{pmatrix}$$

(4) Let A be an $n \times n$ matrix.
(i) Prove that A is invertible if and only if all the eigen values of A are not zero.
(ii) Show if A is invertible the eigen values of A^{-1} are reciprocals of eigen values of A.

(5) Let A be an $n \times n$ matrix and α be a scalar. Find eigen values of $A - \alpha I$ in terms of eigen values of A. Show that A and $A - \alpha I$ and A have eigen vectors.

(6) Let A be an $n \times n$ matrix. Show that A and A^T have same eigen values. Do they have the same eigen vectors ?

(7) Let A be an $n \times n$ matrix,
(i) If A is idempotent i.e. $A^2 = A$, show that eigen values of A are either 0 or 1.
(ii) If A is nilpotent i.e. $A^m = 0$ for some $m \geq 1$, show that all eigen values of A are 0.

(8) Let A be a square matrix with an eigen value λ and let u be the eigen vector corresponding to λ.
(i) Show that λ^k is an eigen value of A^k $(k \geq 1)$ with eigen vector u,
(ii) Suppose $p(x) = a_0 + a_1x + a_2x^2 + ... + a_nx^n$ is polynomial. Define the matrix $p(A) := a_0I + a_1A + a_2A^2 + ... + a_nA^n$. Prove that $p(\lambda)$ is also an eigen value of $p(A)$ with eigen vector u.

(9) Let A and B be square matrices of same order. Prove that characteristic polynomials of AB and BA are same.