$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & 3 & 4 \end{bmatrix}$$

$$\left.\begin{aligned} 2x_1 - x_2 + x_3 &= 0 \\ x_1 + 3x_2 + 4x_3 &= 0 \end{aligned}\right\} \quad \text{I}$$

$$\underbrace{\begin{aligned} x_2 + x_3 &= 0 \\ x_1 + x_3 &= 0 \end{aligned}}$$

$$\downarrow \quad \downarrow \quad \downarrow$$

$$\begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

$$\downarrow$$

$$\begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \checkmark$$

$$x_1 + x_3 = 0$$
$$x_2 + x_3 = 0$$

$$\left.\begin{aligned} x_1 + x_2 + 2x_3 + x_4 &= 5 \\ x_1 + x_2 + 2x_3 + 6x_4 &= 10 \\ x_1 + 2x_2 + 6x_3 + 2x_4 &= 7 \end{aligned}\right\} \quad \text{II}$$

$$\left[\begin{array}{cccc|c} 1 & 1 & 2 & 1 & 5 \\ 1 & 1 & 2 & 6 & 10 \\ 1 & 2 & 6 & 2 & 7 \end{array}\right] \rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 3 \\ 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 4 & \cancel{1}\,0 & \cancel{2}\,1 \end{array}\right]$$

$$\rightarrow \left[\begin{array}{cccc|c} 1 & 0 & -2 & 0 & 3 \\ 0 & 1 & 4 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 \end{array}\right]$$

ROW REDEUCED ECHELON FORM :-

1. If a row contains a nonzero entry, then its first nonzero entry = 1.

(This is called the 'pivot' of the row)

$$\begin{bmatrix} 1 & 1 \\ 0 & 0 \end{bmatrix}$$

2. If a colounmn contains a pivot then all other entries in that colounmn = 0.

3. If $i<j$ and both the rows have pivots then the pivot of the i-th row must lie at the left of the j-th row.

Any matrix satisfying 1-3 listed above is called RRE matrix.

TYPE-I :-

$$A = \begin{bmatrix} - & R_1 & - \\ & \vdots & \\ - & R_n & - \end{bmatrix}_{m\times n}$$

$i, j, \quad i \neq j$

$c \in \mathbb{F}$

$$EA = \begin{bmatrix} - & R_1 & - \\ & \vdots & \\ - & R_{i-1} & - \\ - & R_i + cR_j & - \\ & R_{i+1} & \\ & \vdots & \\ - & R_m & - \end{bmatrix}$$

CONCLUSION

Type I row op. is equiv to multiplying by a type I elementary matrix from the left.

Consider I_m : Apply type - I

$$E = \begin{bmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & & 1 & c & \\ & & & 1 & \\ 0 & & & & \ddots \\ & & & & & 1 \end{bmatrix}, \quad \begin{bmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & c & 1 & & \\ & & & 1 & \\ & & & & \ddots \\ 0 & & & & & 1 \end{bmatrix}$$

$i < j \qquad\qquad i > j$

Elementary matrices of first kind

$$
\begin{bmatrix} b_{k1} & b_{k2} & \cdots & b_{km} \end{bmatrix}_{r \times m}
\underbrace{\begin{bmatrix} a_{11} & \cdots & a_{1n} \\ a_{21} & \cdots & a_{2n} \\ \vdots & & \\ a_{m1} & \cdots & a_{mn} \end{bmatrix}_{m \times n}}_{= A}
$$

$$
= \begin{matrix} b_{k1} a_{11} & b_{k1} a_{12} & \cdots & b_{k1} a_{1n} \\ b_{k2} a_{21} & b_{k2} a_{22} & & b_{k2} a_{2n} \\ + \quad \vdots & \vdots & & \vdots \\ b_{km} a_{m1} & b_{km} a_{m2} & & b_{km} a_{mn} \end{matrix}
$$

$$
\begin{aligned}
& b_{k1} \begin{pmatrix} a_{11} & a_{12} & \cdots & a_{1n} \end{pmatrix} \\
+ \; & b_{k2} \begin{pmatrix} a_{21} & a_{22} & \cdots & a_{2n} \end{pmatrix} \\
& \vdots \\
+ \; & b_{km} \begin{pmatrix} a_{m1} & a_{m2} & \cdots & a_{mn} \end{pmatrix}
\end{aligned}
$$

TYPE-II :-

$$E = \begin{bmatrix} 1 & & & & 0 \\ & \ddots & & & \\ & & c & & \\ & & & \ddots & \\ 0 & & & & 1 \end{bmatrix}$$

$$EA = \begin{bmatrix} - & R_1 & - \\ & \vdots & \\ \cdots & cR_i & - \\ - & R_m & - \end{bmatrix}$$

TYPE-III :-

$$E = \begin{array}{c} \\ \\ i \\ j \\ \\ \\ \end{array} \begin{bmatrix} 1 & & & & & 0 \\ & \ddots & & & & \\ & & 0 & \cdots & 1 & \\ & & 1 & \ddots & 0 & \\ & & & & \ddots & \\ 0 & & & & & 1 \end{bmatrix}$$

CONCLUSION :-