Question 1a - Bit-Flips Amortized Analysis

- 2 Marks: phi(i): number of zeroes in the counter / binary representation of (l - i)
- 2 Marks: phi(0): O(n) i.e. number of bits in counter
- 2 Marks: Explaining that how last k bits change from 0 to 1 if we decrement something and how no of zeroes goes down respect to that
    - eg. 10001 -> 10000
    - No of bit flips = 1 => c
    - Change in phi = phi(i+1) - phi(i) = 4 - 3 = 1 => c
    - Total = 2c

    - Eg. 10000…. -> 01111…. (assuming k bits of 1 in suffix)
    - No of bit flips = k + 1 => (k +1)c
    - Change in phi = phi (i+1) - phi(i) = 1 - (k) = 1-k => (1-k)c
    - Total = 2c
- 1 Mark: Final Answer : O(k) ….. Or something close to that…. Your answer must have k as a parameter O(n) does not work

Question 1b - Bit Counter Amortized Analysis

- 2 Marks: No….. No marks for Yes and (any explanation)
- 2 Marks: Explanation —- If the number in the counter is 1110 (phi(i) = suffix of all 1s = 0) and incrementing leads to 1111 (phi(i) = 4).... And we choose a phi that increases slowly and decreases at a single big step. How will you tackle this big jump in phi…

Question 2a - Job assignment to Max Flow Reduction

- 4 marks deducted if separate vertices are not created for each applicant
- 2 marks deducted for each wrongly mentioned capacity/per type(there are supposed to be 3 types of edges)
- 4 marks deducted if the simple bipartite version of the problem is attempted incorrectly.

Question 2b - Global Minimum Cut Algorithm Design
- 4 marks deducted if the algorithm does not guarantee correct global min-cut.
- 5 marks awarded only if the intuition behind the algorithm is correctly justified.

Question 3a - Upper bound on value(f*) - value(f)
- Marks are awarded **only if** the final upper bound is correct
- **2 marks** for the correct upper bound
- **3 marks** for the correct justification (**partial marking** is allowed for the justification)
- Correct answer:

we Know, value $(f^*) \leq C(A, \bar{A})$
and value $(f) = f_{out}(A) - f_{in}(\bar{A})$

Now, $f_{in}(A) = jc'$
and $f_{out}(A) = C(A, \bar{A}) - ic$

value (f^*) − value $(f) \leq c(A, \bar{A})$ − value (f)

$\leq c(A, \bar{A}) - (c(A, \bar{A}) - ic - jc')$

$\leq ic + jc'$

Question 3b - Shortest Path Augmentation
- Binary grading **(9 or 0)**

- The described graph must be **general** (i.e., not a trivial edge case)
- A justification or approach to support the graph should be provided (**optional** if the graph description is exceptionally clear and undeniably correct)