ASSIGNMENT 1

(1) Show that matrix multiplication is associative i.e. $A(BC) = (AB)C$ whenever the multiplication is defined.

(2) Consider the three equations of planes

$$\begin{aligned} x - y + z &= 1 \\ x + ay - 2z &= -10 \\ 2x - 3y + z &= -b, \end{aligned}$$

where a, b are parameters. Using Row Reduction Method, determine the values of a and b for which the planes

(i) intersect in a single point;

(ii) intersect in a single line;

(iii) intersect (taken two at a time) in three distinct parallel lines.

(3) Suppose A and B are matrices of order $m \times n$ such that $A\bar{x} = B\bar{x}$ for all $\bar{x} \in \mathbb{R}^n$. Prove that $A = B$.

(4) Let $A = (a_{ij})$ be a matrix. Transpose of A, denoted by A^T, is defined to be $A^T = (b_{ij})$ where $b_{ij} = a_{ji}$.

(i) Show that $(A + B)^T = A^T + B^T$, whenever $A + B$ is defined

(ii) Show that $(AB)^T = B^T A^T$, whenever AB is defined.

(5) A square matrix A is said to be *symmetric* if $A = A^T$ and a square matrix A is said to be *skew symmetric* if $A = -A^T$.

Prove that a square matrix can be written as a sum of symmetric and skew symmetric matrix.

(6) A square matrix A is said to be *nilpotent* if $A^n = 0$ for some natural number n.

(i) Give example of non-zero nilpotent matrices,

(ii) Prove that if A is nilpotent then $A + I$ is an invertible matrix, where I is identity matrix.

(7) Trace of a square matrix A, denoted by $Tr(A)$, is defined to be the sum of all diagonal entries.

(i) Suppose A, B are two square matrices of same order. Prove that $Tr(AB) = Tr(BA)$.

(ii) Show that if A is invertible then $Tr(ABA^{-1}) = Tr(B)$.