Problem Set-8
MTH-204, 204A
Abstract Algebra

1. Let R be a ring in which every $x \in R$ satisfies $x^2 = x$. Prove that R must be commutative.

2. Give an example of a ring R and ideals I_1, I_2, I_3 and J such that $J \subseteq I_1 \cup I_2 \cup I_3$ but $J \not\subseteq I_k$ for any $k = 1, 2, 3$.

3. If I is an ideal of R, let $Ann(I) = \{x \in R : xu = 0 \ for \ all \ u \in I\}$. Prove that $Ann(I)$ is an ideal of R.

4. If I is an ideal of R let $[R : I] = \{x \in R : rx \in I \ for \ every \ r \in R\}$. Prove that $[R : I]$ is an ideal of R and that it contains I.

5. Prove that any homomorphism of a field is either one-to-one or takes each element into 0.

6. Determine all ring homomorphisms from $\mathbb{R}$ to $\mathbb{R}$.

7. Find all the units of the ring of Gaussian integers $\mathbb{Z}[i]$.

8. Let R be a commutative ring with 1. If every proper ideal of R is prime, show that R is a field.

9. Let R be a ring. Prove that the set of all matrices over R form a ring under matrix addition and matrix multiplication. What are all the ideals of this ring ?

10. Let R be a ring in which $x^3 = x$ for every $x \in R$. Prove that R is a commutative ring.