MSO205 PRACTICE PROBLEMS SET 1

Question 1. Write down the sample spaces of the following random experiments.

(a) Shuffle a standard deck of cards and draw the first card.

(b) A box contains 3 identical red balls and 2 identical green balls. Draw a ball from the box blindfolded and then check (and note down) the colour of the ball. Put the ball back into the box.

(c) Throw a standard six-sided die two times and add up the two numbers obtained.

Question 2. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be a probability space associated with a random experiment $\mathcal{E}$.

(a) Let $B \in \mathcal{F}$ be such that $\mathbb{P}(B) = 1$. For any event $A \in \mathcal{F}$, show that $\mathbb{P}(A) = \mathbb{P}(A \cap B)$. (Hint: What is $\mathbb{P}(A \cap B^c)$?)

(b) (Boole's inequality) Let $n \geq 2$ be any integer and let $E_1, E_2, \cdots, E_n$ be events in $\mathcal{F}$. Prove that

$$\mathbb{P}(\bigcup_{i=1}^{n} E_i) \leq \sum_{i=1}^{n} \mathbb{P}(E_i).$$

(c) Let $\{E_n\}_n$ be a sequence of events in $\mathcal{F}$. Show that

$$\mathbb{P}(\bigcup_{n=1}^{\infty} E_n) \leq \sum_{n=1}^{\infty} \mathbb{P}(E_n).$$

(Hint: Take $A_1 := E_1$ and for $n \geq 2$, define $A_n := E_n \cap (E_1 \cup E_2 \cup \cdots E_{n-1})^c$. Prove that $\cup_n A_n = \cup_n E_n$. Use the A_n's.)

(d) Let $n \geq 2$ be any integer and let $E_1, E_2, \cdots, E_n$ be events in $\mathcal{F}$. Prove that

$$\mathbb{P}(\bigcap_{i=1}^{n} E_i) \geq \sum_{i=1}^{n} \mathbb{P}(E_i) - (n-1).$$

Question 3. Let $(\Omega, \mathcal{F}, \mathbb{P})$ be the probability space associated with a random experiment $\mathcal{E}$. Let A, B, C be events such that $\mathbb{P}(A) = 0.3, \mathbb{P}(B) = 0.4, \mathbb{P}(A \cap B) = 0.2, \mathbb{P}(C) = 0.1$. Further assume that A, C are mutually exclusive and B, C are mutually exclusive.

Find the probabilities that

(a) exactly one of the events A or B occurs

(b) at least one of the events A, B or C occurs

(c) none of A and B will occur

(d) A occurs, but C does not occur.