## MSO205 PRACTICE PROBLEMS SET 13

<u>Question</u> 1. Refer to Question 3 of problem set 12. Show by an example that the continuous mapping theorem does not hold for converge in r-th mean/moment.

<u>Question</u> 2. Suppose  $X, X_1, X_2, \dots, Y, Y_1, Y_2, \dots$  be RVs defined on the same probability space. Fix  $\alpha, \beta \in \mathbb{R}$ .

- (i) Suppose  $X_n \xrightarrow{n \to \infty} X$  and  $Y_n \xrightarrow{n \to \infty} Y$  in r-th mean for some  $r \ge 1$ . Show that  $\alpha X_n + \beta Y_n \xrightarrow{n \to \infty} \alpha X + \beta Y$  in r-th mean.
- (ii) Suppose  $X_n \xrightarrow[n \to \infty]{\text{a.s.}} X$  and  $Y_n \xrightarrow[n \to \infty]{\text{a.s.}} Y$ . Show that  $\alpha X_n + \beta Y_n \xrightarrow[n \to \infty]{\text{a.s.}} \alpha X + \beta Y$ .

<u>Question</u> 3. Construct an example of a sequence of RVs  $\{X_n\}_n$  converging in law/distribution, but not in probability.

<u>Question</u> 4. Consider a sequence  $\{X_n\}_n$  of RVs with  $X_n \sim N(\frac{1}{n}, 1 - \frac{1}{n}), \forall n$ . Does this sequence converge in law/distribution?

Question 5. Suppose that a continuous RV X has a quantile of order  $\frac{1}{3}$  at 5. Consider a random sample of size 100 from the distribution of X. What is the probability (approximately) that more than 40 sample values are more than 5? Express the approximate value in terms of  $\Phi$ , the DF of N(0,1) distribution.

<u>Question 6</u>. Fix  $\lambda > 0$ . Let  $X_1, X_2, \cdots$  be a sequence of i.i.d. RVs with  $Exponential(\lambda)$  distribution. Consider the sample mean  $\bar{X}_n := \frac{1}{n} \sum_{j=1}^n X_j, \forall n$ . Show that

$$\sqrt{n}\left(\frac{1}{\bar{X}_n} - \frac{1}{\lambda}\right) \xrightarrow[n \to \infty]{d} N\left(0, \frac{1}{\lambda^2}\right).$$

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