$$\left\{\begin{array}{cccc} 2 & -1 & 1 & =0 \\ 1 & 3 & 4 & =0 \end{array}\right\} \quad \underline{\mathrm{I}}$$

$R_1' = R_1 - 2R_2$

$$\left\{\begin{array}{cccc} 0 & -7 & -7 & =0 \\ 1 & 3 & 4 & =0 \end{array}\right\} \quad \underline{\mathrm{II}}$$

$R_1' = \left(-\frac{1}{7}\right) R_1$

$$\left\{\begin{array}{cccc} 0 & 1 & 1 & =0 \\ 1 & 3 & 4 & =0 \end{array}\right\} \quad \underline{\mathrm{III}}$$

$R_2' = R_2 - 3R_1$

$$\left\{\begin{array}{cccc} 0 & 1 & 1 & =0 \\ 1 & 0 & 1 & =0 \end{array}\right\} \quad \underline{\mathrm{IV}}$$

<u>NOTE</u> :- We perform "row operations" on the matrices.

$$\begin{array}{cc} i & \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \end{array} \quad \begin{array}{c} j \\ \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} \end{array}$$

(i,j), $\quad 1 \leq i \leq m$, $\quad 1 \leq j \leq n$

An $m \times n$ matrix is a fn.

$$\{1, \dots, m\} \times \{1, \dots, n\} \longrightarrow \mathbb{F}$$
$$(i,j) \longmapsto a_{ij}$$

$$A = (a_{ij})_{i,j}$$

— DEFINITIONS :-

1. A, B – $m \times n$ matrices with entries / coeffs from $\mathbb{F}$

$$A + B = (a_{ij})_{i,j}$$

2. A – " " " , $\quad c \in \mathbb{F}$

$$cA = (c\,a_{ij})_{i,j}$$

3. A – $m \times n$, $\quad B$ – $n \times k$, $\quad AB = (c_{ij})_{i,j}$, $\quad c_{ij} = a_{i1} b_{1j} + \dots + a_{in} b_{nj}$

$\mathbb{F} = \mathbb{Q}, \mathbb{R}, \mathbb{C}$ $\qquad x_1, x_2, x_3, \ldots, x_n$ — variables

We abbre (∗-1) using matrix not. as follows:-

$a_{ij} \in \mathbb{F}$

$$\left\{ \begin{array}{l} a_{11}x_1 + a_{12}x_2 + \cdots + a_{1n}x_n = b_1 \\ a_{21}x_1 + a_{22}x_2 + \cdots + a_{2n}x_n = b_2 \\ \vdots \\ a_{m1}x_1 + a_{m2}x_2 + \cdots + a_{mn}x_n = b_n \end{array} \right\} \cdots (\ast\text{-}1)$$

$Ax = b$, where

$A = (a_{ij})_{i,j}$

$x = \begin{pmatrix} x_1 \\ \vdots \\ x_n \end{pmatrix}$

$b = \begin{pmatrix} b_1 \\ \vdots \\ b_m \end{pmatrix}$

A soln. of (∗-1) is an n-tuple $(y_1, \ldots, y_n)$ s.t. each $y_i \in \mathbb{F}$. which solves all the eqn. of (∗-1).

If $\underbrace{(y_1, \ldots, y_n)}$ is a soln, $y := \begin{pmatrix} y_1 \\ \vdots \\ y_n \end{pmatrix}$, then

$Ay = b$

Let $c_1, \ldots, c_m \in \mathbb{F}$, an eqn of the form

$$(c_1 a_{11} + c_2 a_{21} + \cdots + c_m a_{m1})x_1 + (c_1 a_{12} + c_2 a_{22} + \cdots + c_m a_{m2})x_2 + \cdots + (c_1 a_{1n} + \cdots + c_m a_{mn})x_n$$
$$= c_1 b_1 + \cdots + c_m b_m.$$ is called a linear comb.

Let

$$\left.\begin{array}{l} d_{11}x_1 + \cdots + d_{1n}x_n = e_1 \\ d_{21}x_1 + \cdots + d_{2n}x_n = e_2 \\ \quad\vdots \qquad\qquad \vdots \qquad\quad \vdots \\ d_{m1}x_1 + \cdots + d_{mn}x_n = e_m \end{array}\right\} \quad (*-2)$$

Assume that each eqn of $(*-2)$ is a linear combn. of those of $(*-1)$.
Then all soln of $(*-1)$ are also soln of $(*-2)$.

Suppose in addition that each eqn of $(*-1)$ is a linear comb. of those of $(*-2)$.
Then $(*-1)$ & $(*-2)$ have exactly same soln.s.

Type 1: Multiplying the i-th row by a scalar and adding that to the j-th row

Type 2: Multiplying the ith row by a scalar

Type 3: Interchanging i-th and j-th rows.