MTH 114: ODE: Assignment-1

1. (**T**)Classify each of the following differential equations as linear, nonlinear and specify the order.

   (i) $y'' + (\cos x)\, y = 0$ (ii) $y'' + x \sin y = 0$ (iii) $y' = \sqrt{1+y}$
   (iv) $y'' + (y')^2 + y = x$ (v) $y'' + xy' = \sin y$ (vi) $(x\sqrt{1+x^2}\, y')' = e^x\, y$.

2. Find the differential equation of each of the following families of plane curves. Here $a, b, c \in \mathbb{R}$ denote arbitrary constants:
   (i) $xy^2 - 1 = cy$ (ii) $cy = c^2x + 5$ (iii)(**T**) $y = ax^2 + be^{2x}$ (iv) $y = ax + b + c$
   (v)(**T**) Circles touching the x-axis with centres on y-axis.
   (vi) $y = a \sin x + b \cos x + b$,
   where a, b and c are arbitrary constants.

3. Verify that the given function in the left is a general solution to the corresponding differential equation in the right.

(i)	$x^3 + y^3 = 3cxy$	$x(2y^3 - x^3)y' = y(y^3 - 2x^3)$
(**T**) (ii)	$y = ce^{-x} + x^2 - 2x + 4$	$y' + y = x^2 + 2$
(iii)	$y = cx - c^2$	$y'^2 - xy' + y = 0$

4. Solve $\frac{dy}{dx} = y^2 - 2y + 2$ by separating variables.

5. (**T**) Verify that $y = \frac{1}{x+c}$ is general solution of $y' = -y^2$. Find particular solutions such that (i) $y(0) = 5$, and (ii) $y(2) = -\frac{1}{5}$. In both the cases, find the largest interval I on which y is defined.

6. Solve the IVP $y\frac{dy}{dx} = e^x,\ y(0) = 1$. Find the largest interval of validity of the solution.

7. For each of the following differential equations, draw several isoclines with appropriate lineal elements. Solve the equations and sketch some solution curves.

   (**T**)(i) $y' = x$ (ii) $y' = -\frac{x}{y}$.

   (http://mathlets.org/mathlets/isoclines/)

8. Find the orthogonal trajectories of the following families of curves:

   (**T**) (i) $e^x \sin y = c$ (ii) $y^2 = cx^3$

9. Find the family of oblique trajectories which intersect the family of straight lines $y = cx$ at an angle of 45^o.

10. Show that the following families of curves are self-orthogonal:
    (**T**) (i) $y^2 = 4c(x + c)$ (ii) $\frac{x^2}{c^2} + \frac{y^2}{(c^2-1)} = 1$

    Draw the families.

11. Draw isoclines, lineal element (slope field) and use them to draw some solution curve of the equation $y' = y^2 - x^2$.