MSO205 PRACTICE PROBLEMS SET 7

Question 1. Let $X \sim Binomial(n, p)$ for some integer $n \geq 3$ and $p \in (0, 1)$. Compute $\mathbb{E}X(X-1)(X-2)$, if it exists.

Question 2. Verify that $\Gamma(\frac{1}{2}) = \sqrt{\pi}$.

Question 3. Let $X \sim N(\mu, \sigma^2)$ for $\mu \in \mathbb{R}, \sigma > 0$. Compute $\mathbb{E}X^k$ for $k = 2, 3, 4$. [Hint: When $X \sim N(0, 1)$, these moments has been computed in the lecture notes.]

Question 4. Fix $\alpha > 0, \beta > 0$ and let $X \sim Beta(\alpha, \beta)$. Compute the MGF of X, if it exists.

Question 5. Let $X \sim Beta(1, 1)$. Does the distribution of X match with any other distribution discussed in the lecture notes?

Question 6. An RV X has the MGF given by the following expressions. Identify the distribution of X.

(a) $M_X(t) = (1 - \frac{t}{2})^{-3}, \forall t < 2$.

(b) $M_X(t) = \frac{1}{3}e^{-t} + \frac{2}{3}, \forall t \in \mathbb{R}$.