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**Quiz 1****MTH302: Set Theory and Mathematical Logic**

(Odd Semester 2024/25, IIT Kanpur)

**Question 1. [3 × 1 Points]**

For each of the following statements, determine whether it is **true or false**. No justification required.

- (i) For every uncountable linear ordering  $(L, <)$ , there exists an infinite  $X \subseteq L$  such that  $(X, <)$  is a well-ordering. \_\_\_\_\_
- (ii) The set of all irrational numbers has the same cardinality as the set of all real numbers. \_\_\_\_\_
- (iii) There exists a sequence of sets  $\langle X_n : n < \omega \rangle$  such that  $|X_{n+1}| < |X_n|$  for every  $n < \omega$ . \_\_\_\_\_

**Question 2. [7 Points]**

- (a) [1 Point] State Schröder-Bernstein theorem.
- (b) [2 Points] State Zorn's lemma.
- (c) [2 Points] Let  $f : \mathbb{R} \rightarrow \mathbb{R}$  be an additive function with  $f(1) = 7$ . Assume  $f$  is continuous at 0. Show that  $f(x) = 7x$  for all  $x \in \mathbb{R}$ .
- (d) [2 Points] Show that the set of all bijections from  $\omega$  to  $\omega$  is uncountable.