Problem Set-6
MTH-204, 204A
Abstract Algebra

1. Prove that a group G is not cyclic if and only if G is a union of proper subgroups.

2. Let H and K be 2 subgroups of a group G. Compute the order of HK using the orbit stabilizer theorem.

3. The number of congugacy classes in S_n is the number of partitions of n.

4. The additive group $\mathbb{R}$ acts on $\mathbb{C}$ by $a.z = e^{ia}z$, for $a \in \mathbb{R}$ and $z \in \mathbb{C}$. Find all orbits and stabilizers for this action.

5. Find all conjugacy classes and verify the class equation for the following groups.

a. D_n b. Q_8 c. A_4

6. Let H, K, N be normal subgroups of a group G. If $NK = HK$ and $N \cap K = H \cap K$, then is $N = K$?

7. If an element has exactly 2 conjugates in a group G then G is not simple.

8. If a nontrivial finite group G acts on a finite set of more than one elements and the action has only one orbit then show that some $g \in G$ has no fixed points.

9. Prove that in a finite group the union of the subgroups conjugate to a proper subgroup do not fill up the whole group.

10. Prove that a group of order 255 is cyclic. https://youtu.be/MejAbEvOJcE?si=WawQlmGeMh29BhWc
https://math.stackexchange.com/a/1565732

11. A group of order 30 has a normal subgroup of order 5.

12. Prove that a group of order 72 is not simple. https://youtu.be/PiTki4Mk7eI?si=se2iNSepQtNGOlfD

13. Prove that a group of order p^2q, where p, q are distinct primes is not simple.

14. Prove that a group of order pqr, where p, q, r are distinct primes is not simple.

https://www.youtube.com/watch?v=n1tK7lJSv_8