Problem Set-7
MTH-204, MTH-204A
ABSTRACT ALGEBRA

1. Let G be a non-abelian group of order pq, where p and q are distinct primes. Show that G is solvable, but not nilpotent.

2. Let G be a nilpotent group of order n. Then show that for every divisor d of n there exists a subgroup of order d.

3. Show that a finite group G is nilpotent if and only if $xy = yx$ for all $x, y \in G$ having relatively prime orders.

4. Show that the additive group $\mathbb{Q}$ has no composition series.

5. Suppose that G is a solvable group with order $n \geq 2$. Show that G contains a normal nontrivial abelian subgroup.

6. Find all the abelian groups of order 720 up to isomorphism.

7. Prove that an abelian group has a composition series if and only if it is finite.

8. For which numbers n are all abelian groups of that order cyclic ?