Question 1 - Bellman Ford Algorithm

a	inf	-3	-3	-4	-6	-6
b	inf	inf	0	-2	-2	-2
c	inf	3	3	3	3	3
d	inf	4	3	3	2	0
e	inf	2	0	0	0	0
s	0	0	0	0	0	0
	0	**1**	**2**	**3**	**4**	**5**

Question 2 - Optimal Triangulation

- 2.5 marks are awarded for writing the correct recursive formulation along with the base case.
- 1 mark is deducted if the base case is missing.
- 1.5 marks are awarded for providing a clear justification.

Question 3 a)- Optimal Subpath Theory

- 1 mark is awarded only if the property is well stated.

Question 3 b)- Graph violating the property

- 4 marks are awarded if the graph shown violates the property.
- 1 mark is deducted if there are two optimal paths reaching ‘y’.

Question 4 - On multiplying adjacency matrix

- 4a) 1 mark awarded for answering “Degree of x”
- 4b) 0.5 mark deducted if it simply mentions C[i,j] is “not equal to zero” if there exists a path of length 2 from i to j .
- 4b)1 mark awarded for answering C[i,j] = shows the “number of paths” of length 2 from i to j
- 4 c)1 mark awarded if the student has misinterpreted the question and has answered ‘2’
- 4 c)2 marks are awarded for floor(n/2) +1

Question 5 - Integrality of Maximum Flow

- 2 marks awarded for stating the correct theorem
- 1 mark deducted for not stating “if all edge capacities are integers”
- 1 mark deducted if it is only implied that for all edges the flow will be integers instead of stating it properly.

Question 6a - Number of Triangles present

- Full marks are awarded for using the blackbox once, applying the correct conditions, and calculating the exact number of triangles.
- 1–2 marks are deducted if there are mistakes in applying the correct conditions or in calculating the exact number of triangles.
- No deduction applies if all steps are correct.

Question 6b - Transitive Reduction

- You must have multiplied the adjacency matrix A with the transitive closure A+ and proceeded correctly from there.
- Any other approach that leads to higher complexity than required (e.g., solving using topological sort) has been awarded 0 marks.

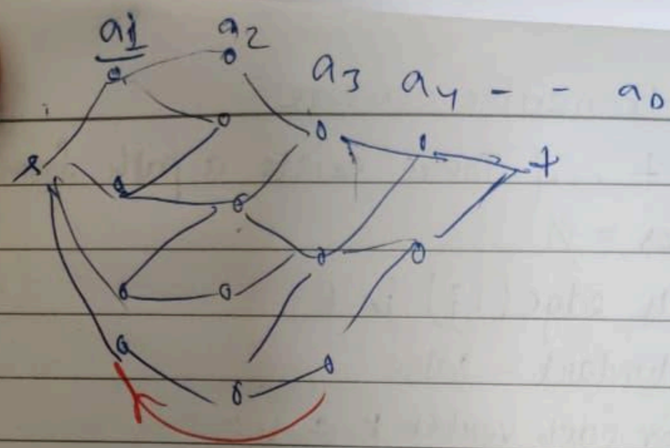
Question 7a - Ford Fulkerson (Time complexity)

- 1 mark is awarded for stating the correct bound O(m f*).
- 1 mark is awarded for providing the correct justification.
- Justification: The flow increases by one unit per iteration, leading to f* iterations, and each iteration takes (m + n) time.

Question 7b - Ford Fulkerson (Proof of Time Complexity)

- 2 marks are awarded for correctly categorizing the vertices in BFS depth order.
- 3 marks are awarded for identifying that the total flow cannot exceed n_(i)×n_(i+1), where n_(i) is the number of vertices at distance iii from the source.
- 7 marks are awarded for correctly applying the AM–GM inequality.
- 8 marks are awarded for providing the complete answer.

Sample Solution for 7b:



every st cut (let it be b/w a_i and a_{i+1})

No of edges

$= \text{at max } a_i \times a_{i+1}$ $\therefore$ This is upper bound

Now sum of all adjacent pairs

$$\sum_{i=0}^{D-1} a_i + a_{i+1}$$

$$= a_0 + 2\sum_{i=1}^{D-1} a_i + a_D$$

$$= 2\sum_{i=0}^{D} a_i - (a_0 + a_D)$$

$$= 2n - (\text{something})$$

$$\leq 2n$$

There will exist one (i) for which (Pigeonhole)

$$\boxed{a_i + a_{i+1} \leq \frac{2n}{D}}$$

For some i

$$\frac{a_i + a_{i+1}}{2} \leq \frac{n}{D}$$

$$\sqrt{a_i\, a_{i+1}} \leq \frac{a_i + a_{i+1}}{2} \leq \frac{n}{D}$$ By AM GM

$$a_i\, a_{i+1} \leq \left(\frac{a_i + a_{i+1}}{2}\right)^2 \leq \left(\frac{n}{D}\right)^2$$

$\{$ Now $D = O(n^{2/3}$

$$(a_i\, a_{i+1}) \leq \left(\frac{n}{n^{2/3}}\right)^2$$

Some cut $\leq n^{2/3}$

$$f^* \leq n^{2/3}$$