ASSIGNMENT 2

(1) Suppose $AX = b$ and $CX = b$ have same solutions for every $b \in \mathbb{R}^m$ prove that $A = C$.

(2) Let $\sigma \in S_4$ be given by

$$\begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 5 & 4 & 1 & 2 & 3 \end{pmatrix}$$

(a) Find sign of σ and sign of σ^{-1},

(b) Find $\sigma^2 = \sigma \circ \sigma$.

(3) Suppose $A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix}$. Calculate A^n. Prove that $A^2 - 2A + I = 0$, where I is identity matrix of order 3. From here, prove that A is invertible and find A^{-1}.

(4) Let A be a square matrix of order n. Show that $det(A) = 0$ if and only if there exists non-zero vector $X = (x_1, x_2, ..., x_n)$ such that $AX^T = 0$.

(5) Let $A = (a_{ij})$ be an invertible matrix and let $B = (p^{i-j} a_{ij})$ where $p > 0$. Find $det(B)$ and inverse of B.

(6) Let A be an invertible square matrix with interger entries. Show that A^{-1} has integer entries if and only if $det(A) = \pm 1$.

(7) (Vandermonde Matrix) Find the determinant of the following matrix:

$$\begin{pmatrix} 1 & x_1 & x_1^2 & ... & x_1^{n-1} \\ 1 & x_2 & x_2^2 & ... & x_2^{n-1} \\ .. & .. & .. & ... & .. \\ 1 & x_n & x_n^2 & ... & x_n^{n-1} \end{pmatrix}$$

(8) The numbers 1375, 1287, 4191 and 5731 are all divisible by 11. Prove that the determinant of the matrix

$$\begin{pmatrix} 1 & 1 & 4 & 5 \\ 3 & 2 & 1 & 7 \\ 7 & 8 & 9 & 3 \\ 5 & 7 & 1 & 1 \end{pmatrix}$$

is also divisible by 11.

(9) Find the determinant of

$$\begin{pmatrix} 1 & 2 & 3 & 4 & ... & n \\ 2 & 2 & 3 & 4 & ... & n \\ 3 & 3 & 3 & 4 & ... & n \\ .. & .. & .. & .. & ... & .. \\ n & n & n & n & ... & n \end{pmatrix}$$