Problem Set-9
MTH-204, 204A
Abstract Algebra

1. Let R be the ring of all the real-valued, continuous functions on $[0, 1]$. Let $M = \{f \in R : f(1/2) = 0\}$. Prove that M is a maximal ideal of R. Every maximal ideal of R is of this form.

2. Prove that the ring $\mathbb{Z}[i]$ is a Euclidean domain.

3. Let K be a field. Prove that the ring $K[x]$ is a PID.

4. Prove that the quotient ring $\mathbb{R}[x]/(x^2 + 1)$ is isomorphic to $\mathbb{C}$.

5. Prove that if $f(x) \in \mathbb{Q}[x]$, then f is divisible by the square of a polynomial if and only if $f(x)$ and $df(x)/dx$ have a greatest common divisor $d(x)$ of positive degree.

6. If $f(x) \in \mathbb{Z}_p[x]$, p a prime, and $f(x)$ irreducible over $\mathbb{Z}_p$ of degree n, prove that $\mathbb{Z}[x]/(f(x))$ is a field with p^n elements.