ASSIGNMENT 5

(1) ***Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation defined by $T(x, y) = (ax + by, cx + dy)$. Find the matrix of T with respect to standard basis of $\mathbb{R}^2$. Now do the same by considering the basis $\{(1.0), (1, 1)\}$ on domain and co-domain of T.

(2) Let V be a finite dimensional vector space. Using Rank-Nullity theorem, a linear transformation $T : V \to V$ is onto if and only if it is injective.

(3) *** Consider the linear map $T : \mathbb{C} \to \mathbb{C}$ defined by $T(z) = iz$. By considering the basis $\{1, i\}$ of $\mathbb{C}$ (over $\mathbb{R}$) on domain and co-domain of T, find the matrix of T.

(4) *** Let $T : V \to V$ be a linear transformation with $Ker(T) = R(T)$, $R(T)$ is range of T. Show that $T^2 = 0$. Give example of such a linear map $T : \mathbb{R}^2 \to \mathbb{R}^2$.

(5) ***Does there exists a linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^4$ such that range of T, $R(T) = \{(x_1, x_2, x_3, x_4) \in \mathbb{R}^4 : x_1 + x_2 + x_3 + x_4 = 0\}$.

(6) *** Let V be a vector space of dimension n and $\{u_1, u_2, ..., u_n\}$ be a basis of V. Suppose $w_1, w_2, ..., w_n$ are n-elements of V with $w_j = a_{1j}u_1 + a_{2j}u_2 + ... + a_{nj}u_n$ ($(a_{1j}, a_{2j}, ..., a_{nj})$ is said to be coordinates of w_j with respect to basis $\{u_1, ..., u_n\}$). Let $A = (a_{ij})$ then show that $\{w_1, w_2, ..., w_n\}$ is a basis of V if and only if A is invertible.

(7) Find the kernel and range of $T(x, y, z) = (x + z, x + y + 2z, 2x + y + 3z)$.

(8) *** Let $<, >$ be an inner product on $\mathbb{R}^n$. Prove that there exists a symmetric matrix A of order n such that $< u, v >= u^T Av$ for all $u, v \in \mathbb{R}^n$.

(9) *** Equip $\mathbb{R}^3$ with usual standard inner product. Using Gram-Schimdt process, transform the set of vectors $\{(1, 1, 1), (1, 0, 2), (0, 1, 2)\}$ into an orthonormal basis of $\mathbb{R}^3$.