MSO205 PRACTICE PROBLEMS SET 5

Question 1. Suppose there are RVs X and Y given by the distribution functions below. Find quantiles of order $p \in (0,1)$. What are the medians, lower and upper quartiles?

(a)

$$F_X(x) := \begin{cases} 0, \text{ if } x < 0, \\ \frac{1}{2}, \text{ if } 0 \leq x < 1, \\ 1, \text{ if } x \geq 1. \end{cases}$$

(b)

$$F_Y(x) := \begin{cases} 0, \text{ if } x < 0, \\ \frac{x}{2}, \text{ if } 0 \leq x < 1, \\ \frac{1}{2}, \text{ if } 1 \leq x < 2, \\ \frac{x-1}{2}, \text{ if } 2 \leq x < 3, \\ 1, \text{ if } x \geq 3. \end{cases}$$

Question 2. Let X be a continuous RV with the p.d.f.

$$f_X(x) := \begin{cases} \exp(-x), \text{ if } x > 0, \\ 0, \text{ otherwise.} \end{cases}$$

Find the support of X.

Question 3. Consider a discrete RV X with the p.m.f.

$$f_X(x) := \begin{cases} \frac{1}{4}\left(\frac{3}{4}\right)^x, \text{ if } x \in \{0,1,2,\cdots\}, \\ 0, \text{ otherwise.} \end{cases}$$

Consider the RV $Y = \frac{X}{X+1}$.

(a) First find the DF F_Y and then compute the p.m.f. f_Y.

(b) First find the p.m.f. f_Y and then compute the DF F_Y.