MSO205 PRACTICE PROBLEMS SET 12

Question 1. Compute the mode of $Binomial(n, p)$ distribution.

Question 2. Let $X \sim Poisson(\lambda)$ for some $\lambda > 0$. Compute the coefficient of skewness and excess kurtosis.

Question 3. Let $c := \sum_{m=1}^{\infty} m^{-3} < \infty$. Then the function $f : \mathbb{R} \to [0, 1]$ given by

$$f(x) = \begin{cases} \frac{1}{c} x^{-3}, & \text{if } x \in \{1, 2, \cdots\} \\ 0, & \text{otherwise} \end{cases}$$

is a p.m.f.. Let X be a discrete RV with this p.m.f. and consider the following sequence of RVs $\{X_n\}_n$ defined by

$$X_n = \begin{cases} X, & \text{if } X \leq n, \\ 0, & \text{otherwise} \end{cases}, \forall n.$$

Show that the sequence of RVs $\{X_n\}_n$ converges in first mean to X, but not in the second mean.

Question 4. Let $\{X_n\}_n$ be a sequence of i.i.d. RVs with finite second moment. Show that:

(1) $\frac{2}{n(n+1)} \sum_{j=1}^{n} j X_j \xrightarrow[n\to\infty]{\mathbb{P}} \mathbb{E}X_1$.

(2) $\frac{6}{n(n+1)(2n+1)} \sum_{j=1}^{n} j^2 X_j \xrightarrow[n\to\infty]{\mathbb{P}} \mathbb{E}X_1$.

Question 5. Let $a, b \in \mathbb{R}$ and let $\{X_n\}_n$ be a sequence of RVs such that $X_n \xrightarrow[n\to\infty]{\mathbb{P}} a$ as well as $X_n \xrightarrow[n\to\infty]{\mathbb{P}} b$. Show that $a = b$.