ASSIGNMENT 3

(1) Let A be a square matrix with entries from the set of complex numbers $\mathbb{C}$. A is said to be Hermitian if $A = \overline{A}^T$ (A is equal to its conjugate transpose). A is said to be skew-Hermitian if $A = -\overline{A}^T$. Which of the following statements are true. Justify your answer.
***(a) set of Hermitian matrices of order n is a vector space over $\mathbb{C}$ under usual matrix addition and scalar multiplication;
***(b) set of Hermitian matrices of order n is vector space over set of real numbers $\mathbb{R}$ under usual matrix addition and scalar multiplication;
(c) set of skew-Hermitian matrices of order n is a vector space over $\mathbb{C}$ under usual matrix addition and scalar multiplication;
(d) set of skew-Hermitian matrices of order n is a vector space over $\mathbb{R}$ under usual matrix addition and scalar multiplication;

(2) *** In $\mathbb{R}$, consider the addition $x \oplus y = x + y - 1$ and the scalar multiplication $\lambda.x = \lambda(x-1)+1$. Prove that $\mathbb{R}$ is a vector space with respect to these operations and the additive identity is 1.

(3) Which of the followings are true:
***(i) $\{(x,y) \in \mathbb{R}^2 : x \geq 0\}$ is a subspace of $\mathbb{R}^2$,
(ii) $\{(x,y) \in \mathbb{R}^2 : y = x^2\}$ is a subspace of $\mathbb{R}^2$,
***(iii) the set $\mathcal{P}_n(X)$ of all single variable polynomials of degree at most n is a subspace of vector space $\mathcal{P}(X)$ of all single variable polynomials.
(iv) $x + y + z = 1$ is a subspace of $\mathbb{R}^3$.
***(v) $\{A \in M_2(\mathbb{R}) : det(A) = 0\}$ is a subspace of $M_2(\mathbb{R})$, where $M_2(\mathbb{R})$ is vector space of real 2×2 matrices under usual matrix addition and scalar multiplication,
(vi) Let $C([0,1])$ be vector space of real valued continuous functions on $[0,1]$ and let $a \in [0,1]$. The set $M_a = \{f \in C([0,1]) : f(a) = 0\}$ is a subspace of $C([0,1])$.
***(vii) the space of all upper triangular matrices of order n is a subspace of $M_n(\mathbb{R})$, where $M_n(\mathbb{R})$ is vector space of real $n \times n$ matrices under usual matrix addition and scalar multiplication
(viii) the set of all orthogonal matrices of order 2 is a subspace of $M_2(\mathbb{R})$. (A square martix A is said to be orthogonal if $AA^T = I$)

(4) Show that $W = \{(x_1, x_2, x_3, x_4) : x_4 - x_3 = x_2 - x_1\}$ is a subspace of $\mathbb{R}^4$ spanned by vectors $(1,0,0,-1), (0,1,0,1), (0,0,1,1)$.

(5) Let W_1 and W_2 be two subspaces of a vector space V such that $W_1 \cup W_2$ is a subspace of V. Prove that either $W_1 \subset W_2$ or $W_2 \subset W_1$.

(6) *** Find all subspaces of $\mathbb{R}^3$.

(7) *** Let V be a vector space over $\mathbb{C}$ and $\{u_1, u_2, ..., u_n\}$ be a linearly independent set of vectors in V. Prove that $\{u_1, u_2, ..., u_n, iu_1, iu_2, ..., iu_n\}$ is a set of linearly independent vectors when considered V as a vector space over $\mathbb{R}$.

(8) Discuss the linear dependence/independence of following set of vectors:
(i) $\{(1, 0, 0), (1, 1, 0), (1, 1, 1)\}$ in $\mathbb{R}^3$,
***(ii) $\{(1, 0, 0, 0), (1, 1, 0, 0), (1, 1, 1, 0), (3, 2, 1, 0)\}$ in $\mathbb{R}^4$,
***(iii) $\{(1, i, 0), (1, 0, 1), (i + 2, -1, 2)\}$, in $\mathbb{C}^3(\mathbb{C})$ ($\mathbb{C}^3$ considered as a vector space over $\mathbb{C}$),
(iv) $\{(1, i, 0), (1, 0, 1), (i+2, -1, 2)\}$, in $\mathbb{C}^3(\mathbb{R})$ ($\mathbb{C}^3$ considered as a vector space over $\mathbb{R}$),
(v) $\{u + v, v + w, w + u\}$ in a vector space V given that $\{u, v, w\}$ is linearly independent.