Name _____

Roll Number _____

Quiz 2

MTH302: Set Theory and Mathematical Logic

(Odd Semester 2024/25, IIT Kanpur)

Question 1. [3 × 1 Points]

For each of the following statements, determine whether it is **true or false**. No justification required.

(i) If $T \vdash (\phi \vee \psi)$, then either $T \vdash \phi$ or $T \vdash \psi$. _____

(ii) $(\mathbb{Z}, +)$ is an elementary submodel of $(\mathbb{Q}, +)$. _____

(iii) If $A \subseteq \omega$ and $\omega \setminus A$ are both c.e., then A is computable. _____

Question 2. [7 Points]

(a) [**2 Points**] Let $\mathcal{L}$ be a first order language and T be an $\mathcal{L}$-theory.
   (i) Define what it means for T to be **consistent**.
   (ii) Define what it means for T to be **complete**.

(b) [**2 Points**] Let $\mathcal{L}$ be the empty language. Show that $T = \{\exists_{\geq n} : n \geq 2\}$ is a complete $\mathcal{L}$-theory.

(c) [**3 Points**] Describe all elementary submodels of $(\mathbb{Q}, +)$.