MIDTERM EXAM

INTRO TO REAL ANALYSIS

The exam consists of five questions, each worth 20 points. You may take up to three hours to complete the exam.

Problem 1. Complete the following definitions. Write in complete English sentences, and avoid use of quantifier symbols like $\forall$ and $\exists$.

(a) A sequence (a_n) *converges* to a real number a if ...
(b) A sequence (a_n) is a *Cauchy sequence* if ...
(c) An infinite series $\sum_{n=1}^{\infty} a_n$ *converges absolutely* if ...

Problem 2. Let us define an *AB-sequence* to be a function from $\mathbb{N}$ to the two-element set $\{A, B\}$. Thus, an AB-sequence can be thought of as an infinite string of the letters A and B.

(a) Is the set of AB-sequences uncountable? Give a proof of your answer.
(b) Define an AB-word to be a finite string of the letters A and B. Is the set of AB-words uncountable? Justify your answer.

Problem 3. Suppose (a_n) and (b_n) are convergent sequences with limits a and b respectively. Prove that $(a_n + b_n)$ converges to $a + b$. (Prove this directly from the definition of convergence, without appealing to the Algebraic Limit Theorem.)

Problem 4. Assume (a_n) is a bounded sequence with the property that every convergent subsequence of (a_n) converges to the same limit $a \in \mathbb{R}$. Show that (a_n) must converge to a.

Problem 5. Prove that a set F is closed if and only if its complement F^c is open. (Recall the definitions of open and closed sets: we say a set $U \subset \mathbb{R}$ is *open* if for any point $x \in U$, there exists a neighborhood $V_\epsilon(x)$ contained in U. And we say a set $F \subset \mathbb{R}$ is *closed* if it contains all its limit points.)

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