FINAL EXAM

INTRO TO REAL ANALYSIS II

The exam consists of five questions, each worth 20 points. You may take up to three hours to complete the exam.

Problem 1. Suppose A and B are both open subsets of $\mathbb{R}^n$, and that $f : A \to B$ is a one-to-one, onto function of class C^r. Suppose $g : B \to A$ is a differentiable inverse to f. Prove directly that g is of class C^r (in particular do not use the Inverse Function Theorem).

Problem 2. Suppose α is a real root of multiplicity 1 of the real polynomial $a_0 + a_1 x_1 + \ldots + a_n x^n$. Show that for any $\epsilon > 0$ there exists a $\delta > 0$ such that the polynomial $a'_0 + a'_1 x_1 + \ldots + a'_n x^n$ has a root in $(\alpha - \epsilon, \alpha + \epsilon)$, provided $|a'_j - a_j| < \delta$ for all j.

Problem 3. Let $p_1, \ldots, p_m$ be m points in $\mathbb{R}^n$. Show that $\sum_{i=1}^m ||x - p_i||^2$ achieves its absolute minimum at $x = \frac{1}{m} \sum_{i=1}^m p_i$.

Problem 4. Fix an element $a \in \mathbb{R}^n$. Use the method of Lagrange multipliers to find the extreme values of $f(x, y) = x^2 - y^2$ on $S := \{x^2 + y^2 = 1\}$.

Problem 5. Consider a complex function $f : \mathbb{C} \to \mathbb{C}$. Show that if

$$f'(a) := \lim_{\Delta z \to 0} \frac{f(a + \Delta z) - f(a)}{\Delta z}$$

exists and equals $\alpha + i\beta$, then

$$Df(a) = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}.$$

Date: April 26, 2018.