

PROBLEM SET 1

ANALYSIS II

Problem 1. Consider $f(x) = 1/x$ over the interval $[1, 4]$. Let P be the partition consisting of the points $\{1, \frac{3}{2}, 2, 4\}$.

- (a) Compute $L(f, P)$, $U(f, P)$, and $U(f, P) - L(f, P)$.
- (b) What happens to the value of $U(f, P) - L(f, P)$ when we add the point 3 to the partition?
- (c) Find a partition P' of $[1, 4]$ for which $U(f, P') - L(f, P') < \frac{2}{5}$.

Problem 2. Suppose f_n is an integrable function on $[a, b]$ for each natural number n . If $(f_n) \rightarrow f$ uniformly on $[a, b]$, prove that f is also integrable on this set. (We will see that this conclusion can fail to hold if convergence is merely pointwise.)

Problem 3. Let $f : [a, b] \rightarrow \mathbb{R}$ be monotone increasing on the set $[a, b]$. Show that f is integrable on $[a, b]$.

*All questions taken from *Understanding Analysis: 2nd Edition* by Stephen Abbott.