

NOTES ON THURSTON PAGE 181

Let (X, G) be a model geometry. Let G' be the connected component of the identity in G . Let's assume for the time being that G' also acts transitively on X .

Now let $x \in X$ be any point. There is an injective and surjective map $G'/G'_x \rightarrow X$ defined by $[g] \mapsto [g]x$. If we instead quotient by $(G'_x)_0 \subset G'_x$, we may lose injectivity, but will still have a surjective map $\pi : G'/(G'_x)_0 \rightarrow X$ defined by $[g] \mapsto [g]x$, with fiber isomorphic to $G'_x/(G'_x)_0$. This latter set is discrete (this will be the case for any group modulo the connected component of the identity). By compactness of G'_x , the set $G'_x/(G'_x)_0$ is therefore finite. It follows that π is a finite-sheeted covering space map.

Now, since X is simply connected, π must be a trivial covering map, collapsing finitely many copies of X to X . And since $G'/(G'_x)_0$ is connected, π must be one to one. It follows that the fiber $G'_x/(G'_x)_0$ is a single point, and hence that $G'_x = (G'_x)_0$.