Tutorial No - 6

Q1)

a) Since this is a bernoulli distribution,
Pr (k tails in the first k tosses, then 1 head) $= (1-\lambda)^k \lambda$

b) Let M be the number of the tosses required to get the first head and
Let $S = E[M]$.
Given that tosses are independent, and expectation is additive:

$$S = \lambda \times 1 + (1-\lambda) * (S+1)$$

Solving for S gives ~~S = 1/λ~~:

$$S = \cancel{\lambda} * 1 + S + 1 - \lambda S - \cancel{\lambda}$$

$$\therefore \boxed{S = \frac{1}{\lambda}}$$

Q2) a) By definition of variance:

$$E[(X - E[X])^2] = E[X^2 - 2XE[X] + E[X]^2]$$

$$= E[X^2] - 2E[XE[X]] + E[X]^2$$

$$= E[X^2] - 2E[X]^2 + E[X]^2$$

$$= E[X^2] - E[X]^2$$

PAGE No.

DATE / /

b) $Var(X) = E[X^2] - E[X]^2 = 1$

If $Y = a + bX$,

$E[Y^2] = E[(a+bX)^2] = E[a^2 + 2abX + b^2X^2]$

$= a^2 + 2ab\,E[X] + b^2 E[X^2] = a^2 + b^2$

$E[Y] = E[a+bX] = a + bE[X] = a$

$Var(Y) = E[Y^2] - E[Y]^2 = a^2 + b^2 - a^2 = b^2$

Q3) Let T be event that "Aku predicts that black beauty is a winning horse".
Let ~T be event "Aku predicts that "black beauty" is not winning horse."

Let W be event that black beauty horse wins.
Let ~W be event that black beauty horse does not wins.

a) Given a horse, probability that it wins is :

$$P(W) = P(W,T) + P(W,\sim T)$$
$$= P(W|T)\,P(T) + P(W|\sim T)\,P(\sim T)$$
$$= 0.99 \times 10^{-5} + (1 - 0.99999) \times (1 - 10^{-5})$$
$$\approx 1.99 \times 10^{-5}$$

b) Probability that Aku correctly predicts the winning horse :

PAGE NO.
DATE / /

$$P(T|W) = \frac{P(T,W)}{P(W)} = \frac{P(W|T)\,P(T)}{P(W)}$$

$$= \frac{0.99 \times 10^{-5}}{0.99 \times 10^{-5} + (1-0.99999) \times (1-10^{-5})}$$

$$\boxed{P(T|W) \approx 0.497}$$

$\Rightarrow$ Hypothesis :

Ex-1