Tutorial 4

Ex 1: $\text{Var}(x) = \langle (x - \langle x \rangle)^2 \rangle$

$= \langle x^2 \rangle - \langle x \rangle^2$

$= \left\langle \left(\sum_i w_i s_i \right)^2 \right\rangle - \left\langle \sum_i w_i s_i \right\rangle^2$

$= \left\langle \left(\sum_i w_i s_i \right)^2 \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right)^2$

$= \left\langle \left(\sum_i w_i s_i \right) \left(\sum_j w_j s_j \right) \right\rangle - \left(\sum_i w_i \langle s_i \rangle \right) \left(\sum_j w_j \langle s_j \rangle \right)$

$= \left\langle \sum_{i,j} w_i w_j s_i s_j \right\rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$

$= \sum_{i,j} w_i w_j \langle s_i s_j \rangle - \sum_{i,j} w_i w_j \langle s_i \rangle \langle s_j \rangle$

$= \sum_{i,j} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) + \sum_{\substack{i,j \\ i \neq j}} w_i w_j \left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right)$

$\left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) = \text{Var}(s_i) = 1$

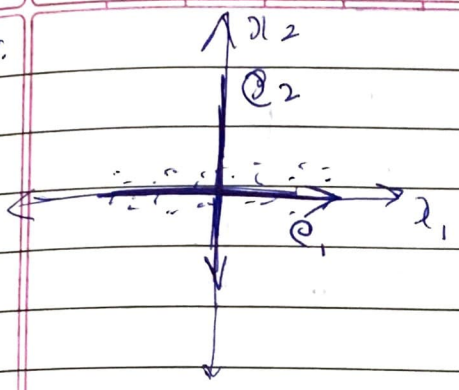
$\left(\langle s_i s_j \rangle - \langle s_i \rangle \langle s_j \rangle \right) = 0$

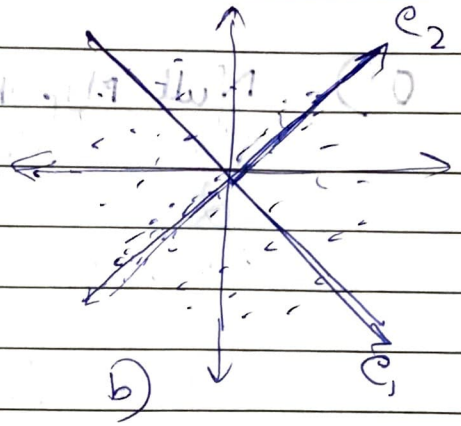
$= \sum_i w_i^2$

∴ For unit variance constraint

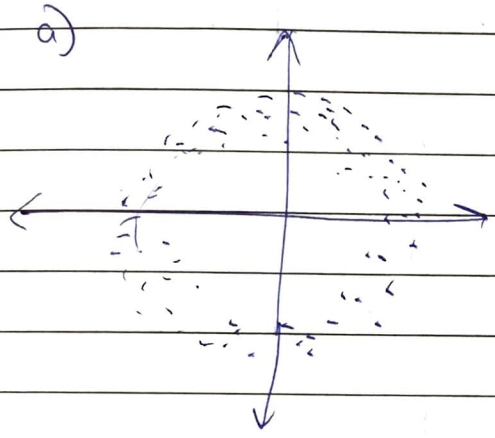
$$\boxed{\sum_i w_i^2 = 1}$$

Ex. 2:





a)



Vanishing is not possible