DS

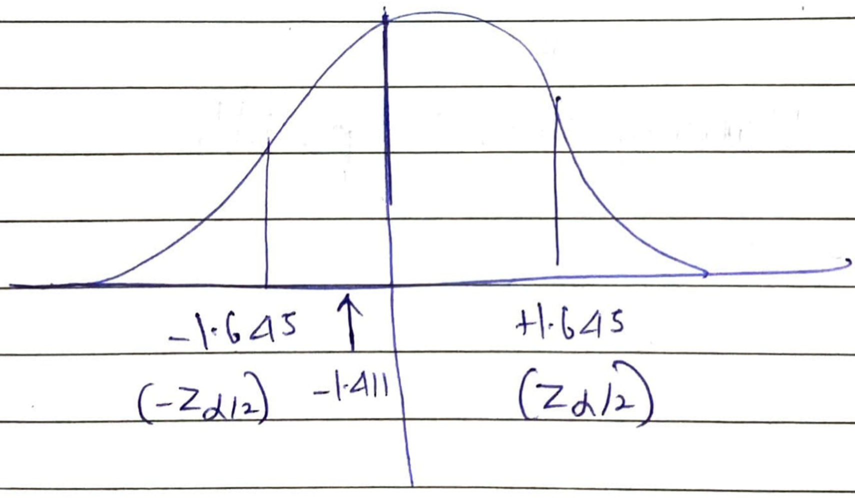
Tutorial 5

Q1) Null Hypothesis : $H_0 : p = 0.7$
Alternate Hypothesis : $H_1 : p > 0.7$
$\alpha = 0.1$, two tailed test.
Critical Value $= \pm 1.645 = Z_{\alpha/2}$

Test statistic $= Z = \frac{\hat{p} - p}{\sqrt{pq/n}} \sim N(0,1)$

$\hat{p} = 8/15 = 0.533$

$$Z = \frac{0.533 - 0.7}{\sqrt{0.7 \times 0.3 / 15}} = -1.411$$



$\because Z > -Z_{\alpha/2}$ and $Z < Z_{\alpha/2}$
the NULL hypothesis CANNOT be Rejected.

Q2)

$H_0 : p = 0.6$

$H_1 : p > 0.6$

$\alpha = 0.05$

Critical ~~Region~~ Value $Z_{\alpha/2} = \pm 1.96$

$$Z = \frac{\hat{p} - p}{\sqrt{pq/n}}$$

$\therefore \hat{p} = 70/100 = 0.7$

$$\therefore Z = \frac{0.7 - 0.6}{\sqrt{0.6 \times 0.4/100}} = 2.04$$

$\therefore Z > Z_{\alpha/2}$, it lies in the rejection region

$\therefore$ Reject ~~NULL~~ NULL Hypotheses.

Q3) Let the proportion of Mumbai voter is p_1

Let the porportion of surrounding voter is p_2.

$\hat{p}_1 = 120/200 = 0.6$

$\hat{p}_2 = 240/500 = 0.48$

$$\hat{p}_p = \frac{120 + 240}{200 + 500} = 0.5143$$

$H_0 : p_1 \leq p_2$

$H_1 : p_1 > p_2$

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Given $\alpha = 0.05$

$Z_\alpha = Z_{0.05} = 1.65$

$$Z = \frac{\hat{p}_1 - \hat{p}_2}{\sqrt{\hat{p}_p(1-\hat{p}_p)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}}$$

$$= \frac{0.6 - 0.48}{\sqrt{0.5143\left(1-0.5143\right)\left(\frac{1}{200}+\frac{1}{500}\right)}}$$

$= 2.8697$

The Rejection Region is $Z > 1.65$

$\therefore$ We reject the NULL Hypothesis.

Ex 4:

a) Null Hypothesis is
$H_0 : p = 0.2$

Alternate Hypothesis : $H_1 : p > 0.2$
The critical region is in right tail.

b) Null Hypothesis $H_0 : \mu = 3$
Alternate Hypothesis $H_1 : \mu_1 \neq 3$

The critical region is in both tails.

c) Null Hypothesis: $H_0 : p = 0.15$
Alternate Hypothesis: $H_1 : p < 0.15$

Critical Region is in left tail

d) Null Hypothesis: $H_0 : \mu = 500$
Alternate Hypothesis $H_1 : \mu > 500$

Critical Region is in right tail

e) $H_0 : \mu = 15$
$H_1 : \mu \neq 15$
The critical region is in both tails

Q5) Let μ_1 and μ_2 be Population mean for company A and company B.

$H_0 : \mu_1 = \mu_2$
$H_1 : \mu_1 \neq \mu_2$
at significance level $\alpha = 0.05$

Finding sample means and variances,

$$\bar{X}_1 = \frac{1}{n_1} \sum_{i=1}^{n_1} x_{1i} = \frac{79.5}{10} = 7.95$$

$$\bar{X}_2 = \frac{1}{n_2} \sum_{i=1}^{n_2} x_{2i} = \frac{102.6}{10} = 10.26$$

$$S_1^2 = \frac{1}{n_1 - 1} \left[\sum_{i=1}^{n_1} x_{1i}^2 - n_1 \bar{X}_1^2 \right] = \frac{10.865}{9}$$
$$= 1.207$$

$$S_2^2 = \frac{1}{n_2-1}\left[\sum_{i=1}^{n_2} x_{2i}^2 - n_2 \bar{X}_2^2\right] = \frac{2.924}{9} = 0.325$$

Since sample variances are quite different, we cannot assume that the population variances are equal, so we will use the unpooled t-test

DoF:

$$v = \frac{\left(\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{S_1^2}{n_1}\right)^2 + \frac{1}{n_2-1}\left(\frac{S_2^2}{n_2}\right)^2}$$

$$= \frac{\left(\frac{1.207}{10} + \frac{0.325}{10}\right)^2}{\frac{1}{10-1}\left(\frac{1.207}{10}\right)^2 + \frac{1}{10-1}\left(\frac{0.325}{10}\right)^2}$$

$$= 10.3$$

Test statistic:

$$T = \frac{\bar{X}_1 - \bar{X}_2 - (\mu_1 - \mu_2)}{\sqrt{\frac{S_1^2}{n_1} + \frac{S_2^2}{n_2}}}$$

$$= \frac{7.95 - 10.26}{\sqrt{\frac{1.207}{10} + \frac{0.325}{10}}} = -5.9$$

p-value is doubled area under density curve of t distribution with 10 degree of freedom, $|t| = |-5.9| = 5.9$

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p-value $= 2 \cdot P(T \geq |t|) = 2\, P(T \geq 5.9)$

$\boxed{\text{p-value} \leq 0.001}$

$\because t_{0.0005}(10) = 4.587$

since $|t| = 5.9$ is greater than $P(T \geq 5.9) < 0.0005$

$\because$ P value is less than level of significance, we reject NULL hypothesis.