Comments on the December 2010 Skills papers

Below you will find some comments on the questions on last year's Physics Skills papers. These are NOT solutions. Acceptable solutions will comprise coherent explanations and reasoning, give definitions of symbols appearing in equations etc. Here the idea is simply to sketch out the ideas so that you can explore your notes, review your thoughts etc. in an efficient way.

As well as thinking about the specific questions, try to develop a feel for the *kind* of question posed, and the type of explanations, arguments, calculations etc. they should provoke. This is more important than possessing solutions for particular questions.

Don't despair - ALL students find the Physics Skills exam hard. And you should not expect to know everything as your curriculum may have not included all topics. But note that some knowledge and skills are tested in an unfamiliar context (e.g. haystacks and chickens) and you should not be put off by that. Squeeze everything you can from the knowledge and skills you have.

*****************COMMON QUESTIONS*****************

1. *Estimate* the energy (in eV) of photons required for studying the crystal structure of solids.

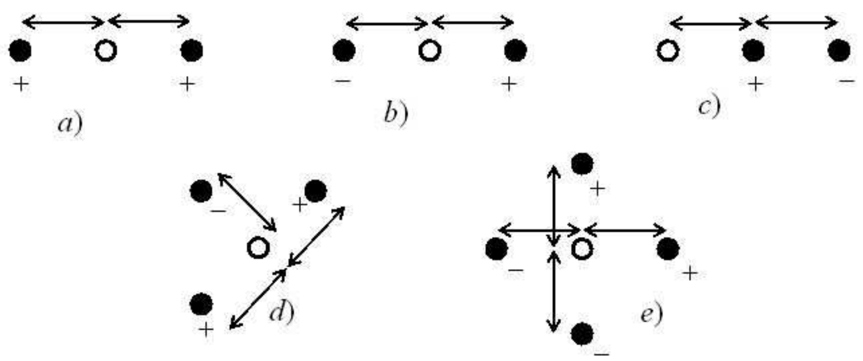
What energy electrons could be used for this purpose? **[5]**

Comments:
Key concept: we determine structures (i.e. atomic positions) by observing the *diffraction* of incident waves, for which we require the wavelength to match the atomic spacing. A sensible estimate of the size of an atom is required, and equating this to the photon wavelength allows the photon energy to be deduced, since

$$E = hc/\lambda.$$

Getting the units right should lead to $E \sim 4000$ eV. For electrons we use the de Broglie relation to obtain p and the Newtonian dispersion relation $E = p^2/2m$. You will see from your answer that this is a non-relativistic electron, but it would not be incorrect to use the relativistic energy-momentum relationship. (*If you can't see immediately how to obtain the answer, start by stating the key concepts - you will get partial credit for this, and you may be able to get further than you first thought.*)

2. A test charge (white) is placed in the 5 different environments in turn, labelled *a*) to *e*) in the illustration below. In each case the arrows indicate a single unit of distance.



Rank these environments (in ascending order) according to the magnitude of electrostatic force experienced by the test charge. **[5]**

Comments:
Key concept: Coulomb's law (and vector addition). Keep track of the direction of the force due to each charge. The order is a, c, d, b, e. (*Don't just write down your proposed ranking, state that you are using Coulomb's law. The cover sheet on the front of the exam paper insists that "Reasoning must be given", and this makes it easier to award partial credit when you go wrong.*)

3. You are pulling a box along a rough, flat surface at constant velocity by applying a horizontal force with a rope. The coefficient of friction between the box and the surface is μ. Determine the factor by which the pulling force must change if the rope makes an angle θ with the horizontal and the box continues to move with constant velocity. **[5]**

Comments:
In both cases the forces on the box are the tension in the rope, $\underline{T}$, the gravitational force, $m\underline{g}$, the normal reaction force, $\underline{N}$, and friction, $\underline{F}$ (with magnitude μN). Of these we can only be sure that the gravitational force is independent of the rope angle, so we give the other forces either a 0 or θ subscript depending upon whether we have in mind the horizontal or inclined rope case. (*The physics in this question is reasonably simple - think carefully and keep your head.*)
Take the first case (i.e. horizontal rope) and make a force diagram. Since there is no acceleration the forces must balance (key concept). Vertically we have $N_0 = mg$ while horizontally $T_0 = F_0 = \mu N_0 = \mu mg$. Again for the inclined rope case we resolve forces horizontally and vertically:

$$T_\theta \cos\theta = \mu N_\theta \qquad N_\theta + T_\theta \sin\theta = mg.$$

Combining these to eliminate $\underline{N}_\theta$ we can find T_θ, and hence deduce that the tension changes by a factor $1/(\cos\theta + \mu \sin\theta)$.

4. A 100 W tungsten lamp has a luminous efficiency of $\sim 2\%$. How many visible photons of average wavelength 550 nm does such a lamp emit per second?

Given that the melting point of tungsten is 3695 K explain briefly why the luminous efficiency is so small. **[5]**

Comments:
Dividing 100 J by the photon energy hc/λ and multiplying by the efficiency we get 5.5×10^{18} photons per second.
We know that the peak of the black-body emission spectrum occurs when the photon energy is approximately equal to $k_b T$. Evaluating $k_b T$ for tungsten near to its melting point we arrive at a wavelength of several microns, i.e. photons in the IR region, where most of the energy of an incandescent bulb is released.

5. A point light source is placed a distance h below the surface of a liquid with refractive index n. Show that the light emerges into the air above the liquid's surface through a circle of diameter d, where

$$d = \frac{2h}{\sqrt{n^2 - 1}}.$$

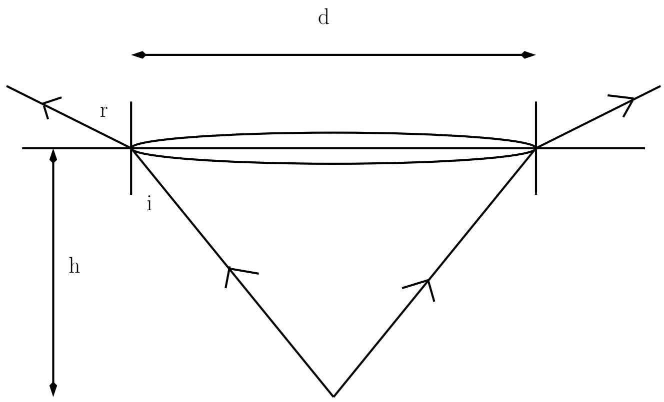
[5]

Comments:

When light crosses a boundary between two media we use Snell's law to figure out how the direction of propagation changes:

$$\frac{\sin i}{\sin r} = \frac{1}{n}$$

where we have taken the refractive index of the air to be 1. *(If you can't remember a formula, at least state its name and say briefly what it is about.)* The maximum angle of refraction is 90 degrees, so we can use the above expression to obtain the incidence angle beyond which light suffers total internal reflection. Knowing this critical angle we deduce d by geometry/trigonometry.



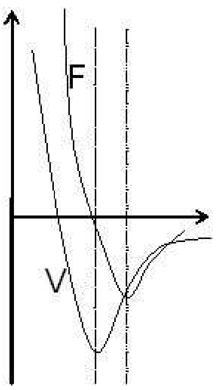
6. The interaction energy of two Kr atoms has the form

$$V = 4V_0\left[\left(\frac{\ell}{r}\right)^{12} - \left(\frac{\ell}{r}\right)^{6}\right]$$

where r is their separation and V_0 and ℓ are constants. Sketch the variation of interatomic *force* with r and comment briefly on the physical significance of the main features of your sketch. [5]

Comments:

Key idea: $F = -dV/dr$.
The force vanishes at the equilibrium position and is linear nearby in accordance with Hooke's law.
Strong repulsion is observed at low r where matter becomes incompresible due to Pauli repulsion.
There is a maximum attractive force. Application of a greater applied force would break a molecule apart.
(Don't overlook the "physical significance" instruction - do what the question asks.)



7. To summon my dog, I blow a whistle that has a frequency of 500 Hz. My dog runs towards me with a speed of $10\,\mathrm{m\,s^{-1}}$. Unfortunately he sees another dog in the distance and runs past me. What is the change in frequency that the dog perceives upon passing me?

[The speed of sound in air is $330\,\mathrm{m\,s^{-1}}$.] [5]

Comments:
Using the standard Doppler formula the frequency perceived by the dog is

$$f_D = f_0\,(1 \pm v_d/v_s)$$

where v_d and v_s are the speeds of dog and sound, we see that the change in frequency is thus

$$\Delta f = 2f_0\left(\frac{v_d}{v_s}\right) = 2 \times \frac{10}{330} \times 500 \approx 30\,\mathrm{Hz}.$$

(If you couldn't remember the Doppler formula perhaps you could have figured it out: Let T_d be the time between two crests of the sound wave for the dog. In this time he moves a distance v_dT_d. Thus the

distance between crests, i.e. the wavelength, is

$$\lambda = v_s T_d \pm v_d T_d$$

where the + sign applies to the case where the dog runs away from the whistle. Hence

$$f_d = \frac{1}{T_d} = \frac{v_s \pm v_d}{\lambda} = f_0 \frac{v_s \pm v_d}{v_s}$$

the formula we needed.) (Stating without derivation was expected, but some people prefer to remember less and derive more.)

8. In the rest frame of inertial observer A the velocity vector of a photon makes angle θ with the x axis. Find an expression for the corresponding angle, θ', measured by inertial observer B who is moving along the x direction with speed v.

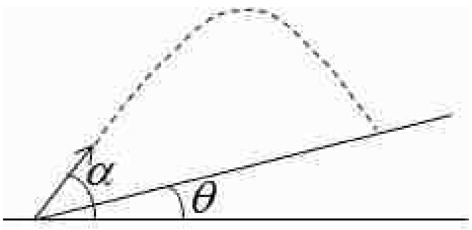
If $\theta = 45^\circ$ and $\theta' = 90^\circ$ find the speed of B in the rest frame of A. [5]

<u>Comments</u>:
The x-component velocity transformation is:

$$c' \cos\theta' = \frac{c\cos\theta - v}{1 - \cos\theta\, v/c}.$$

Setting $c' = c$ and rearranging leads to $v/c = 0.7$.
(Again, if you couldn't remember the velocity transformation you should at least give the Lorentz transformation. You may then see how to derive the velocity formula.)

9. A particle is fired with velocity $\underline{u}$ at angle α to the horizontal, as illustrated below.



Find expressions for the horizontal and vertical co-ordinates of the particle at time t.

The particle lands on a plane inclined at angle θ to the horizontal. Express d, the horizontal distance travelled by the particle before it lands, in terms of α, θ, u and g (the acceleration due to gravity). [5]

<u>Comments</u>:
The co-ordinates of the particle are:

$$x = ut\cos\alpha \qquad y = ut\sin\alpha - \frac{1}{2}gt^2.$$

It hits the plane when $y = x\tan\theta$. Substituting for x and y from the first equation gives the time at which the particle hits the plane. This value can be put back into our x equation to obtain d.

10. Show that the energy of a free particle with mass m and speed v is

$$E = mc^2 + \frac{1}{2}mv^2$$

in the non-relativistic limit.

The uranium isotope ${}_{92}\mathrm{U}^{238}$ undergoes α decay to thorium, ${}_{90}\mathrm{Th}^{234}$. Using the non-relativistic result above, find the velocity of the α particle, assuming that the uranium nucleus decays from rest. [5]

[The masses of ${}_{92}\mathrm{U}^{238}$, ${}_{90}\mathrm{Th}^{234}$ and the α particle are 238.0508 a.m.u., 234.0436 a.m.u. and 4.0026 a.m.u. respectively.]

Comments:
The relativistic energy is γmc^2. Making a binomial expansion for γ and keeping only the first order term in v^2/c^2 gives:

$$E = mc^2\left[1-\left(\frac{v}{c}\right)^2\right]^{-\frac{1}{2}} \approx mc^2\left[1+\frac{1}{2}\left(\frac{v}{c}\right)^2\right] = mc^2 + \frac{1}{2}mv^2.$$

Using the above result we have energy conservation

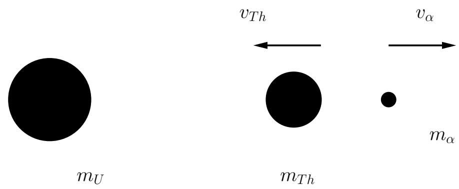
$$m_U c^2 = m_{Th}c^2 + \frac{1}{2}m_{Th}v_{Th}^2 + m_\alpha c^2 + \frac{1}{2}m_\alpha v_\alpha^2$$

together with momentum conservation

$$m_{Th}v_{Th} = m_\alpha v_\alpha$$

Thus $v_{Th} = m_\alpha / m_{Th} v_\alpha$ which gives

$$v_\alpha = c\sqrt{2\,\frac{m_{Th}}{m_\alpha}\,\frac{m_U - m_{Th} - m_\alpha}{m_{Th}+m_\alpha}}$$

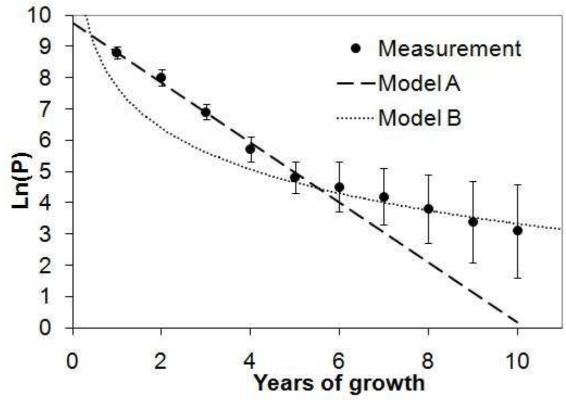


Note that the approximation we are instructed to use is accurate to order v^2/c^2. Using the exact formula for the relativistic momentum, γmv, would include terms to all orders. For a consistent approach we are entitled to neglect terms of order v^3 and higher. That means using the Newtonian momentum formula. Putting in numbers (and note we do not need to know what an atomic mass unit is) gives $v_\alpha/c \sim 0.0475$.

In last year's exam Questions 2, 7 and 8 were found to be slightly easier than average (as defined by their average score) while Questions 1, 3 and 4 were done slighty less well than average.

*****************BSc QUESTIONS*****************

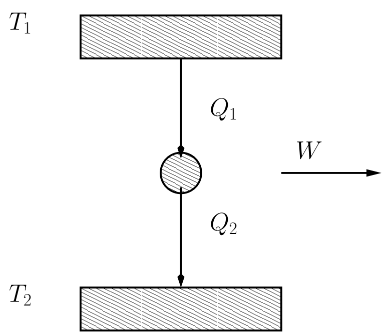
1. A bio-fuels company is evaluating the yield, Y from a new strain of algae after a number of years of growth. The graph below shows their measured data, plotted as the natural logarithm of Y against number of years of growth. Two researchers have proposed theoretical models of the data, plotted below as dashed and dotted curves. State whether the company should adopt either of these models, carefully explaining your reasoning. [5]



Comments:
Superficially, model B appears to match the shape of the data. While the model intersects two thirds of the error bars, however, a number of the remaining data points lie many error bars away from the fit (key concept), making the model statistically inconsistent with the experiment. And since the y-axis is logarithmic, the model is very wrong in the high-yield (and therefore more important) region. Therefore model B should be rejected.
Model A also intersects two thirds of the error bars, but is also less than 2 sigma away from the remaining data points. It also works very well in the high-yield regime. It should be preferred.

2. In the car showroom a car salesman tries to sell me a used car with an engine that he claims is 70% efficient. Upon questioning him I am told that the hottest part of the engine operates at 500° C. Show that the salesman is not telling the truth. [5]

Comments:



The most efficient cycle is the Carnot cycle, so the car engine cannot be better than this. For a Carnot cycle there is no change in entropy (key concept) so,

$$\Delta S = S_2 - S_1 = \frac{Q_2}{T_2} - \frac{Q_1}{T_1} = 0$$

giving

$$\eta = \frac{Q_1 - Q_2}{Q_1} = \frac{T_1 - T_2}{T_1} = 1 - \frac{T_2}{T_1}$$

(with $W = Q_1 - Q_2$). What value of T_2 is implied? And is this value reasonable?

3. *Estimate* the mass of snow on the roof of a typical 3 bedroom house in Edinburgh at the height of the recent bad weather. [5]

Comments:
A good physicist can make reasonable estimates. For roof area and snow thickness let's take $A \sim 100$ m^2, $t \sim 0.4$ m, giving a volume of 40 m^3.
The density of water at s.t.p. is 10^3 kg m^{-3} (a good physicist remembers a few basic numbers such as this), and that of ice is a little lower, say 900 kg m^{-3}. However snow comprises ice crystals which are far from close packed. Its density varies from one type of snow to another, but 200 kg m^{-3} is a reasonable estimate.
The mass follows some multiplication. (*The answer is perhaps higher than one might expect. If you are*

worried by the answer, say so - knowing when smelling a rat is a skill. Also check your units.)

*****************MPhys4 QUESTIONS*****************

1. Show that the scalar product of two 4-vectors is Lorentz invariant. [5]

Comments:
We write the scalar product:

$$a.b = -a^0 b^0 + a^1 b^1 + a^2 b^2 + a^3 b^3.$$

Lorentz transform the a and b vectors and repeat.

2. At temperature T, one mole of ideal gas has a total mean energy $E = \frac{3}{2}RT$ and a constant volume heat capacity $C_V = \frac{3}{2}R$, where R is the ideal gas constant. Estimate the typical fractional fluctuation in the energy, $\overline{(\Delta E)^2}^{1/2}/\overline{E}$, at a temperature of $T = 300$ K. [5]

Comments:

$$\overline{(\Delta E)^2} = kT^2 C_V \quad \Rightarrow \quad \frac{\overline{(\Delta E)^2}^{1/2}}{\overline{E}} = \frac{k^{1/2} T C_V^{1/2}}{\overline{E}} = \frac{k^{1/2} T (\frac{3}{2})^{1/2} N_A^{1/2} k^{1/2}}{\frac{3}{2} N_A k T} = \frac{1}{\left(\frac{3}{2} N_A\right)^{1/2}} \sim 10^{-12}.$$

3. The population of Edinburgh is almost 500000. *Estimate* how many teachers are teaching in Edinburgh in primary and secondary schools.

Considering only the greatest source of uncertainty, calculate the error on your estimate. [5]

Comments:
Fraction of population in school can be estimated as the ratio of the number of years in school to average life expectancy, which gives $12/70 = 0.17$. This ignores child mortality and birth rate fluctuations, immigration etc. An error estimate of (25% error) seems reasonable. Using this estimate, the number of school children is $\sim 85,000$. Children per class is 25 ± 5 (20% error) implying 3,400 teachers.
Largest error is 25% so we estimate $3.4 \pm 0.8 \times 10^3$ teachers.

*****************PHYS/CP QUESTIONS*****************

1. A pipe is inserted into a hole drilled straight through the Earth, connecting Edinburgh to an island off the coast of New Zealand. Show that the gravitational field of the Earth causes an object dropped into the pipe to execute simple harmonic motion, and calculate its period. [5]

[You may assume the Earth is a sphere of uniform density with mass $M_e = 6.0 \times 10^{24}$ kg and radius $r_e = 6.4 \times 10^6$ m.]

Comments:
The gravitational force is

$$F(r) = -\frac{GM(r)m}{r^2}$$

where $M(r)$ is the Earth's mass that lies within radius r. Assuming uniform density, this mass scales with the volume within radius r, and hence with r^3. If M_e is the total mass of the Earth, we deduce

$$M(r) = M_e \frac{r^3}{r_E^3}.$$

Remember that SHM means $\ddot{x} = -x\omega^2$, where ω is the angular frequency of the oscillations (equal to 2π divided by the period).

2. State Heisenberg's position-momentum uncertainty relation.

Estimate the minimum kinetic energy in MeV of a nucleon (with mass $m \approx 1.7 \times 10^{-27}\,\mathrm{kg}$) in a nucleus of radius $R \approx 10^{-15}\,\mathrm{m}$. **[5]**

Comments:
The Heisenberg uncertainty principle states that

$$\Delta x\, \Delta p_x \geq \frac{1}{2}\hbar\,.$$

In the case at hand $\Delta x \sim \Delta y \sim \Delta z \sim 2R$ roughly. The minimum kinetic energy that the nucleon can have is when there is only "zero point motion", i.e. $p_x^{min} \sim \Delta p_x \geq \frac{1}{2}\frac{\hbar}{\Delta x}$ etc. This gives T^{min}.

3. A farmer wishes to measure the width of a road. He releases all the chickens in a flock of 100 simultaneously from one side of the road. Chickens are known to cross roads directly at an average speed of $1\ \mathrm{m\,s^{-1}}$ and standard deviation of $0.2\ \mathrm{m\,s^{-1}}$. After 5 seconds the experiment is abruptly ended by the sudden arrival of a large lorry, at which point only 17 chickens have crossed the road.

Estimate the width of the road, and the error on your estimate. **[5]**

Comments:
(First, don't be put off by questions that sound strange.) 17% of chickens have speed more than one standard deviation from the mean. Thus the last crossing bird can be assumed to move at 1.2 m/s. Therefore the road is $5 \times 1.2 = 6$ m across.
Then we need the relationship between the standard deviation of the speed distribution and the standard error of the mean. (There are 17 independent measurements.)

*****************ASTRO 5 QUESTIONS*****************

1. How brightly does the full moon illuminate the night sky? Express your answer in terms of the equivalent electric wattage of a light bulb at a distance of 2 m, assuming a standard 100 W light bulb produces a radiant luminosity of 2 W. **[5]**

[The radius of the moon is 1.74×10^6 m, the separation of sun and moon is 1.5×10^{11} m, and the separation of Earth and moon is 3.8×10^8 m. The luminosity of the sun is 3.8×10^{26} W.]

Comments:
Key concept: the flux density F at distance D from a point source of luminosity L is $L/(4\pi D^2)$. So the flux density at moon's location is

$$F_{sm} = \frac{L_\odot}{4\pi D_{sm}^2}$$

where D_{sm} is the separation between sun and moon. *(If you can't get far on a question, at least mention your understanding of quantities given - in this case the luminosity.)* The reflected power from the full moon is then

$$L_m = F_{sm}\pi R_m^2.$$

The moon re-radiates into 2π steradians, so the flux density at the Earth of light reflected from the moon is

$$F_m = \frac{L_m}{2\pi D_{me}^2}.$$

Substituting from above we find $F_m = 0.014\ \mathrm{W\,m^{-2}}$.
Putting it all together, and repeating for the easier case of the light bulb at 2 m, you should find that the full moon is equivalent to a 35 W electric bulb at 2 m.

2. A ball is dropped from the edge of a platform on a perfectly vertical tower 30 m high situated on the Earth's equator. Neglecting air resistance and assuming it is a perfectly calm day, how far from the base of the tower will the ball land, and in which direction on the compass? (Neglect the width of the platform.) [5]

Comments:
At $t = 0$ the top of the tower is travelling at speed $(r_E + h)\omega$ where ω is the angular velocity of the rotating Earth. The ground below has speed $r_E\omega$. Thus the ball starts with horizontal velocity $h\omega$ (directed East), relative to the ground. The time taken for the ball to fall a distance h is $t = \sqrt{2h/g}$. During this time the ball also travels East.

3. The ionising radiation from an O star impinges on the surface of a large hydrogen cloud in the interstellar medium. If F_{inc} is the incident flux of ionising photons per second per area on the surface of the cloud, show that the cloud will re-emit ionising photons with flux $F_{\mathrm{re-emit}} \simeq (1/2)F_{\mathrm{inc}}$ leaving its surface. In this way, the cloud acts as a 'half-silvered' mirror. [5]

Comments:
Consider ionisation balance in a box of area A and depth l. No. of incident photons = $F_{inc}A$ = No of recombinations = $e_e n_H \alpha_{rec} A l$ so

$$F_{inc} = n_e n_H \alpha_{rec} l.$$

No of diffuse photons generated per unit time = $n_e n_H \alpha_{rec} l A$. These leave through layer with surface area $2A$ (top and bottom).

**************MATHEMATICAL PHYSICS QUESTIONS**************

1. During the time interval $0 \leq t \leq 2T$ a clock has velocity

$$v = c\sin\left[\frac{\pi(t-T)}{4T}\right].$$

It leaves the origin at time at time $t = 0$, returning there at time $t = 2T$. What time is displayed on the clock upon its return to the origin? [5]

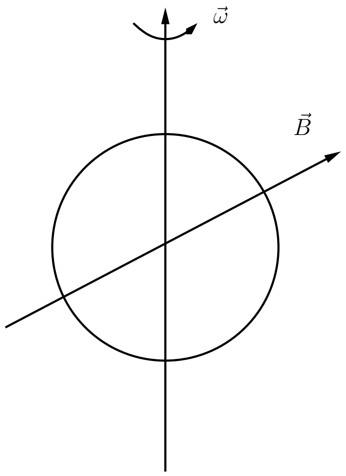
Comments:
Key concept: the moving clock appears to be running slowly by a factor γ due to time dilation:

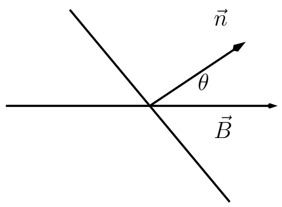
$$\tau = \int_0^{2T} \frac{dt}{\gamma} = \int_0^{2T} \sqrt{1 - v^2/c^2}\,dt.$$

2. A metal ring of radius R rotates with constant angular velocity ω about a diameter. Perpendicular to the rotation axis is a constant magnetic induction field $\vec{B}$. Find the EMF induced in the ring as a function of time.

If the ring has conductivity σ, what current flows around it if one assumes that the current is uniformly distributed over the ring cross-section A? [5]

Comments:





[looking down]

Let $\theta = \omega t$. Then

$$\Phi = \int d\vec{S} \cdot \vec{B} = B\pi R^2 \cos \omega t \,,$$

giving [2]

$$\mathcal{E}_{ind} = -\frac{\partial}{\partial t}\Phi = B\pi R^2 \omega \sin \omega t$$

Furthermore [1]

$$\mathcal{E}_{ind} = \oint_{ring} \vec{E} \cdot d\vec{r} = \frac{1}{\sigma} \oint_{ring} \vec{j} \cdot d\vec{r} \,.$$

$\vec{j}$ is in the $\vec{e}_\theta$ direction.

3. State Bohr's angular momentum quantisation condition.

A non-relativistic particle moves in a circular orbit under the influence of an attractive force

$$F(r) = -kr \qquad k > 0 \,.$$

Assuming the Bohr quantisation condition, show that

$$v^2 = \frac{n\hbar}{m}\sqrt{\frac{k}{m}} \,, \qquad r^2 = \frac{n\hbar}{k}\sqrt{\frac{k}{m}} \,,$$

where m is the mass of the particle and n is an integer. Hence show that the total energy of the particle is

$$E_n = n\hbar\sqrt{\frac{k}{m}} \,.$$

[5]

Comments:
The Bohr quantisation condition is $L = n\hbar$ where L is angular momentum and n is an integer. We have

$$kr = \frac{mv^2}{r} \,, \qquad L = rmv = n\hbar \,.$$

Two equation with two unknowns $(v,\, r)$. Combining them gives the required results.
To get the potential energy, remember that $F = -\partial U/\partial r = -kr$.