Diffraction Physics – Interference-III

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Anti-reflection coatings-I

Need e_1 and e_0 be of opposite sign, so $g_1 = \pi$, giving that

$$n_1\, t_1 = \frac{\lambda}{4} \quad \text{so that} \quad t_1 = \frac{\lambda}{4\, n_1}$$

optical path-length of *quarter of a wavelength* , and also we must have that

$$r_1 = r_0 \quad \text{so that} \quad \frac{1-n_1}{1+n_1} = \frac{n_1 - n_0}{n_1 + n_0}$$

which has the solution that

$$n_1 = \sqrt{n_0}$$

so giving us both the thickness and the refractive index of the coating for *one* particular wavelength. Important derivation
This looks simple, but *Lowest* refractive index is $\mathrm{Mg\,F_2}$ with $n = 1.38$. Most glasses are $n_0 \approx 1.5 \rightarrow 1.7$, so a problem!

High Reflective Coatings

Complementary problem of trying to make a *high reflectivity*, alternate layers of *high* and *low* index. The amplitude reflection at to the n_h/n_l and n_l/n_h boundaries,

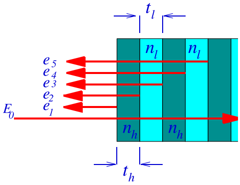
$$r_{h,l} = \frac{n_h - n_l}{n_h + n_l} \quad \text{and} \quad r_{l,h} = \frac{n_l - n_h}{n_l + n_h}$$

so if we write $r_{h,l} = r$, then

$$r_{l,h} = -r \quad \text{or alternatively} \quad r_{l,h} = r\ \exp(i\pi)$$

The light travels through each layer *twice*, to the phase shift is

$$g_l = \frac{4\,\pi\,t_l\,n_l}{\lambda} \quad \text{and} \quad g_h = \frac{4\,\pi\,t_h\,n_h}{\lambda}$$



High Reflective Coatings I

If we illuminate with a beam of amplitude E_0, then ,

$$
\begin{aligned}
e_2 &= E_0\, r\, \exp(i\, g_h) \\
e_3 &= E_0\, r\, \exp\left(i\left(\pi + g_l + g_h\right)\right) \\
e_4 &= E_0\, r\, \exp\left(i\left(2g_h + g_l\right)\right) \\
e_5 &= E_0\, r\, \exp\left(i\left(\pi + 2g_h + 2g_l\right)\right)
\end{aligned}
$$

We want all of these to be *in phase*, which we can obtain by setting

$$g_l = g_h = \pi$$

so we get *quarter wave* thicknesses with,

$$t_l = \frac{\lambda}{4\, n_l} \quad \text{and} \quad t_h = \frac{\lambda}{4\, n_h}$$

e_1 refection from the front air/n_h interface, will also add *in phase*.
If all reflections *in phase*, then as the number of layers increase then $r_t \rightarrow -1$.

High Reflective Coatings II

It *can be shown* , that for a

$$[\text{air}]\ (n_h \,|\, n_l)^m\ [\text{glass}]$$

structure with m double layers, the amplitude reflectivity,

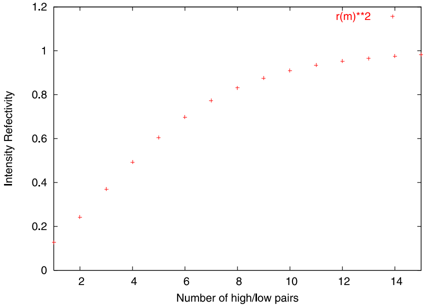
$$r_t = \frac{1 - n_0\left(\frac{n_h}{n_l}\right)^{2m}}{1 + n_0\left(\frac{n_h}{n_l}\right)^{2m}}$$

where n_0 is the refractive index of the glass.
$r_t \rightarrow -1$ independent of the values for n_h, n_l or n_0.
The most common n_l and n_h materials are:

	n
MgF_2	1.38
Al_2O_3	1.62

High Reflective Coatings III

The intensity reflectance for $n_l = 1.38$, $n_h = 1.63$ and $n_0 = 1.51$



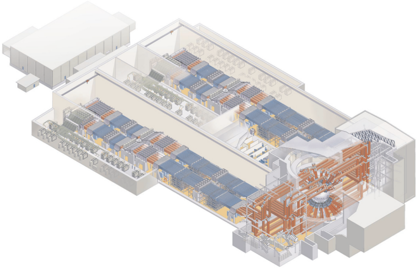
If this film is designed to be used in the visible region, then it will typically gave high reflectivity over $\approx \pm 20\,\mathrm{nm}$.

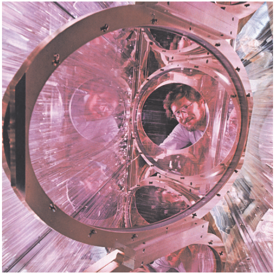
Applications

- Useful for high power mirrors A silvered mirror works by the light inducing currents in the metal. These dissipate heat and in terrawatt applications the heat would explode the mirror.
- Because the reflectivity can be tuned and is much higher than for a silvered mirror, multilayer coatings a very useful for the Fabry-Perot which we will meet in the next lecture.

National ignition facility

Uses 500 terrawatt pulses to compress and 'ingnite' fusion.

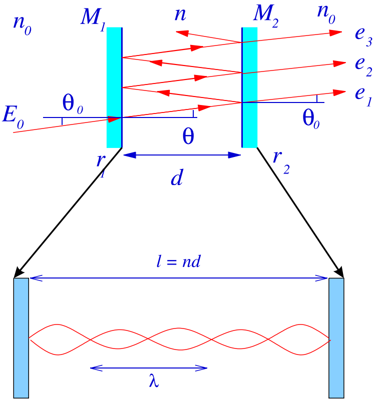




Fabry-Perot Interferometer

Fabry-Perot consists of two parallel flat mirrors with a medium of refractive index n

Rays make multiple bounces within the cavity. The reflectivity of the surfaces and the need to fit a whole number of half wavelengths into the cavity make this a wavelength selector.



Model for the Fabry Perot

Intensity reflectance are R_1 and R_2 then, *intensity* transmittances are (assuming no losses),

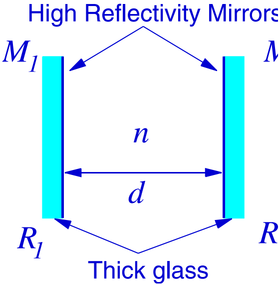
$$T_1 = 1 - R_1 \quad \text{and} \quad T_2 = 1 - R_2$$

We will take the *amplitude* reflectance (of the $\vec{E}$ field)to be

$$r_1 = \sqrt{R_1} \quad \text{and} \quad r_2 = \sqrt{R_2}$$

we ignore $\pm\pi$ phase shifts, these will cancel in the final result.
Similarly we have the *amplitude* transmittance of the mirrors being

$$t_1 = \sqrt{T_1} \quad \text{and} \quad t_2 = \sqrt{T_1}$$



Fabry-Perot Cavity

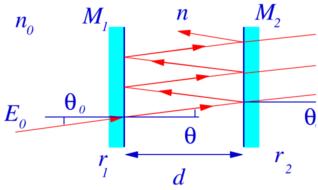
Illuminate with a collimated beam of amplitude E_0 so intensity

$$I_0 = \frac{1}{2}|E_0|^2$$

Inside the cavity the beam direction is θ where,

$$n_0 \sin\theta_0 = n \sin\theta$$

In most applications $n_0 = n = 1$. Cavity is an air gap.



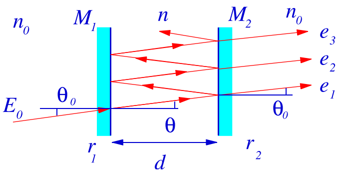
Fabry-Perot Cavity-I

High reflectivity mirrors we get a series of transmitted beams, the first being

$$e_1 = E_0\, t_1\, t_2 \exp(i\phi)$$

where ϕ is the phase delay introduced by the mirrors and one transit of the cavity.
Dielectric block of thickness d and refractive index n, phase introduced by one cavity *round trip* is

$$\delta = \frac{4\,\pi\, d\, n}{\lambda} \cos\theta$$



Fabry-Perot Cavity-II

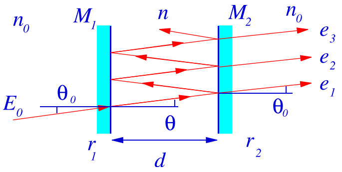
The second transmitted beam is,

$$e_2 = E_0\, t_1\, t_2 \exp(i\phi)\, r_1\, r_2\, \exp(i\delta) = e_1\, r_1\, r_2\, \exp(i\delta)$$

We can them repeat this for other beams, giving that

$$e_{m+1} = e_m\, r_1\, r_2 \exp(i\delta)$$

where e_m is the complex amplitude of the m^{th} beam.
The output complex amplitude will be a sum of these components, so can be written as



$$A = E_0\, t_1\, t_2\, \exp(i\phi) \left[1 + r_1 r_2\, \exp(i\delta) + (r_1 r_2\, \exp(i\delta))^2 + \cdots\right]$$

Fabry Perot Cavity-III

$$A = E_0\, t_1\, t_2\, \exp(i\phi) \left[1 + r_1 r_2\, \exp(i\delta) + (r_1 r_2\, \exp(i\delta))^2 + \cdots \right]$$

Term inside [] is a geometric series, which sums to,

$$A = E_0\, t_1\, t_2\, \exp(i\phi)\, \frac{1}{1 - r_1 r_1\, \exp(i\delta)}$$

The transmitted intensity is therefore

$$I = \frac{1}{2}|A|^2 = I_0\; T_1\, T_2 \frac{1}{1 - 2\, r_1 r_2 \cos\delta + (r_1 r_2)^2}$$

which *with a bit of work* , we can express in terms of r_1 and r_2 only, also using the identity that $2\sin^2\delta/2 = 1 - \cos\delta$, to get that

$$I = I_0 \frac{(1 - r_1^2)(1 - r_2^2)}{(1 - r_1 r_2)^2 + 4\, r_1\, r_2 \sin^2\left(\frac{\delta}{2}\right)}$$

Fabry-Perot Cavity-III

Special case of both the mirrors have the same *intensity* reflectance R, so

$$r_1^2 = r_1^2 = r_1 r_2 = R$$

we finally get the more manageable formula that,

$$I = I_0 \frac{(1-R)^2}{(1-R)^2 + 4\,R\,\sin^2\left(\frac{\delta}{2}\right)}$$

The expression is clearly a *maximum* with $I_{\max} = I_0$ which is independent of R, when,

$$\delta = \frac{4\pi\, n\, d}{\lambda} \cos\theta = 2m\,\pi$$

Fabry-Perot Cavity-IV

$$I = I_0 \frac{(1-R)^2}{(1-R)^2 + 4\,R\,\sin^2\left(\frac{\delta}{2}\right)}$$

Has a *minimum* of

$$I_{\min} = I_0 \frac{(1-R)^2}{(1+R)^2} \quad \text{when} \quad \delta = \frac{4\pi\,n\,d}{\lambda}\cos\theta = 2(m+1)\pi$$

so $I_{\min} \to 0$ as $R \to 1$, which is the condition for bright and dark fringes.

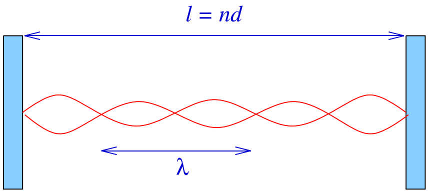
The contrast between bright and dark is tuned by the reflectivity of the surfaces

Wavelength Selection in Fabry-Perot

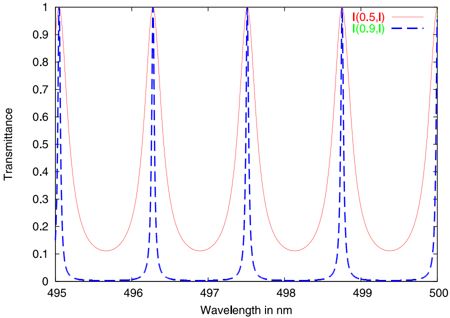
If the beam contains a range of wavelengths, $\theta = 0$, so transmission when,

$$\lambda_m = \frac{2\, n\, d}{m}$$

integer number of half-wavelength in the cavity



so expect a series of *peaks* at λ_m, The full output in plotted with $d = 100\,\mu\mathrm{m}$, for two mirrors with $R = 0.5$ and $R = 0.9$.



Series of transmission peaks with, very sharp, with little transmission between. Note the scale!.

The Fabry-Perot is thus operating as a very narrow band pass-filter.

Wavelength Selection in Fabry-Perot II

Consider frequency rather than wavelength. Transmission is a series of peaks at frequency

$$\nu_m = \frac{m\,c}{2\,n\,d}$$

thus the separation between the peaks is then constant, being

$$\Delta\nu_p = \frac{c}{2\,n\,d}$$

so the larger *nd* is, the *closer* the peaks are together.
See workshop question for an expression for the width of each peak.
Since we can control the spacing of the transmitted peaks by changing plate separation and their width by changing the reflectivity we have a tunable filter

Applications of the Fabry-Perot

$$\nu_m = \frac{m\,c}{2\,n\,d}$$

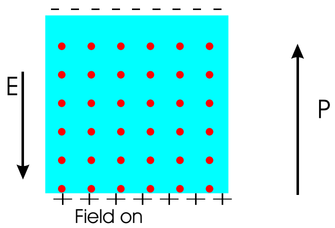
Fabry-Perot with large $d \approx \mathrm{cms}$, and high reflectivity mirrors $R > 98\%$, gives an extremely good descrimination between wavelengths

A high resolution spectrometer capable of splitting spectral lines, see

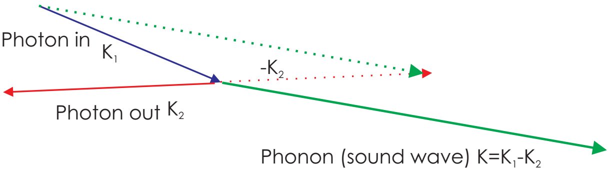
1. fine and hyper-fine structure content
2. Zeeman effect (Splitting of spectral lines by external magnetic field).
3. Brillouin Spectroscopy

Brillouin Spectroscopy

Brillouin Spectroscopy aims to measure the energy (and hence the velocity) of sound waves in solids and liquids.
Recall that the electric field polarises a solid
If the polarisability of the solid changes with density (and it usually does) then sound waves (density oscillations) can couple to light (electric field oscillations).

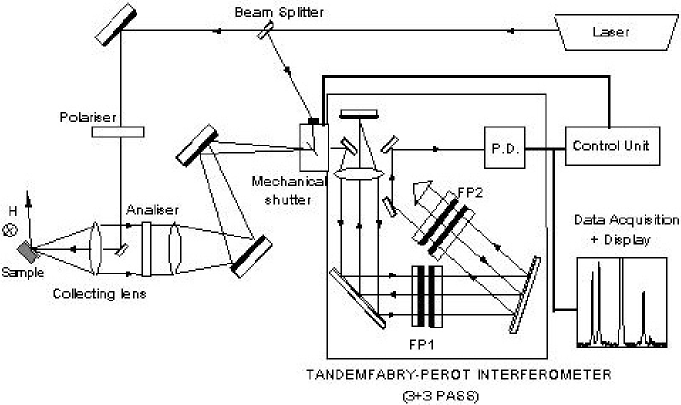


Conservation Rules

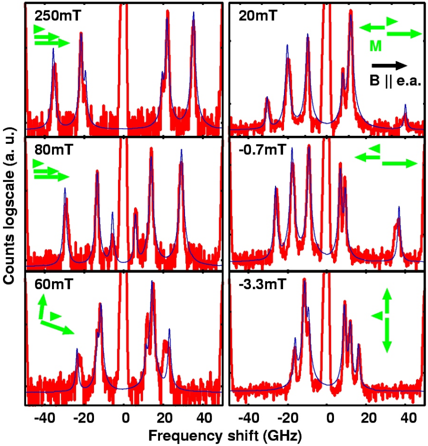


Have to conserve both momentum (wavevector) $\vec{k}_1 = \vec{k}_2 + \vec{K}$ and energy $E_{photonin} = E_{photonout} + E_{phonon}$
Problem is that photon energies are $\sim 1eV$ and phonon energies are $\sim 10^{-3}eV$.
The scattered light will be very close in energy to the exciting (input) line.

Practical Brillouin Spectrometer



Brillouin Spectra



Why Measure Sound Velocities this Way

It is generally much easier to measure velocities by the transit method.

Hit one end of a bar and see how long sound takes to travel to the other end

Use Brillouin when you can easily do this.

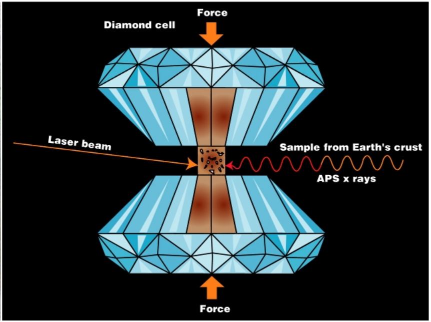
For example in a pressure cell.

Generating Pressure

The way to generate pressure is to squeeze the sample between two hard things

55 bc

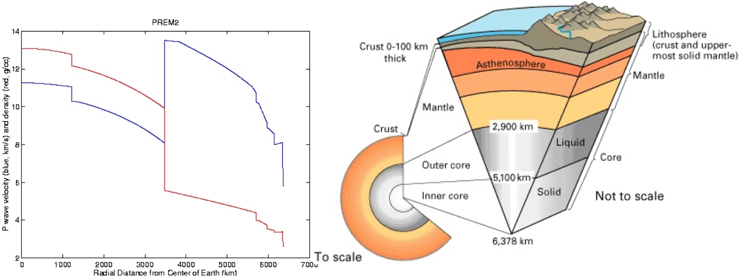
The diamond anvil cell



Diamond anvil cell can achieve pressures of 3 Mbar (same as the centre of the Earth)

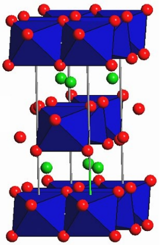
The Earth

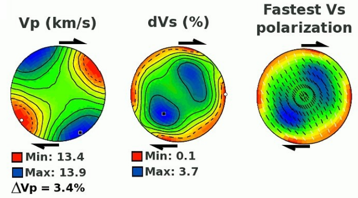
We know very little for sure about the interior of the Earth. We know its density and the velocity of sound.

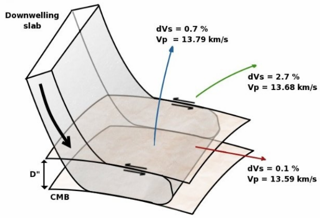


Measurements of the velocity of sound in various minerals (rocks) using DACs and Brillouin Spectroscopy can tell us what is below us.

The Earth-I







Contribution of silicate p-Pv to seismic anisotropy in D" after 20% deformation in shear

Key Points

- By tuning the separation of the Fabry-Perot we can tune the wavelength of the light it passes
- By tuning the reflectivity we can tune the range of wavelengths that are passed

The End

Here endeth Interference.

- Interference between two coherent waves is

$$2\sqrt{I_1 I_2}(\cos\delta)$$

- Identifying the phase difference δ is the key to evaluating interference

Key Points

- Interfaces give partial reflection which leads to interference.
- The reflectivity of an raw interface is determined by the properties of the materials.
- We can use interference and multiple interfaces to tune the raw reflectivity.