UNIVERSITY OF MORATUWA, SRI LANKA
Faculty of Engineering
Department of Electronic and Telecommunication Engineering
Semester 4 (Intake 2020)

BM2102 Analysis of physiological systems
Assignment 2
Branched Cylinders: Dendritic Tree Approximations

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Question 1

The Membrane Potential of each branch at steady state satisfies the equation,

$$\frac{d^2V}{dX^2} = V$$

The general solutions are,

$$V_1(X) = A_1e^{-x} + B_1e^{x} \qquad 0 \leq X \leq L_1$$
$$V_{21}(X) = A_{21}e^{-x} + B_{21}e^{x} \qquad L_1 \leq X \leq L_{21}$$
$$V_{22}(X) = A_{22}e^{-x} + B_{22}e^{x} \qquad L_1 \leq X \leq L_{22}$$

Considering boundary condition 1 - Applied current at beginning of parent cylinder,

$$V_1(X) = A_1e^{-x} + B_1e^{x} \qquad 0 \leq X \leq L_1$$

Differentiating w.r.t. X and Substituting X=0,

$$\frac{dV_1}{dX} = -A_1e^{-x} + B_1e^{x}$$
$$-(r_i\lambda_c)_1I_{app} = -A_1 + B_1$$
$$(r_i\lambda_c)_1I_{app} = A_1 - B_1$$

Considering the remaining 2 boundary conditions – Daughter terminals held at rest,
When X = L_{21}

$$V_{21}(X) = A_{21}e^{-x} + B_{21}e^{x} \qquad L_1 \leq X \leq L_{21}$$
$$V_{21}(L_{21}) = A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0$$
$$A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} = 0$$

When X = L_{22}

$$V_{22}(X) = A_{22}e^{-x} + B_{22}e^{x} \qquad L_1 \leq X \leq L_{22}$$
$$V_{22}(L_{22}) = A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$
$$A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$

Considering Continuity of Membrane Potential at Nodes,

$$V_1(L_1) = V_{21}(L_1)$$
$$A_1e^{-L_1} + B_1e^{L_1} = A_{21}e^{-L_1} + B_{21}e^{L_1}$$
$$A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0$$

And

$$V_{21}(L_1) = V_{22}(L_1)$$
$$A_{21}e^{-L_1} + B_{21}e^{L_1} = A_{22}e^{-L_1} + B_{22}e^{L_1}$$
$$A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0$$

By Conservation of Current,

$$\frac{-1}{(r_i\lambda_c)_1}(-A_1e^{-L_1} + B_1e^{L_1}) = \frac{-1}{(r_i\lambda_c)_{21}}(-A_{21}e^{-L_1} + B_{21}e^{L_1}) + \frac{-1}{(r_i\lambda_c)_{22}}(-A_{22}e^{-L_1} + B_{22}e^{L_1})$$
$$\frac{-Ae^{-L_1}}{(r_i\lambda_c)_1} + \frac{B_1e^{L_1}}{(r_i\lambda_c)_1} + \frac{A_{21}e^{-L_1}}{(r_i\lambda_c)_{21}} - \frac{B_{21}e^{L_1}}{(r_i\lambda_c)_{21}} + \frac{A_{22}e^{-L_1}}{(r_i\lambda_c)_{22}} - \frac{-B_{22}e^{L_1}}{(r_i\lambda_c)_{22}} = 0$$

Equations in (7) verified.

Question 2

$$Ax = B$$

$$\begin{pmatrix} 1 & -1 & 0 & 0 & 0 & 0 \\ 0 & 0 & e^{-L_{21}} & e^{L_{21}} & 0 & 0 \\ 0 & 0 & 0 & 0 & e^{-L_{22}} & e^{L_{22}} \\ e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} & 0 & 0 \\ 0 & 0 & e^{-L_1} & e^{L_1} & -e^{-L_1} & -e^{L_1} \\ -e^{-L_1}/(r_i\lambda_c)_1 & e^{L_1}/(r_i\lambda_c)_1 & e^{-L_1}/(r_i\lambda_c)_{21} & -e^{L_1}/(r_i\lambda_c)_{21} & e^{-L_1}/(r_i\lambda_c)_{22} & -e^{L_1}/(r_i\lambda_c)_{22} \end{pmatrix} \begin{pmatrix} A_1 \\ B_1 \\ A_{21} \\ B_{21} \\ A_{22} \\ B_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} A_1 - B_1 \\ A_{21}e^{-L_{21}} + B_{21}e^{L_{21}} \\ A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} \\ A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} \\ A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} \\ -e^{-L_1}A_1/(r_i\lambda_c)_1 + e^{L_1}B_1/(r_i\lambda_c)_1 + e^{-L_1}A_{21}/(r_i\lambda_c)_{21} - e^{L_1}B_{21}/(r_i\lambda_c)_{21} + e^{-L_1}A_{22}/(r_i\lambda_c)_{22} - e^{L_1}B_{22}/(r_i\lambda_c)_{22} \end{pmatrix} = \begin{pmatrix} (r_i\lambda_c)_1 I_{app} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Comparing each element of 2 matrices,

$$A_{22}e^{-L_{22}} + B_{22}e^{L_{22}} = 0$$
$$A_1e^{-L_1} + B_1e^{L_1} - A_{21}e^{-L_1} - B_{21}e^{L_1} = 0$$
$$A_{21}e^{-L_1} + B_{21}e^{L_1} - A_{22}e^{-L_1} - B_{22}e^{L_1} = 0$$
$$-e^{-L_1}A_1/(r_i\lambda_c)_1 + e^{L_1}B_1/(r_i\lambda_c)_1 + e^{-L_1}A_{21}/(r_i\lambda_c)_{21} + -e^{L_1}B_{21}/(r_i\lambda_c)_{21} + e^{-L_1}A_{22}/(r_i\lambda_c)_{22} + -e^{L_1}B_{22}/(r_i\lambda_c)_{22} = 0$$

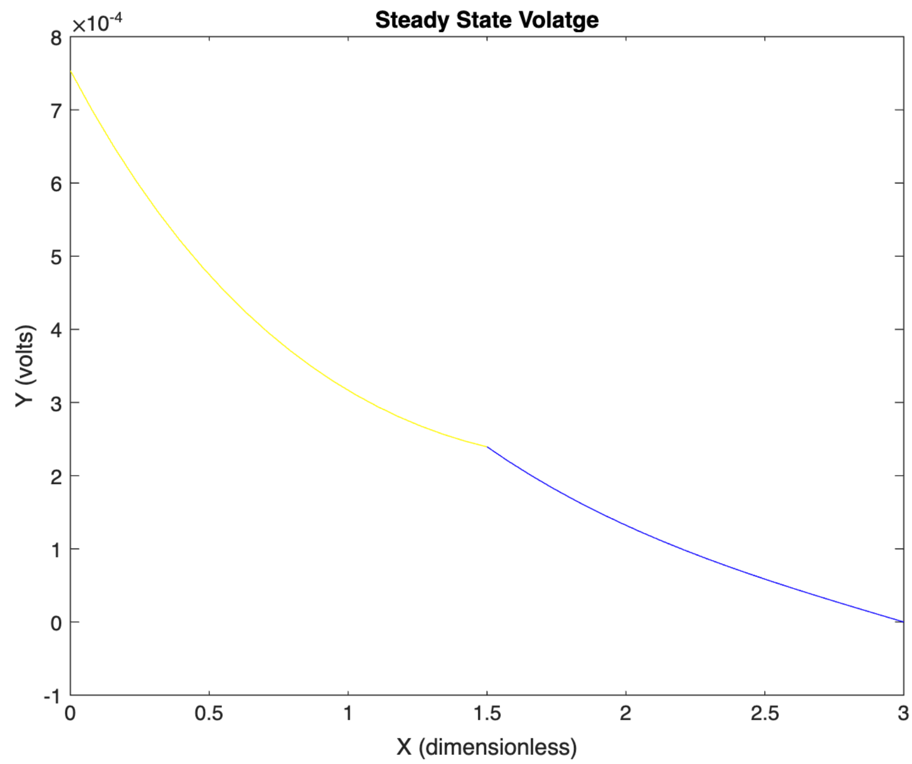
The equations are same as those in (7).

Question 3

From Matrix X,

A1 = 0.0007	A21 = 0.0011	A22 = 0.0011
B1 = 0	B21 = 0	B22 = 0

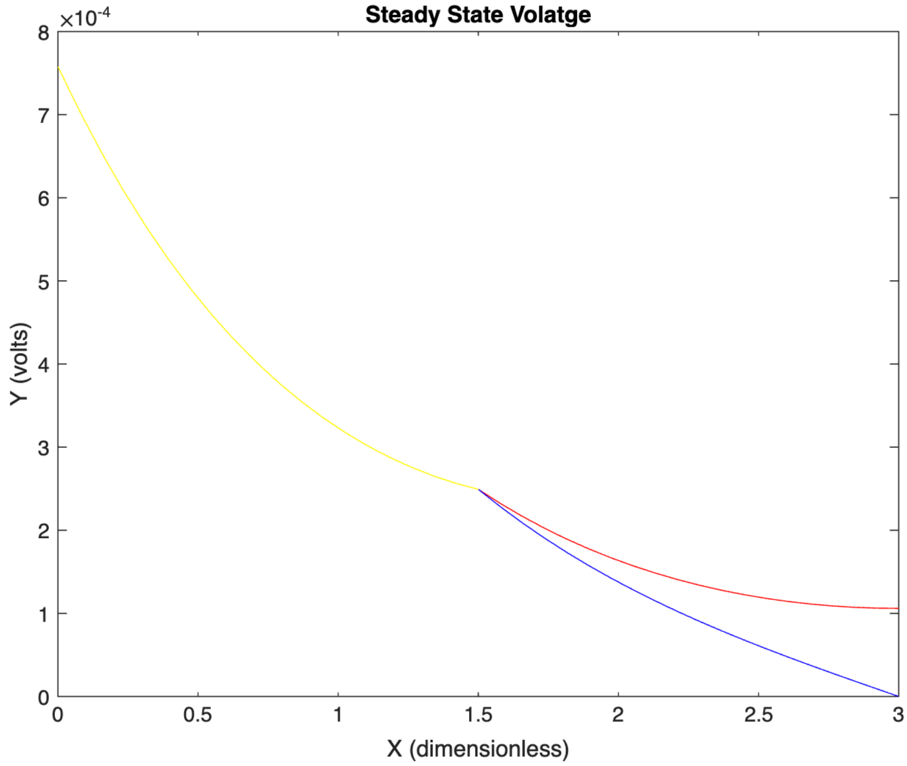
Question 4



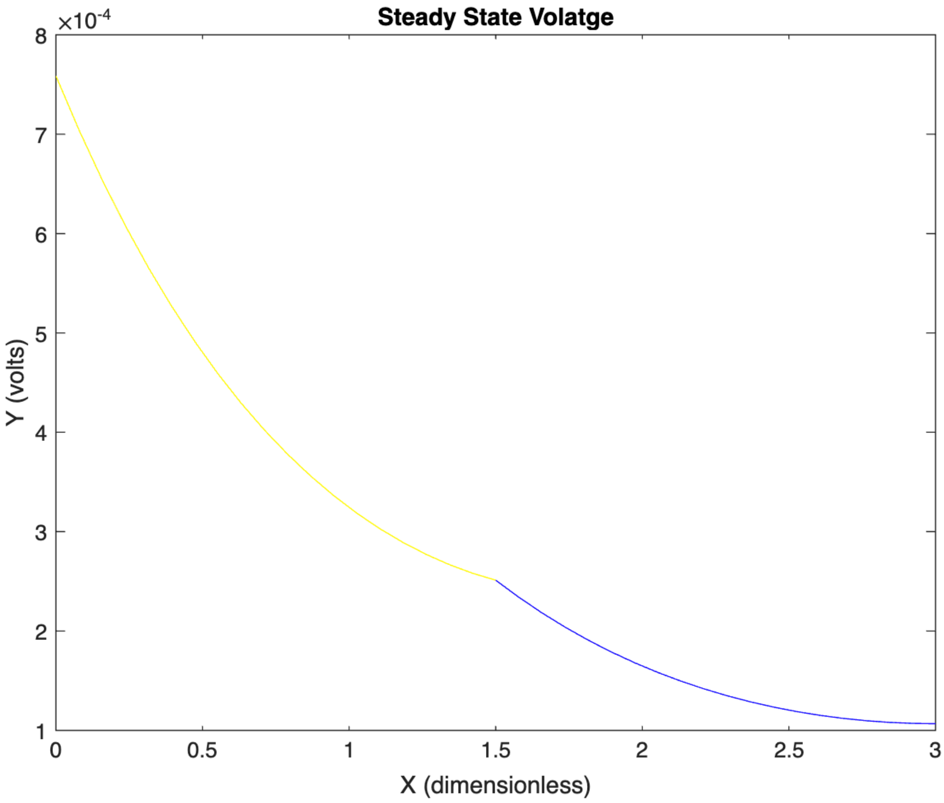
In the code, the steady state voltage profile of the two daughter branches was plotted using red and blue lines. However, due to the overlapping of the blue line over the red line, the red line is not visible. This indicates that the steady state voltage profiles of the two daughter branches coincide and are therefore equal.

Steady State Voltage Profiles for different boundary conditions

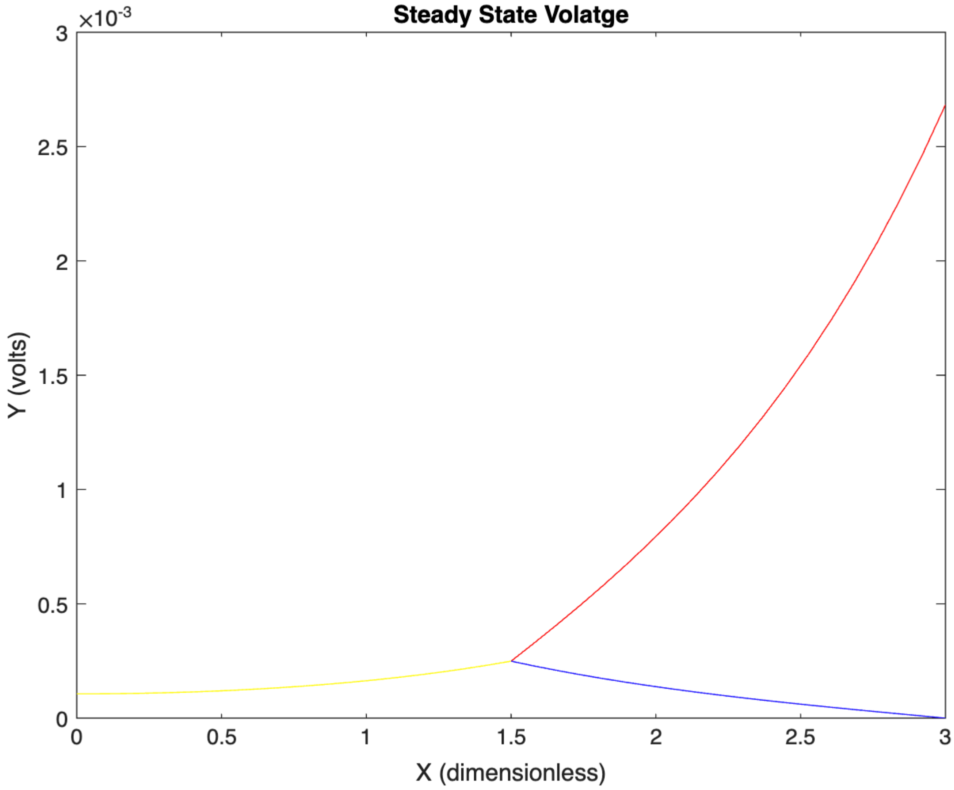
2(a)



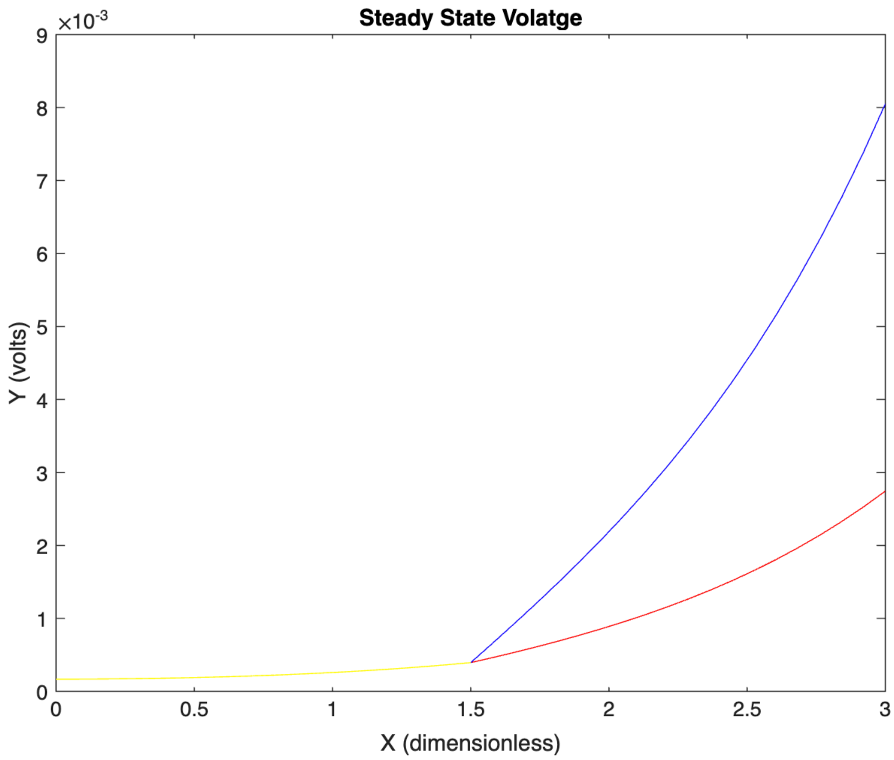
2(b)



2(c)



2(d)



Question 5

In the leftmost node of the parent cell, an electrical impulse is received, resulting in a negative voltage gradient. Conversely, in the rightmost nodes of the two daughter branches, the electrical impulse is transmitted to neighboring neurons or branches, resulting in a positive voltage gradient.

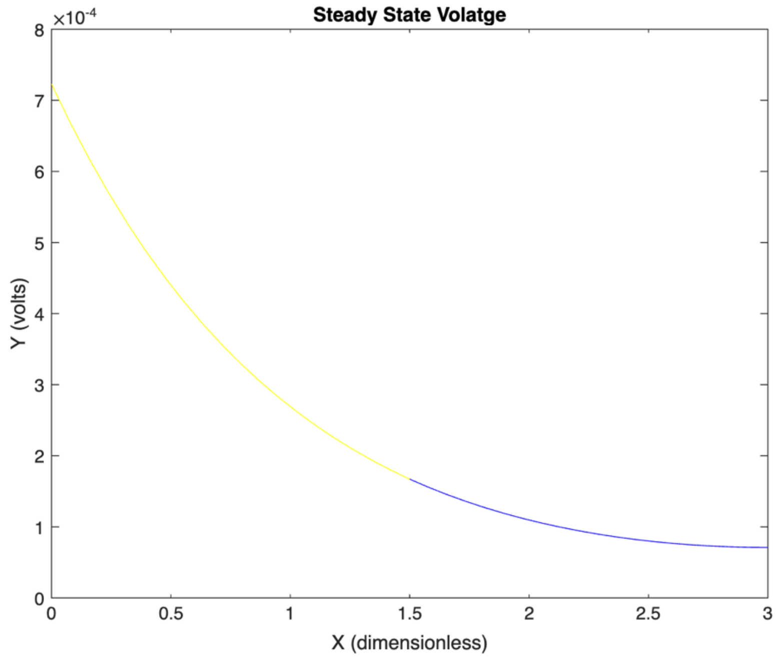
Question 6

Recalculated Coefficients for $d_{21} = d_{22} = 47.2470 \times 10^{-4}$ cm,

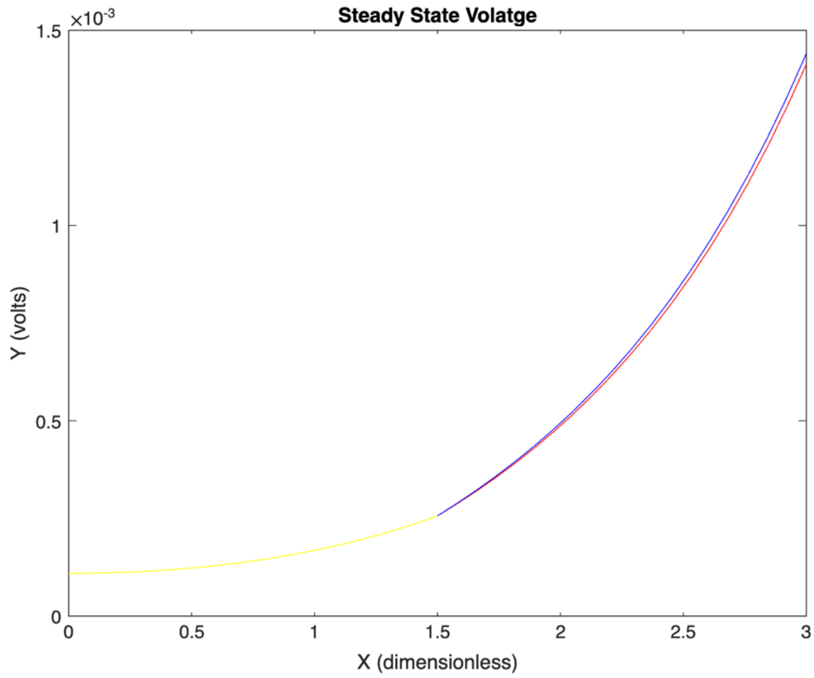
From Matrix X,

$A1 = 0.7216 \times 10^{-3}$	$A21 = 0.7132 \times 10^{-3}$	$A22 = 0.7132 \times 10^{-3}$
$B1 = 0.0013 \times 10^{-3}$	$B21 = 0.0018 \times 10^{-3}$	$B22 = 0.0018 \times 10^{-3}$

2(b)



2(d)



The updated graphs exhibit continuous differentiability, unlike the previous graphs. Additionally, in graph 2(d), the Steady State Voltages of the two daughter branches are nearly identical due to their equal diameters.