Karnaugh map - (K - Map) -

→ Provides a systematic method for simplifying Boolean expression and if properly used, will provide/ produce the simplest SOP or POS expression possible, Known as the minimum expression.

→ Gray code representation is used in K-map.

→ Two, Three and four variable K-map.

A \ B	0	1
0	$\bar{A}\bar{B}$ 0	$\bar{A}B$ 1
1	$A\bar{B}$ 2	AB 3

A \ BC	00	01	11	10
0	0	1	3	2
1	4	5	7	6

AB \ CD	00	01	11	10
00	0	1	3	2
01	4	5	7	6
11	12	13	15	14
10	8	9	11	10

SOP Minimization :-

→ 1 is placed on the K-map. for each product term in the expression.

Ex- $\bar{A}\bar{B}C + \bar{A}B\bar{C} + AB\bar{C} + ABC$

ex- $\bar{A}\bar{B}CD + \bar{A}B\bar{C}\bar{D} + AB\bar{C}D + ABCD + AB\bar{C}\bar{D} + \bar{A}\bar{B}\bar{C}D + A\bar{B}C\bar{D}$

K-map simplification of SOP expression :-

Three steps.

(1)- Grouping the 1's

(2)- determining the product term for each group

(3)- summing the resulting product terms.

→ A group must contain either 1, 2, 4, 8, 16, cells, which are all powers of two.

→ In the case of 3-variable map $2^3 = 8$ cells is the maximum group.

→ Each cell in a group must be adjacent to one or more cells in the same group. But all cells in the group do not have to be adjacent to each other.

→ Always include the largest possible no. of 1's in a gp.

SOP example -

① - $f(A,B) = \Sigma m(0,1,3)$

A \ B	0	1
0	1	1
1		1

$f(A,B) = \bar{A} + B$

② - $f(A,B) = \Sigma m(0,2,3)$

③ - $f(A,B) = \Sigma m(0,1,2,3)$

A \ B	0	1
0	1	1
1	1	1

$f = 1$

④ - $f(A,B,C) = \Sigma m(0,1,3,5,7) = C + \bar{A}\bar{B}$

⑤ - $f(A,B,C) = \Sigma m(1,3,6,7) = \bar{A}C + AB$

A \ BC	00	01	11	10
0		1	1	
1			1	1

BC is redundant term.

(6) - $f(A,B,C) = \Sigma m\,(0,2,3,6,7) = B + \bar{A}\bar{C}$

(7) -

$A \backslash BC$	00	01	11	10
0	1	1		
1		1	1	1

$AB + \bar{A}\bar{B} + AC$

or

$AB + \bar{A}\bar{B} + \bar{B}C$

NOTE :- K-map will provide minimized logical expⁿ But not necessarily unique.

(8) - $f(A,B,C,D) = \Sigma m(0,1,2,3,5,7,8,9,11,13,15)$

$= D + \bar{A}\bar{B} + \bar{B}\bar{C}$

Problems with don't care :

→ In SOP don't care (X) is used then it is = 1

→ " " not " " = 0

(9) - $f(A,B) = \Sigma m(1,3) + \Sigma d(2)$

$A \backslash B$	0	1
0		1
1	X	1

$f(A,B) = B$

(10) - $f(A,B) = \Sigma m(0,3) + \Sigma d(2)$

$A \backslash B$	0	1
0	1	
1	X	1

$f = A + \bar{B}$

(11) -

$A \backslash B$	0	1
0	1	X
1	1	X

$f = 1$

(12) -

A \ BC	00	01	11	10
0	1	1	X	
1		X	1	1

$\bar{A}\bar{B}$ C   AB

C ↳ Redundant

$f(A,B) = \bar{A}B + AB$

(13) -

A \ BC	00	01	11	10
0	1	1	X	X
1		X	1	1

$f = \bar{A} + B$

(14) -

A \ BC	00	01	11	10
0	1	1		1
1	1	X	X	1

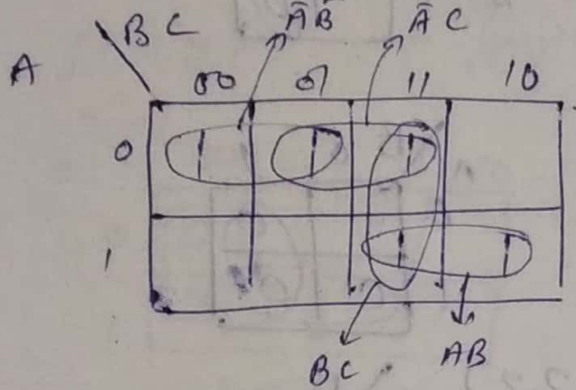
$= \bar{B} + \bar{C}$

A → Redundant

→ Implicant

→ Prime implicant

→ Essential Prime implicant



$\bar{A}\bar{B} + AB + \bar{A}C$

$\bar{A}\bar{B} + AB + BC$

Common in Both so EPI

→ each product term or min term is known as implicant = 5

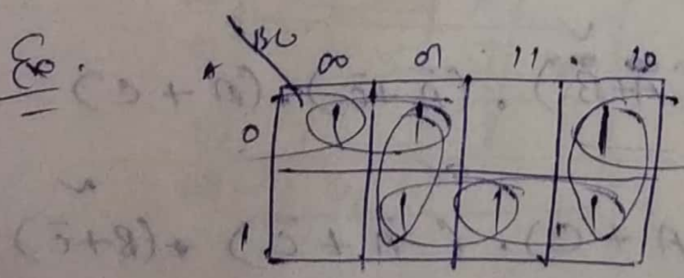
→ Prime implicants is obtained by combining maximum possible adjacent ones -

$\boxed{PI = 4}$ $\bar{A}\bar{B}$, $\bar{A}C$, BC, AB

→ EPI is a PI which is obtain by combining only one way and there is no alternative way.

$\boxed{EPI = 2}$ $\bar{A}\bar{B}$, AB

Ex:

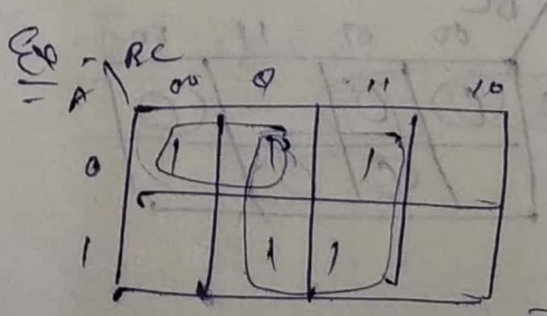


I = 6

PI = 6

EPI = 0

Ex



$C + \bar{A}\bar{B}$

I = 5

PI = 2

EPI = 2

POS minimization :-

① $f(A,B) = \Pi m(0,1,3)$

A \ B	0 (B)	1 ($\bar{B}$)
0 (A)	0	0
1 ($\bar{A}$)	1	0

$= A \cdot \bar{B}$

② $f(A,B) = \Pi m(1,2,3)$

A \ B	0	1
0		0
1	0	0

$= \bar{A} \cdot \bar{B}$

③ $f(A,B) = \Pi m(0,1,2,3)$

A \ B	0	1
0	0	0
1	0	0

$f = 0$

④ $f(A,B) = \Pi m(0,2) + \Pi d(1)$

A \ B	0	1
0	0	X
1	0	

$= B$

⑤

A \ B	0	1
0	0	X
1	1	0

$= A \cdot \bar{B}$

⑥ $f(A,B,C) = \Pi m(0,1,2,5,7)$

A \ BC	00	01	11	10
0	0	0		0
1		0	0	

$= (\bar{A}+\bar{B}) \cdot (\bar{A}+\bar{C}) \cdot (A+C)$ ✓

or

$(A+C) \cdot (\bar{A}+\bar{C}) \cdot (B+\bar{C})$

Q.

A \ BC	$B+C$	$B+\bar{C}$	$\bar{B}+\bar{C}$	$\bar{B}+C$
A	0	0		0
$\bar{A}$	0	0	0	0

$= \bar{A} \cdot B \cdot C$

Q→ $f(A,B,C) = \pi M(0,1,3,5) + \pi d(4,6)$

A \ BC	00	01	11	10
0	0	0	0	
1	X	0		X

$= B \cdot (A+\bar{C})$

$f(A,B,C,D) = \pi M(0,1,3,4,6,7,8,13,15)$

AB \ CD	00	01	11	10
00	0	0	0	
01	0		0	0
11		0	0	
10	0			

$f = (\bar{A}+\bar{B}+\bar{D}) \cdot (A+\bar{C}+\bar{D})$
$(A+\bar{B}+D) \cdot (A+B+C)$
$(B+C+D)$

Q→ $(A,B,C,D) = \pi M(0,1,5,7,9,13,15)$
$+ \pi d(3,6,8)$

AB \ CD	00	01	11	10
00	0	0	X	
01		0	0	X
11		0	0	
10	X	0		

$= (\bar{B}+\bar{D}) \cdot (B+C)$

five variable K-map

$f(A,B,C,D,E) = \Sigma m(0,1,4,8,12,13,15,16,17,23,29,31)$

A = 0

BC \ DE	00	01	11	10
00	1 (0)	1 (1)	(3)	(2)
01	1 (4)	(5)	(7)	(6)
11	1 (12)	1 (13)	1 (15)	(14)
10	1 (8)	(9)	(11)	(10)

A = 1

BC \ DE	00	01	11	10
00	1 (16)	1 (17)	(19)	(18)
01	(20)	(21)	1 (23)	(22)
11	(28)	1 (29)	1 (31)	(30)
10	(24)	(25)	(27)	(26)

$f(A,B,C,D,E) = \bar{B}\bar{C}\bar{D} + BCE + \bar{A}\bar{D}\bar{E} + ACDE$

Q→ $Y = \bar{A}\bar{B}\bar{C}\bar{D}\bar{E} + \bar{A}\bar{B}C\bar{D}\bar{E} + \bar{A}BC\bar{D}\bar{E} + \bar{A}B\bar{C}\bar{D}\bar{E} + A\bar{B}\bar{C}\bar{D}\bar{E}$
$+ A\bar{B}C\bar{D}\bar{E} + ABC\bar{D}\bar{E} + AB\bar{C}\bar{D}\bar{E} + \bar{A}\bar{B}\bar{C}D\bar{E} + \bar{A}\bar{B}CD\bar{E}$
$+ \bar{A}BCD\bar{E} + \bar{A}B\bar{C}D\bar{E} + A\bar{B}\bar{C}D\bar{E} + A\bar{B}CD\bar{E} + ABCD\bar{E}$
$+ AB\bar{C}D\bar{E}$

Sol^n →

BC \ DE	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

A = 0

BC \ DE	00	01	11	10
00	1			1
01	1			1
11	1			1
10	1			1

A = 1

$Y = \bar{E}$

Quine - McCluskey or Tabular Method of minimization of logical function :-

Step - I - A set of all ~~variable~~ prime implicants of the function must be obtained.

II - From the set of all prime implicants, a set of essential prime implicants must be determined by preparing a prime implicant chart.

III - The minterms which are not covered by the essential implicants are taken into consideration and a minimum cover is obtained from the remaining prime implicants.

Selecting Prime Implicants :-

(i) - Each minterm should be expressed by its binary representation.

(ii) - The minterms should be arranged in increasing index (index can be defined as the no. of 1's in a minterm). Separate each set of minterm possesing the same index by lines.

(iii) - Each term of index 'n' should be compared with each term of 'n+1'. If two term differ in only one variable, that variable should be removed and dash (–) placed at that position, thus a new term with one less literal is formed.

(iv) - The next stage of elemination or matching process should be repeated for the new terms. Two terms can be combined only when they have dashes in the same position.

(v) - The cycle have to be contenued untill no new list can be found.

(vi) - All terms which remain unchecked during the process are considered to be P.I..

Prime Implicant Chart:-

(i) - The PI should be represented in Rows and each minterm of the f^n in a Column.

(ii) - Crosses should be placed in each row to show the composition of minterms that makes the PI.

(iii) - A Completed PI table should be inspected for column containing only a single cross. PI that cover minterm with a single cross in their column are called EPI.

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Q→ Find the Minimal Sum of Product for the f^n. (14)

$$f = \Sigma(1, 2, 3, 7, 8, 9, 10, 11, 14, 15)$$

Sol^n →

Minterms	Variables			
	A	B	C	D
1	0	0	0	1
2	0	0	1	0
3	0	0	1	1
7	0	1	1	1
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1
14	1	1	1	0
15	1	1	1	1

No. of 1's	minterm	Variable			
		A	B	C	D
1	1 ✓	0	0	0	1
	2 ✓	0	0	1	0
	8 ✓	1	0	0	0
2	3 ✓	0	0	1	1
	9 ✓	1	0	0	1
	10 ✓	1	0	1	0
3	7 ✓	0	1	1	1
	11 ✓	1	0	1	1
	14 ✓	1	1	1	0
4	15 ✓	1	1	1	1

Combination	Variables A	B	C	D
(1,3) ✓	0	0	–	1
(1,9) ✓	–	0	0	1
(2,3) ✓	0	0	1	–
(2,10) ✓	–	0	1	0
(8,9) ✓	1	0	0	–
(8,10) ✓	1	0	–	0
(3,7) ✓	0	–	1	1
(3,11) ✓	–	0	1	1
(9,11) ✓	1	0	–	1
(10,11) ✓	1	0	1	–
(10,14) ✓	1	–	1	0
(7,15) ✓	–	1	1	1
(11,15) ✓	1	–	1	1
(14,15) ✓	1	1	1	–

Combination	A	B	C	D
(1, 3, 9, 11)	–	0	–	1
(2, 3, 10, 11)	–	0	1	–
(8, 9, 10, 11)	1	0	–	–
(3, 7, 11, 15)	–	–	1	1
(10, 11, 14, 15)	1	–	1	–

Prime Implicant	minterm									
	1	2	3	7	8	9	10	11	14	15
(1,3,9,11)*	X		X			X		X		
(2,3,10,11)*		X	X				X	X		
(8,9,10,11)*					X	X	X	X		
(3,7,11,15)*			X	X				X		X
(10,11,14,15)*							X	X	X	X
	✓	✓		✓	✓				✓	

$$\boxed{f = \bar{B}D + \bar{B}C + A\bar{B} + CD + AC}$$

Q→ $f(w, x, y, z) = \sum(1,3,4,5,9,10,11) + \sum\phi(6,8)$

minterm	Variables			
	w	x	y	z
1	0	0	0	1
3	0	0	1	1
4	0	1	0	0
5	0	1	0	1
6	0	1	1	0
8	1	0	0	0
9	1	0	0	1
10	1	0	1	0
11	1	0	1	1

No. of 1's	minterm	w	x	y	z
1	1 ✓	0	0	0	1
	4 ✓	0	1	0	0
	8 ✓	1	0	0	0
2	3 ✓	0	0	1	1
	5 ✓	0	1	0	1
	6 ✓	0	1	1	0
	9 ✓	1	0	0	1
	10 ✓	1	0	1	0
3	11 ✓	1	0	1	1

Combination	w	x	y	z
(1, 3) ✓	0	0	–	1
(1, 5)	0	–	0	1
(1, 9) ✓	–	0	0	1
(4, 5)	0	1	0	–
(4, 6)	0	1	–	0
(8, 9) ✓	1	0	0	–
(8, 10) ✓	1	0	–	0
(3, 11) ✓	–	0	1	1
(9, 11) ✓	1	0	–	1
(10, 11) ✓	1	0	1	–

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Combinations	w	x	y	z
(1,3,9,11)	–	0	–	1
(8,9,10,11)	1	0	–	–

P.I.	Minterms						
	1	3	4	5	9	10	11
(1, 3, 9, 11)*	X	X			X		X
(8,9,10,11)*					X	X	X
(4,5)*			X	X			
	✓	✓	✓	✓		✓	

$$f = \bar{x} \cdot z + w\bar{x} + \bar{w}x\bar{y}$$