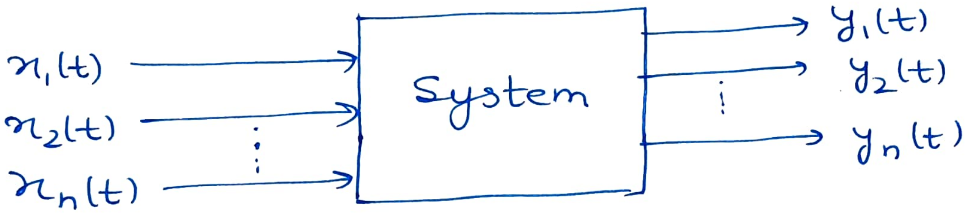
Signals & Systems

Signal : It is defined as a function of one or more variable, which provides information on the nature of a physical phenomenon.

Systems : It's a entity that takes an input signal and produces an output signal.



→ It is a combination and interconnection of several components to perform a desired task.

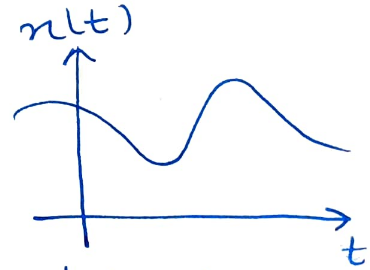
Classification of Signals

1) Continous time and discre time signals →

Continous time Signals

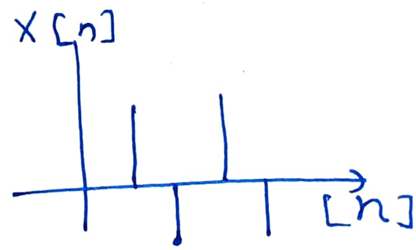
→ defined at each point of time

→ mathematically solved by using differential equation which is further solved by L.T.



Discrete time Signals

→ Defined at particular point of time



→ Mathematically solved by using difference equation which is further solved by Z-transform.

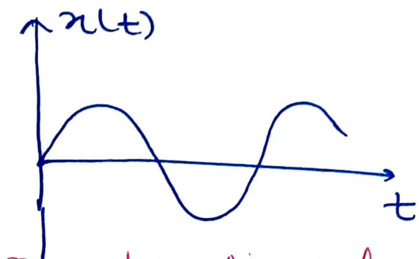
2. Periodic and Non-periodic Signals

Periodic Signals → A Signal $f(t)$ is periodic if $f(t) = f(t \pm nT)$ — ①

$n \rightarrow$ positive integer

T – period of $f(t)$

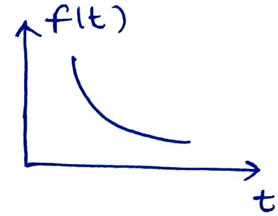
eg.: $\sin t$, $\cos t$



* A periodic signal must start at $t = -\infty$ and continue forever

Non-Periodic Signals – A signal not satisfying eq'n ① is Non-periodic signal.

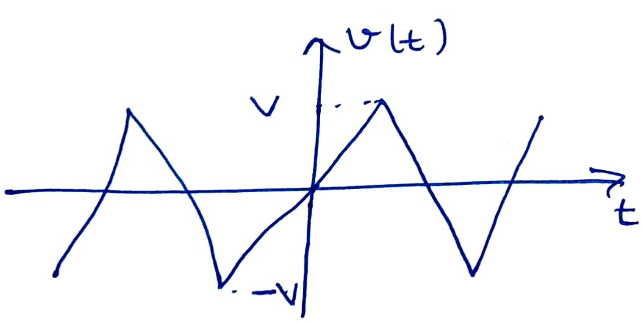
eg.: e^t, t^2, t etc.

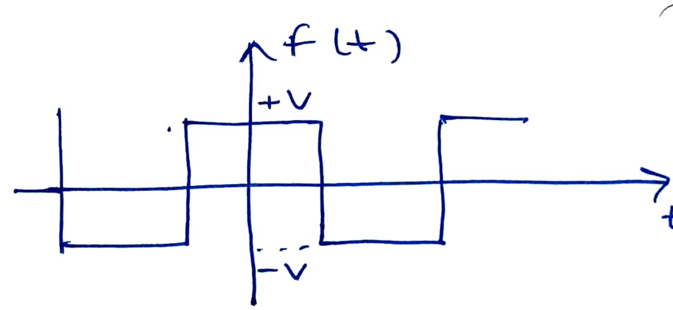


3. Even and odd Signals

Even Signal → A Signal $f(t)$ is said to be even if $f(-t) = f(t)$; eg.: $\cos t$, $\sin^2 t$

otherwise if $f(-t) = -f(t)$; signal is said to be odd Signal. eg.: $\sin t$





Decomposition of a Signal into odd and even Component →

Let $f_o(t)$ = odd component of $f(t)$

$f_e(t)$ = even component of $f(t)$

$$f(t) = f_o(t) + f_e(t) \quad \text{— ②}$$

$$f(-t) = f_o(-t) + f_e(-t) \quad \text{— ③}$$

* Even Signals are symmetrical about the vertical axis or origin whereas odd signals are symmetrical about horizontal axis

On adding eq'n no. (2) & (3)

$$f_e(t) = \frac{1}{2}[f(t) + f(-t)] \quad \text{——} (4)$$

On Subtracting eq'n (3) from eq'n (2)

$$f_o(t) = \frac{1}{2}[f(t) - f(-t)] \quad \text{——} (5)$$

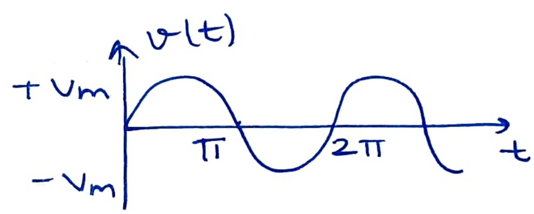
Properties

1> Sum of even function = Even function
2> Sum of odd function = odd function
3> Multiplication of even and even fn = Even fn
4> Multiplication of odd and odd fn. = odd fn
5> Multiplication of even and odd fn = odd fn
6> Sum of even and odd function = neither even nor odd

Standard Signals

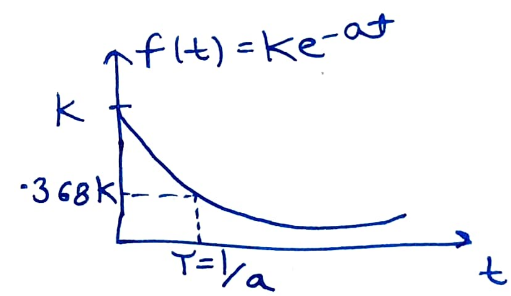
1> Sinusoidal Signals

$$v(t) = V_m \sin\omega t$$



2> Exponential Signal

$$f(t) = 0 \quad ; \quad t < 0$$
$$= Ke^{-at} \quad ; \quad t \geq 0$$



K and a are constant.

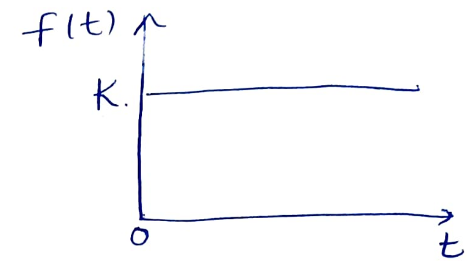
Reciprocal of a has the dimension of time and is known as time constant ($T = 1/a$)

Time Constant is the time taken to reach 63.2% of the total change from initial to final value.

Singularity Signals → can be obtained from one another by successive differences or integration.

1> Step Signal

$$f(t) = \begin{cases} 0 & ; t < 0 \\ k & ; t > 0 \end{cases}$$



If $k = 1$

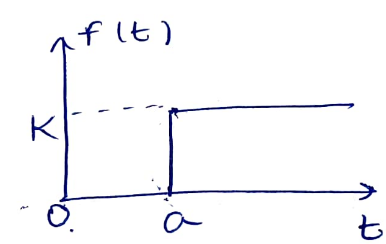
$$u(t) = \begin{cases} 0 & ; t < 0 \\ 1 & ; t > 0 \end{cases}$$

$u(t) \rightarrow$ Unit Step signal

$$f(t-a) = \begin{cases} 0 & ; t < a \\ k & ; t > a \end{cases}$$

↓

Shifted Step Signal



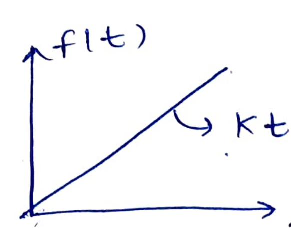
$$u(t-a) = \begin{cases} 0 & ; t < a \\ 1 & ; t > a \end{cases}$$

↓

Unit Shifted Step Signal

2> Ramp Signal

$$f_r(t) = \begin{cases} 0 & ; t < 0 \\ kt & t > 0 \end{cases}$$



If $k = 1$ then Signal is unit Ramp Signal

$$f_r(t-a) = \begin{cases} 0 & ; t < a \\ kt & ; t > a \end{cases}$$

Shifted Ramp signal

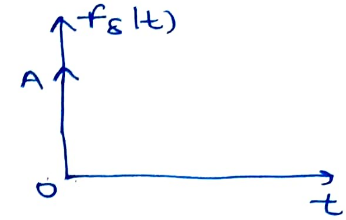
If $k = 1$ then signal is unit shifted ramp signal

3> Impulse Signal.

→ Also known as dirac delta signal

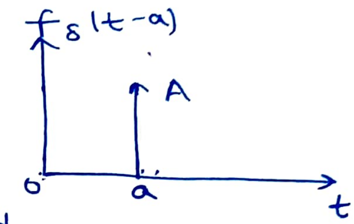
→ Represented by $\delta(t)$

$$f_\delta(t) = \begin{cases} 0 ; & t \neq 0 \\ A ; & t = 0 \end{cases}$$



If $A = 1$ then ~~of~~ it is defined as unit Impulse signal

$$f_\delta(t-a) = \begin{cases} 0 ; & t \neq a \\ A ; & t = a \end{cases}$$



If $A = 1$ then it is defined as shifted unit Impulse Signal.

Area of unit Impulse signal is defined as

$$\text{Area} = \int_{-\infty}^{\infty} \delta(t)\,dt = 1$$

Relationship between Standard Signals

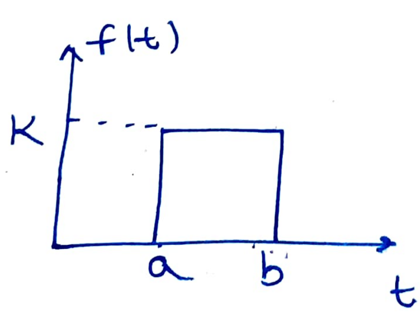
1> Derivative of Step Signal = Impulse Signal

2> Derivative of Ramp Signal = Step Signal

3> Integration of Impulse Signal = Step Signal

4> Integration of Step Signal = Ramp Signal.

Gate Function



$$f(t) = K\,[u(t-a) - u(t-b)]$$

Direct formula or K.M. formula

$$f(t) = \sum_{T=-\infty}^{\infty} (A_f - A_i)\, u(t-T)$$

A_f = final value at the corresponding time instant

A_i = Initial value at the corresponding time instant

T = Time instant at which function changes its value.

Different types of Systems

1> Time-invariant and time varying system. →

A system is time invariant if the behaviour or characteristics of the systems do not change with time otherwise system is time varying.

Time invariant systems are modeled with constant coefficient equations.

$$\left.\begin{aligned} x(t) &\rightarrow y(t) \\ x(t-T) &\rightarrow y(t-T) \end{aligned}\right\} \text{Time Invariant}$$

eg. $y(t) = t\,x(t) + 4$; Is this time invariant?

for input $x_1(t)$

output $y_1(t) = t\,x_1(t) + 4$ — (1)

for input $x_1(t-T)$

output $y_2(t) = t\,x_1(t-T) + 4$ — (2)

from the condition of time invariant

$y_1(t-T) = (t-T)\,x_1(t-T) + 4$ — (3)

$$y_2(t) \neq y_1(t-T)$$

Not time invariant

2> Linear and Non-linear System →

A system is said to be linear if it follow the principal of superposition and homogenity.

Superposition $x_1(t) \rightarrow y_1(t)$

$x_2(t) \rightarrow y_2(t)$

$x_1(t) + x_2(t) \rightarrow y_1(t) + y_2(t)$

Homogenity $x(t) \rightarrow y(t)$

$Kx(t) \rightarrow Ky(t)$

3> Instantaneous (static) and Dynamic Systems →

A system is said to be static if output at specific time depends on the present value of input only. On the other hand a dynamic system is one whose output depends on the past or future value of input in addition to the present time.

eg. Instantaneous System – A circuit containing R

$v(t) = Ri(t)$

Dynamic System – A circuit containing L and C

$$v(t) = L\frac{di(t)}{dt}$$

$$v(t) = \frac{1}{C}\int i(t)\,dt$$

4> Casual and Non-Casual System

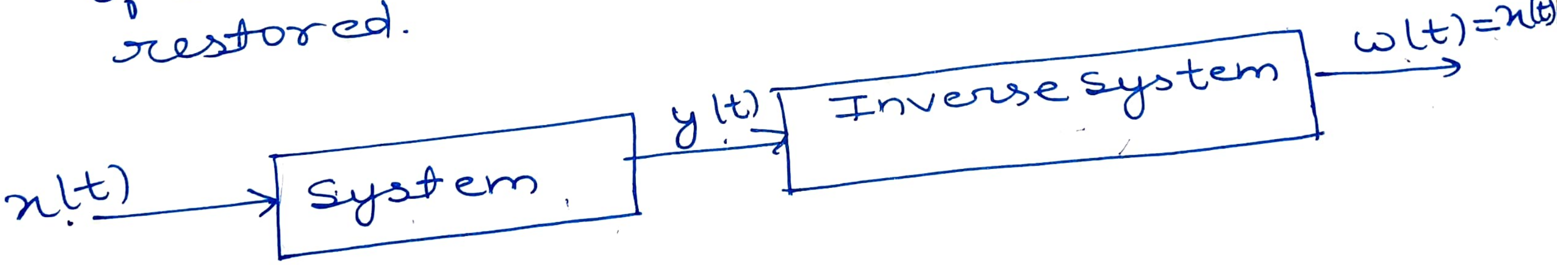
A system is said to be casual if the output of the system depends only on the input at the present time / or in the past, but not the future value of the input. It is also known as non-anticipative.

5> Active and Passive System

6> Stable and Unstable Systems –
A stable is one where the output does
not diverge as long as the input does
not diverge. It is a BIBO system
[Bounded input Bounded output]
otherwise system is Unstable system.

7> Invertible and Non-Invertible Systems-
A System is said to be invertible if
1> Distinct input lead to distinct output
2> The input can be recovered from the
output.

eg. – Design of communication systems
when a transmitted signal propogate
through a communication channel, it
become distorted due to physical char.
of the channel. An equalizer is connected
in cascade with the channel in the
receiver to compensate this distortion.
By designing the equalizer to be inverse
of the channel, the transmitted signal is
restored.



Examples

Eg. 1 Check whether the system $y(t) = \sin[x(t)]$ is time invariant.

Sol. for I/P $x_1(t)$, o/p $y_1(t) = \sin[x_1(t)]$ — ①

for I/P $x_1(t-T)$ o/p $y_2(t) = \sin[x_1(t-T)]$ — ②

from the condition of time invariance

$$y_1(t-T) = \sin[x_1(t-T)] \quad — ③$$

from (2) and (3)

$$y_2(t) = y_1(t-T)$$

Hence system is time invariant.

Eg. 2 Check whether the given systems are linear or not.

(a) $y(t) = x^2(t)$

$y_1(t) = x_1^2(t)$

$y_2(t) = x_2^2(t)$

for an input $\{k_1x_1(t) + k_2x_2(t)\}$

$$y_3(t) = [k_1x_1(t) + k_2x_2(t)]^2 \quad — ①$$

from the condition of linearity, output should be

$$k_1y_1(t) + k_2y_2(t) = k_1x_1^2(t) + k_2x_2^2(t) \quad — ②$$

① $\neq$ ② Non linear

(b) $y(t) = mx(t) + c$

$y_1(t) = mx_1(t) + c$

$y_2(t) = mx_2(t) + c$

for I/P $x_1(t) + x_2(t)$; $y_3(t) = m\{x_1(t) + x_2(t)\} + c$ — ①

from the condition of linearity, output should be

$$y_1(t) + y_2(t) = m\{x_1(t) + x_2(t)\} + 2c \quad — ②$$

① $\neq$ ② Non linear

(c) $y(t) = t\,x(t)$

$y_1(t) = t\,x_1(t)$

$y_2(t) = t\,x_2(t)$

for I/P $\{x_1(t)+x_2(t)\}$ $y_3(t) = t\{x_1(t)+x_2(t)\}$

from the condition of linearity

$y_1(t)+y_2(t) = t\,x_1(t) + t\,x_2(t)$

$= t\{x_1(t)+x_2(t)\}$

Linear System

Eg. 3 Check system is stable or not

(a) $y(t) = t\,x(t)$

as $t \to \infty$ $\quad y(t) \to \infty$

Unstable System

(b) $y(t) = x(t) \sin 100\pi t$

$x(t)$ is multiplied by $\sin 100\pi t$.

Value of sine varies between -1 and 1

$y(t)$ is bounded as long as $x(t)$ is bounded.

Hence System is stable.

Eg. 4 Check systems are casual or non-casua

(a) $y(t) = x(t) \cos(t+1)$

$y(t)$ depends on present input $x(t)$

So system is casual.

(b) $y(t) = x(-t)$

If $t = -3$

$y(-3) = x(3)$

O/P depends on future input. Non Casual

(c) $y(t) = x(2t)$

If $t = 5$

$y(5) = x(10)$

O/P depends on future I/P. Non Casual

(d) $\frac{dy(t)}{dt} + 10y(t) + 5 = x(t)$

$y(t)$ depends on present value of $x(t)$

Casual.

(e) $y(t) = \int_{-\infty}^{\infty} x(t)\,dt$

$y(t)$ depends on present and past value of $x(t)$ but not on future value. Casual

Eg. 5 Check System is Invertible or not

(a) $y(t) = 10x(t)$

Inverse Sys'm will be $w(t) = \frac{1}{10} y(t)$

$$x(t) \longrightarrow \boxed{\text{System}} \xrightarrow{y(t)=10x(t)} \boxed{\text{Inv. System}} \xrightarrow{w(t)=\frac{1}{10}y(t)}$$

Eg. 6 Check systems are static or not.

(a) $y(t) = e^{x(t)}$

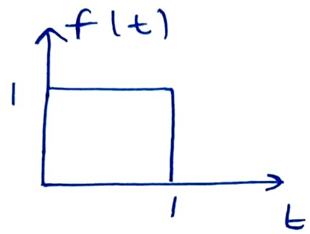
o/p depends on present input only. Static

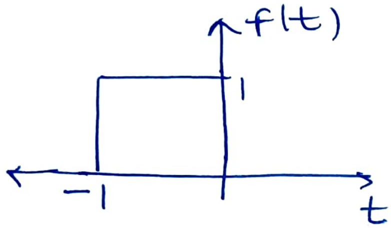
(b) $y(t) = \frac{d}{dt} x(t)$

o/p depends on differentiation of I/P. Calculat of differentiation depends on present as well as past value. Dynamic

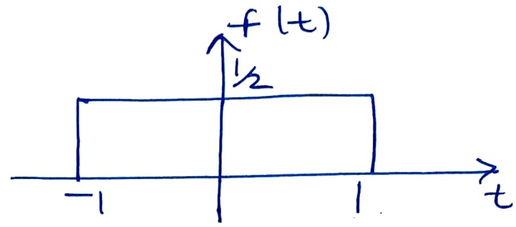
Eg. Determine Even and odd component of the following signals

(a)

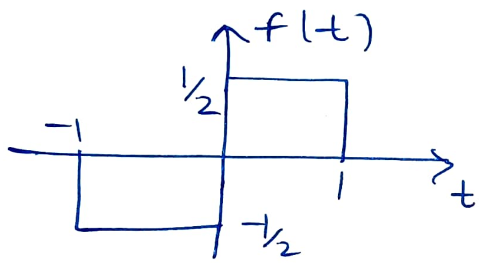




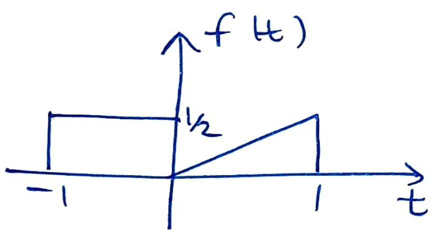
folded Signal

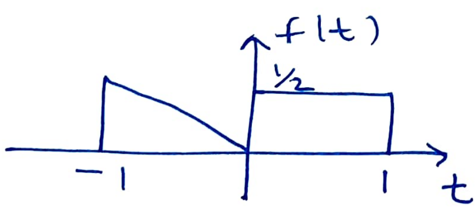


Even Component

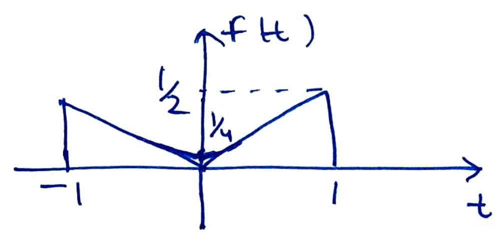


(b)

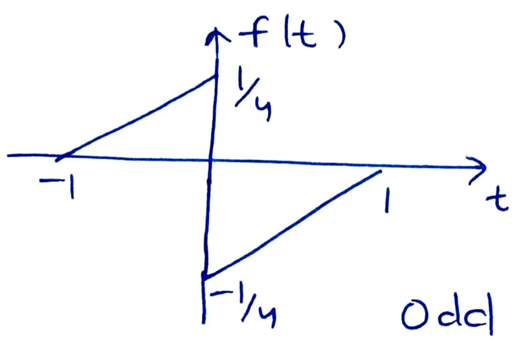




folded Signal

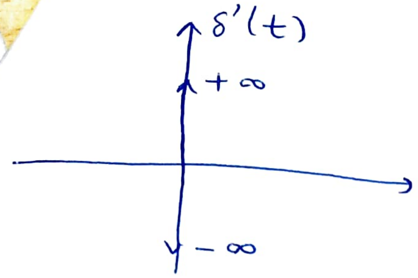


Even Componen

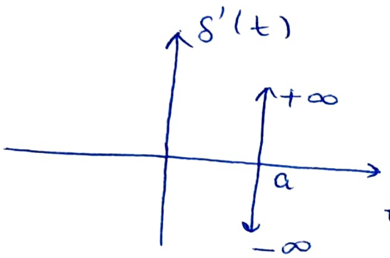


Odd Componen

Unit Doublet Signal



$$\delta'(t) = \frac{d}{dt}[\delta(t)] = +\infty \text{ and } -\infty \quad ; t=0$$
$$= 0 \quad ; t \neq 0$$



$$\delta'(t-a) = +\infty \text{ and } -\infty \quad ; t = a$$
$$\delta'(t) = 0 \quad ; t \neq a$$

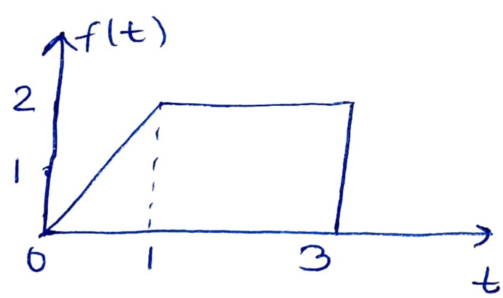
$$\mathcal{L}[\delta'(t)] = L\left[\frac{d}{dt}[\delta(t)]\right] = s\,L[\delta(t)] = s$$
$$\mathcal{L}[\delta'(t-a)] = L\left[\frac{d}{dt}\delta(t-a)\right] = s\,L[\delta(t-a)] = s e^{-as}$$

Derivative of Unit ~~Doublet~~ Impulse Signal = Unit Doublet Signal

Integral of Unit Doublet Signal = Unit Impulse Signal.

Synthesize the Given Waveforms:

1)

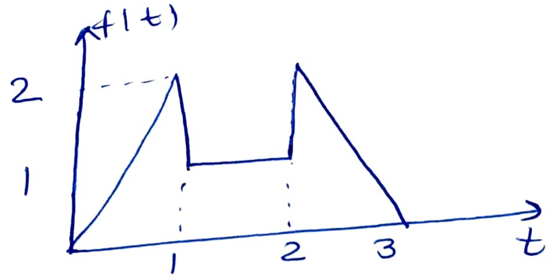


$$f(t) = G_{0,1}(t) + G_{1,3}(t)$$
$$= 2t[u(t) - u(t-1)] + 2[u(t-1) - u(t-3)]$$
$$= 2t\,u(t) - 2t\,u(t-1) + 2u(t-1) - 2u(t-3)$$

In terms of Standard Signals

$$\Rightarrow 2t\,u(t) - 2(t-1+1)\,u(t-1) + 2u(t-1) - 2u(t-3)$$
$$\Rightarrow 2t\,u(t) - 2(t-1)\,u(t-1) \cancel{- 2u(t-1)} \cancel{+ 2u(t-1)} - 2u(t-3)$$
$$= 2r(t) - 2r(t-1) - 2u(t-3)$$

2)



$$f(t) = G_{0,1}(t) + G_{1,2}(t) + G_{2,3}(t)$$

$$= 2t\,[u(t) - u(t-1)] + 1\,[u(t-1) - u(t-2)] - 2(t-3)\,[u(t-2) - u(t-3)]$$

$$= 2t\,u(t) - 2t\,u(t-1) + u(t-1) - u(t-2) - 2(t-3)\,u(t-2) + 2(t-3)\,u(t-3)$$

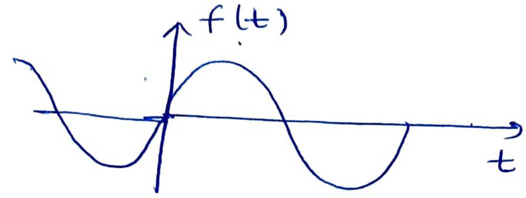
$$= 2t\,u(t) - 2(t-1+1)\,u(t-1) + u(t-1) - u(t-2) - 2(t-2-1)\,u(t-2) + 2(t-3)\,u(t-3)$$

$$= 2t\,u(t) - 2(t-1)\,u(t-1) - 2u(t-1) + u(t-1) - u(t-2) - 2(t-2)\,u(t-2) + 2u(t-2) + 2(t-3)\,u(t-3)$$

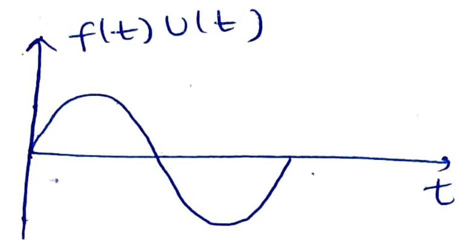
$$= 2r(t) - 2r(t-1) - u(t-1) - u(t-2) - 2r(t-2) + 2u(t-2) + 2r(t-3)$$

9. If $f(t) = \sin \omega t$ then draw the following signals

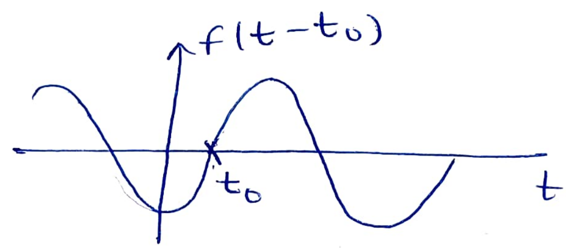
(a) $f(t)$



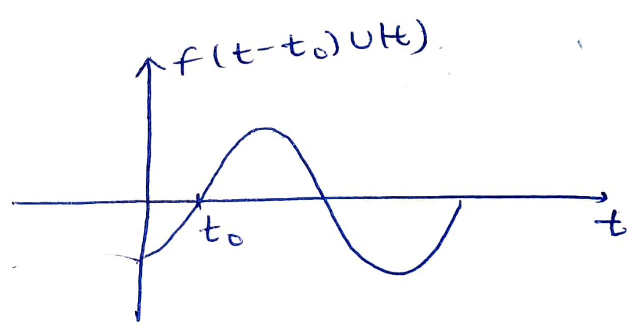
(b) $f(t)\,u(t)$



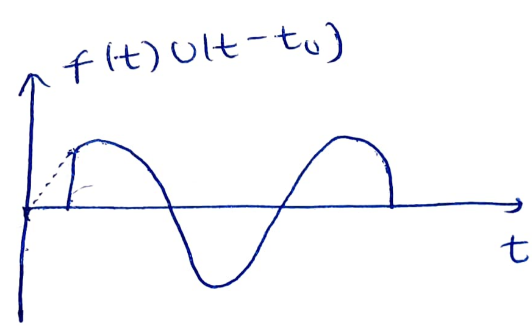
(c) $f(t-t_0)$



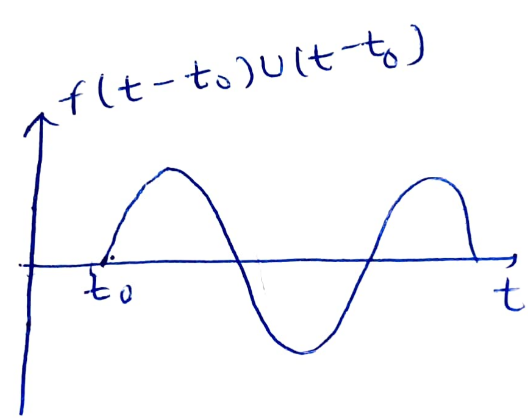
(d) $f(t-t_0)\,u(t)$



(e) $f(t)\,u(t-t_0)$

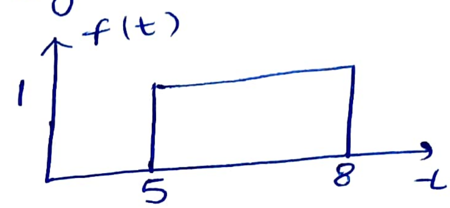


(f) $f(t-t_0)\,u(t-t_0)$

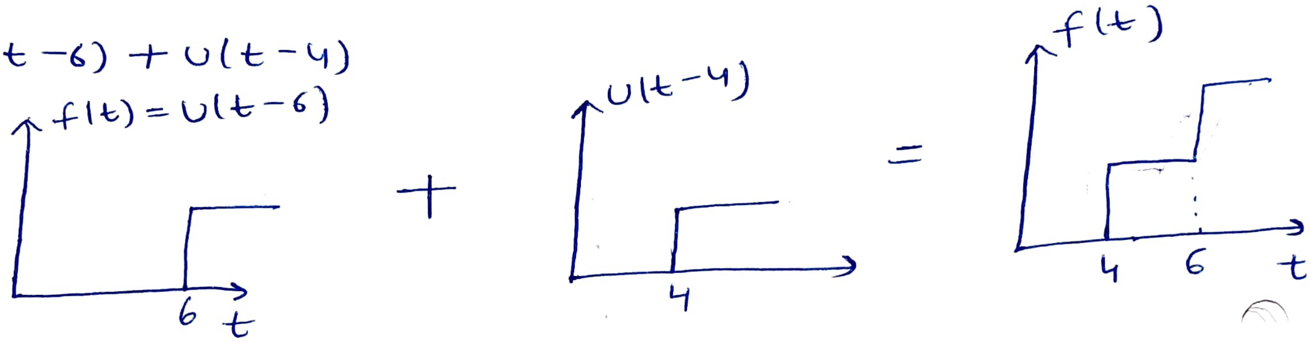


Sketch the following signals

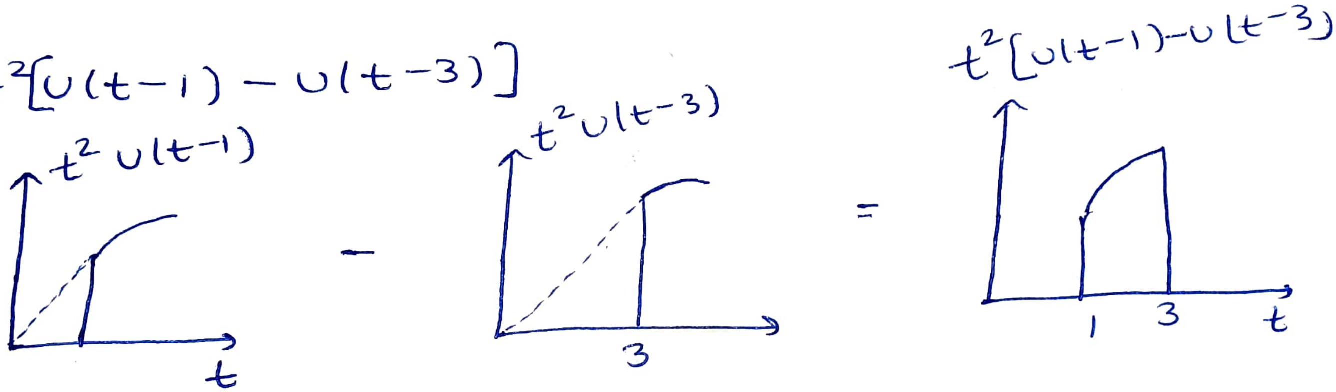
1) $u(t-5) - u(t-8)$



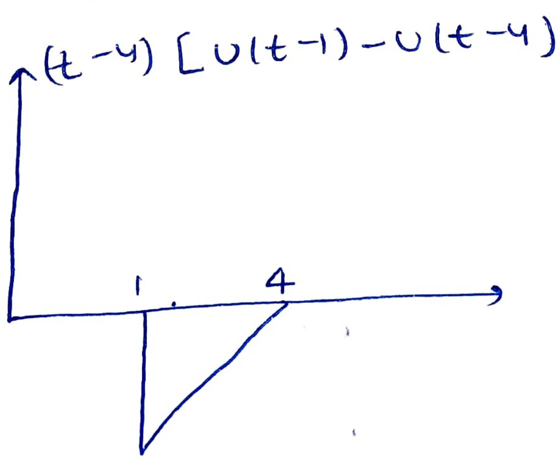
2) $u(t-6) + u(t-4)$



3) $t^2[u(t-1) - u(t-3)]$



4) $(t-4)\,[u(t-1) - u(t-4)]$



Power and Energy Signals

→ An energy signal is a signal with finite energy and zero average power, Energy signals are Non-periodic in nature. $E_\infty < \infty$

→ Power Signals is a signal with infinite energy and finite average power. Power signals are periodic in nature. $0 < P_\infty < \infty$

→ A signal can not be both an energy signal and a power signal

→ Eg. of Energy Signals – Exponentially decaying or increasing signals

Eg. of Power Signals- Sinusoidal, Unit Step etc.

→ Total Energy of the Signal is given by

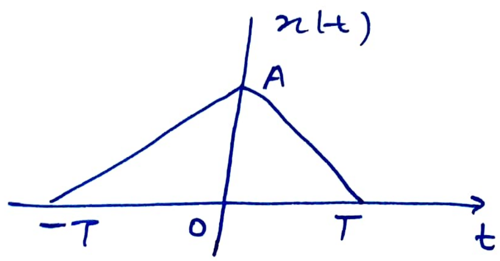
$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 \, dt = \int_{-\infty}^{\infty} |x(t)|^2 \, dt$$

→ Average Power of the Signal is given by

$$P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_{-T}^{T} |x(t)|^2 \, dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 \, dt$$

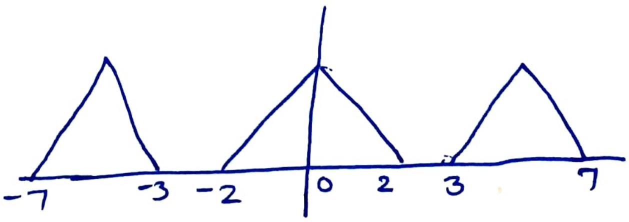
→ Energy Signals are time limited whereas power signals exist for infinite time.

Energy Signal



A Single Triangular pulse

Power Signal



A periodic triangular pulse

→ Mostly signals have finite energies and therefore energy signals.

→ A Power signals must necessarily have infinite duration

eg. Check whether the given signals are energy signal or power signals. Also calculate energy and power for each signal.

(i) $x(t) = ke^{-at}u(t)$; $a > 0$

$$E_\infty = \lim_{T\to\infty} \int_{-T}^{T} |x(t)|^2 dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_\infty = \int_0^\infty |Ke^{-at}|^2 dt = K^2 \int_0^\infty e^{-2at} dt = K^2 . \frac{e^{-2at}}{-2a}\Big|_0^\infty$$

$$= \frac{K^2}{-2a} [e^{-\infty} - e^0) = \frac{K^2}{2a}$$

Therefore $x(t)$ is an energy signal so $P_\infty = 0$

(ii) $x(t) = Ku(t)$

$$E_\infty = \int_0^\infty K^2 dt = K^2 t\Big|_0^\infty = K^2(\infty - 0) = K^2$$

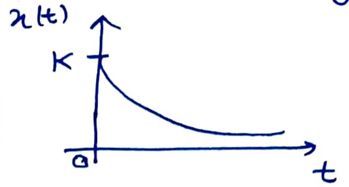
$$E_\infty = \infty$$

so $x(t)$ is a power signal

$$\text{Average Power } P_\infty = \lim_{T\to\infty} \frac{1}{2T} \int_0^T K^2 dt$$

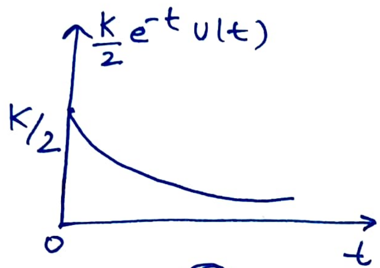
$$= \lim_{T\to\infty} \frac{1}{2T} K^2 t\Big|_0^T = \frac{1}{2T} . K^2 T = \frac{K^2}{2}$$

find and plot even and odd part of the signal
$x(t) = ke^{-t}u(t)$

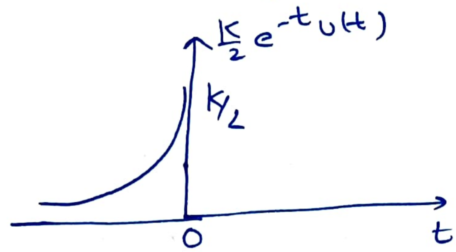


Sol. $x_e(t) = \frac{1}{2}\left[ke^{-t}u(t) + ke^{t}u(-t)\right]$

$x_o(t) = \frac{1}{2}\left[ke^{-t}u(t) - ke^{t}u(-t)\right]$

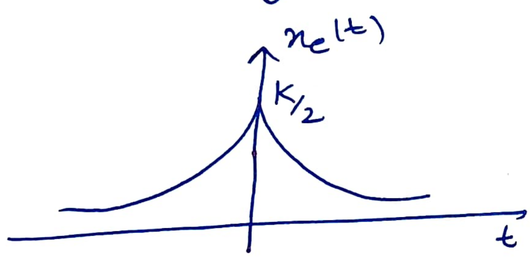


①

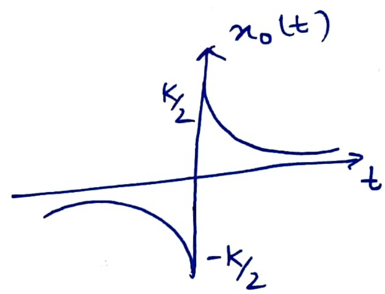


②

On adding fig. (1) & (2)



On Subtracting fig.(1) &(2)



eg. Determine whether or not each of the following signal is periodic. If it is periodic find its fundamental period.

(a) $x_1(t) = je^{j10t}$ (b) $x_2(t) = 4\sin(t/2 + \phi)$

(c) $x_3(t) = \cos\sqrt{2}\pi t$ (d) $x_4(t) = 2e^{j(t+\pi/4)}u(t)$

Sol. We know that fundamental time period of continous time signal is $2\pi/\omega$

(a) $T = \frac{2\pi}{\omega} = \frac{2\pi}{10} = \frac{\pi}{5}$

(b) $T = \frac{2\pi}{1/2} = 4\pi$

(c) $T = \frac{2\pi}{\sqrt{2}\pi} = 1.414$

(d) Unperiodic, as it does not exist from $-\infty$ to ∞.

eg. Check whether the given system is linear or not.

$y(t) = t^2\, x(t-1)$

for I/P $x_1(t) \rightarrow y_1(t) = t^2 x_1(t-1)$ — ①

for I/P $x_2(t) \rightarrow y_2(t) = t^2 x_2(t-1)$ — ②

for I/P $x_1(t) + x_2(t) \rightarrow y_3(t) = t^2 [x_1(t-1) + x_2(t-1)]$ — ③

On adding ① & ② we have

$y_4(t) = t^2 (x_1(t-1) + x_2(t-1)]$ ⟶ ④

eq'n (3) & (4) are same, hence system is linear.

eg. Consider a continous time system

$$y(t) = t^2 x(t-1)$$

Determine whether the system is time-invariant.

Sol. for I/P $x_1(t) \rightarrow y_1(t) = t^2 x_1(t-1)$

Now consider another I/P which is shifted by t_0

$$x_2(t) = x_1(t-t_0)$$

for I/P $x_2(t) \rightarrow y_2(t) = t^2 x_2(t-1)$

$$= t^2 x_1(t - t_0 - 1)$$

If $y_1(t)$ is shifted by t_0, then

$$y_1(t-t_0) = (t-t_0)^2 x(t-t_0-1)$$

$$y_2(t) \neq y_1(t-t_0)$$

Hence system is time varying.

eg. Check whether the given systems are instantaneous or not.

(i) $y(t) = 3x(t) - x^2(t)$ ⟶ Instantaneous

(ii) $y(t) = 4x(t) + x(t-1)$ ⟶ dynamic

(iii) $y[n] = 2x[n]$ ⟶ Instantaneous

(IV) $y(t) = K \int_{-\infty}^{t} x(t)\,dt$ ⟶ dynamic eg. check whether the given systems are casual or not

(a) $y(t) = x(t) + x(t-1) \rightarrow$ Casual

(b) $y(t) = 2x^2(t) \rightarrow$ Casual

(c) $y(t) = x(t-1) - x(t-2) \rightarrow$ Casual

(d) $y(t) = 2x(t) + x(t+1) \rightarrow$ Non Casual

(e) $y(t) = x(t+1) \rightarrow$ Non Casual

(f) $y[n] = x^2[n-1] \rightarrow$ Casual

(g) $y[n] = 3x[-n] \rightarrow$ Casual only for positive value of n.

eg. check whether the given systems are stable or not

(a) $y(t) = x(t) \cos \omega(t)$

$\because |\cos \omega(t)| \leq 1$

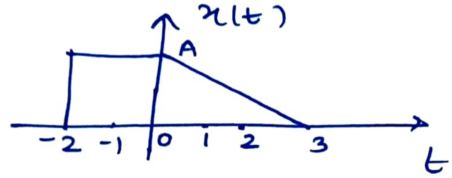
$|y(t)| = |x(t) \cos \omega t| \leq |x(t)|$

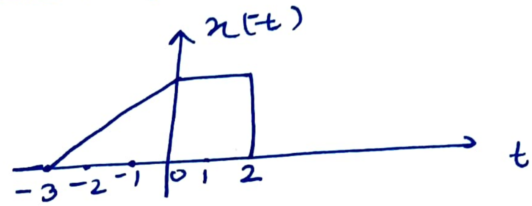
hence Stable.

Transformation of Independent Variable

It consist 3 basic operations

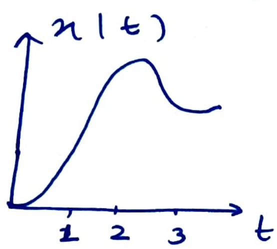
1. Time Reversal (Reflection)

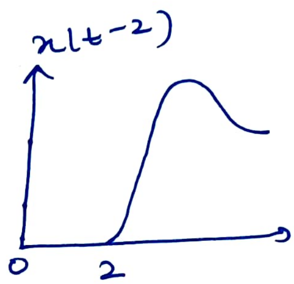


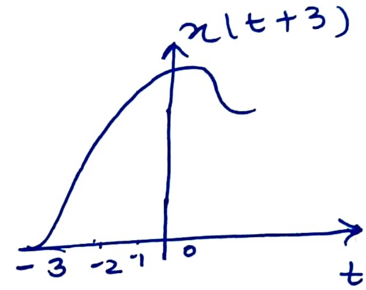


2. Time Shifting

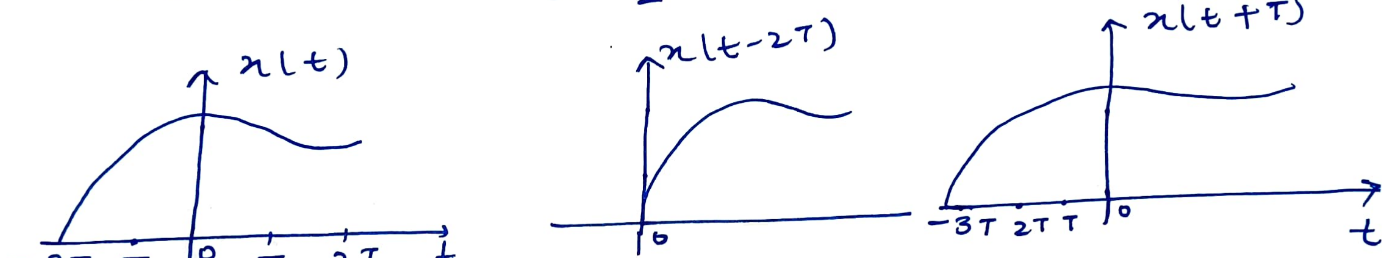
(i)



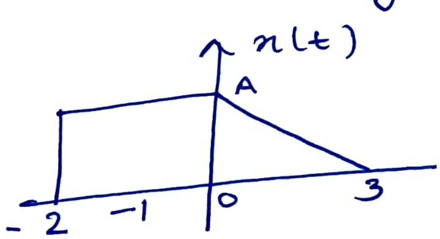


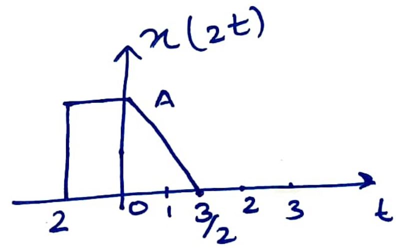


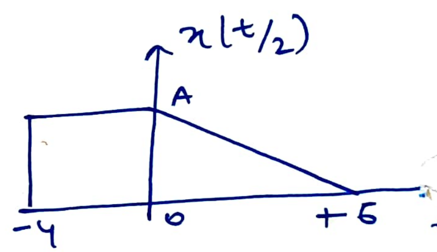
(ii)



3. Time Scaling







In this $x(t)$ is replaced by $x(\alpha t)$

If $\alpha > 1$ the Signal $x(\alpha t)$ is compressed version of $x(t)$ on time axis

If $0 < \alpha < 1$ the Signal $x(\alpha t)$ is expanded version of $x(t)$ on time axis.