Lecture-3
Propositional Equivalences

1.2 Propositional Equivalences

Tautologu, Contradiction & Contingency

DEFINITION 1

A compound proposition that is always true, no matter what the truth values of the propositions that occurs in it, is called a *tautology*. A compound proposition that is always false is called a *contradiction*. A compound proposition that is neither a tautology or a contradiction is called a *contingency*.

Examples of a Tautology and a Contradiction.

p	$\neg p$	$p \vee \neg p$	$p \wedge \neg p$
T	F	T	F
F	T	T	F

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DEFINITION 2

The compound propositions p and q are called *logically equivalent* if $p \leftrightarrow q$ is a tautology. The notation $p \equiv q$ denotes that p and q are logically equivalent.

- Compound propositions that have the same truth values in all possible cases are called logically equivalent.
- Example: Show that $\neg p \vee q$ and $p \rightarrow q$ are logically equivalent.

Truth Tables for $\neg p \vee q$ and $p \rightarrow q$.

p	q	$\neg p$	$\neg p \vee q$	$p \rightarrow q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

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- In general, 2^n rows are required if a compound proposition involves *n* propositional variables in order to get the combination of all truth values.

1.2 Logical Equivalences

TABLE 6 Logical Equivalences.

Equivalence	*Name*
$p \wedge \mathbf{T} \equiv p$ $p \vee \mathbf{F} \equiv p$	Identity laws
$p \vee \mathbf{T} \equiv \mathbf{T}$ $p \wedge \mathbf{F} \equiv \mathbf{F}$	Domination laws
$p \vee p \equiv p$ $p \wedge p \equiv p$	Idempotent laws
$\neg(\neg p) \equiv p$	Double negation law
$p \vee q \equiv q \vee p$ $p \wedge q \equiv q \wedge p$	Commutative laws
$(p \vee q) \vee r \equiv p \vee (q \vee r)$ $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$	Associative laws
$p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$	Distributive laws
$\neg(p \wedge q) \equiv \neg p \vee \neg q$ $\neg(p \vee q) \equiv \neg p \wedge \neg q$	De Morgan's laws
$p \vee (p \wedge q) \equiv p$ $p \wedge (p \vee q) \equiv p$	Absorption laws
$p \vee \neg p \equiv \mathbf{T}$ $p \wedge \neg p \equiv \mathbf{F}$	Negation laws

1.2 Logical Equivalences involving conditioal and Biconditional Statements

TABLE 7 Logical Equivalences Involving Conditional Statements.

$p \to q \equiv \neg p \vee q$
$p \to q \equiv \neg q \to \neg p$
$p \vee q \equiv \neg p \to q$
$p \wedge q \equiv \neg(p \to \neg q)$
$\neg(p \to q) \equiv p \wedge \neg q$
$(p \to q) \wedge (p \to r) \equiv p \to (q \wedge r)$
$(p \to r) \wedge (q \to r) \equiv (p \vee q) \to r$
$(p \to q) \vee (p \to r) \equiv p \to (q \vee r)$
$(p \to r) \vee (q \to r) \equiv (p \wedge q) \to r$

TABLE 8 Logical Equivalences Involving Biconditionals.

$p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$
$p \leftrightarrow q \equiv \neg p \leftrightarrow \neg q$
$p \leftrightarrow q \equiv (p \wedge q) \vee (\neg p \wedge \neg q)$
$\neg(p \leftrightarrow q) \equiv p \leftrightarrow \neg q$

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- Example: Show that $\neg(p \to q)$ and $p \wedge \neg q$ are logically equivalent.

  Solution:

  $\neg(p \to q) \equiv \neg(\neg p \vee q)$

  $\equiv \neg(\neg p) \wedge \neg q$ *by the second De Morgan law*

  $\equiv p \wedge \neg q$ *by the double negation law*

- *Example: Show that* $(p \wedge q) \to (p \vee q)$ is a tautology.

  Solution: To show that this statement is a tautology, we will use logical equivalences to demonstrate that it is logically equivalent to T.

  $(p \wedge q) \to (p \vee q) \equiv \neg(p \wedge q) \vee (p \vee q)$

  $\equiv (\neg p \vee \neg q) \vee (p \vee q)$ *by the first De Morgan law*

  $\equiv (\neg p \vee p) \vee (\neg q \vee q)$ *by the associative and communicative law for disjunction*

  $\equiv T \vee T$

  $\equiv T$

Questions on Logical equivalences

- Show that $\neg(p \lor (\neg p \land q))$and $\neg p \land \neg q$. are logically equivalent by developing a series of logical equivalences.

- Show that $\neg(p \lor (\neg p \land q))$and $\neg p \land \neg q$. are logically equivalent using truth table.

Normal forms for compound propositions

- A literal is a propositional variable or the negation of a propositional variable.
- A term is a literal or the conjunction (and) of two or more literals.
- A clause is a literal or the disjunction (or) of two or more literals.

Definition

A compound proposition is in **disjunctive normal form** (DNF) if it is a term or a disjunction of two or more terms. (i.e. an OR of ANDs).

A compound proposition is in **conjunctive normal form** (CNF) if it is a clause or a conjunction of two or more clauses. (i.e. and AND of ORs)

Conjunctive normal form (CNF)

	x	y	z	$x \vee y \rightarrow \neg x \wedge z$
1	F	F	F	T
2	F	F	T	T
3	F	T	F	F
4	F	T	T	T
5	T	F	F	F
6	T	F	T	F
7	T	T	F	F
8	T	T	T	F

The formula is **not** satisfied by the truth assignment in row 3 **and** in row 5 **and** in row 6 **and** in row 7 **and** in row 8. So:, it is logically. equivalent. to:

$\neg(\neg x \wedge y \wedge \neg z) \wedge \neg(x \wedge \neg y \wedge \neg z) \wedge \neg(x \wedge \neg y \wedge z) \wedge \neg(x \wedge y \wedge \neg z) \wedge \neg(x \wedge y \wedge z)$

apply DeMorgan's law to obtain its CNF:

$(x \vee \neg y \vee z) \wedge (\neg x \vee y \vee z) \wedge (\neg x \vee y \vee \neg z) \wedge (\neg x \vee \neg y \vee z) \wedge (\neg x \vee \neg y \vee \neg z)$

Disjunctive normal form (DNF)

	x	y	z	$x \vee y \rightarrow \neg x \wedge z$
1	F	F	F	T
2	F	F	T	T
3	F	T	F	F
4	F	T	T	T
5	T	F	F	F
6	T	F	T	F
7	T	T	F	F
8	T	T	T	F

The formula is satisfied by the truth assignment inrow 1 **or** by the truth assignment inrow 2 **or** by the truth assignment inrow 4. So, its DNF is :
$(\neg x \wedge \neg y \wedge \neg z) \vee (\neg x \wedge \neg y \wedge z) \vee (\neg x \wedge y \wedge z)$

STEPS TO FIND CNF OR DNF

1. Remove Negation in the Statement

2. Convert Conditional & Biconditional Statements in the Form of “And” & “OR”

3. Apply Algebraic Laws to convert them in the form SOP or POS

STEPS TO FIND PCNF OR PDNF

1. Remove Negation in the Statement

2. Convert Conditional & Biconditional Statements in the Form of “And” & “OR”

3. Apply Algebraic Laws to convert them in the form SOP or POS

4. If there is a missing term in the compound proposition ,introduce ($p \wedge \neg p$) *for PDNF and* ($p \vee \neg p$) *in case of PCNF.*

Find DNF of the following

1. $(\neg p \lor \neg q) \rightarrow (p \leftrightarrow \neg q)$
2. $q \land (p \lor \neg q)$.

Find CNF of the following

1. $p \land (p \rightarrow q)$
2. $\neg(p \lor q) \leftrightarrow (p \land q)$

Find PDNF of the following

1. $((p \wedge q) \rightarrow r) \vee ((p \wedge q) \rightarrow \neg r)$.
2. $P \rightarrow (Q \wedge P) \wedge (\neg P \rightarrow (\neg Q \wedge \neg R))$.
3. $(q \vee (p \wedge r)) \wedge ((p \vee r) \wedge q)$.

4. Show that $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P) = (P \vee Q \vee R) \wedge (P \vee \neg Q \vee R) \wedge (P \vee \neg Q \vee \neg R) \wedge (\neg P \vee Q \vee R) \wedge (\neg P \vee Q \vee \neg R)$

Thank You