Lecture-1
Propositional Logic

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Introduction to FCS

- Why we call this subject Foundation of Computer Science?
- Which topics will be covered in this subject.
- Why discrete mathematics is important in the field of Computer Science.
- Application of discrete mathematics in Computer Science.

List of Text books & Reference Books

- [T1] - Norman L. Biggs, “Discrete Mathematics”, Oxford, second edition.
- [T2] Kenneth H. Rosen, “Discrete Mathematics and Its Applications”, TMH, seventh edition.
- [R1]Elements of Discrete Mathematics by C L LIU,TMH,2000
- [R2] Discrete Mathematics by T.Veerarajan ,TMH,2006

1.1 Propositional Logic

- A **proposition** is a declarative sentence (a sentence that declares a fact) that is either true or false, but not both.
- Are the following sentences propositions?
  - Delhi is the capital of India. (Yes)
  - Read this carefully. (No)
  - 1+2=3 (Yes)
  - x+1=2 (No)
  - What time is it? (No)

1.1 Propositional Logic

- Are the following sentences propositions? What is the truth value of the proposition.
  - is it raining today ?
  - Where are you going?
  - 1+3>2
  - 1*Y=2
  - There is no pollution in Delhi.
  - Today is monday.

1.1 Propositional Logic

- Propositional Logic – the area of logic that deals with propositions
- Propositional Variables – variables that represent propositions: *p*, *q*, *r*, *s*
  - E.g. Proposition *p* – “Today is Friday.”
- Truth values – T, F

1.1 Propositional Logic

DEFINITION 1

Let p be a proposition. The negation of p, denoted by $\neg p$, is the statement "It is not the case that p."

The proposition $\neg p$ is read "not p." The truth value of the negation of p, $\neg p$ is the opposite of the truth value of p.

- Examples
  - Find the negation of the proposition "Today is Friday." and express this in simple English.

    Solution: The negation is "It is not the case that *today is Friday*."
    In simple English, "Today is not Friday." or "It is not Friday today."
  - Find the negation of the proposition "At least 10 inches of rain fell today in Delhi." and express this in simple English.

    Solution: The negation is "It is not the case that *at least 10 inches of rain fell today in Delhi*."
    In simple English, "Less than 10 inches of rain fell today in Delhi."

1.1 Propositional Logic

Find the negation of following proposition

1. The summer in Delhi is hot and sunny.
2. Today is Tuesday.
3. $2+1=3$
4. $2^n > 100$
5. There is lots of pollution in Delhi.

1.1 Propositional Logic

DEFINITION 2

Let p and q be propositions. The *conjunction* of p and q, denoted by $p \wedge q$, is the proposition "p and q". The conjunction $p \wedge q$ is true when both p and q are true and is false otherwise.

- Examples
  - Find the conjunction of the propositions p and q where p is the proposition "Today is Friday." and q is the proposition "It is raining today.", and the truth value of the conjunction.

  Solution: The conjunction is the proposition "Today is Friday and it is raining today." The proposition is true on rainy Fridays.

1.1 Propositional Logic

> DEFINITION 3
>
> Let p and q be propositions. The *disjunction* of p and q, denoted by $p \vee q$, is the proposition "p or q". The conjunction $p \vee q$ is false when both p and q are false and is true otherwise.

- Note:

  inclusive or : The disjunction is true when at least one of the two propositions is true.

  - E.g. "Students who have taken calculus or computer science can take this class." – those who take one or both classes.

  exclusive or : The disjunction is true only when one of the proposition is true.

  - E.g. "Students who have taken calculus or computer science, but not both, can take this class." – only those who take one of them.
- Definition 3 uses *inclusive or*.

1.1 Propositional Logic

DEFINITION 4

Let p and q be propositions. The *exclusive or* of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and is false otherwise.

The Truth Table for the Conjunction of Two Propositions.

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

The Truth Table for the Disjunction of Two Propositions.

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

The Truth Table for the Exclusive *Or* (*XOR*) of Two Propositions.

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

1.1 Propositional Logic

Conditional Statements

DEFINITION 5

Let p and q be propositions. The *conditional statement* $p \rightarrow q$, is the proposition "if p, then q." The conditional statement is false when p is true and q is false, and true otherwise. In the conditional statement $p \rightarrow q$, p is called the *hypothesis* (or *antecedent* or *premise*) and q is called the *conclusion* (or *consequence*).

- A conditional statement is also called an implication.
- Example: "If I am elected, then I will lower taxes." $p \rightarrow q$

implication:

elected, lower taxes.	T	T	T
not elected, lower taxes.	F	T	T
not elected, not lower taxes.	F	F	T
elected, not lower taxes.	T	F	F

1.1 Propositional Logic

Conditional Statements

A variety of terminology is used to express $p \rightarrow q$

- If P then Q
- P only if Q
- P is sufficient condition for Q
- Q is necessary for P
- Q if P
- Q follows from P
- Q unless $\neg$P

1.1 Propositional Logic

- Example:
  - Let p be the statement "Maria learns discrete mathematics." and q the statement "Maria will find a good job." Express the statement $p \longrightarrow q$ as a statement in English.

  Solution: Any of the following -

  "If Maria learns discrete mathematics, then she will find a good job.

  "Maria will find a good job when she learns discrete mathematics."

  "For Maria to get a good job, it is sufficient for her to learn discrete mathematics."

  "Maria will find a good job unless she does not learn discrete mathematics."

Write each of the following statements in the form “if p then q” in English refer to the list of common ways to provide for the conditional statements

a) It is necessary to wash the boss's car to get promoted.
b) Winds from the south imply a spring thaw.
c) A sufficient condition for the warranty to be good is that you bought the computer less than a year ago.
d) Willy gets caught whenever he cheats.
e) You can access the website only if you pay a subscription fee.

1.1 Propositional Logic

DEFINITION 6

Let p and q be propositions. The *biconditional statement* $p \leftrightarrow q$ is the proposition "p if and only if q." The biconditional statement $p \leftrightarrow q$ is true when p and q have the same truth values, and is false otherwise. Biconditional statements are also called *bi-implications*.

- $p \leftrightarrow q$ has the same truth value as $(p \rightarrow q) \land (q \rightarrow p)$
- "*if and only if*" can be expressed by "*iff*"
- Example:
  - Let p be the statement "You can take the flight" and let q be the statement "You buy a ticket." Then $p \leftrightarrow q$ is the statement

    "You can take the flight if and only if you buy a ticket."

    Implication:

    If you buy a ticket you can take the flight.

    If you don't buy a ticket you cannot take the flight.

1.1 Propositional Logic

The Truth Table for the Biconditional $p \leftrightarrow q$.		
p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

1.1 Propositional Logic

Determine whether each of these conditional statements is true or false.

a) If $1 + 1 = 2$, then $2 + 2 = 5$.
b) If $1 + 1 = 3$, then $2 + 2 = 4$.
c) If $1 + 1 = 3$, then $2 + 2 = 5$.
d) If monkeys can fly, then $1 + 1 = 3$.

1.1 Propositional Logic

Determine whether these biconditionals are true or false.

a) $2 + 2 = 4$ if and only if $1 + 1 = 2$.
b) $1 + 1 = 2$ if and only if $2 + 3 = 4$.
c) $1 + 1 = 3$ if and only if monkeys can fly.
d) $0 > 1$ if and only if $2 > 1$.

Thank You

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Any Questions ?