ASSIGNMENT 1

1. State Dirichlet's conditions for convergence of Fourier series and check whether the function $f(x)=\frac{1}{3-x}$, $0<x<2\pi$ satisfy Dirichlet's conditions or not?
2. Find a Fourier series to represent $x-x^2$ from $-\pi$ to π. Hence show that
$$\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+.....=\frac{\pi^2}{12}$$
3. Express $f(x)=|x|$, $-\pi<x<\pi$ as Fourier series.
4. Express $f(x)=x^2$ as a half-range cosine series for $0<x<2$.
5. Obtain the Fourier sine series for $f(x)$ containing three non-zero terms where $f(x)$ is given in the following table:

x	0	1	2	3	4	5
$f(x)$	0	10	15	8	5	3

6. Obtain Fourier sine integral of the function $f(x)=\begin{cases} x\,, & 0<x<1 \\ 2-x, & 1<x<2 \\ 0\,, & x>2 \end{cases}$
7. Find the Fourier transform of $f(x)=e^{-|x|}$.
8. Find the Fourier cosine transform of $f(x)=\frac{1}{a^2+x^2}$. Hence derive Fourier sine transform of $\phi(x)=\frac{x}{a^2+x^2}$.
9. Find the inverse Fourier transform of the function $\frac{1}{(4+\omega^2)}$.
10. The temperature u in the semi-infinite rod $0\leq x<\infty$ is determined by the differential equation $\frac{\partial u}{\partial t}=k\frac{\partial^2 u}{\partial x^2}$ subject to the conditions

    (i) $u=0$ when $t=0,\ x\geq 0$ (ii) $\frac{\partial u}{\partial x}=-\mu$ (a constant) when $x=0,\ t>0$.

    Show that $u(x,t)=\frac{2\mu}{\pi}\int_0^\infty \frac{(1-e^{k\omega^2 t})}{\omega^2}\cos\omega x\, d\omega$.

ASSIGNMENT 1 (ANSWERS)

1. No
2. $f(x) \sim \dfrac{-\pi^2}{3} + 2\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^2}(2\cos nx + n\sin nx)$
3. $f(x) \sim \dfrac{\pi}{2} - \dfrac{4}{\pi}\sum_{n=1}^{\infty} \dfrac{\cos(2n-1)x}{(2n-1)^2}$
4. $f(x) \sim \dfrac{4}{3} - \dfrac{16}{\pi^2}\sum_{n=1}^{\infty} \dfrac{(-1)^{n+1}}{n^2}\cos\dfrac{n\pi x}{2}$
5. $f(x) \sim 10.607\sin\dfrac{\pi x}{6} + 4.907\sin\dfrac{2\pi x}{6} + 1.667\sin\dfrac{3\pi x}{6}$
6. $f(x) = \dfrac{2}{\pi}\int_0^{\infty} \dfrac{(2\sin\omega - \sin 2\omega)\sin\omega x}{\omega^2}\, d\omega$
7. $\dfrac{2}{1+\omega^2}$
8. $\dfrac{\pi}{2a}e^{-a\omega}$, $\dfrac{\pi}{2}e^{-a\omega}$
9. $\dfrac{1}{4}e^{-2t}$ if $t>0$; $\dfrac{1}{4}e^{2t}$ if $t<0$; $\dfrac{1}{2}$ if $t=0$.