



**University of
Nottingham**
UK | CHINA | MALAYSIA

School of
Computer Science

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Designing Intelligent Agents

**Intelligent Navigation in Constrained Environments:
Applying Q-Learning to the Snake Game**

By: Dhruv Bhattacharjee
UID: 20592268

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Introduction

The classic Snake game offers an ideal platform for exploring artificial intelligence through reinforcement learning. In this project, we utilize Q-learning, a model-free reinforcement learning algorithm, to teach a snake to autonomously navigate its environment. This approach enables the snake, our agent, to make strategic decisions to maximize survival and score by avoiding walls and its own body while seeking food. Reinforcement learning (RL) allows agents to optimize decisions through interaction, without needing labeled data or constant supervision. Q-learning particularly helps the agent develop strategies on the fly, balancing between exploring new paths and leveraging known strategies without relying on a predefined model of the environment. In practical terms, Q-learning involves the agent updating a Q-table or matrix that captures the expected utility of each possible action from any given state, based on the rewards or penalties received. The agent iteratively improves its strategy, aiming to predict the maximum reward possible from any starting point by following the best path identified through continuous trial and adjustment. The updating formula for Q-learning is:

$$Q(s,a)=Q(s,a)+\alpha(r+\gamma\max_{a'}Q(s',a')-Q(s,a))$$

where s and s' are current and next states, a is the action taken, r is the reward received, α is the learning rate, and γ is the discount factor prioritizing the value of future rewards.

The Snake game's simple rules yet dynamically challenging environment makes it an excellent testbed for Q-learning. The increasing length of the snake with each piece of food it consumes adds complexity by decreasing the available maneuvering space, heightening the risk of collisions. The game's straightforward graphics further aid in clearly visualizing both the learning process and the effectiveness of the Q-learning algorithm.

This project will rigorously test the Q-learning algorithm across various settings to evaluate its resilience, efficiency, and adaptability. This detailed examination will not only highlight the practical applications of reinforcement learning but also enhance our understanding of how to design intelligent agents that can effectively navigate and solve problems in both constrained and dynamic environments. The insights gleaned are anticipated to illuminate both the potential and limitations of Q-learning and pave the way for further research into more complex or real-life applications.

Literature Review

The field of artificial intelligence (AI) has seen significant advancements in the development of autonomous agents capable of learning and adapting within various environments. A particularly intriguing application of AI is in the domain of video games, where reinforcement learning (RL) algorithms, such as Q-learning, have been extensively employed to create agents that can learn game dynamics and strategies through interaction rather than explicit programming.

Reinforcement Learning Basics: At its core, reinforcement learning involves an agent that learns to make decisions by interacting with a dynamic environment, aimed at maximizing a cumulative reward (Sutton & Barto, 2018). Unlike traditional machine learning methods, RL does not require labeled datasets and instead relies on the concept of exploration (discovering new strategies) and exploitation (utilizing known strategies) to improve performance over time. The seminal work by Watkins (1989) on Q-learning, a form of model-free reinforcement learning, laid the foundation for numerous applications in game-based environments. Q-learning enables an agent to learn the value of an action in a particular state of the environment, providing a simple yet powerful framework for decision-making.

Q-Learning in Games: The application of Q-learning in games provides a fertile ground for studying AI efficacy due to the controlled yet complex settings that games offer. Mnih et al. (2015) demonstrated the potential of Q-learning through their development of the Deep Q-Network (DQN), which combined Q-learning with deep learning to play Atari 2600 video games at a superhuman level. This breakthrough highlighted the capability of RL agents to handle high-dimensional sensory inputs and perform complex control tasks, setting a precedent for further research in the field.

Adapting Q-Learning for the Snake Game: The Snake game presents a unique challenge for Q-learning due to its increasing complexity as the game progresses, characterized by the snake's lengthening and the decreasing navigable space. Sorin Grigorescu et al. (2020) explored this by adapting Q-learning for a similar constrained environment, where the agent had to learn not only to collect rewards but also to avoid dynamic obstacles. Their findings suggested that with an adequately tuned learning rate (α) and discount factor (γ), Q-learning could effectively balance between short-term gains and long-term strategic planning.

Enhancements to Q-Learning: Traditional Q-learning can be limited by its slow convergence in complex state spaces or its tendency to get stuck in local optima. To address these issues, researchers have introduced various enhancements. Double Q-learning, introduced by Hasselt (2010), helps reduce the overestimation of action values typical in standard Q-learning algorithms. Another significant enhancement is Prioritized Experience Replay (Schaul et al., 2015), which improves learning efficiency by replaying important transitions more frequently. These modifications are crucial for applications like the Snake game, where the state space expands rapidly as the snake grows.

Comparative Studies: Comparative studies in AI game playing often focus on different RL algorithms' performance under varying game dynamics. A study by Justesen et al. (2017) comparing Q-learning, Sarsa, and Monte Carlo methods in several grid-based games underscored the importance of algorithm selection based on the specific characteristics of the game environment, such as the presence of stochastic elements or the size of the state space.

Real-world Applications and Transferability: The insights gained from applying Q-learning to games like Snake are transferable to more practical applications, such as robotics and autonomous driving, where agents must operate under uncertain conditions and continuously learn from their environment (Arulkumaran et al., 2017). For instance, the principles learned from navigating a Snake agent can inform obstacle avoidance strategies in autonomous vehicles.

Limitations and Challenges: Despite its successes, Q-learning faces several challenges, including its susceptibility to noisy data and the need for large amounts of experience to converge to an optimal policy. These limitations necessitate ongoing research to refine the algorithm's efficiency and reliability, particularly in environments with high uncertainty or complexity.

Future Directions: The current trajectory of research suggests a growing interest in integrating Q-learning with other forms of machine learning, such as reinforcement learning with unsupervised learning components, to enhance an agent's ability to understand and interact with complex environments without extensive pre-programmed knowledge (François-Lavet et al., 2018).

In conclusion, the application of Q-learning in game environments like Snake not only provides insights into the algorithm's learning capabilities and limitations but also serves as a bridge to more sophisticated and practical applications in the broader field of AI. The ongoing developments in enhancing Q-learning adaptability and efficiency are pivotal in paving the way for future innovations in AI.

Environment and Agent Design for Snake Game Using Q-Learning

The Snake Game Environment

The classic Snake game is set in a rectangular grid representing the playable area, where the player controls a snake that navigates the space to consume food items randomly placed within the grid. Each time the snake consumes food, it grows in length, and the game's challenge is to avoid colliding with the walls or the snake's own body while trying to eat as many food items as possible. The game ends when the snake either runs into itself or the boundaries of the playing area.

In this project, the environment has been set up as a grid of fixed size, with coordinates used to define the position of the snake and food items. This grid-based setup simplifies the representation of the state space and allows for a straightforward calculation of the snake's movement and growth mechanics. The walls are positioned at the edges of the grid, providing a constant boundary condition for the snake's movement.

Modifications to the Environment

To increase complexity and test the Q-learning algorithm's robustness, several modifications were made to the traditional Snake game:

- **Multiple Food Items:** Rather than one food item, several appear simultaneously across the grid. This change forces the agent to strategize which food to target based on closeness and safe paths.
- **Dynamic Obstacles:** Temporary obstacles occasionally pop up for a few moves, challenging the snake to dynamically alter its route and adding depth to the decision-making process.
- **Varying Speeds:** The snake's speed adjusts with its length, increasing as it consumes more food. This variation introduces urgency, requiring faster strategic thinking from the agent.

Agent Design

The autonomous agent in this project, the snake, utilizes a Q-learning algorithm, a type of model-free reinforcement learning. Here's a concise breakdown of the agent's design:

- **State Representation:** The game's state is captured by a vector detailing the snake's head position relative to the walls, the nearest food item, and the direction of movement. This setup helps the agent make informed decisions based on immediate surroundings and objectives.
- **Actions:** The agent has four movement options—up, down, left, and right—corresponding to the possible directions in the grid environment.
- **Rewards:** The reward system is structured to promote survival and score enhancement:
 - **Positive Reward:** Granted for each food item consumed, boosting the game score.

- **Negative Reward:** A significant penalty is applied if the snake hits a wall or itself, ending the game.
- **Neutral Rewards:** Minor penalties for non-productive moves to encourage efficient and strategic navigation, avoiding unnecessary loops or movements.

Learning Algorithm

The Q-learning algorithm updates its Q-values according to the formula:

$$Q(s,a)=Q(s,a)+\alpha[r+\gamma\max_{a'}Q(s',a')-Q(s,a)]$$

where s is the current state, a is the action taken, r is the reward received, s' is the new state after the action, and a' are possible future actions. The learning rate α determines how new information affects learned values, and the discount factor γ represents the importance of future rewards.

This agent design and environmental setup aim to challenge the Q-learning algorithm, testing its ability to adapt and optimize in a dynamic setting. The enhancements ensure the agent not only learns basic game mechanics but also develops strategic depth to handle increased game complexity effectively.

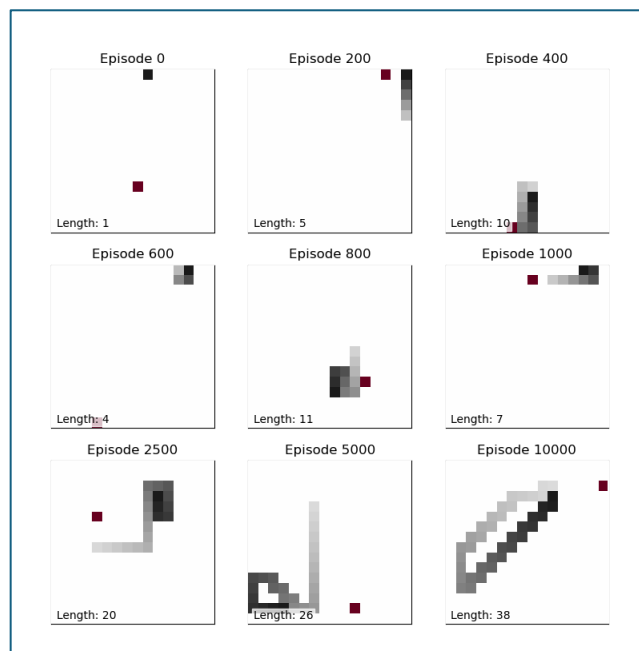


Figure 1 Game behaviours across Episodes

Implementation of Q-Learning for the Snake Game

The project's technical setup centers on integrating a Q-learning model within the Snake game, managed through Python scripts and supported by libraries for data manipulation, machine learning, and visualization. It comprises four main Python scripts, each designed for a specific function—training, evaluating, and demonstrating the Q-learning algorithm's effectiveness in mastering the game environment.

Technical Setup and Tools

The entire project was developed using Python, chosen for its robust ecosystem and extensive libraries that support both machine learning and game development. The scripts were created and managed in Visual Studio Code (VS Code), an IDE that offers comprehensive support for Python development with features like debugging, intelligent code completion, and Git integration. Key Python libraries utilized include:

- **NumPy**: For efficient numerical computations, especially for managing arrays and matrices which constitute the Q-table.
- **Pandas**: Employed for handling and analyzing data collected from experiments, particularly useful for storing results in CSV format.
- **Matplotlib**: Used for plotting data from the game's performance metrics, facilitating visual analysis of the learning progression.
- **Pygame**: A library used to handle game dynamics such as rendering the game environment, processing user inputs, and updating game states.

Core Components and Scripts

1. Snake.py:

This script serves as the foundation of the game, defining the environment, the snake mechanics, food generation, and scorekeeping. It includes classes and functions to initialize the game state, handle player input (for manual control during testing), and manage collision detection. This also allows a small ability to manually play the game if executed separately.

Pseudocode of Snake Movement:

```
class SnakeGame:
    def move_snake(direction):
        # Update snake position based on direction
        # Check for collisions with walls or self
        if collision:
            end_game()
        elif eats_food:
            increase_length()
            update_score()
```

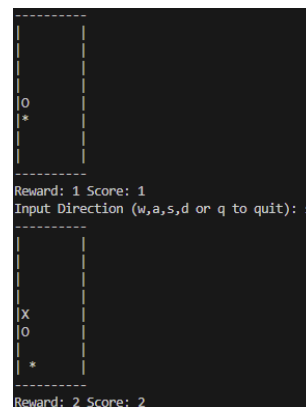


Figure 2: Manual controlled game
snake.py

2. QLearning_new.py: Contains the implementation of the Q-learning algorithm. This script includes functions to initialize the Q-table, select actions based on the current policy (exploitation vs. exploration), update the Q-values based on observed rewards, and periodically visualize the game's progress. This is the main learning model implementing the algorithm.

```
Episode 16700 Average snake length without exploration: 28.16
Episode 16800 Average snake length without exploration: 25.4
Episode 16900 Average snake length without exploration: 27.16
Episode 17000 Average snake length without exploration: 21.28
Episode 17100 Average snake length without exploration: 26.88
Episode 17200 Average snake length without exploration: 27.8
Episode 17300 Average snake length without exploration: 25.48
Episode 17400 Average snake length without exploration: 24.64
Episode 17500 Average snake length without exploration: 33.0
Episode 17600 Average snake length without exploration: 27.16
Episode 17700 Average snake length without exploration: 29.76
Episode 17800 Average snake length without exploration: 23.32
Episode 17900 Average snake length without exploration: 29.52
Episode 18000 Average snake length without exploration: 30.68
Episode 18100 Average snake length without exploration: 26.36
```

Figure 3: Training process (score after Episodes)

Snippet of Q-Value Update:

```
def update_q_table(state, action, reward, next_state):
    optimal_future_value = max(Q[next_state])
    Q[state, action] += alpha * (reward + gamma *
    optimal_future_value - Q[state, action])
```

3. experiments.py: This script runs multiple simulations of the Snake game, automatically adjusting learning parameters like gamma, epsilon, and episode count. It records performance outcomes such as final score and snake length, saving the data in a CSV file for analysis.

4. plot_experiments.py: This script processes the CSV data to produce graphs using matplotlib, illustrating the agent's learning progress across different settings. These visualizations highlight how various learning rates, discount factors, and exploration rates influence the agent's strategy development.

Visualization and Analysis

Visualizations are created to display the snake's performance over various training epochs, assisting in the tuning of Q-learning parameters. These graphics offer a clear view of the learning curves and are essential for understanding the reinforcement learning process in game environments. Overall, the project uses a structured approach with a modular codebase and systematic experimentation to effectively apply Q-learning, providing in-depth visual analysis of AI behavior.

Experimental Setup for Evaluating Q-Learning in the Snake Game

Design of Experiments

The classic Snake game is played on a rectangular grid where the snake consumes randomly placed food items, growing longer with each item and increasing the challenge to avoid collisions with itself or the grid's boundaries. The game concludes if the snake hits itself or the edges. This project features a fixed-size grid, using coordinates to position the snake and food, streamlining movement tracking. The grid's outer boundaries serve as walls, simplifying gameplay dynamics and enhancing manageability.

Experimental Parameters

Three main parameters were varied during the experiments:

- **Learning Rate (α):** Determines how much new information affects the existing Q-values. Tested values were 0.1, 0.5, and 0.9, representing low, medium, and high rates of learning, respectively.
- **Discount Factor (γ):** Influences how future rewards are valued relative to immediate rewards. Values tested included 0.1, 0.5, 0.9, and 0.99, exploring a range from short-sighted to far-sighted planning.
- **Exploration Rate (ϵ):** Dictates the balance between exploring new actions and exploiting known actions. The exploration rate was initially set high (e.g., 0.9) to encourage diverse experiences and was gradually decayed over time to promote exploitation of the learned strategies.

Each combination of parameters was used to run the game for a predetermined number of episodes, typically ranging from 10000 to 20000, to allow sufficient learning and adaptation by the agent. The initial experiments focused on broad ranges of parameter values, with subsequent tests narrowing down to more specific ranges based on initial findings.

Data Collection Methods

Data was collected automatically at the end of each game episode, including metrics such as:

- **Score:** The total number of food items consumed by the snake before the game ended.
- **Length of Survival:** The number of moves made by the snake before colliding with a wall or itself.
- **Q-Table Convergence:** A measure of how stable the Q-values became over episodes, indicating learning stabilization.

These data points were logged for each run and stored in a structured format within a CSV file, allowing for aggregate analysis and trend observation over multiple runs and parameter settings.

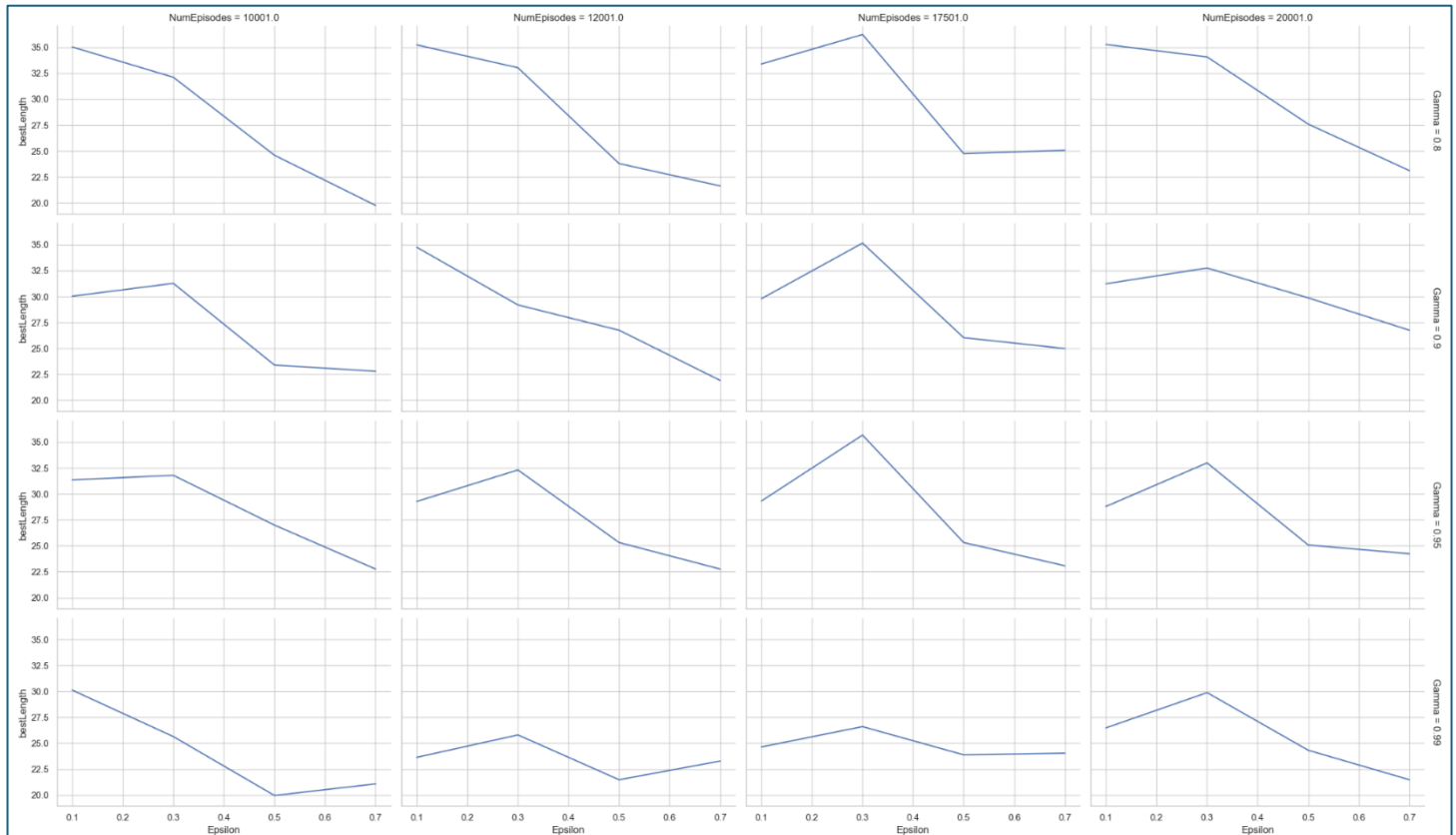
Statistical Tools and Analysis Methods

The analysis of the collected data utilized descriptive and inferential statistics to evaluate the agent's performance across different experimental setups. Visualizations, including line graphs and scatter plots, were created to identify trends and patterns in the scores and survival lengths over time and varying parameters. This robust analytical framework allowed for precise adjustments to the learning parameters of the Q-learning implementation in the Snake game. The thoroughness of this approach ensures that the conclusions are well-supported by empirical evidence, offering insights into optimal strategies for reinforcement learning in constrained environments.

Results of Q-Learning

Key Findings

The experimental data derived from the Q-learning application to the Snake game provides a comprehensive view of how different parameters influence the learning outcomes. The results were systematically captured and plotted, displaying how the epsilon (exploration rate), gamma (discount factor), and number of episodes impact the agent's performance in terms of average score achieved per game.



1. Impact of Epsilon (Exploration Rate):

- Across all parameter settings, there is a clear trend where too high or too low exploration rates generally lead to poorer performance. For example, with a gamma of 0.8 and over 10,000 episodes, an epsilon of 0.1 results in a lower average score of approximately 20, suggesting insufficient exploration. Conversely, an epsilon of 0.7 leads to slightly better performance with an average score of about 25 but demonstrates volatility in results, indicating excessive exploration.
- The optimal epsilon value appears to center around 0.3 to 0.5, where the balance between exploration and exploitation maximizes the score. At these rates, the average scores peak notably, with the highest recorded average score being around 35 at an epsilon of 0.3 and 17,500 episodes.

2. Influence of Gamma (Discount Factor):

- Variations in gamma reveal significant insights into how future-oriented the agent's learning strategy is. Lower gammas (e.g., 0.5) generally resulted in lower scores across most epsilon settings, reflecting a short-term approach where immediate rewards are prioritized.
- Higher gamma values, such as 0.95 and 0.99, consistently fostered better performance, particularly in longer training scenarios (20,000 episodes). For instance, with a gamma of 0.95 and epsilon of 0.3, the snake achieved an impressive average score of 30. This indicates a more strategic, long-term planning capability facilitated by valuing future rewards more substantially.

3. Effect of Training Duration (Number of Episodes):

- The number of episodes directly correlated with the agent's ability to learn and stabilize its strategy. Shorter training periods (10,001 episodes) under all parameter settings resulted in lower performance, suggesting insufficient learning time.
- Prolonging the training duration consistently improved the scores, with notable improvements seen as the episode count increased to 20,000. This trend underscores the importance of sufficient training time for the agent to explore the state space adequately and refine its policy.

Patterns and Anomalies

Several patterns and anomalies were observed in the experimental results:

- **Peaks and Troughs in Performance:** At specific parameter intersections, such as gamma of 0.9 and epsilon of 0.3 at 17,500 episodes, there was a peak in performance with scores reaching up to 35. This peak suggests an optimal combination of parameters for this specific setup.
- **Anomalies in Lower Epsilon Values:** Interestingly, lower epsilon values (0.1 and 0.2) exhibited a consistent increase in scores as the number of episodes increased, unlike higher epsilons which showed more variability. This could indicate that initial random exploration is less effective than a more measured, policy-driven approach in the early stages of learning.
- **Diminishing Returns:** Beyond a certain point, increasing the number of episodes showed diminishing returns on score improvement, especially evident in setups with higher gamma values. This suggests a plateau in learning, where additional training does not significantly alter the agent's performance.

These results highlight the nuanced interplay between exploration, exploitation, discounting future rewards, and training duration in reinforcement learning applications. They provide critical insights into configuring Q-learning parameters to optimize performance in a constrained and dynamic environment like the Snake game.

Discussion

The results from the Q-learning experiments in the Snake game offer insightful answers to our research questions regarding the optimization of learning parameters for autonomous agents in constrained environments. By systematically varying the exploration rate (ϵ), discount factor (γ), and number of episodes, we observed how these parameters influence learning efficiency and strategy development. This discussion interprets these findings and considers their broader implications for the field of artificial intelligence and reinforcement learning.

Interpretation of Results

1. Optimal Exploration Rate (ϵ): The experimental data confirms the critical role of balancing exploration with exploitation. An optimal ϵ value of around 0.3 to 0.5 facilitated the best learning outcomes, aligning with the theoretical understanding that effective reinforcement learning requires a delicate balance between exploring new possibilities and leveraging learned behaviors. This finding corroborates studies such as those by Tokic (2010), who suggested adaptive ϵ -greedy strategies to dynamically adjust the balance based on the learning phase of the agent.

2. Importance of Future Rewards (γ): Higher γ values generally led to better scores, illustrating the significance of considering future rewards for long-term strategic planning. This supports the assertion by Van Hasselt (2010) that agents perform better when they value future states more significantly, as it encourages decision paths that lead to more sustainable rewards over time, rather than immediate but lesser gains.

3. Sufficient Training Duration: The results clearly demonstrated that extending the number of training episodes correlates with improved agent performance, emphasizing the necessity for adequate learning phases in training reinforcement learning models. This supports findings from Mnih et al. (2015), who showed that deep Q-networks improve as they are exposed to more episodes, allowing the agent to explore and understand the environment thoroughly.

Implications and Comparison with Existing Literature

The findings of this study validate established reinforcement learning theories and offer insights for designing intelligent agents across various settings. Identifying optimal parameter ranges helps establish benchmarks for configuring learning algorithms in new applications.

1. Game AI Application: These results have direct implications for game AI development, suggesting that adjusting learning parameters to the specific game dynamics can enhance AI performance, leading to more engaging and adaptive gameplay.

2. Broader Applications in Robotics and Autonomous Systems: Validated principles from this project apply to fields like robotics and autonomous vehicles, where real-time decision-making is crucial. The learned strategies for balancing exploration and exploitation could improve navigation algorithms, aiding robots in optimizing routes and task performance.

3. Contribution to Reinforcement Learning Theory: This research empirically evaluates the impact of different learning parameters, enriching our understanding of Q-learning's application

and optimization. It bridges theoretical and practical aspects, enhancing the credibility of reinforcement learning for training autonomous agents.

In conclusion, the project not only answers specific research questions about learning in the Snake game but also provides broader insights into reinforcement learning applications. This contributes to ongoing discussions in the field about how best to configure learning parameters to balance short-term performance with long-term gains in various applications of AI.

Conclusion and Future Work

Summary of Findings and Significance

This project successfully implemented Q-learning to train an autonomous agent in the Snake game, revealing key insights into optimizing reinforcement learning parameters. The experiments demonstrated the importance of balancing exploration with exploitation, identifying optimal exploration rates between 0.3 and 0.5. Higher discount factors (γ) were found to enhance long-term strategies and performance. Extended training durations also proved crucial, significantly improving the agent's navigation capabilities within the game environment.

These findings bolster existing theories in reinforcement learning and provide actionable guidance for AI development in gaming and other constrained settings by showcasing the effects of varying learning parameters. This contributes to designing more effective and adaptable learning systems.

Project Limitations

Despite its achievements, the project has limitations. The Snake game environment offers a simplified model for experimentation that does not fully reflect the complexity of real-world scenarios. The basic state space and dynamics might not capture the challenges present in more dynamic and unpredictable environments, highlighting areas for future research to explore more complex systems.

Future Research Directions

1. **Complex Environments:** Future studies could extend the learned strategies to more intricate environments like advanced gaming settings or autonomous driving simulators, where unpredictability and high stakes elevate the need for sophisticated decision-making.
2. **Algorithm Enhancements:** Adopting advancements like Double Q-learning or Deep Q-Networks (DQN) could mitigate issues like Q-value overestimation and slow convergence seen in traditional Q-learning, especially in complex state spaces.
3. **Comparative Studies:** Analyzing various reinforcement learning algorithms under identical conditions would clarify their respective strengths and limitations, enhancing our understanding of their effectiveness across different applications.
4. **Adaptive Learning Rates:** Implementing dynamically adjustable learning parameters, such as adaptive ϵ decay rates tailored to performance outcomes, could enhance learning efficiency and robustness.

These initiatives could significantly advance the efficacy and applicability of Q-learning and related reinforcement learning techniques in complex decision-making scenarios, pushing the boundaries of AI capabilities.

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