a) Derivative not necessary

direct result of choosing min dist

$$\sum_{k=1}^{k} \pi_k \| X_i - \mu_k \|^2$$

$$\pi_{ik} = \begin{cases} 1 & \text{if } k = \underline{\text{argmin}}_j \|x_i - \mu_j\|^2 \\ 0 & \underline{\text{otherwise}} \end{cases}$$

we are choosing min dist - always

b) $$F(\mu_k) = \sum_{i=1}^{n} \pi_{ik} \|x_i - \mu_k\|^2$$

$$\frac{\partial f(\mu_k)}{\partial \mu_k} = -2 \sum_{i=1}^{n} \pi_{ik} (x_i - \mu_k)$$

But is it a minima?

$$\frac{\partial^2 f(\mu_k)}{\partial \mu_k^2} = +2 \sum \pi_{ik} (x_i - \mu_k)$$

$\therefore$ first derivative is a minima.

$$\frac{\partial f(\mu_k)}{\partial \mu_k} = 0$$

$$\sum \pi_{ik} x_i = \sum \pi_{ik} \mu_k$$

$$\boxed{\mu_k = \frac{\sum \pi_{ik} x_i}{\sum \pi_{ik}}}$$

c) K-Means has to stop

Why?

a) There is a min dist blw μ_k and π_i
   ↳ finite value ↳ E step

b) Function is monotonically decreasing.