Information Theory

Lecture Notes for Self Learning

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Preface

In this lecture notes, I will try to establish a ground work for information theory, then would work of it's philosophy, and applications in mathematics and quantum physics. By reading this leacture note one would essentially learn information theory in it's complete form. This edition is updated until July 1, 2023. For any question or corrections please contact me *Thisismeamir@outlook.com*.

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1 Introduction to Information Theory

1.1 Some People and Some History

The concept of information, was hidden in Boltzmann view on entropy and statistical mechanics. Boltzmann earns the title of founding father, without doubt. Then it was mathematics that found the potential in the concept, gave it a formalism, which was sufficient enough to be used all over the science. Shannon proposed the basic definition which is still valid. Charles Babbage, Ada Lovelace, and Alan Turing, pioneered the idea of a computer and foundations of its mathematical description. Kurt Godel, though never talked about information explicitly, but it is undisputed by now that it is the key to understanding his Incompleteness Theorem.

In physics the idea was developed within theoreticians who recognized the correspondence between information theory and quantum mechanics. John Von Neumann brought the idea of Von-Neumann entropy. Leo Szilard reduced Maxwell's demon to its very essence, a single-bit decision, bringing out for the first time the equivalence of entropy and information. In a sense the theory of information was all along a key concept for our understandings of nature and physical theories.

1.2 Three Dimensions of Information

§ **Syntactic Information** refers to the quantity and formal structure of messages. It counts signs, circumscribes their repretoire and their mutual relations. Syntax defines rules and restrictions how to compose legitimate sequences of signs, often in a hierarchial fashion. The syntactic aspect of information is relevant for physics and chemistry, for structural sciences such as mathematics and informatics, and for technology rellated to communication and computation.

§ **Semantic Information:** It is important to keep in mind that meaning is excluded in syntactic information. The meaning of signs, and more generally, communication as an exchange of messages, enter the scene only with semantic information. Semantic is dedicated to relationship between symbols and their meaning. Obviously, to apply, semantic information requires a language to exist. In the context of semantics, the concepts of sender and reciever acuire the additional aspect of understanding, not contained in syntactic information. Only on

this level, it makes sense talking about the *truth*.

§ **Pragmatic Information** finally takes into account that senders may have intentions emitting messages, and receivers may or may not react as desired by these intentions. With pragmatics, norms come into the play, the dichotomy of true and false if complemented by that of good and evil.

It is a fascinating question to be addressed on passing, exactly where inthe history of nature a phenomenon to be called "meaning" emerged for the first time. As far as we know till now, symbols, that is, objects encoding for others, did not exist in any sense before the advent of life on Earth. With the genetic code, a systematic relationship arose between one class of long-lived molecules (DNA and RNA), serving as memory, and another class of very reactive but short-lived molecules (proteins), capable of affecting and alternating the enviorment. In this context, a terminology should be introduced that will become indispensable in the discussion of self reference below: **Symbols**, irrespective of the acuired function in communication, continue forming part of objective reality, and as such can be referred to by other symbols. Symbols whose meaning is itself a symbol can be associated to a distinct, higher layer of language, called *metalanguage*, eachone forming a *metasymbol* and so on.