# Lecture on Solid State


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## 1 First Lecture

In this lecture we are going to discuss the early state of solid state physics. At first let us recall the description of Heat capacity in a volume:

$$C_v = \frac{\partial u}{\partial T}|_V \tag{1}$$

In reality the heat constant is made of two parts:

$$C_v = C_{\text{elec}} + C_{\text{lattice}} \tag{2}$$

The lattice is the network of nuckeuses, in this section we are going to look into the lattice part. The energy of the lattice is described as below:

$$u_{\text{lattice}} = \sum_k \sum_p u_{kp} \tag{3}$$

where $k$ is the network properties and $p$ is the polarization. Assuming that there exists a wave in the lattice there is a graph for possible $\omega$ in a $k = \frac{2\pi}{\lambda}$ since the wave can be longitudinal and transversive we can have polarizations.

In our simplest models we can write the equation for energy as a collection of simple harmonic oscillator:

$$u_{\text{lattice}} = \sum_{k,p} < n_{k,p} \hbar\omega_{k,p} \tag{4}$$

$n_{k,p}$ is the average heat number of occupation, meaning that in temperature $T$ how many heat equilibrium do we have. and $\hbar\omega_{k,p}$ is the energy of oscillation with momentum $k$ and polarization $p$. calculation of occupation number is to be left for lateer but the answer is of the form:

$$< n >= \frac{1}{e^{\hbar\omega} k_B T - 1} \tag{5}$$

therefore the energy would be:

$$u = \sum_{k,p} \frac{\hbar\omega_{k,p}}{e^{\beta\hbar\omega_{k,p}} - 1} \tag{6}$$

If the number of nodes in the lattice is big enough we can transform the summation into integral with a transformation multiplication:

$$\int d_k \frac{dk}{d\omega} d\omega \tag{7}$$

So the work is to find $d_k$. This is also equivalent with writing:

$$\int D_p(\omega) d\omega \tag{8}$$

Here $D(\omega)$ the phonon density of state, therefore we write:

$$u = \sum_p \int d\omega D_p(\omega) \frac{\hbar\omega}{e^{\frac{\hbar\omega}{k_B T}} - 1} \tag{9}$$

Then we have the heat capacity of lattice as:

$$C_{\text{lattice}} \equiv \frac{\partial u}{\partial T} = k_B \sum_p \int_0^\infty d\omega D(\omega) \frac{x^2 e^x}{(e^x - 1)^2} \tag{10}$$

where $x = \frac{\hbar\omega}{k_B T}$. Now let us find the phonon density let us write again:

$$D(\omega) d\omega = d_k \frac{dk}{d\omega} d\omega \tag{11}$$

We would use the periodic boundary conditions, which means that:

$$u(x) = u(x + L) \tag{12}$$

Since tha lattice has a reoccuring shape this is a good boundary condition for first investigation. fFor this to happen the first thing we can write is of the form:

$$u_0 e^{kx - \omega t} = u_0 e^{k(x+L) - \omega t} \tag{13}$$

This means simply:

$$k = \frac{2\pi}{L} n \tag{14}$$

here $n$ is $0, \pm1, \pm2, \ldots, \pm N$ where $N$ is the number of cells. One can find the allowed $k$s of $k$-space (here 1 dimensional) with:

$$\text{Number of allowed } \ k \equiv \frac{\text{Volume of } \ k}{\text{Distance of nodes}} = \frac{K}{\frac{2\pi}{L}} \tag{15}$$

We can easily write this for a 3 dimensional lattice, by just having:

$$u(\vec{x}) = u(\vec{x} + \vec{L}) \tag{16}$$

and for three dimensions the number of allowed $k$ are:

$$N_k = \frac{\frac{4}{3}\pi k^3}{\left(\frac{2\pi}{L_x}\right)\left(\frac{2\pi}{L_y}\right)\left(\frac{2\pi}{L_z}\right)} = \frac{\frac{4}{3}\pi k^3}{\frac{8\pi}{V}} \tag{17}$$

It is obvious that since the number of modes with a polarization $p$ between $\omega$ and $\omega + d\omega$ is as follows:

$$D(\omega) d\omega \tag{18}$$

leading to the formula:

$$D(\omega) = \frac{dN}{d\omega} \to dN = d\omega D(\omega) \tag{19}$$

Thus we can write :

$$D(\omega) = \frac{dN}{d\omega} \tag{20}$$

$$= \frac{dN}{dk} \frac{dk}{d\omega} \tag{21}$$

$$= \frac{4\pi k^2}{\left(\frac{2\pi}{L}\right)^3} \frac{dk}{d\omega} \tag{22}$$

$$D(\omega) = \frac{V k^2}{2\pi^2} \frac{dk}{d\omega} \tag{23}$$

to have $dk/d\omega$ we have to use approximation: 1. Debay approximation 2. Einsteins approximation.

## 1.1 Debay Approximation

debay noticed that The first third Dispersion Curve can be approximated linearly:

$$\omega \propto k \tag{24}$$
$$\omega = vk \tag{25}$$

therefore $\frac{dk}{d\omega} = \frac{1}{v}$, and we can find the phonon density:

$$D(\omega) = \frac{Vk^2}{2\pi^2}\frac{1}{v} \tag{26}$$

since this is an approximation we don't go over all $k$s, instead we would go until a known $k$ called debay's $k$, or $k_D$. therefor ethe number of all modes would be:

$$N = \frac{\frac{4}{3}\pi k_D^3}{(2\pi)^3/V} \tag{27}$$

And the energy would become:

$$u = \int d\omega D(\omega) < n(\omega) > \hbar\omega = \int_0^{\omega_D} d\omega \left(\frac{V\omega^2}{2\pi^2 v^3}\right) \frac{\hbar\omega}{e^{\beta\hbar\omega} - 1} \tag{28}$$