TAIWAN TECH

Welcome to

EMI 2021 SUMMER PROJECT FOR FRESHMAN YEAR

WHOA!

LET'S GET TO KNOW EACH OTHER A LITTLE BIT MORE!

MORRIS

Sophomore (2nd year)
Electrical Engineering
and Computer Science

JUDY

Sophomore (2nd year)
IATP,
Civil and Construction Engineering

INTRODUCTION

This session is created in order for the freshman students to be familiar with the usage of english in classroom situation along with the essential Calculus-related vocabularies that all will be facing during the 1st year in college.

TABLE OF CONTENTS
CLASS AUG 17

00

ICE BREAKING

Let's break the ice!

I GOT YOUR BACK (支援前線)

BASIC RULES：Bring things we ask as fast as possible, the loser will have to hold the next round.

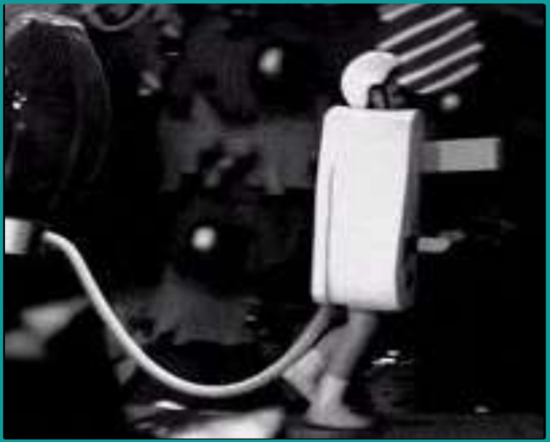
BEFORE WE ALL BEGIN..

- ➔ The students are encouraged to **open their cameras** from time to time so we all could get to know each other more.
- ➔ All the students are encouraged to use **English** in this class. There is no right or wrong!
- ➔ **Do not hesitate to ask any questions!** Whether it is in Chinese or English but bear in mind that English is more preferable.
- ➔ The questions can be asked either by students speaking or leaving them in the chatroom.
- ➔ If there are any extra question regarding the class you all can contact us via LINE or any other platforms!

01

CLASSROOM ENGLISH

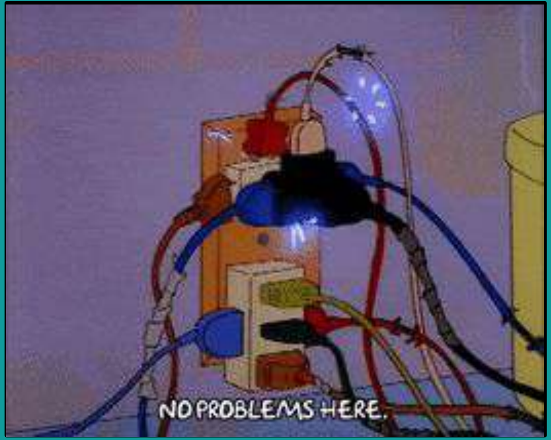
P.10~16



Plug in / Plug out

Ex:

1. Please plug into the socket.
2. Please pull the socket out.



Extension Cord

Ex: Do you have an extension cord?

Projector

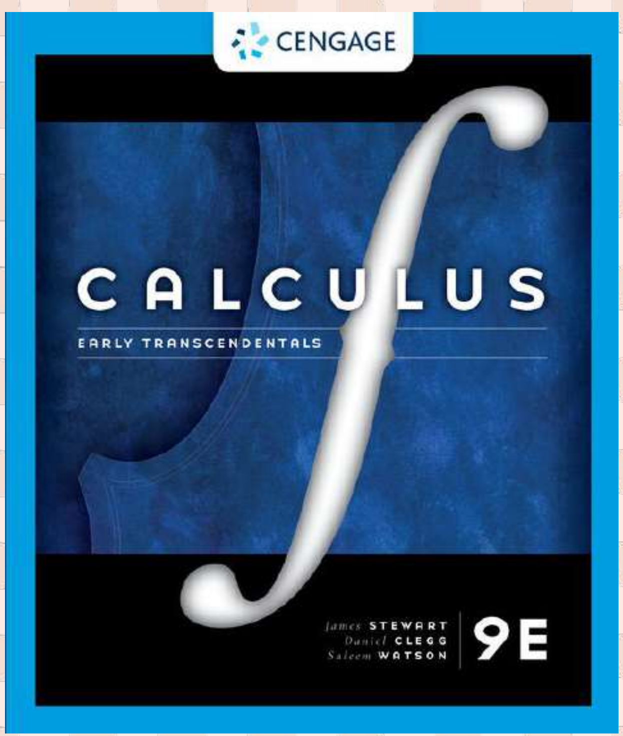
Ex:

1. Please switch on / off the **projector.**
2. The image on the projection screen is blurry (模糊).
3. The bulb of the projector has blown (燒壞).
4. Please connect the laptop to the projector.
5. We have to adjust the focus(焦距) of the projector.

02

CALCULUS VOCABS

World-list 1 - 20



TEXT BOOK

Calculus: Early Transcendentals, Metric Edition, 9th Edition,

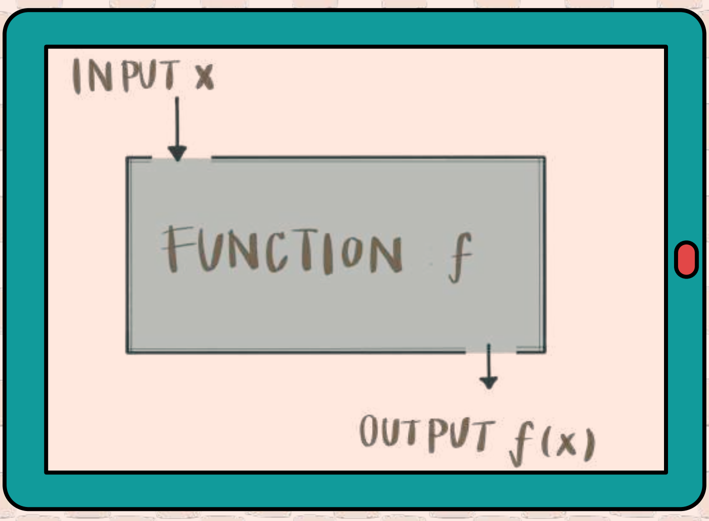
by James Stewart, Daniel K. Clegg, Saleem Watson, Lothar Redlin, 2020
Cengage Learning

1.

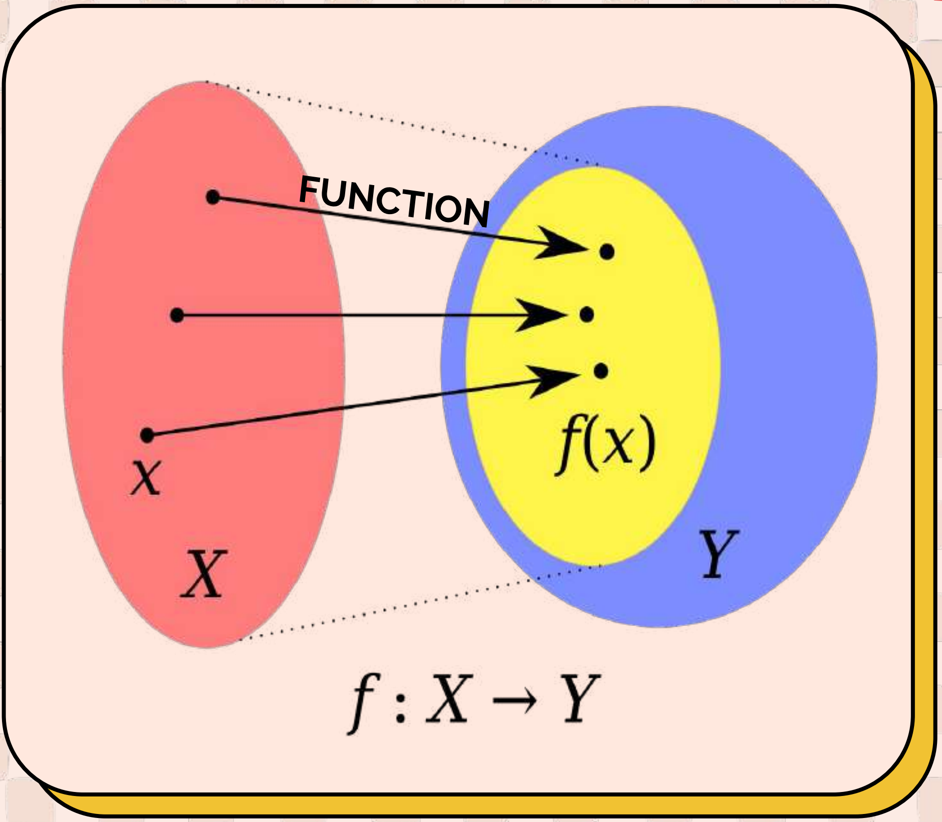
CALCULUS

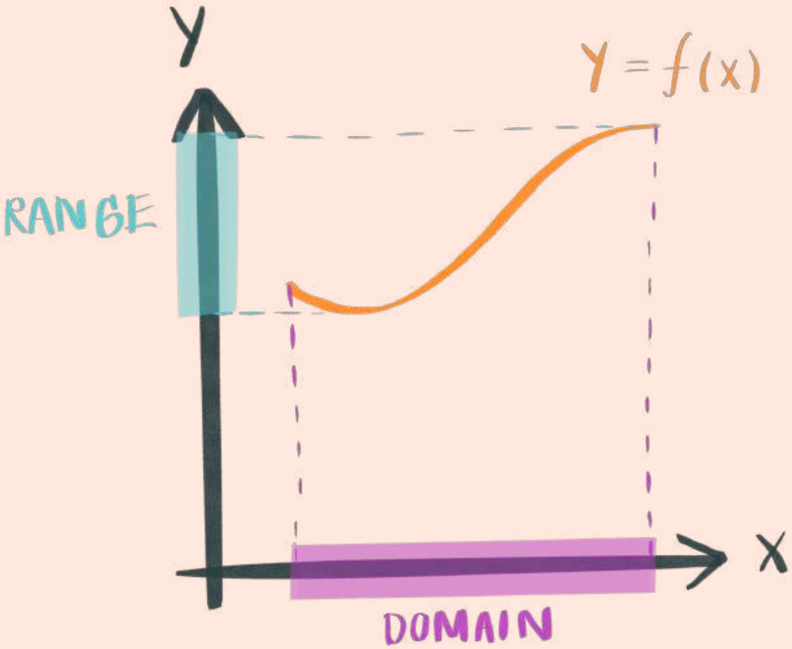
The branch of mathematics that deals with the finding and properties of **derivatives and integrals of functions**, by methods originally based on the summation of infinitesimal differences. The two main types are differential calculus and integral calculus.

FUNCTION



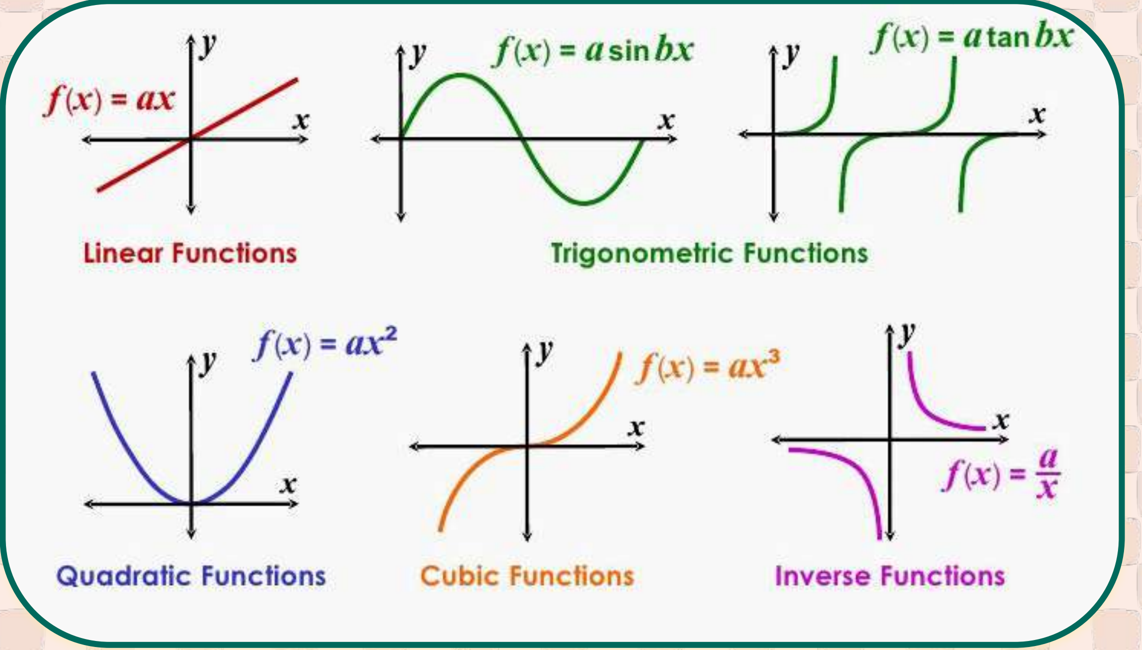
DOMAIN & RANGE





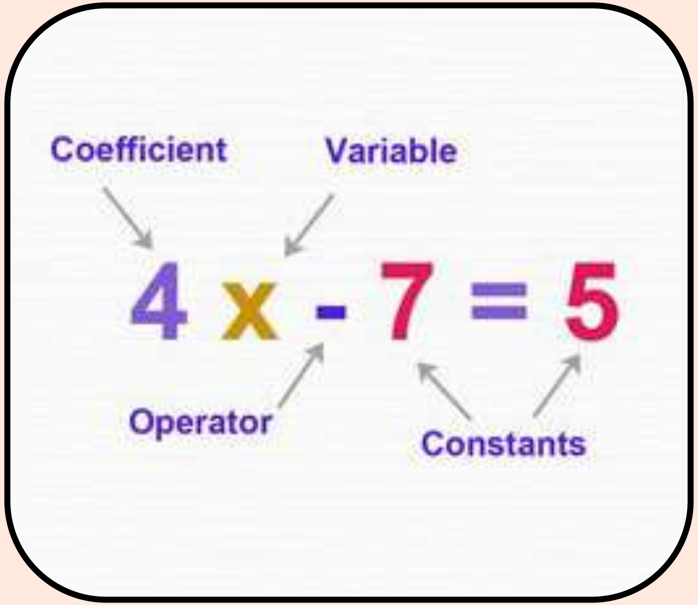
DOMAIN & RANGE IN GRAPH

GRAPH

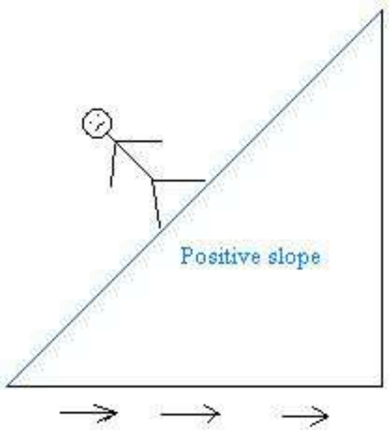


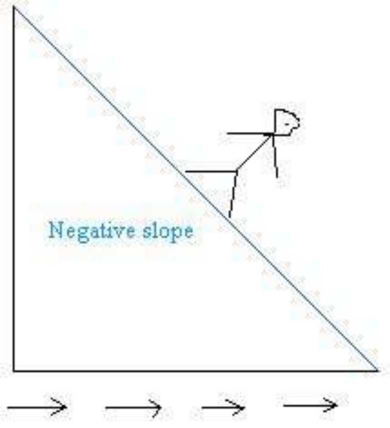
$f(x) = ax$
Linear Functions
$f(x) = a \sin bx$
$f(x) = a \tan bx$
Trigonometric Functions
$f(x) = ax^2$
Quadratic Functions
$f(x) = ax^3$
Cubic Functions
$f(x) = \frac{a}{x}$
Inverse Functions

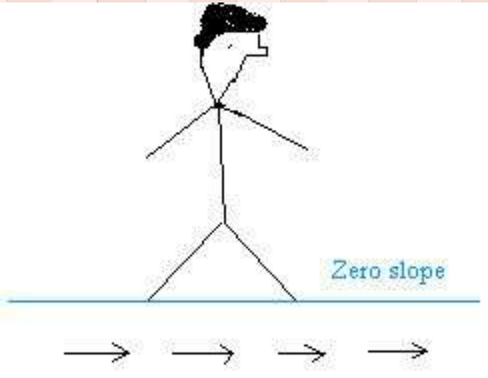
VARIABLE

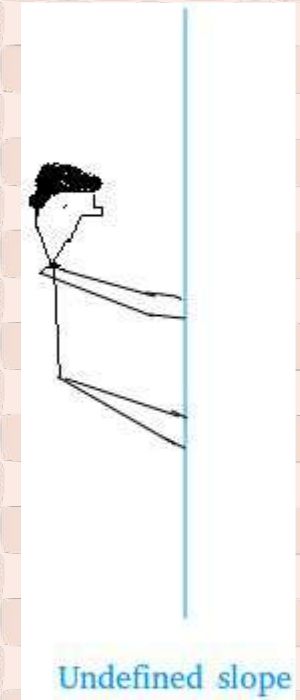


SLOPE

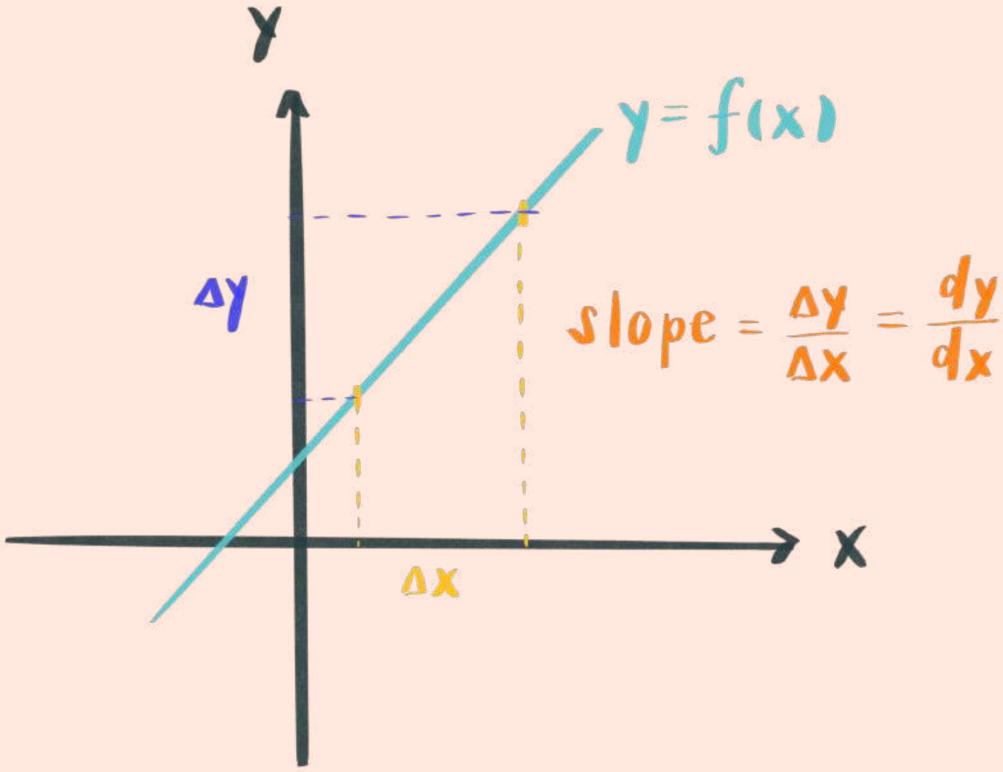






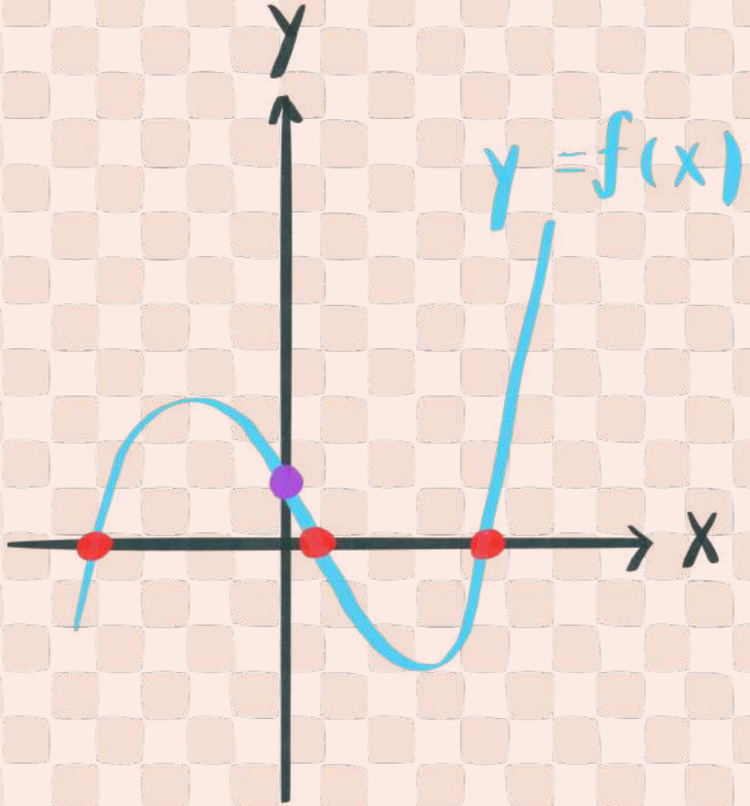


slope is defined as you move from left to right.



y
y= f(x)
Δy
slope = $\frac{\Delta y}{\Delta x} = \frac{dy}{dx}$
Δx
x

INTERCEPT



X-INTERCEPT

Y-INTERCEPT

ABSOLUTE VALUE

$$|x| = \begin{cases} -x, & if\ x < 0 \\ x\,, & if\ x \geq 0 \end{cases}$$

$$|x|$$

CHECK-UP

CALCULUS

GRAPH

FUNCTION

DOMAIN

INTERCEPT

SLOPE

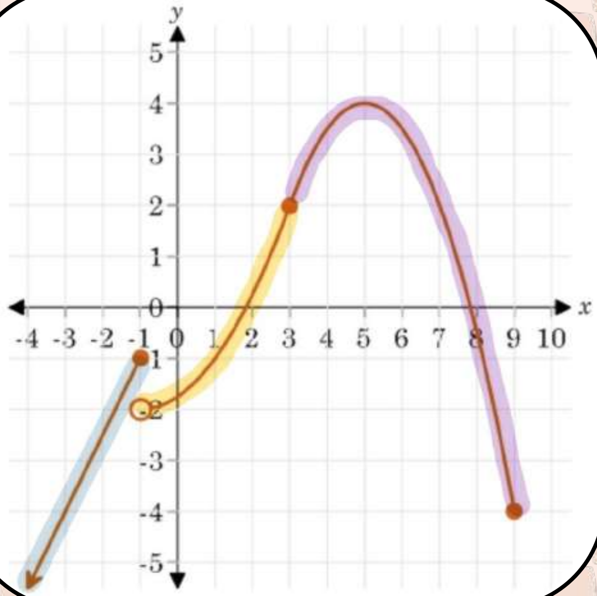
VARIABLE

RANGE

ABSOLUTE VALUE

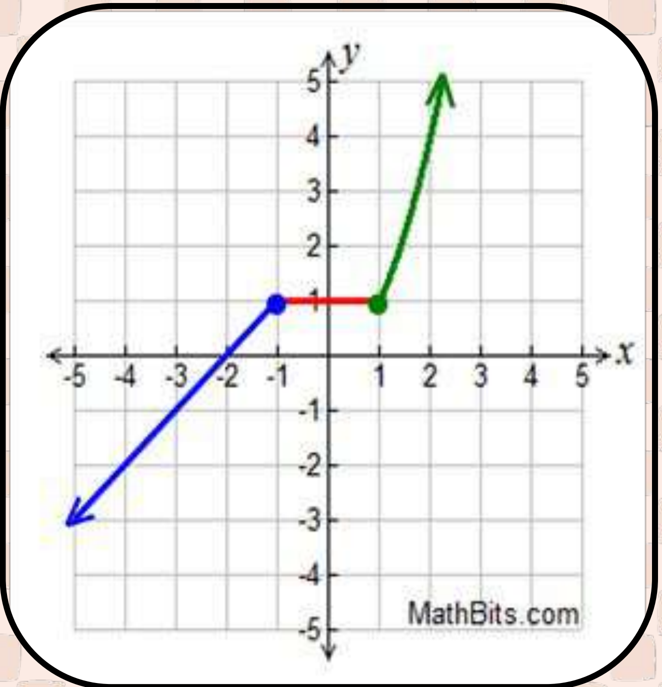
PIECEWISE FUNCTION

$$f(x)=\begin{cases} \frac{3}{2}x+\frac{1}{2}, & x\le -1 \\ \frac{1}{4}(x+1)^2-2, & -1<x\le 3 \\ -\frac{1}{2}(x-3)^2+4, & 3<x\le 9 \end{cases}$$



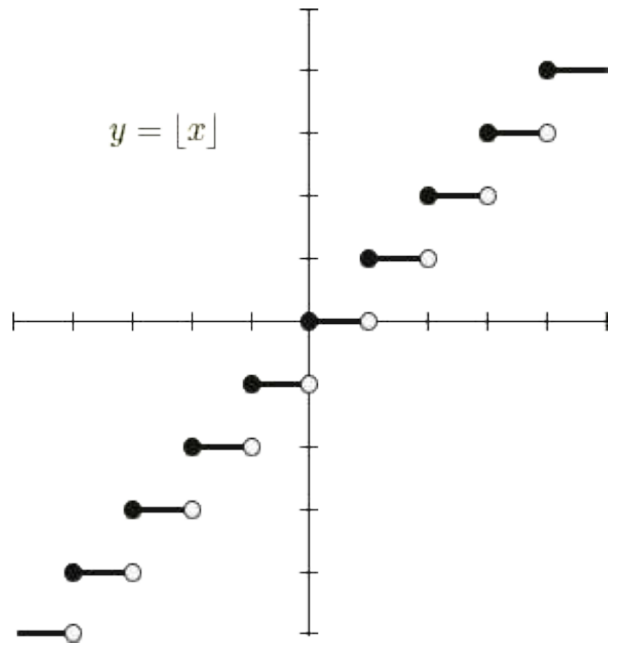
PIECEWISE DEFINED FUNCTION

$$f(x)=\begin{cases} x+2; & x\leq -1 \\ 1; & -1<x<1 \\ x^2; & x\geq 1 \end{cases}$$

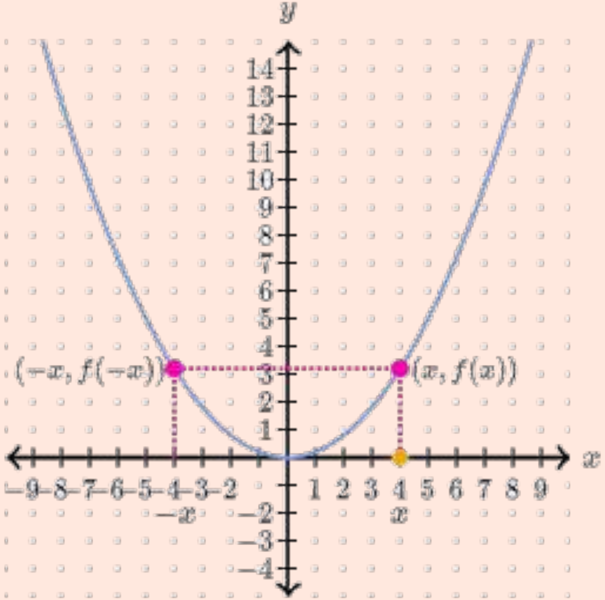


STEP FUNCTION

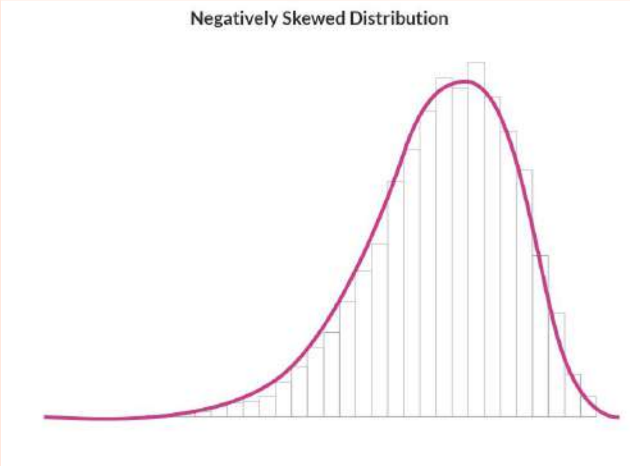
$$f(x) = \begin{cases} -2, & x < -1 \\ 0, & -1 \leq x \leq 2 \\ 2, & x > 1 \end{cases}$$



SYMMETRY VS. ASYMMETRY

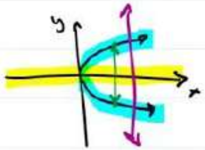


SYMMETRIC

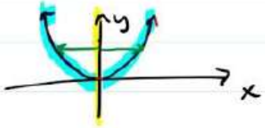


ASYMMETRIC

1. Symmetry about the x-axis

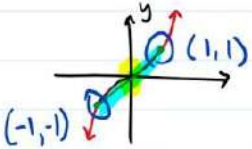


2. Symmetry about the y-axis



(these are Even functions)

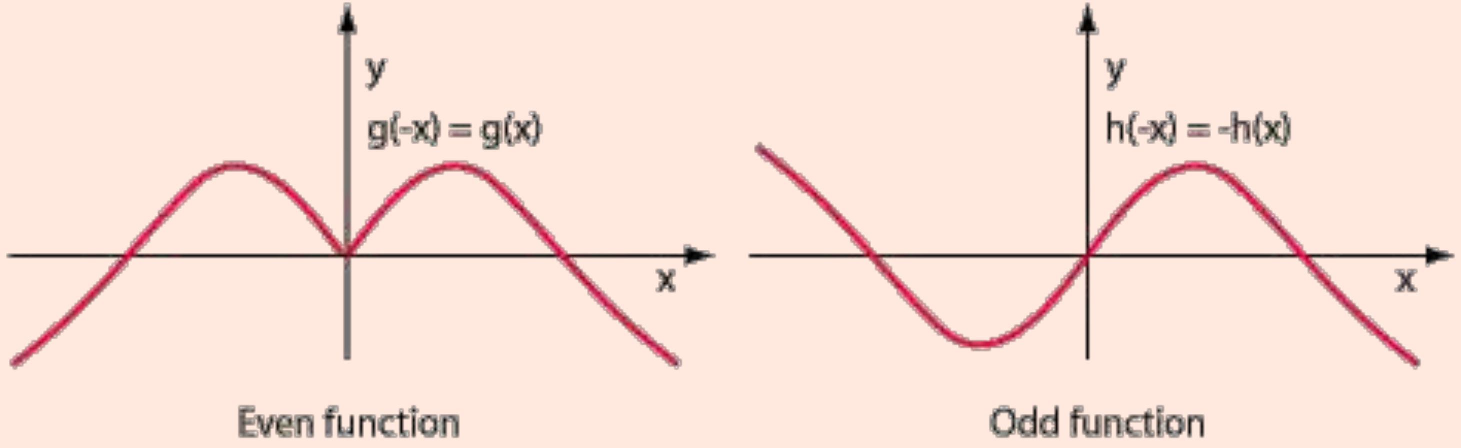
3. Symmetry about the origin



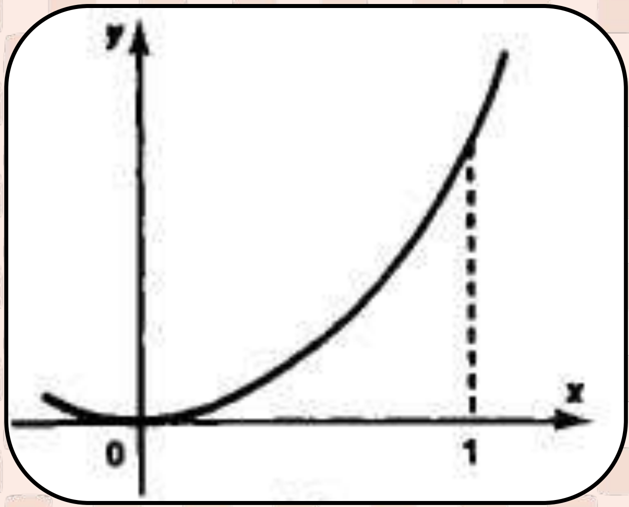
(these are Odd functions)

3 TYPES OF FUNCTION SYMMETRY

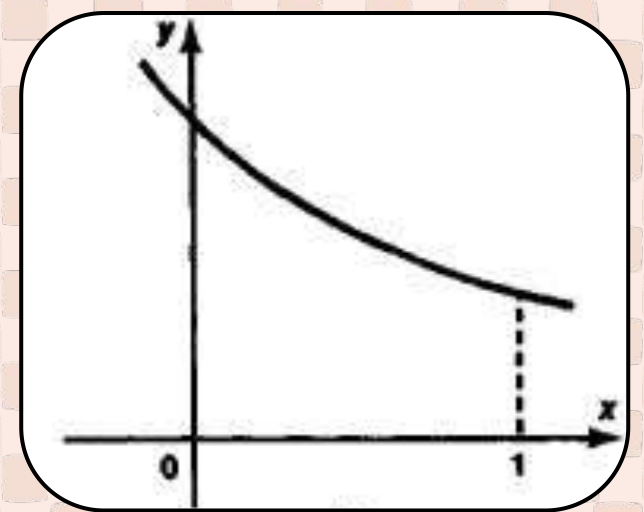
EVEN&ODD FUNCTIONS



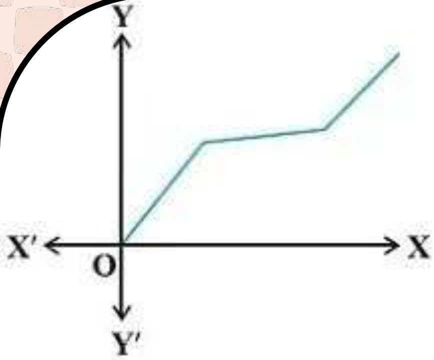
Increasing vs. Decreasing Function



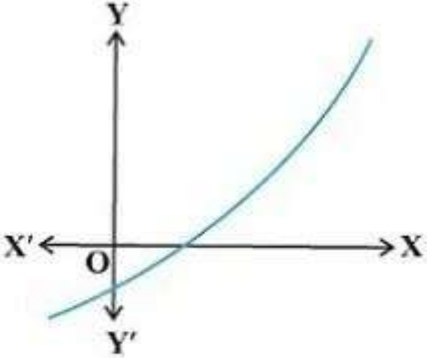
Increasing



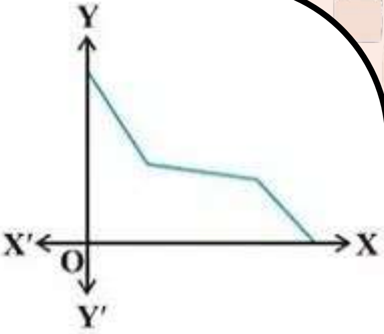
Decreasing



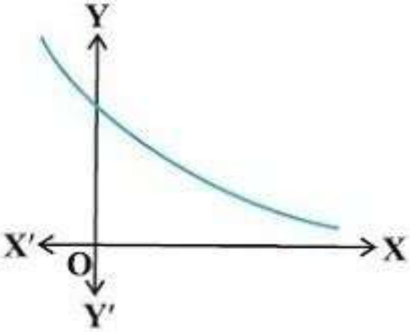
Increasing function
(i)



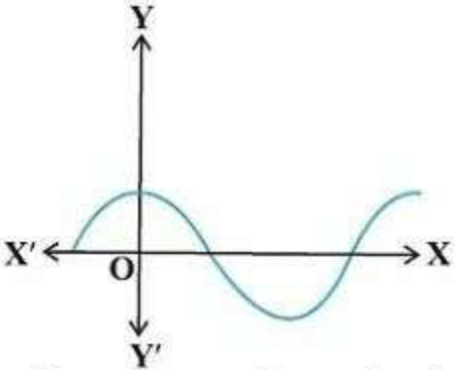
Strictly Increasing function
(ii)



Decreasing function
(iii)



Strictly Decreasing function
(iv)



Neither Increasing nor Decreasing function
(v)

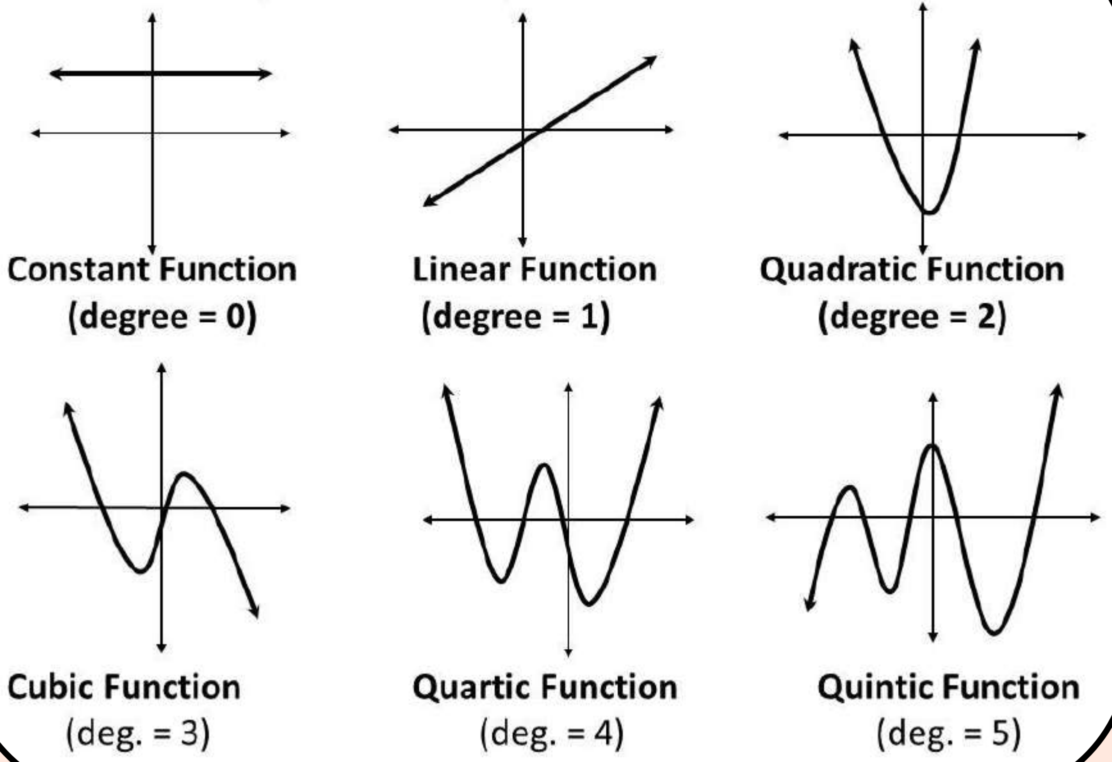
POLYNOMIAL FUNCTION

Leading coefficient Degree

$$f(x) = \underbrace{a_n x^n}_{\text{Leading term}} + \ldots.. + a_2 x^2 + a_1 x + a_0$$

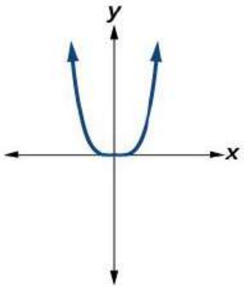
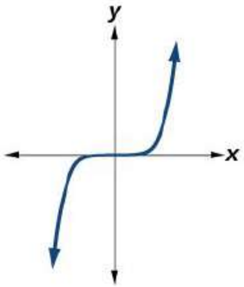
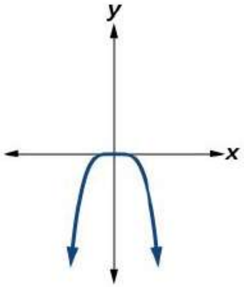
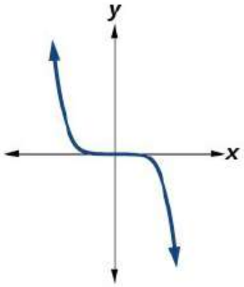
leading term contains the highest power of the variable, or the term with the highest degree.

Graphs of Polynomial Functions:



POWER FUNCTION

$$f(x) = kx^p$$

	Even power	Odd power
Positive constant $k > 0$	 $x \to -\infty$, $f(x) \to \infty$ and $x \to \infty$, $f(x) \to \infty$	 $x \to -\infty$, $f(x) \to -\infty$ and $x \to \infty$, $f(x) \to \infty$
Negative constant $k < 0$	 $x \to -\infty$, $f(x) \to -\infty$ and $x \to \infty$, $f(x) \to -\infty$	 $x \to -\infty$, $f(x) \to \infty$ and $x \to \infty$, $f(x) \to -\infty$

LINEAR FUNCTION

- $f(x) = x$
- $f(x) = 2x - 2$
- $f(x) = x + 1$

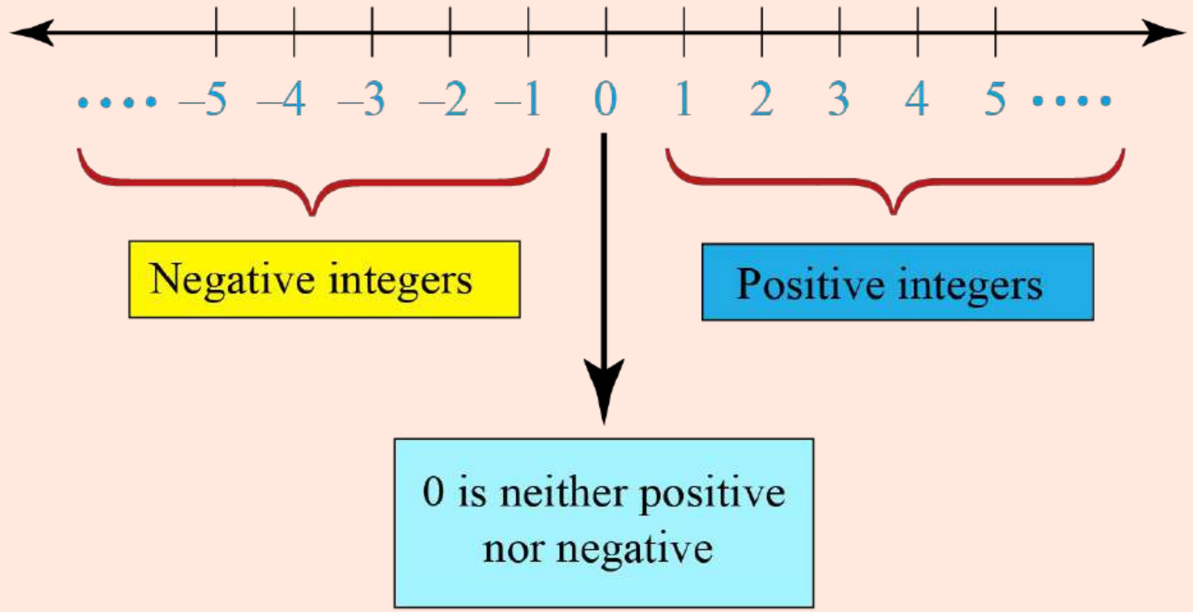
QUADRATIC FUNCTION

- $f(x) = x^2$
- $f(x) = ax^2 + bx + c$

CUBIC FUNCTION

- $f(x) = ax3 + bx2 + cx + d$

INTEGER

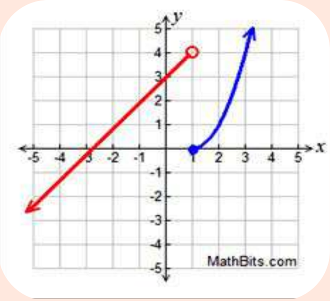


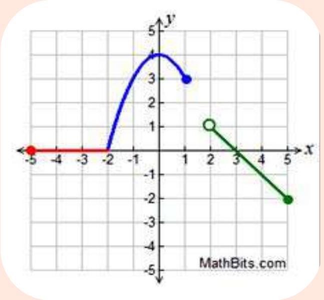
.... –5 –4 –3 –2 –1 0 1 2 3 4 5 ....
Negative integers
Positive integers
0 is neither positive nor negative

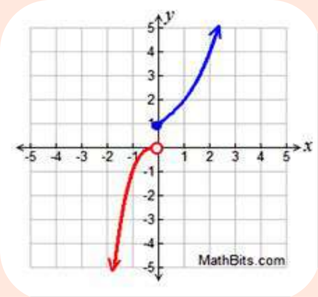
SUM-UP!

Feel free to ask us via LINE!

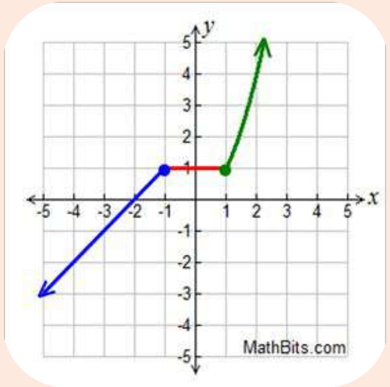
PIECEWISE FUNCTION



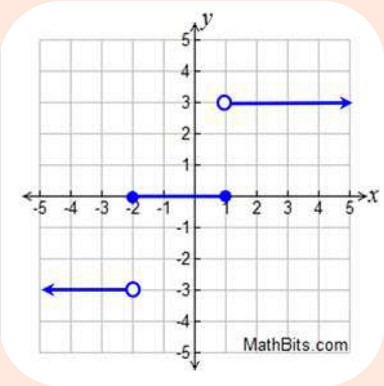




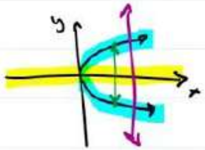
PIECEWISE DEFINED FUNCTION



PIECEWISE STEP FUNCTION

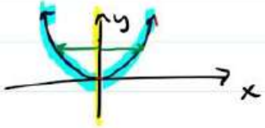


1. Symmetry about the x-axis



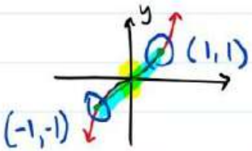
2. Symmetry about the y-axis

(these are Even functions)

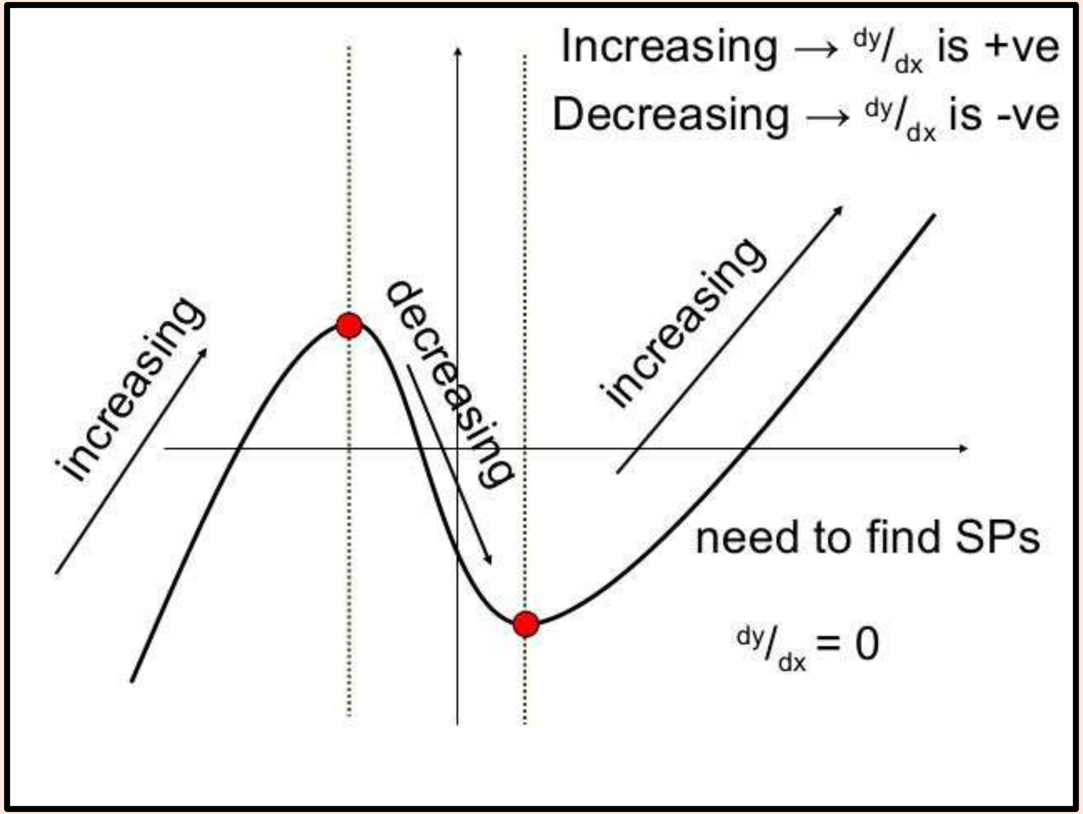


3. Symmetry about the origin

(these are Odd functions)



FUNCTION SYMMETRY & ODD-EVEN


Increasing → dy/dx is +ve
Decreasing → dy/dx is -ve
increasing
decreasing
increasing
need to find SPs
dy/dx = 0

POLYNOMIAL FUNCTION

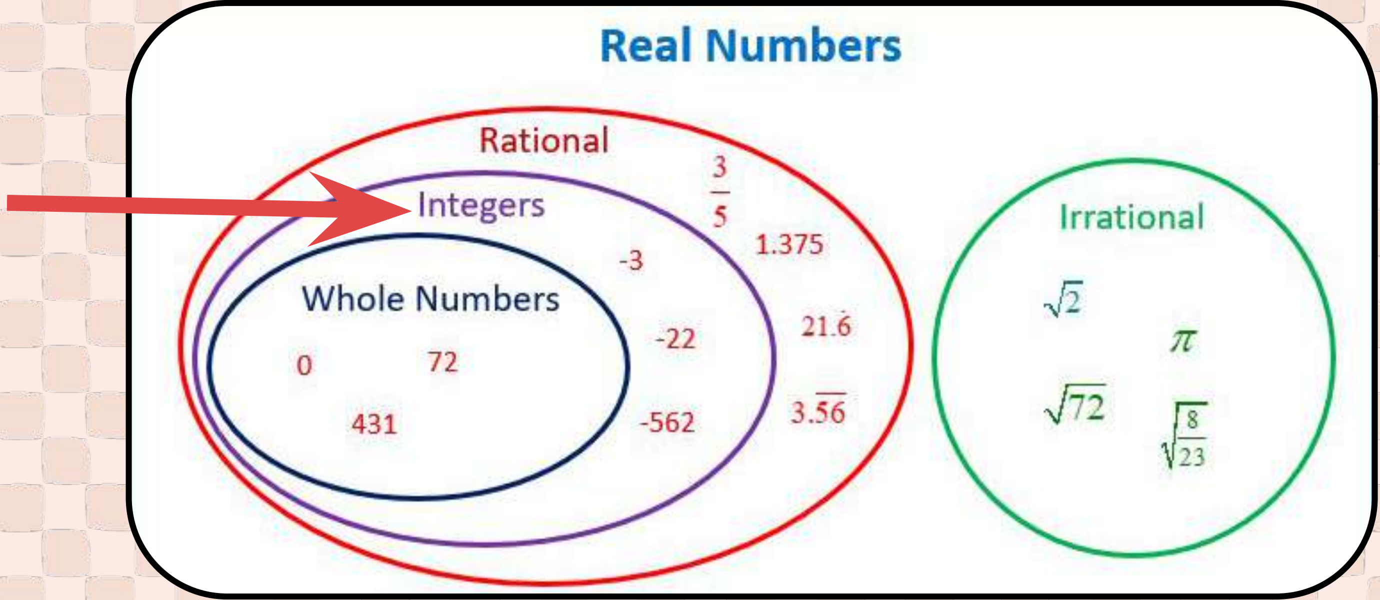
$f(x_i)=a_i x^I$

Degree	Name	Example
0	Constant	5
1	Linear	$2x+3$
2	Quadratic	x^2-x+5
3	Cubic	$3x^3+23$
4	Quartic	x^4+2x^3-3
5	Quintic	$x^5+2x^3-.5x+3$

POWER FUNCTION

$f(x) = kx^p$

Degree	Name	Example
0	Constant	23
1	Linear	$2x$
2	Quadratic	x^2
3	Cubic	$3x^3$
4	Quartic	x^4
5	Quintic	x^5



Real Numbers
Rational
Integers
Whole Numbers
0
72
431
-3
-22
-562
3/5
1.375
21.6
3.56
Irrational
√2
π
√72
√(8/23)

AWESOME JOB FOR TODAY!

TAIWAN TECH

Welcome to

EMI 2021 SUMMER PROJECT

2nd session

01

CLASSROOM ENGLISH

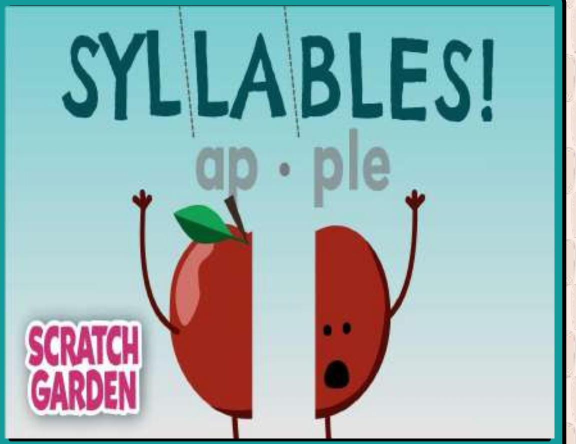
P.17~21

EXPLANATION
ENGLISH

P.17~21

As I said earlier.

就如我上次所說



The stress is on the first syllable.

重音在第一音節

We'll take that up in the next class.

我們下堂課在討論這一點

有聽懂嗎？

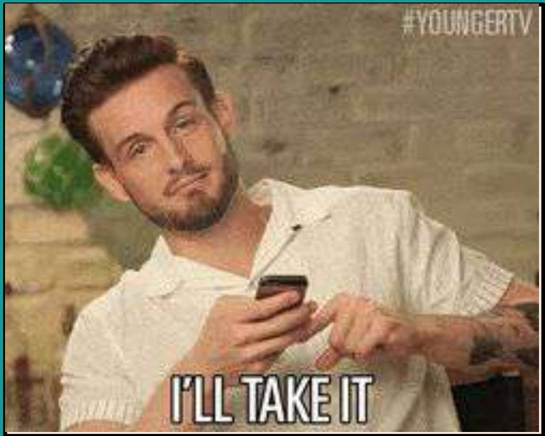
Can you follow me?
Are you with me?
Do you get it?
So far so good?

你的想法呢？

Whats do you have in mind?

What do you reckon?

做個調查



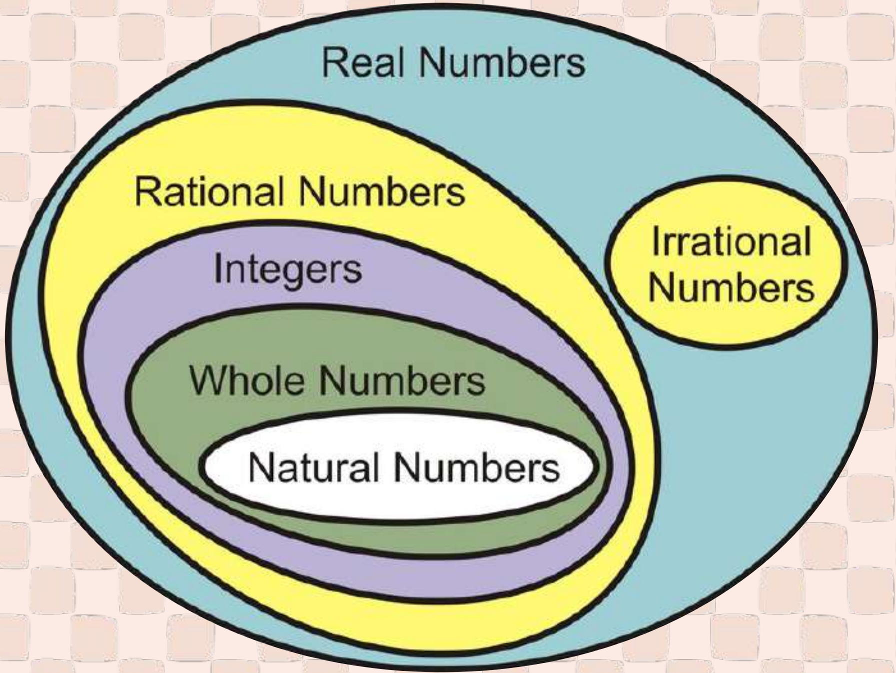
Let's take a poll / survey.

02

CALCULUS VOCABS

World-list 21 - 40

REAL NUMBER

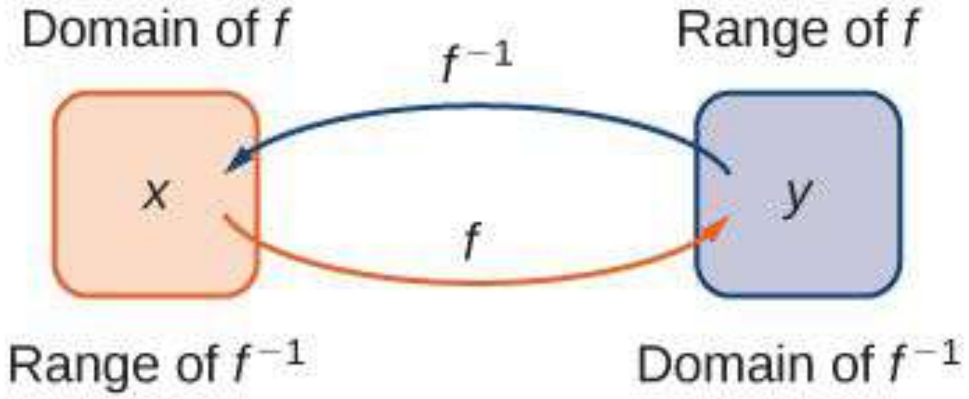


Real Numbers
Rational Numbers
Integers
Whole Numbers
Natural Numbers
Irrational Numbers

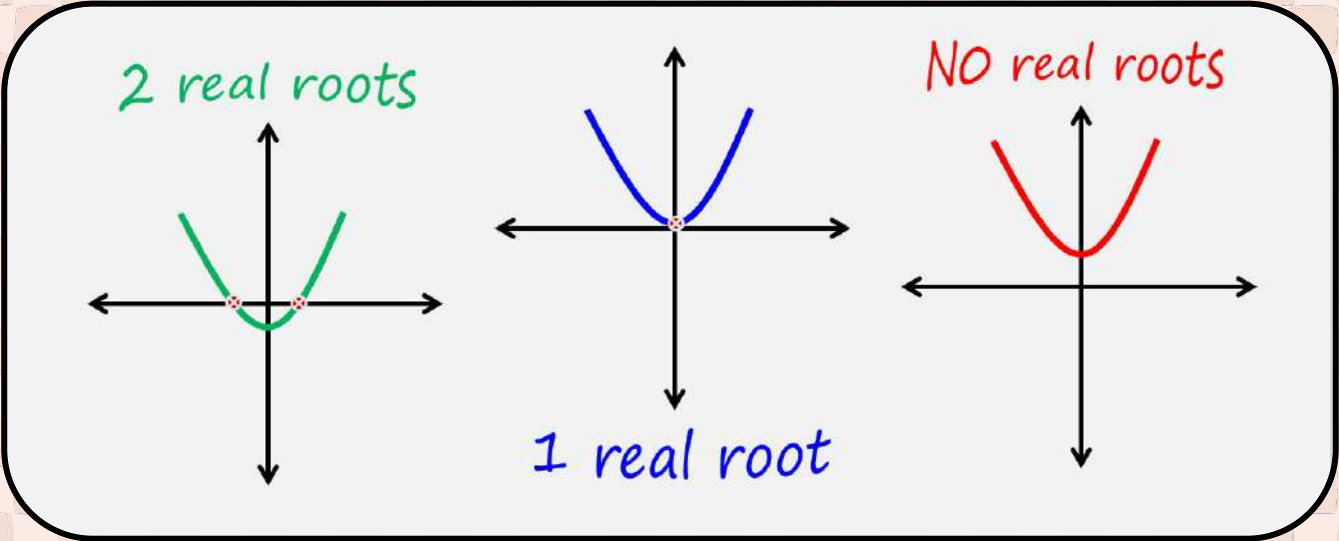
ELEMENTARY FUNCTION

Types	Algebraic	Trigonometric	Exponential & Logarithmic
Examples	Polynomial	Sine	
	Radical	Cosine	
	Rational	Tangent	
		Cotangent	
		Secant	
		Cosecant	

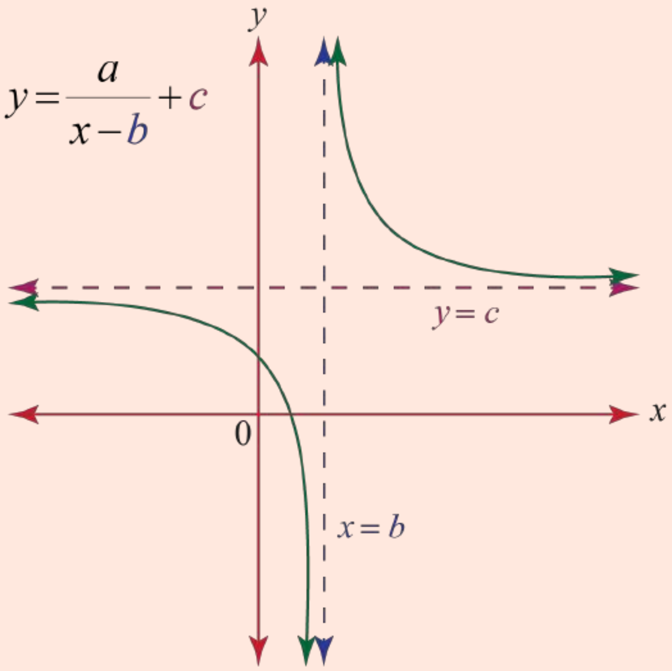
INVERSE FUNCTION



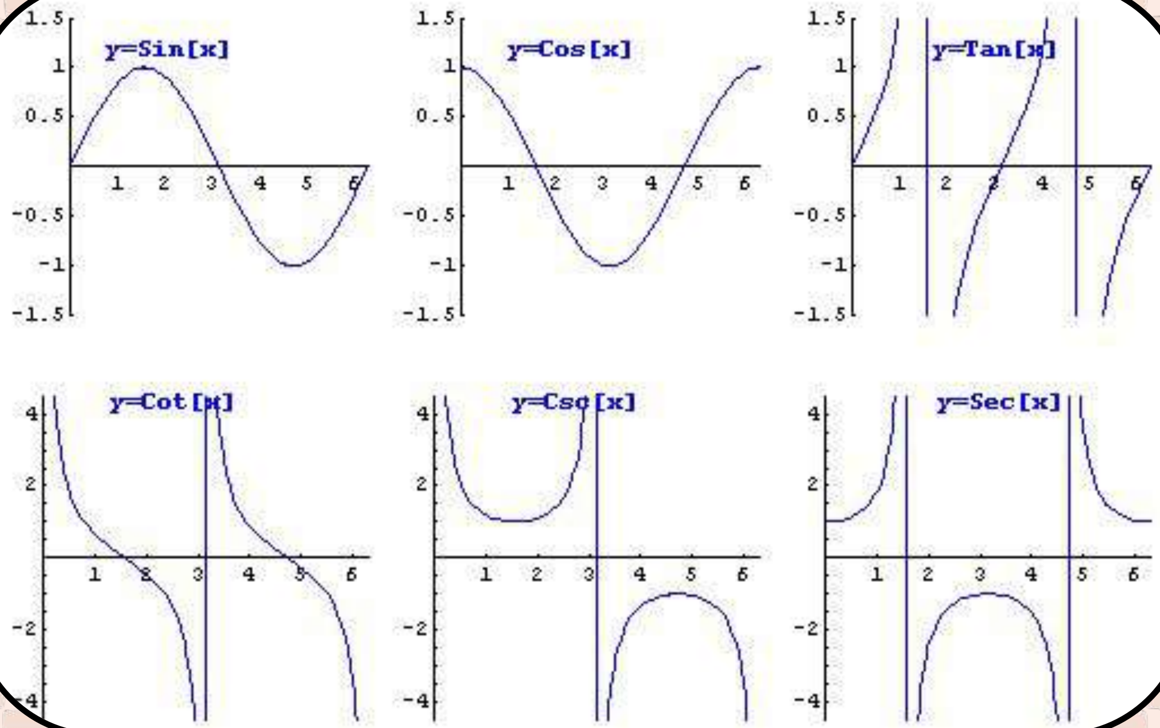
ROOT FUNCTION



RATIONAL FUNCTION

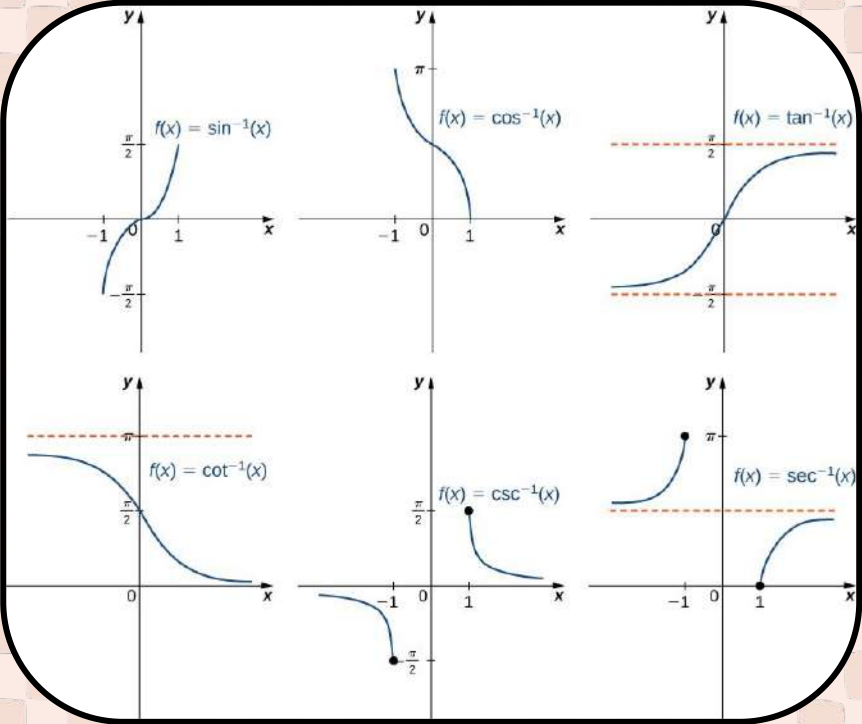


TRIGONOMETRIC FUNCTIONS

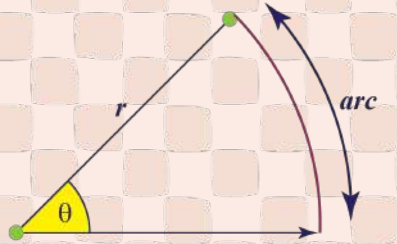


INVERSE TRIGONOMETRIC FUNCTION

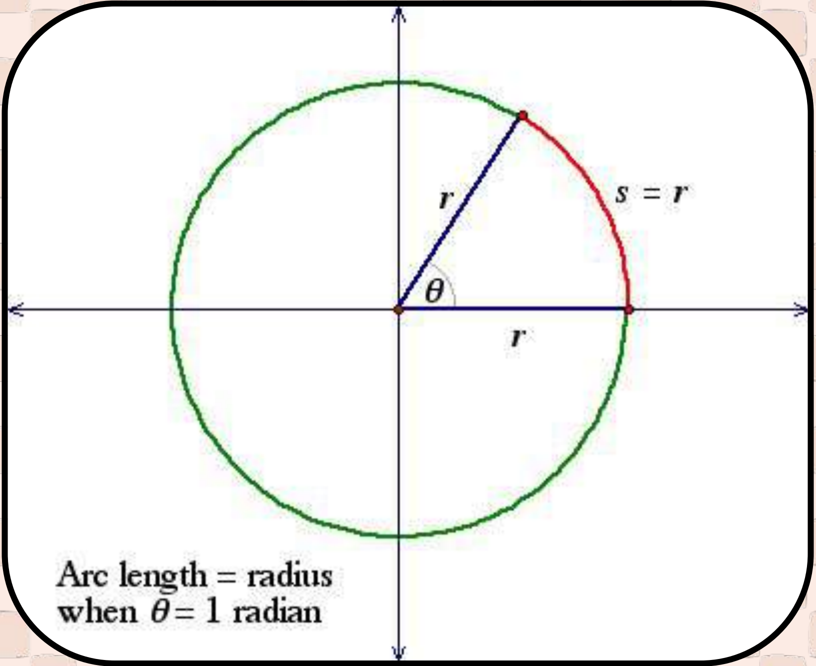
Or 'Arc Function'



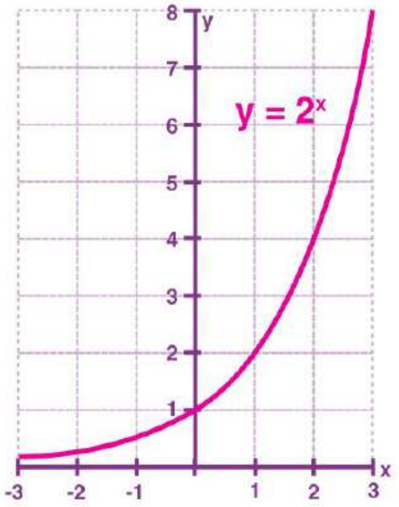
RADIAN

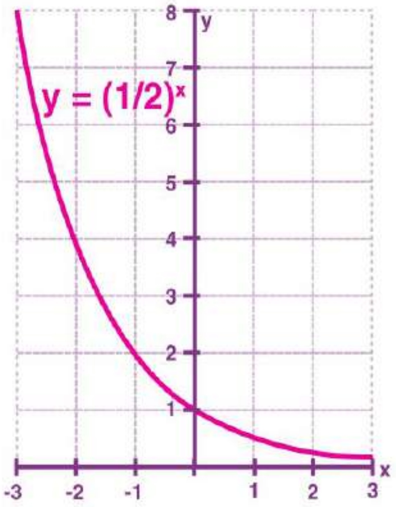


$$\theta = \frac{\text{arc}}{\text{radius}}$$



EXPONENTIAL FUNCTION



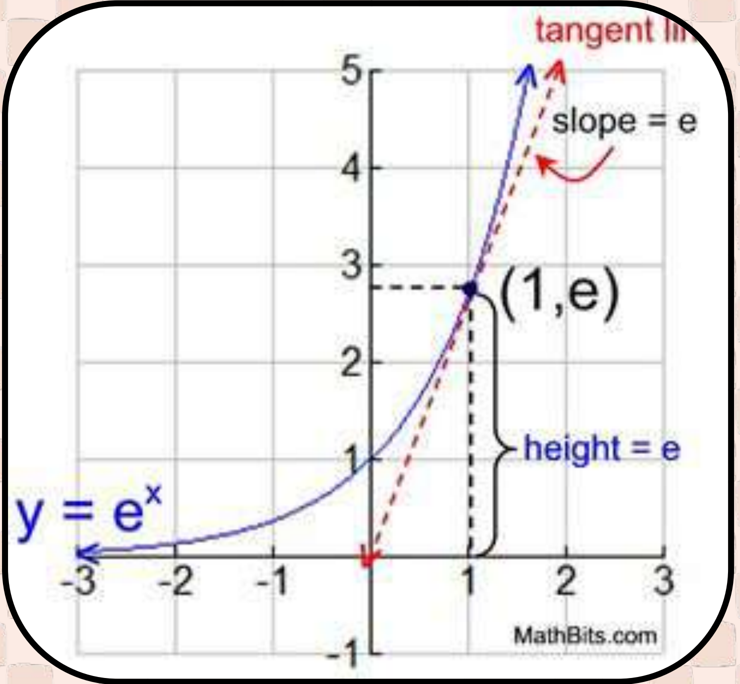


$$f(x) = b^x$$

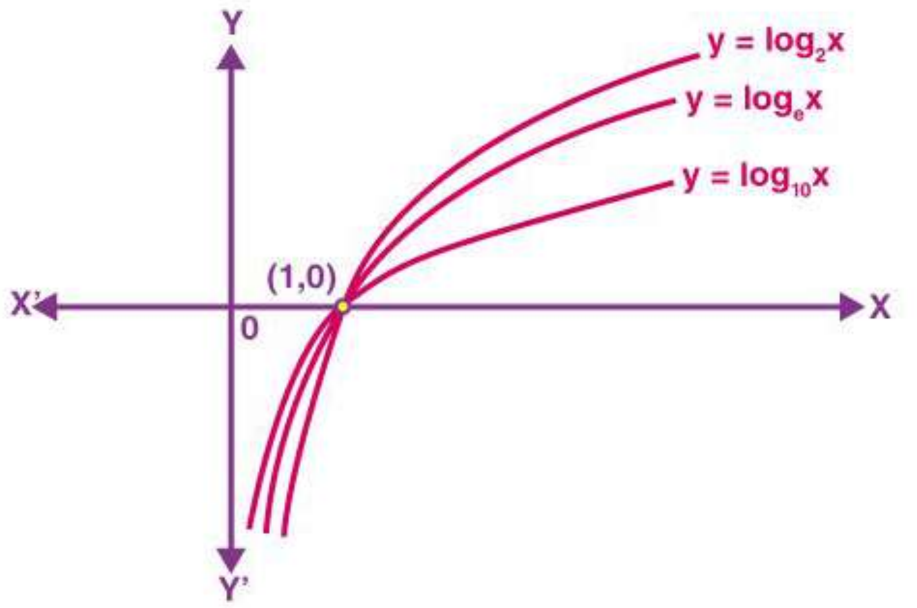
NATURAL EXPONENTIAL FUNCTION

$f(x) = e^x$

e = 2.71828

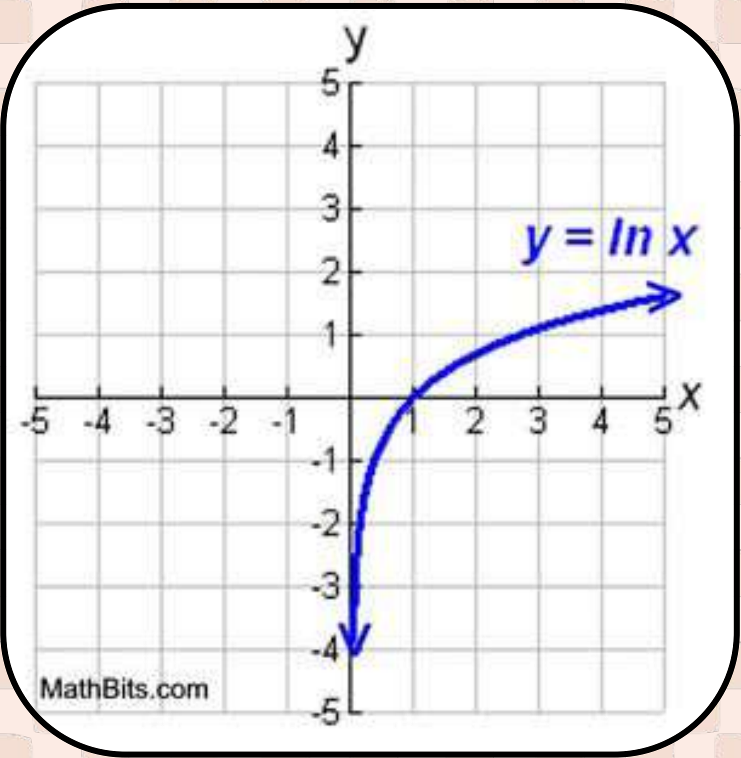


LOGARITHM FUNCTION



NATURAL LOGARITHM FUNCTION

$f(x) = \log_e x$
or more commonly
$f(x) = \ln x$



COMPOSITE FUNCTION

$$\boxed{(f \circ g)(x) = f(g(x))}$$

$$f(x) = 3x + 2 \qquad [g(x) = 4x - 5]$$

$$\begin{aligned} f[g(x)] &= 3[g(x)] + 2 \\ &= 3(4x - 5) + 2 \\ &= 12x - 15 + 2 \\ &= 12x - 13 \end{aligned}$$

LIMIT

$$\lim_{\underbrace{x \to a}_{\text{"As you approach } a \text{ along the x-axis"}}} \overbrace{f(x)}^{\text{function}} = \underbrace{L}_{\text{"What is the y-value getting closer to?"}}$$

ONE-SIDED LIMIT

LEFT-HANDED LIMIT

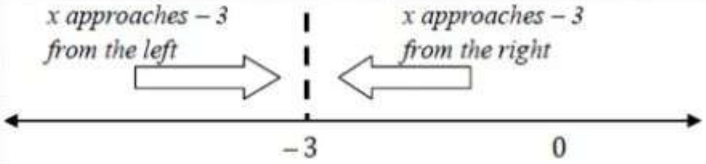


Figure 2.1 x approaching - 3

$$\lim_{x\to -3^-} f(x)? \qquad \lim_{x\to -3^+} f(x)?$$

RIGHT-HANDED LIMIT

INFINITE LIMIT

The “lim” tells us we’re looking for a limit value, not a function value.

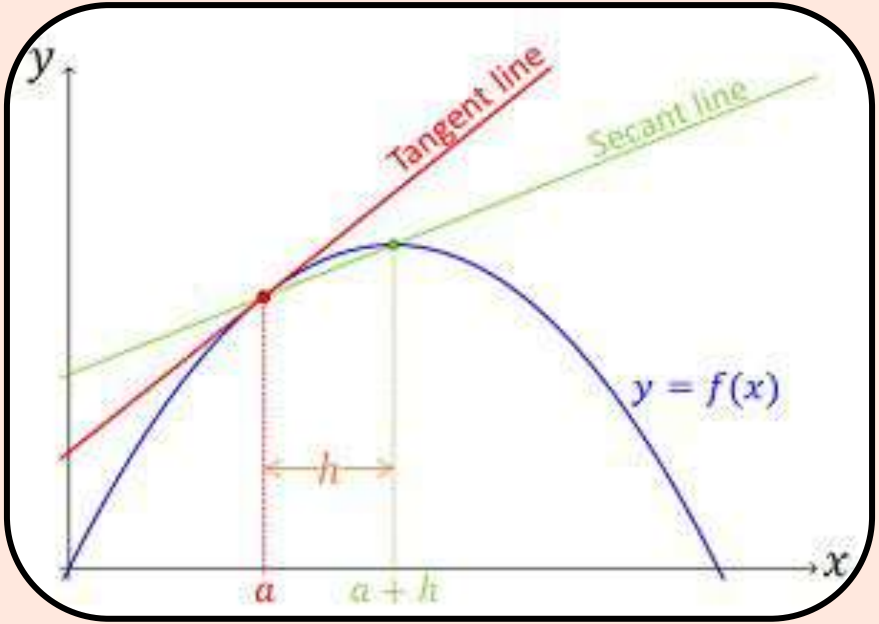
This tells us which function we’re working with.

$$\lim_{x\to a} f(x) = \infty$$

This tells us what the variable is, and what it is approaching.

The limit does not exist, but we know the function is growing infinitely large.

TANGENT & SECANT LINE



DERIVATIVE

$$\frac{dy}{dx} = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

$$m = \lim_{x \to a} \frac{f(x) - f(a)}{x \to a}$$

QUICK CHECK-UP!

REAL NUMBER

a number which can be expressed using decimal expansion

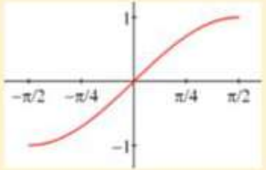
ELEMENTARY FUNCTION

power, exponential, and **trigonometric** functions, their inverses, and finite combinations of them

INVERSE FUNCTION

function f^{-1} with domain R and range D such that $f^{-1}(y)=x$

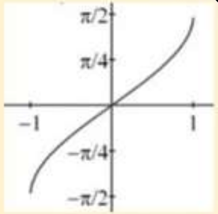
TRIGONOMETRIC & ITS INVERSE



y = sin x

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

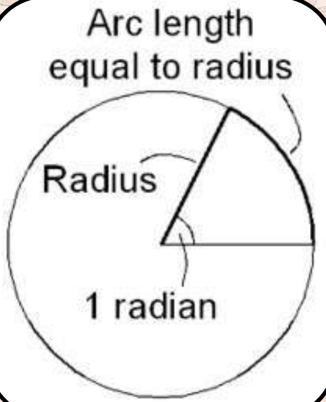
Range: [-1, 1]

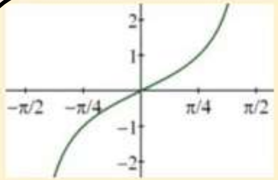


y = arcsin x

Domain: [-1, 1]

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

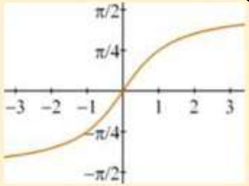




y = tan x

Domain: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$

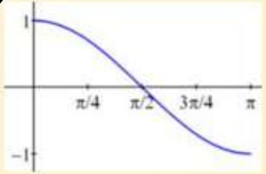
Range: $[-\infty, \infty]$



y = arctan x

Domain: $[-\infty, \infty]$

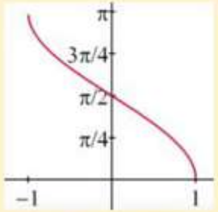
Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



y = cos x

Domain: $[0, \pi]$

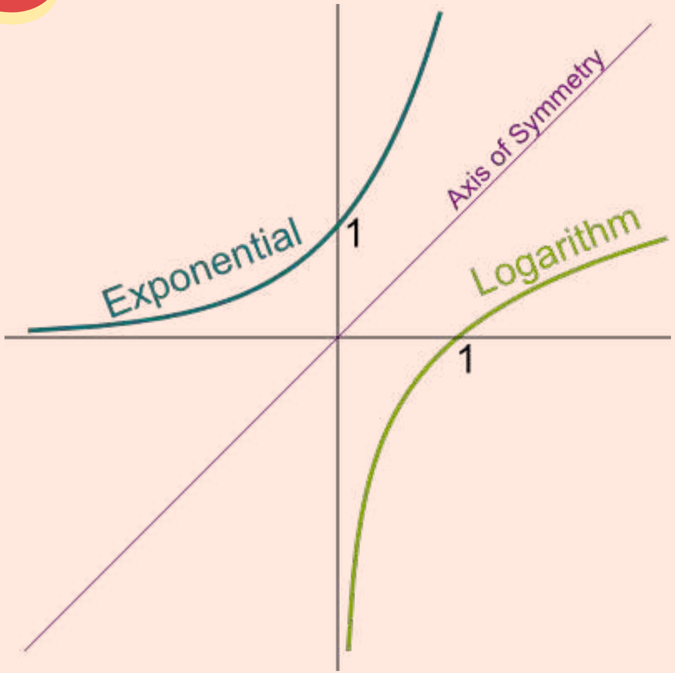
Range: [-1, 1]

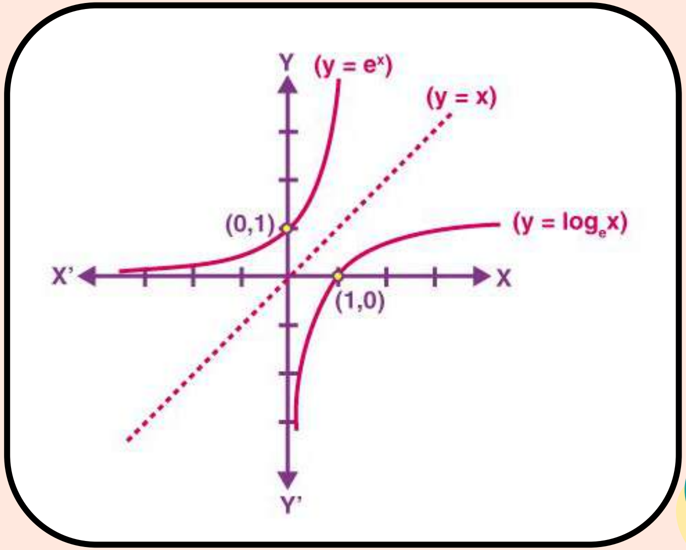


y = arccos x

Domain: [-1, 1]

Range: $[0, \pi]$





logarithmic function is an inverse function to exponential function

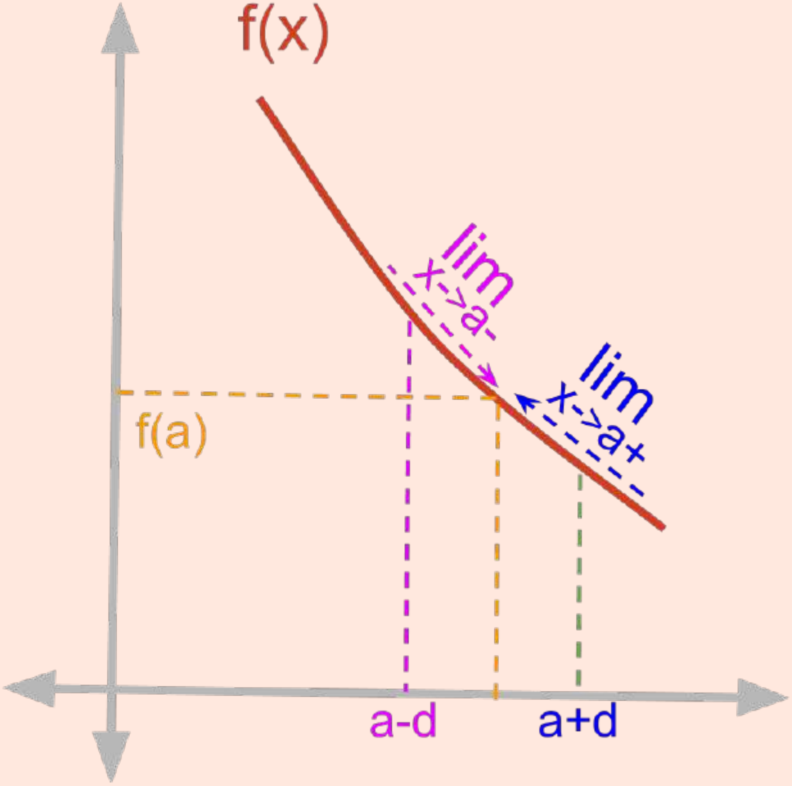
RATIONAL FUNCTION

$$f(x) = \frac{p(x)}{q(x)},$$

$q(x) \neq 0$

COMPOSITE FUNCTION

$$(f \circ g)(x) = f[g(x)]$$



LIMIT

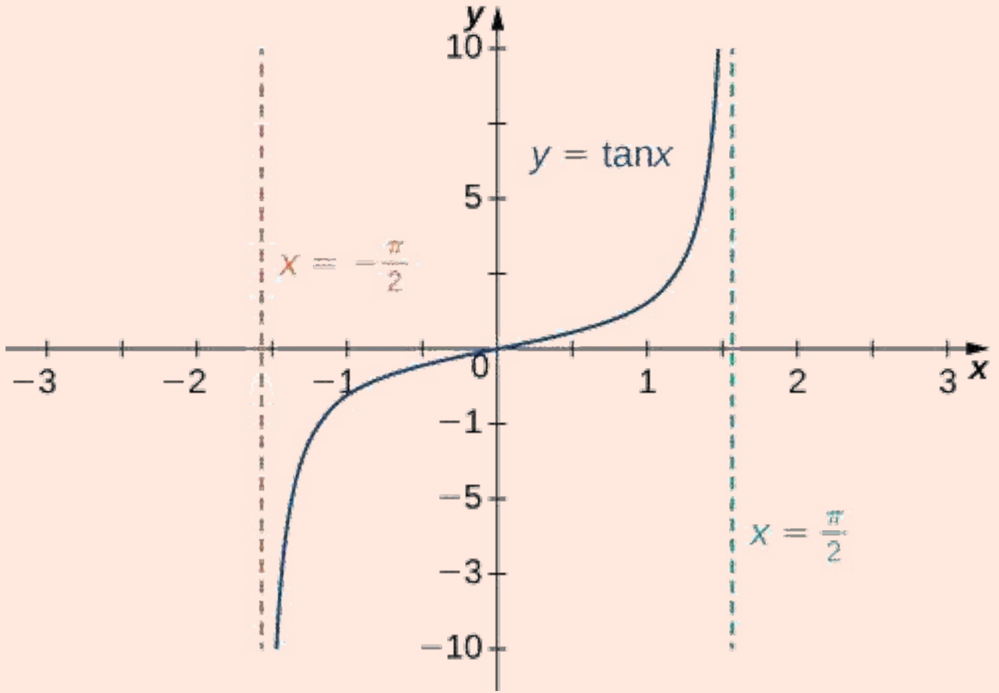
ONE-SIDED LIMIT

LEFT-HAND LIMIT

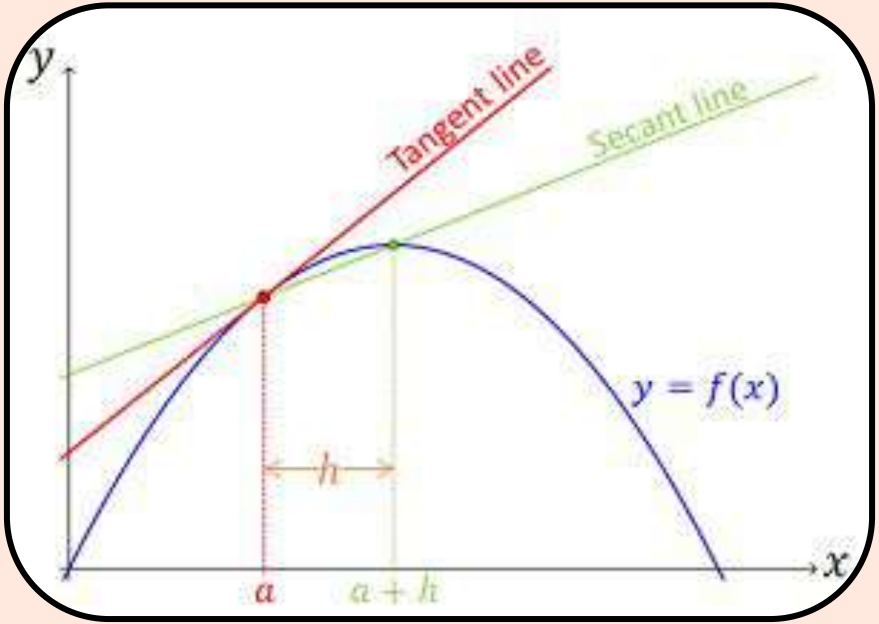
$$\lim_{x \to a^-} f(x) = L$$

RIGHT-HAND LIMIT

$$\lim_{x \to a^+} f(x) = L$$



INFINITE LIMIT



TANGENT & SECANT LINE

DERIVATIVE

slope of a line that is tangent to the function

AWESOME JOB FOR TODAY!

TAIWAN TECH

Welcome to

EMI 2021 SUMMER PROJECT

3nd session

01

CLASSROOM ENGLISH

P.17~21

EXPLANATION
ENGLISH

P.22~27

做得好

Terrific!
Wonderful!
Marvelous!
Fantastic!

太棒了

I’m impressed.

awesome!

進步

You’ve made a lot of progress.

You’ve improved a lot.

差一點就答對了

You're halfway there.

You're on the right lines.

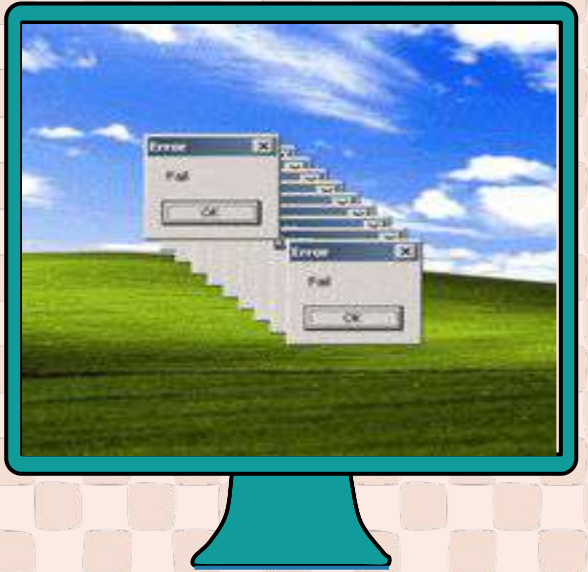
You've almost got it.

某種程度上是(大概)

It might be, I suppose.

Sort of.

Error 錯誤



This is a very common error.

Clue 線索

Do you have any clue?

Settle down

坐下來、安頓

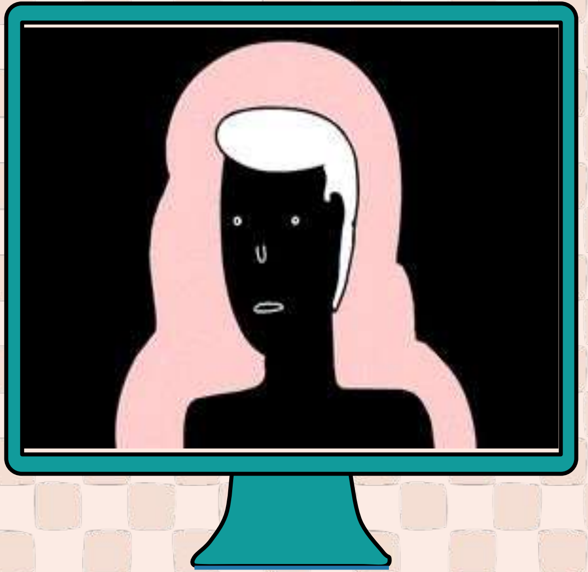
Tidy up

整理乾淨

Listen up

聽好了、注意

Wander off 發呆



Don’t wander off on your own.

發講義

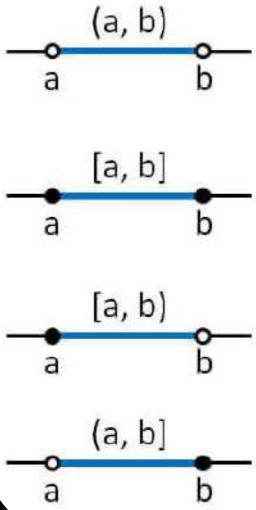
Please hand out the worksheet/copies.

02

CALCULUS VOCABS

World-list 41 - 60

OPEN VS. CLOSED INTERVAL



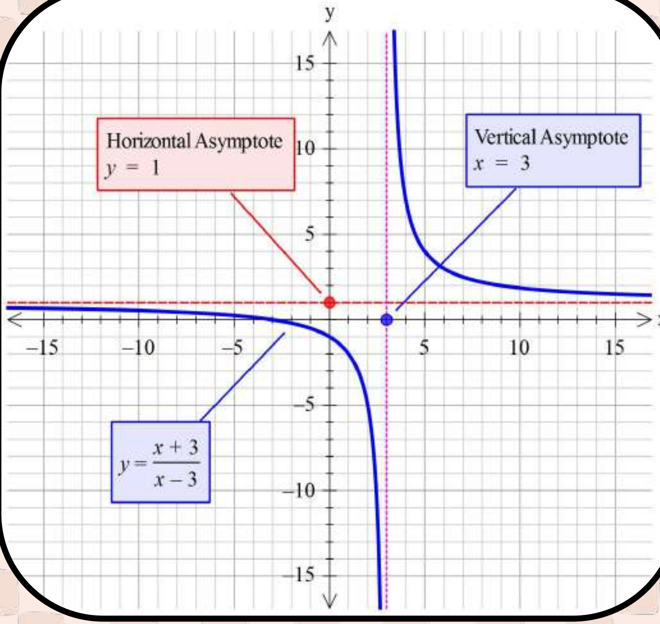
$a < x < b$, Open interval

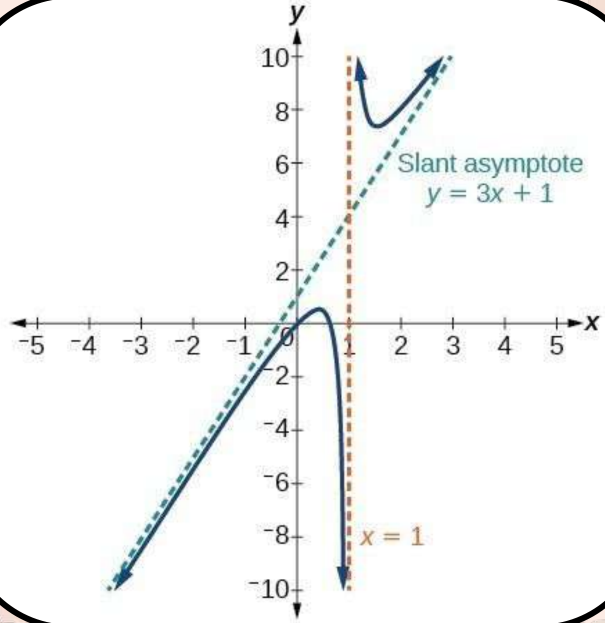
$a \leq x \leq b$, Closed interval

$a \leq x < b$, Semi Open interval

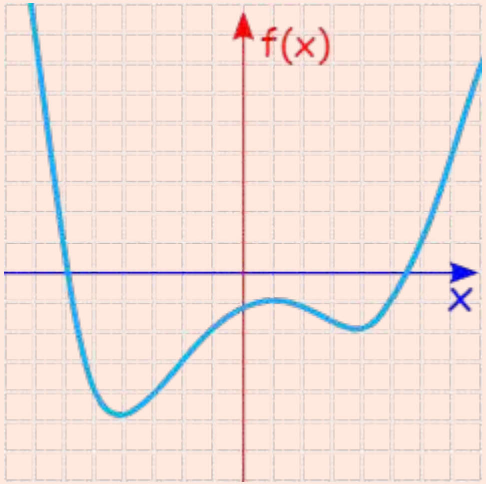
$a < x \leq b$, Semi Open interval

VERTICAL/ HORIZONTAL/ SLANT ASYMPTOTE

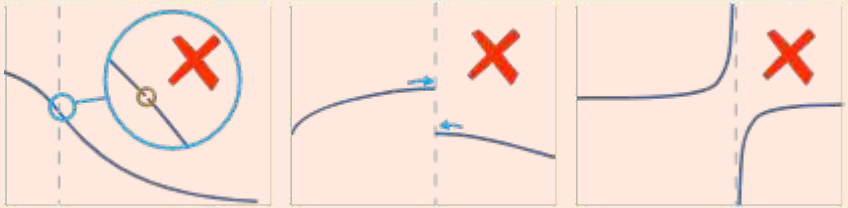




CONTINUOUS FUNCTION



DISCONTINUOUS



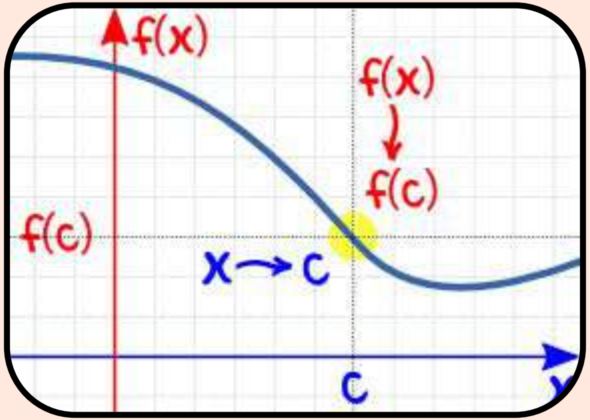
HOLE

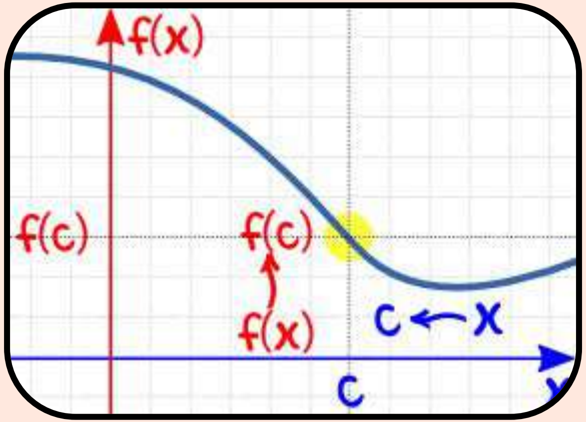
JUMP

VERTICAL ASYMPTOTE

CONTINUITY

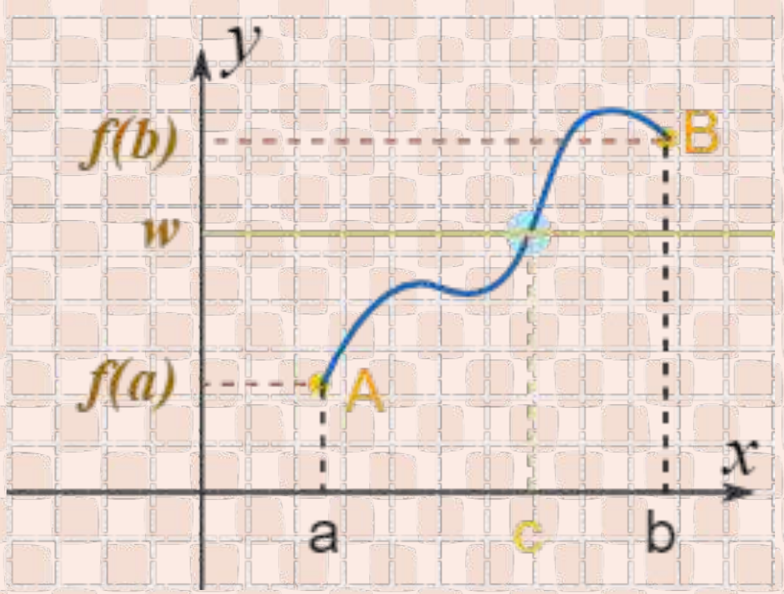
DISCONTINUITY



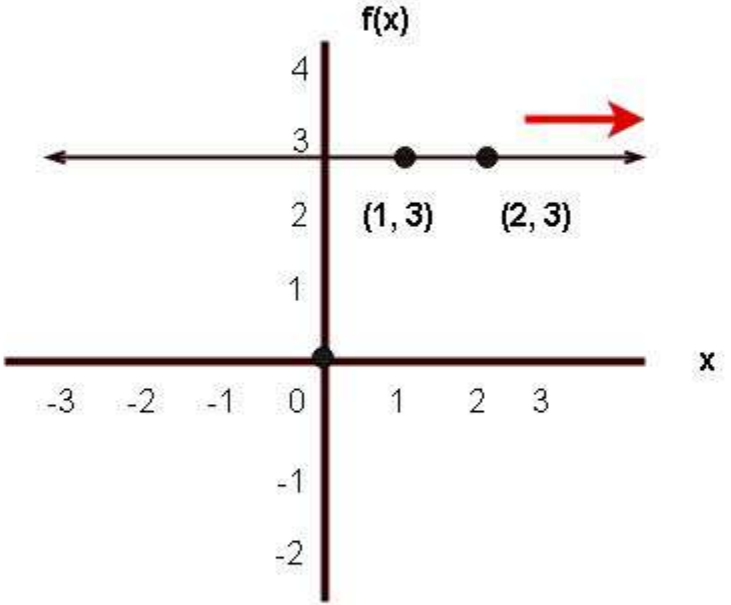


$$\lim_{x \to c} f(x) = f(c)$$

INTERMEDIATE VALUE THEOREM



CONSTANT FUNCTION



MONOTONIC FUNCTION

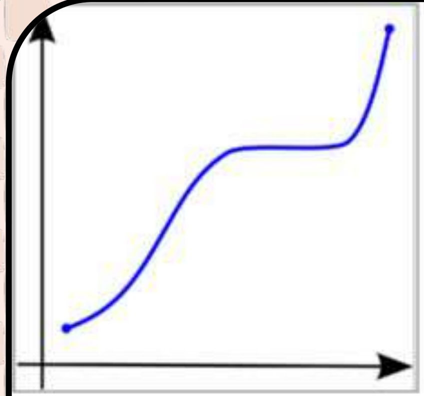


Figure 1 - A monotonically increasing function

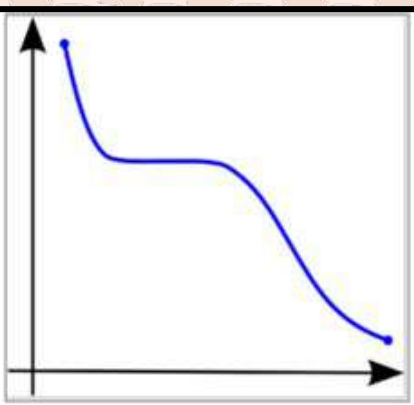


Figure 2 - A monotonically decreasing function

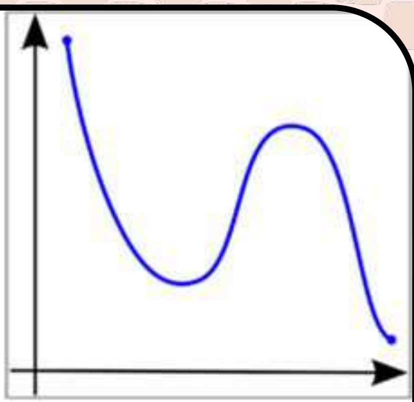
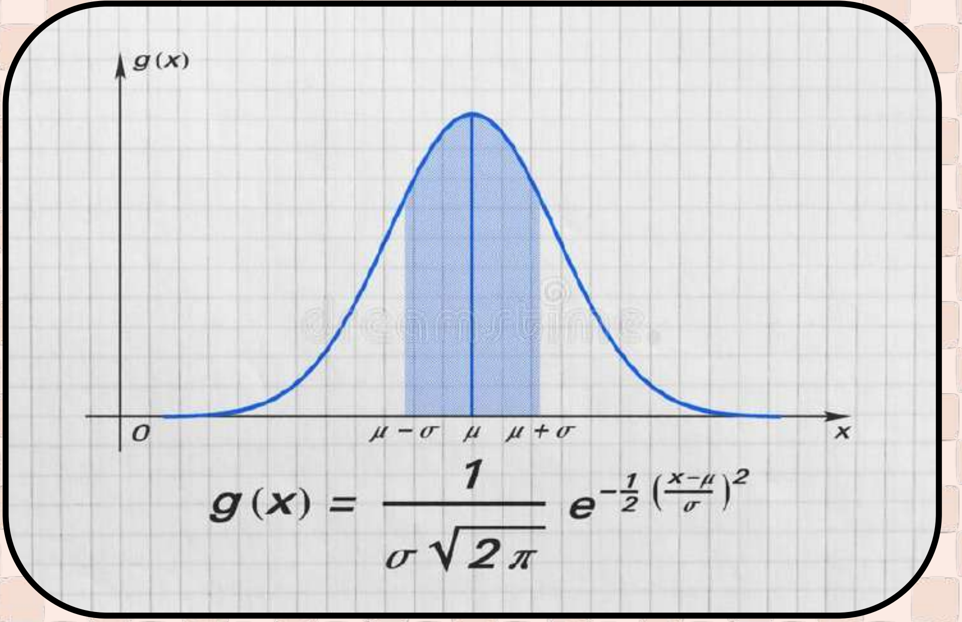


Figure 3 - A function that is not monotonic

GAUSS FUNCTION

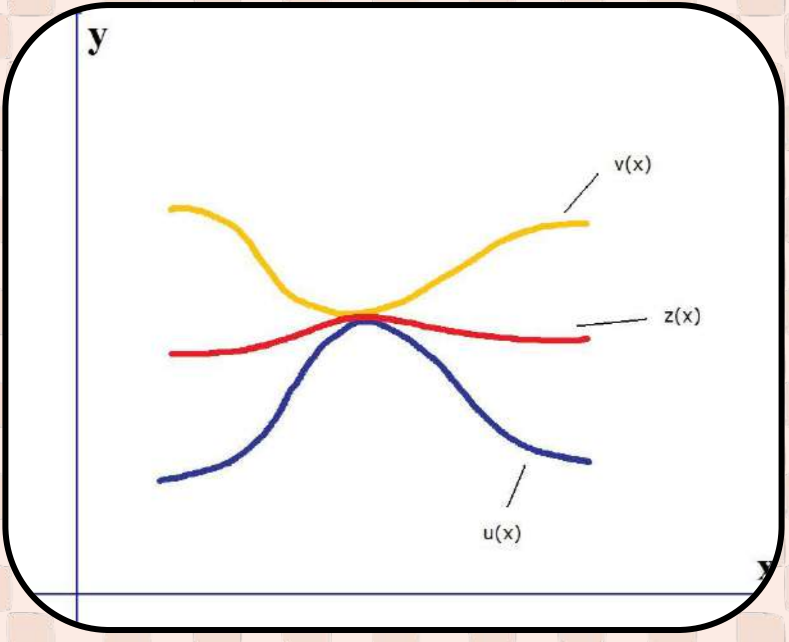


SQUEEZE (SANDWICH) THEOREM

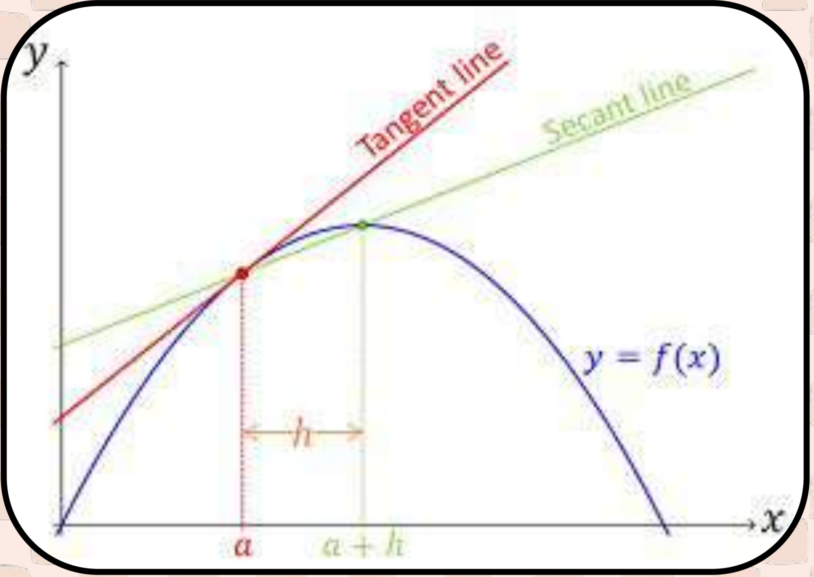
$$u(x) \le z(x) \le v(x)$$

$$\lim_{x \to a} u(x) = L$$

$$\lim_{x \to a} v(x) = L$$



$$\lim_{x \to a} z(x) = L$$



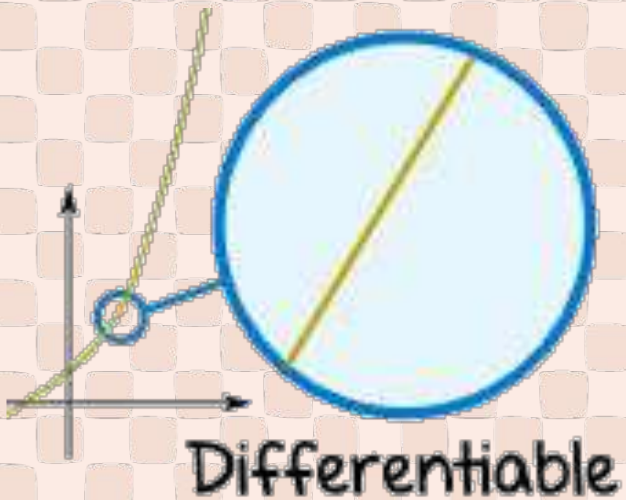
RATE OF CHANGE

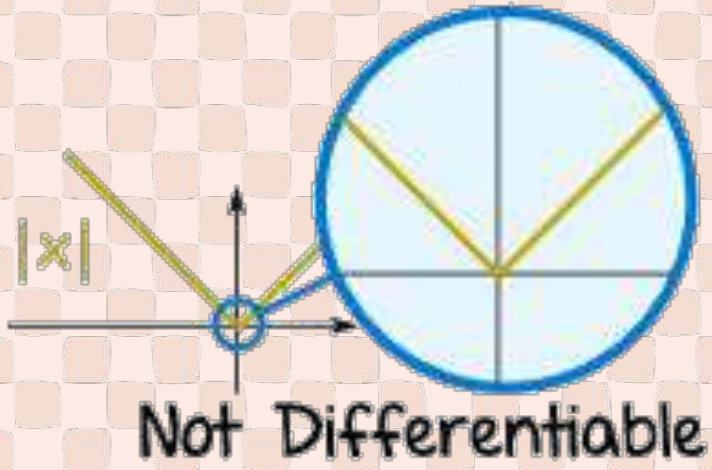
AVERAGE VS INSTANTANEOUS

DIFFERENTIATION

$$\begin{aligned}\frac{df}{dx} &= \lim_{\Delta x \to 0} \frac{f(x+\Delta x) - f(x)}{\Delta x} \\ &= \lim_{h \to 0} \frac{f(x+h) - f(x)}{h} \\ &= \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}.\end{aligned}$$

DIFFERENTIABLE





PRODUCT RULE

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

QUOTIENT RULE

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

CHAIN RULE

If f and g are both differentiable and $F(x)$ is the composite function defined by $F(x) = f(g(x))$ then F is differentiable and F' is given by the product

$$F'(x) = f'(g(x))\, g'(x)$$

Differentiate outer function

Differentiate inner function

IMPLICIT & EXPLICIT FUNCTION

- Explicit functions are functions where one variable is clearly expressed as a function of another such as $y = 3x^2 - 5$ or $f(x) = 2x - 6$

- Implicit functions are only implied by an equation, and may be difficult to express explicitly such as $x^2 - 2y^3 + 4y = 2$

IMPLICIT DIFFERENTIATION

$$x^3 y^3 - y = x$$

$$x^3 [3y^2 \frac{dy}{dx}] + y^3 [3x^2] - \frac{dy}{dx} = 1$$

$$3x^3y^2 \frac{dy}{dx} + 3x^2y^3 - \frac{dy}{dx} = 1$$

$$3x^3y^2 \frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$

$$\frac{dy}{dx} (3x^3y^2 - 1) = 1 - 3x^2y^3$$

$$\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

IMPLICIT FUNCTION THEOREM

It may not always be possible to solve implicit functions of the form $g(x,y)=0$ for unique explicit functions of the form $y = f(x)$

- Mathematicians have derived the necessary conditions
- In many economic applications, these conditions are the same as the second-order conditions for a maximum (or minimum)

RELATED RATE

$$\frac{dx}{dt} = 3$$

$$\frac{dh}{dt} = 0$$

$$x^2 + y^2 = h^2$$

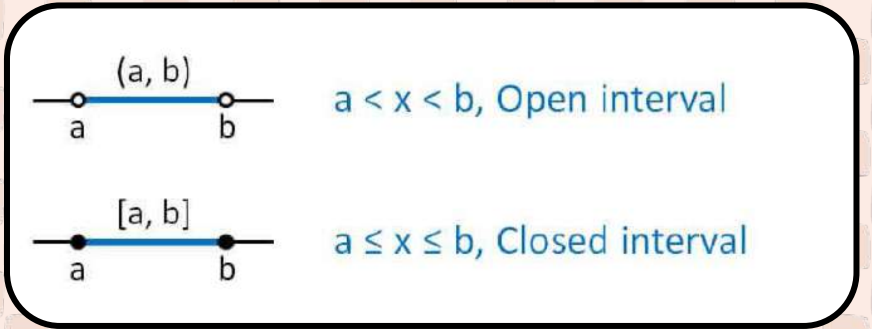
$$\frac{dy}{dt} = ?$$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(h^2)$$

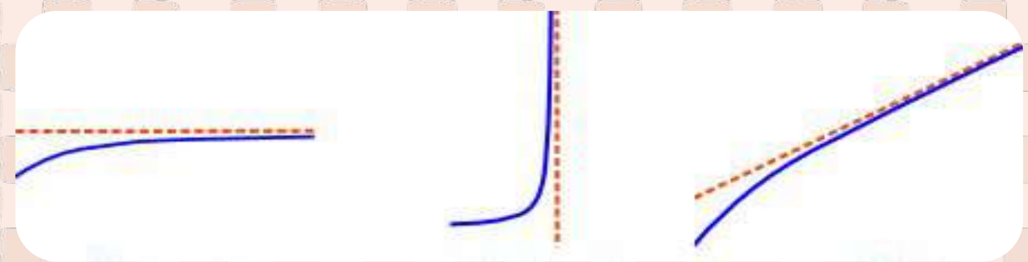
QUICK

CHECK-UP!

OPEN & CLOSED INTERVAL

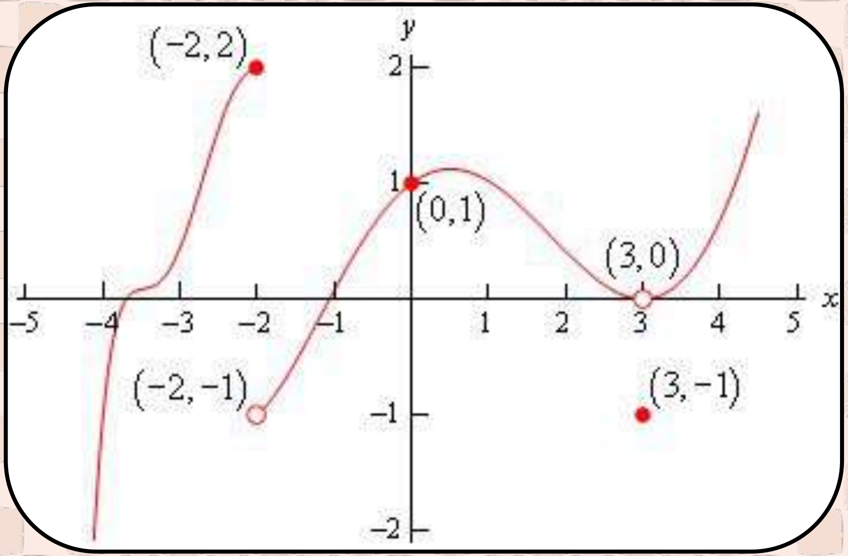


ASYMPTOTES



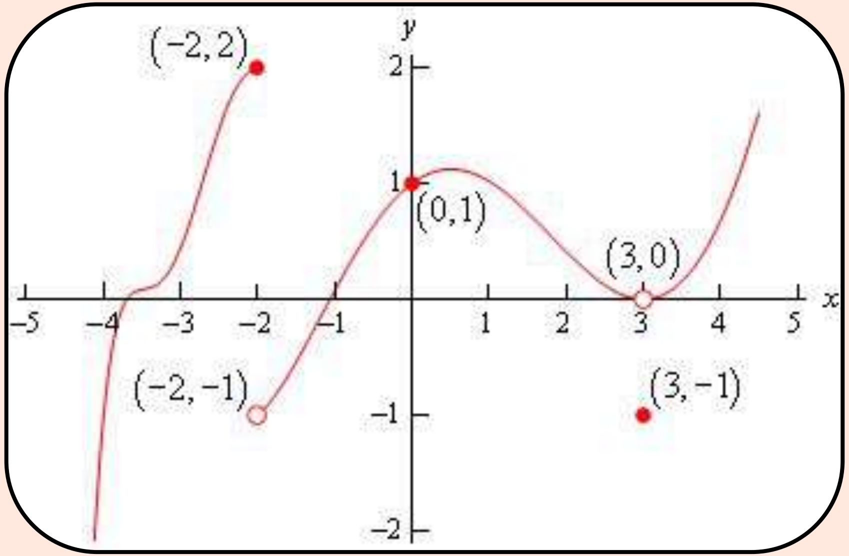
HORIZONTAL VERTICAL SLANT

Is this graph a Continuous Function?



CONTINUITY VS DISCONTINUITY

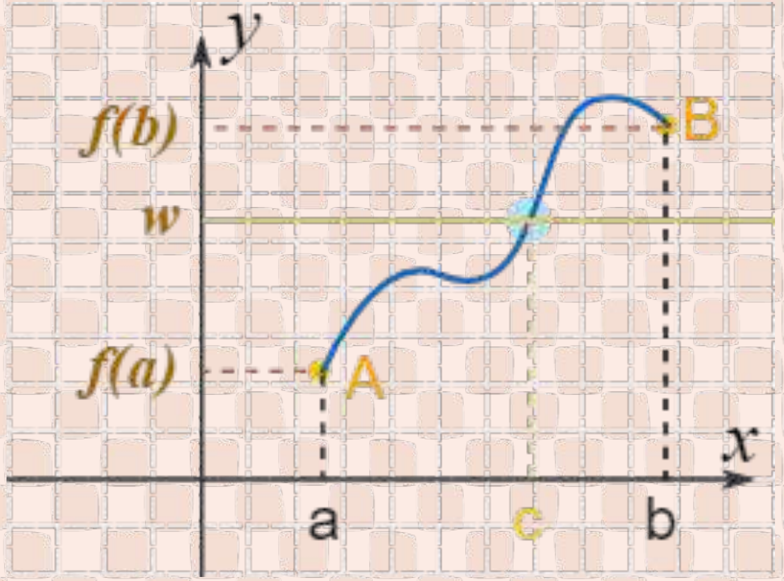
X = -2
Jump Discontinuity



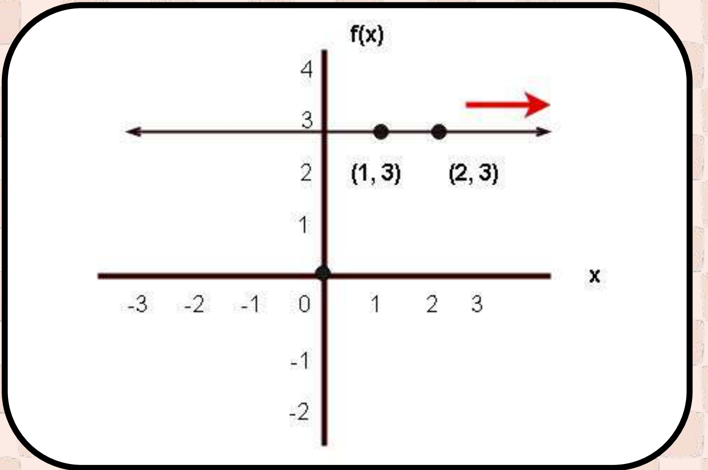
X = 3
Removable or Hole Discontinuity

How many discontinuities are there in this graph? Where and What kind is it?

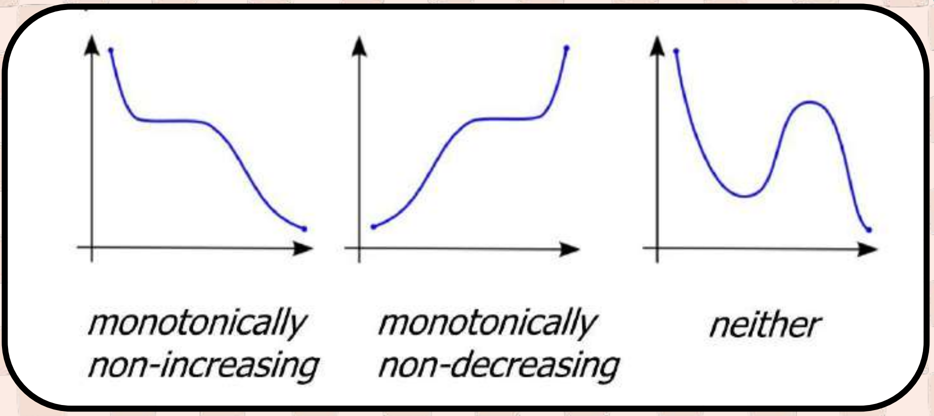
INTERMEDIATE VALUE THEOREM



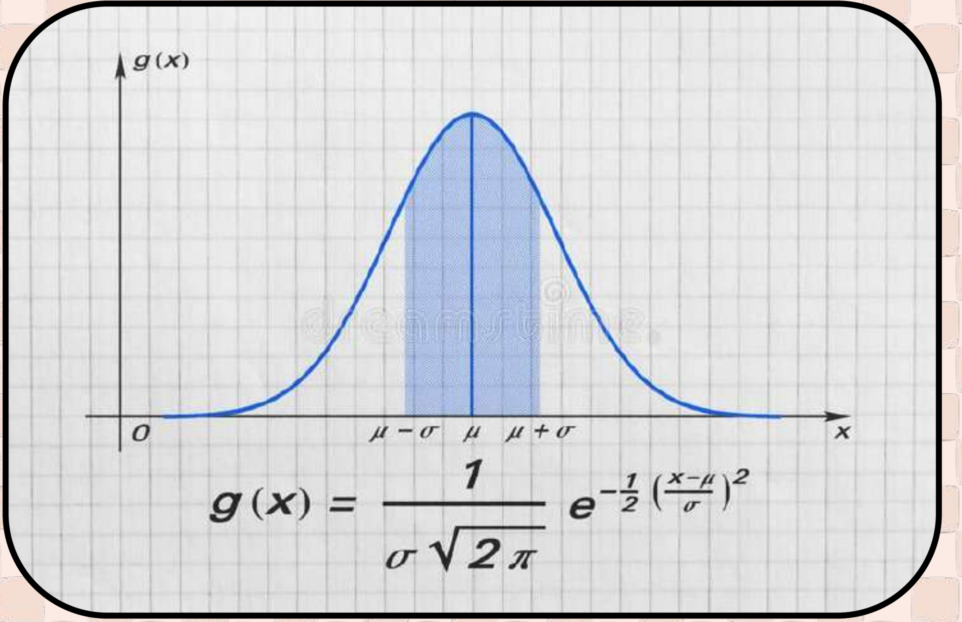
CONSTANT FUNCTION



MONOTONIC FUNCTION

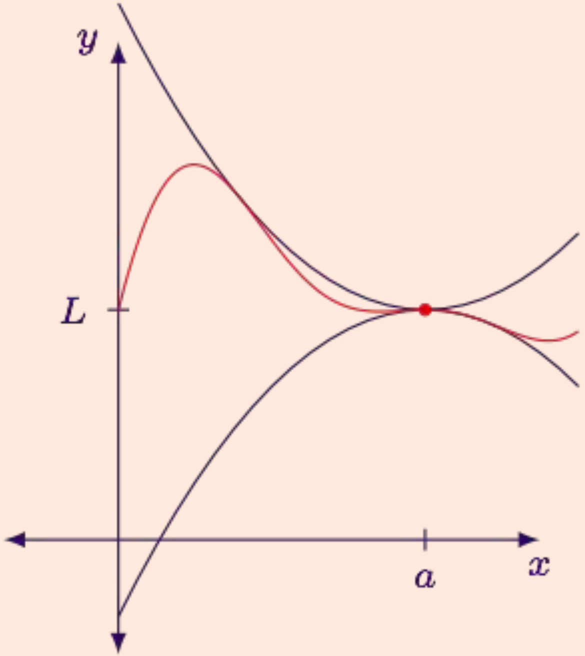


GAUSS FUNCTION



SQUEEZE (SANDWICH) THEOREM

the middle function is **squeezed** to L as x approaches a



RATE OF CHANGE

AVERAGE

$$\text{Average rate of change} = \frac{\text{change in y}}{\text{change in x}} = \frac{f(b) - f(a)}{b - a}$$

INSTANTANEOUS

$$f'(a) = \lim_{h \to 0} \frac{f(a+h) - f(a)}{h},$$

DIFFERENTIATION

RELATED RATE

$$\frac{dx}{dt} = 3$$

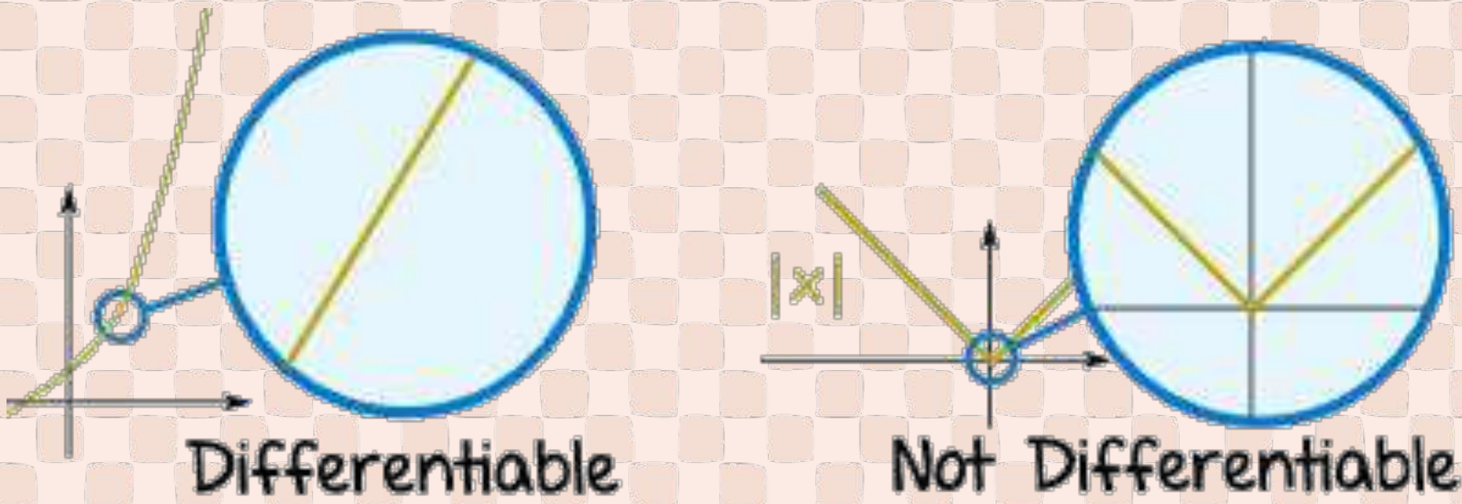
$$\frac{dh}{dt} = 0$$

$$x^2 + y^2 = h^2$$

$$\frac{dy}{dt} = ?$$

$$\frac{d}{dt}(x^2) + \frac{d}{dt}(y^2) = \frac{d}{dt}(h^2)$$

DIFFERENTIABLE



IMPLICIT FUNCTION

$$x^2 - 2y^3 + 4y = 2$$

Difficult to expressed y interim of x

EXPLICIT FUNCTION

$$y = 3x^2 - x + 2$$

Expressed in 1 variable

IMPLICIT FUNCTION THEOREM

$$x^3 y^3 - y = x$$

$$x^3 [3y^2 \frac{dy}{dx}] + y^3 [3x^2] - \frac{dy}{dx} = 1$$

$$3x^3y^2 \frac{dy}{dx} + 3x^2y^3 - \frac{dy}{dx} = 1$$

$$3x^3y^2 \frac{dy}{dx} - \frac{dy}{dx} = 1 - 3x^2y^3$$

$$\frac{dy}{dx} (3x^3y^2 - 1) = 1 - 3x^2y^3$$

$$\frac{dy}{dx} = \frac{1 - 3x^2y^3}{3x^3y^2 - 1}$$

using **Implicit Differentiation**

PRODUCT RULE

$$\frac{d}{dx}[f(x)g(x)] = f(x)g'(x) + g(x)f'(x)$$

QUOTIENT RULE

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$$

CHAIN RULE

$$\frac{d}{dx}\left[(f(x))^n\right] = n(f(x))^{n-1} \cdot f'(x)$$

$$\frac{d}{dx}[f(g(x))] = f'(g(x))g'(x)$$

AWESOME JOB FOR TODAY!

TAIWAN TECH

Welcome to

EMI 2021 SUMMER PROJECT

4nd session

01

CLASSROOM ENGLISH

P.28~32

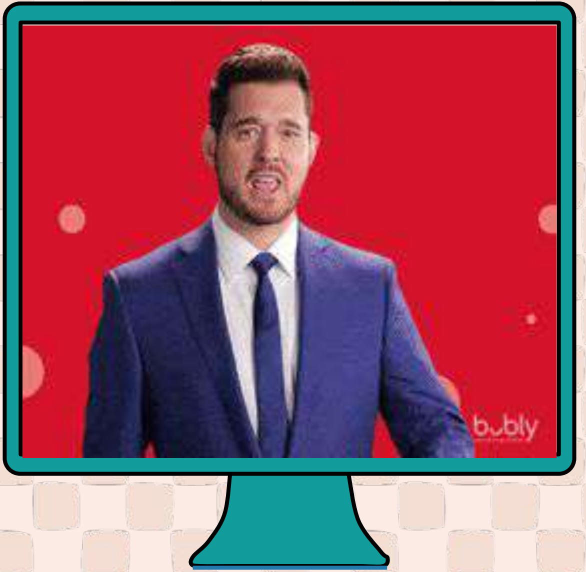
骰子

Roll/Throw the Dice

粉筆

Filled the margins(空白處) of the book with colored chalk.

訂正



Let’s
- go over
- go through
- run through
- check

the answers.

時態 tense

Past 過去
Present 現在
Future 未來
Present perfect 現在完成
Present continuous for the future. 未來完成

優缺點

Advantages and disadvantages.

Pros and cons.

分組

Group of xxx

Ex: group of six 六人一組

02

CALCULUS VOCABS

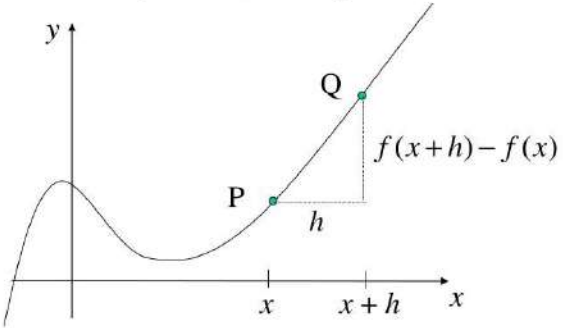
World-list 61 - 80

DIFFERENTIAL

$$dy = f'(x)\,dx,$$

The derivative, or derived function of $f(x)$ denoted $f`(x)$ is defined as

$$f`(x) = \lim_{h \to 0}\left(\frac{f(x+h)-f(x)}{h}\right)$$

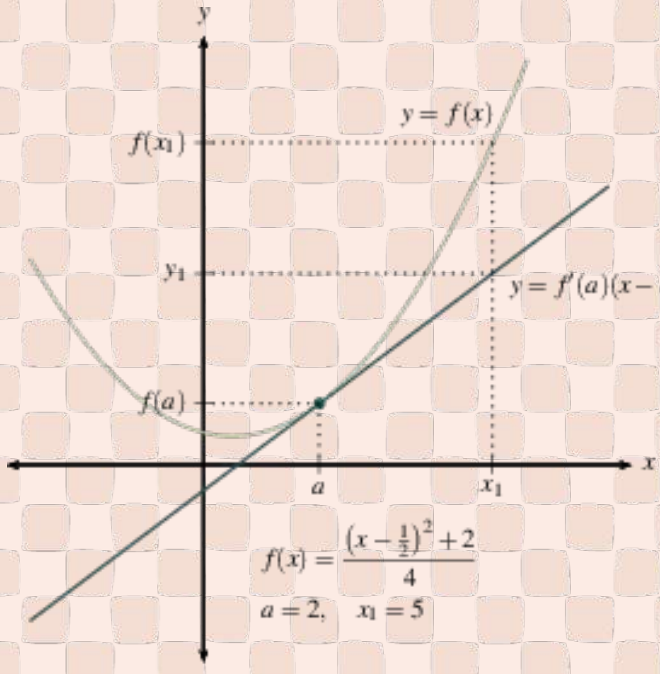


$$m_{PQ} = \frac{f(x+h)-f(x)}{h}$$

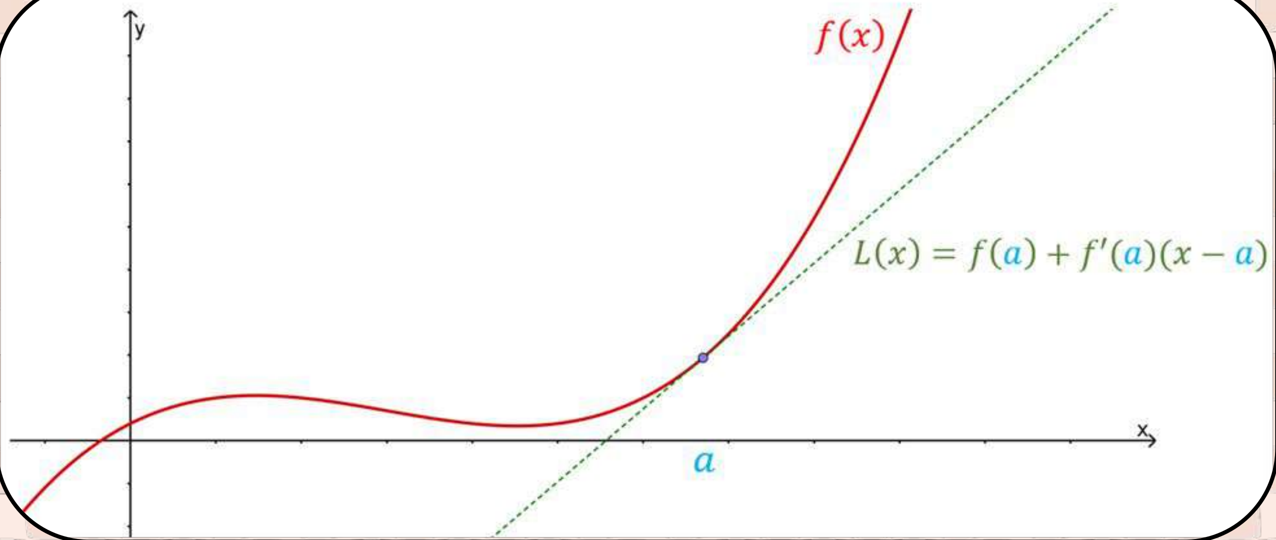
Leibniz Notation: $f`(x) = \lim_{h \to o}\left(\frac{\delta y}{\delta x}\right) = \frac{dy}{dx}$

LINEARIZATION/ LINEAR APPROXIMATION

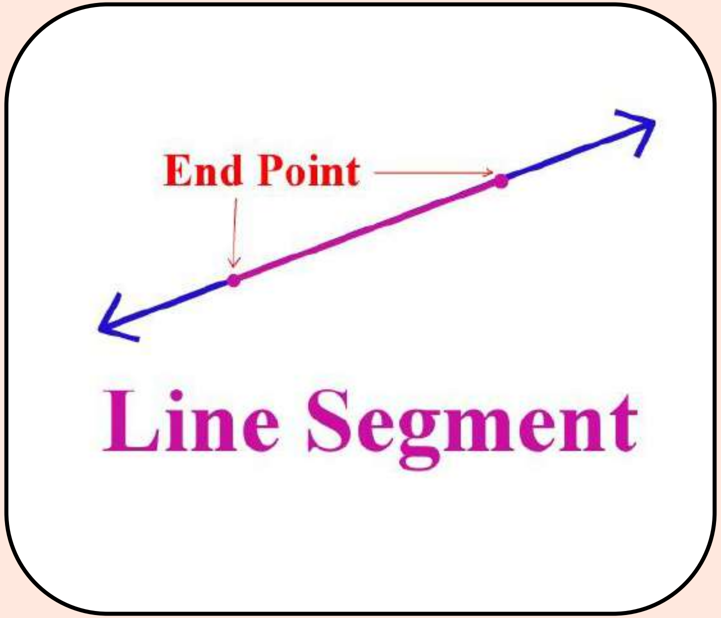
an approximation of a general function using a linear function



$$
\begin{aligned}
f(x) \text{ near } a \;\approx\; & \text{ tangent line at } a \\
&= f(a) + f'(a)(x-a)
\end{aligned}
$$

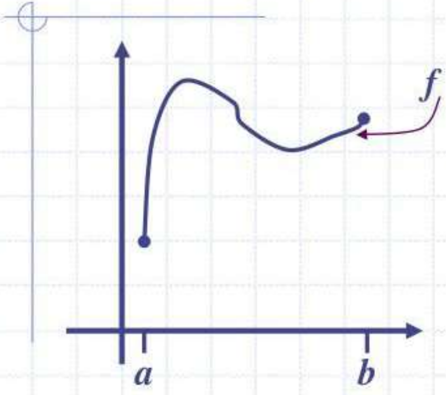


ENDPOINT



Any of the two furthest points on a line segment.

MEAN VALUE THEOREM



If: f is continuous on $[a, b]$,

differentiable on (a, b)

Then: there is a c in (a, b) such that

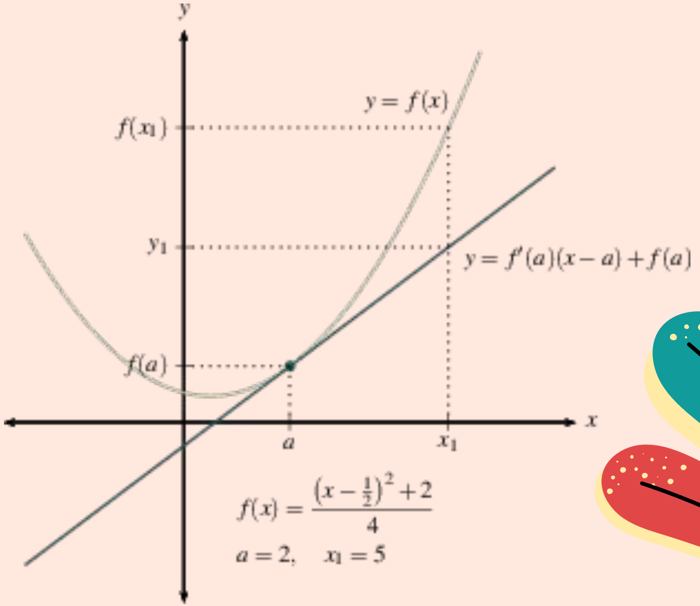
$$f'(c) = \frac{f(b) - f(a)}{b - a}$$

CHECK-UP

DIFFERENTIAL

$$dy = f'(x)\,dx,$$

LINEARIZATION/ LINEAR APPROXIMATION



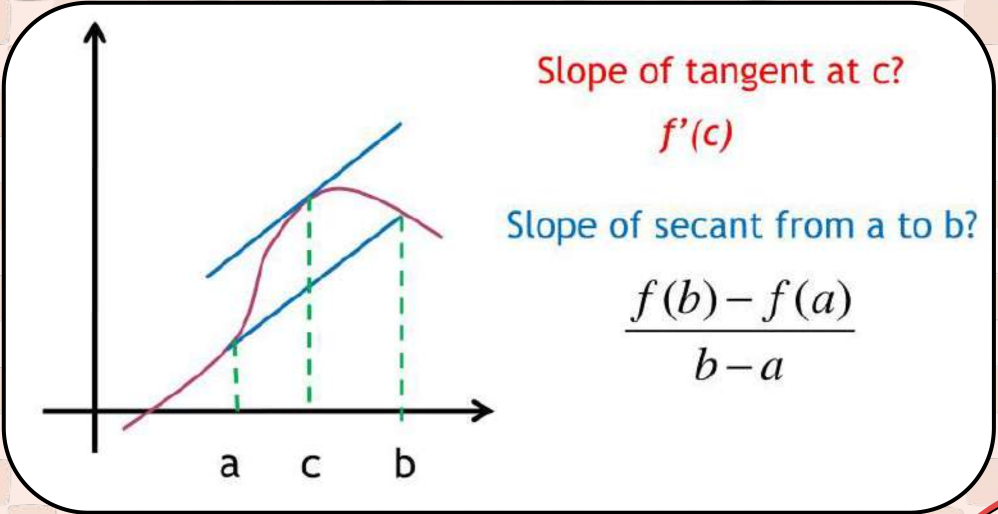
an approximation of a general function using a linear function

END POINT

a point at the **end of a line segment** or ray

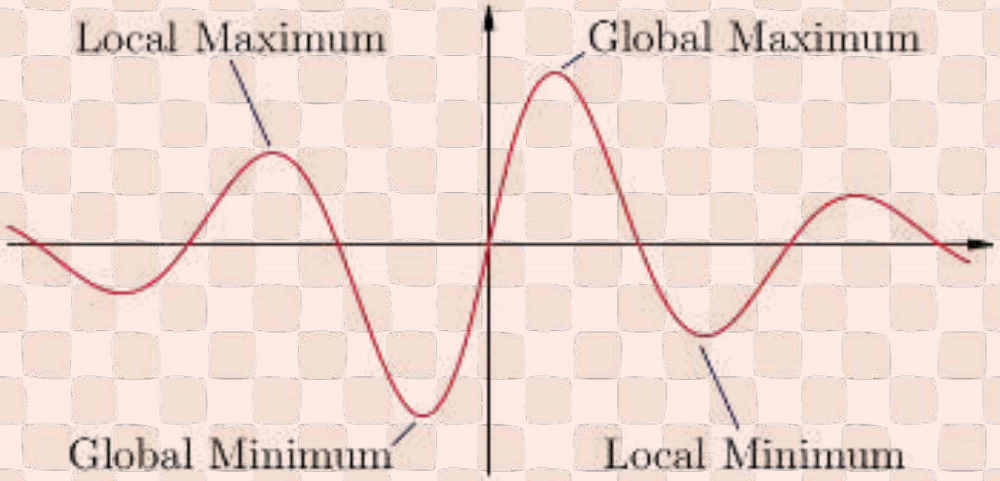
MEAN VALUE THEOREM

the relationship between the **average rate of change and derivative**



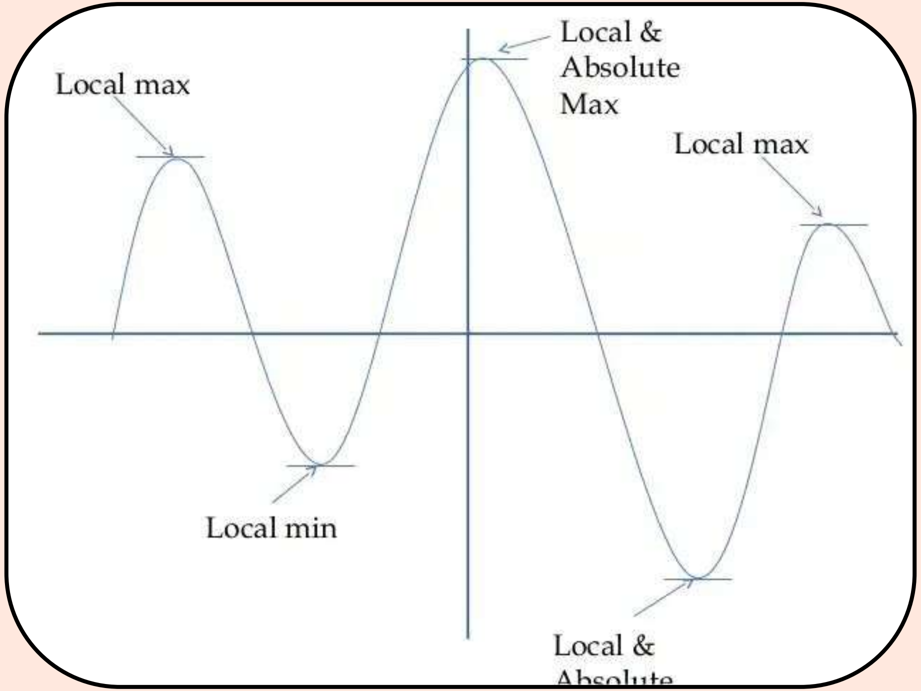
EXTREMA

MAXIMUM VALUE



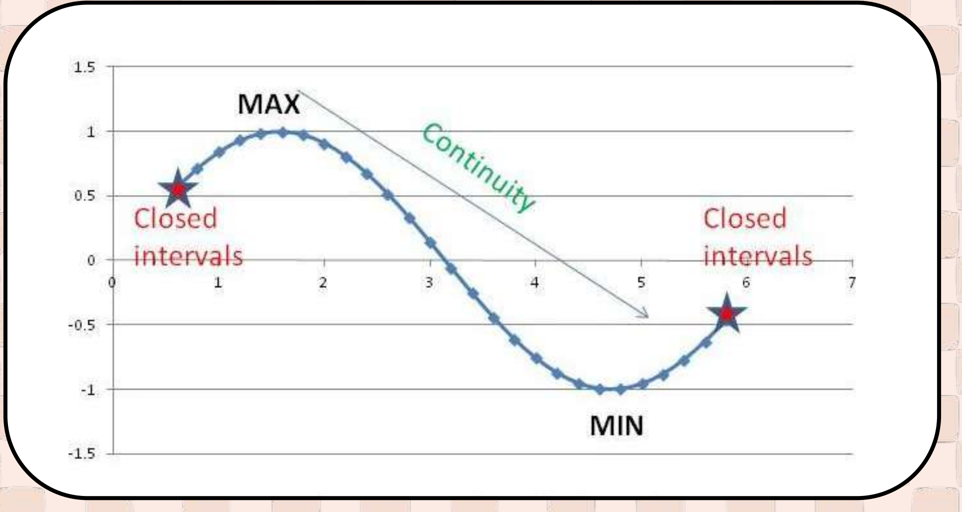
MINIMUM VALUE

ABSOLUTE EXTREMA



LOCAL/ RELATIVE EXTREMA

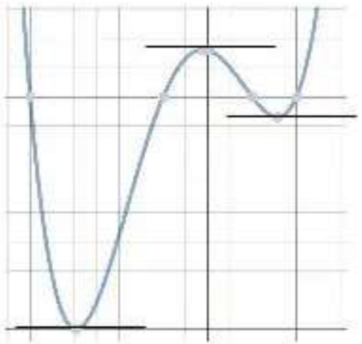
EXTREME VALUE THEOREM

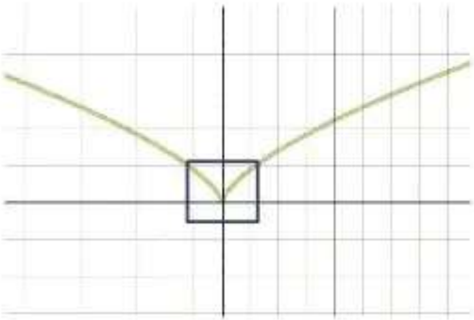


if a function is continuous on a closed interval [a,b], then the function must have a maximum and a minimum on the interval

CRITICAL POINT

- A number c in the domain of f is called a **critical point** if *either* $f'(c) = 0$ or $f'(c)$ is undefined.





CHECK-UP

EXTREMA

MAXIMUM

Highest point/ Largest value

MINIMUM

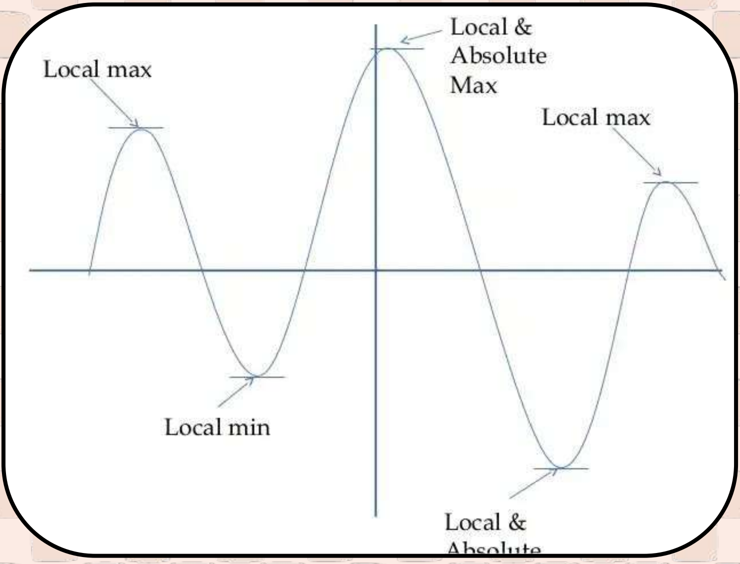
Lowest point/ Lowest value

ABSOLUTE

found throughout the interval of a function

LOCAL/ RELATIVE

the function is bigger or smaller there than points around it



CRITICAL POINT

$$\text{when } \frac{dy}{dx} = 0 \text{ or is undefined}$$

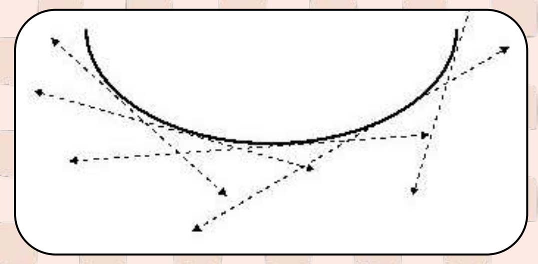
FIRST DERIVATIVE TEST

y' is positive	→	Curve is rising.
y' is negative	→	Curve is falling.
y' is zero	→	Possible local maximum or minimum.

SECOND DERIVATIVE TEST

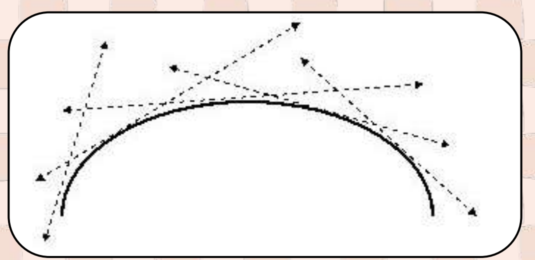
y'' is positive	→	Curve is concave up.	
y'' is negative	→	Curve is concave down.	
y'' is zero	→	Possible inflection point (where concavity changes).	

CONCAVITY



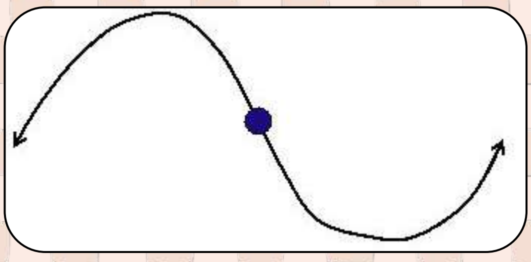
CONCAVE UPWARD

slopes of segments are increasing throughout the interval



CONCAVE DOWNWARD

slopes of segments are decreasing throughout the interval



INFLECTION POINT

a point on the graph of a function at which the **concavity changes**

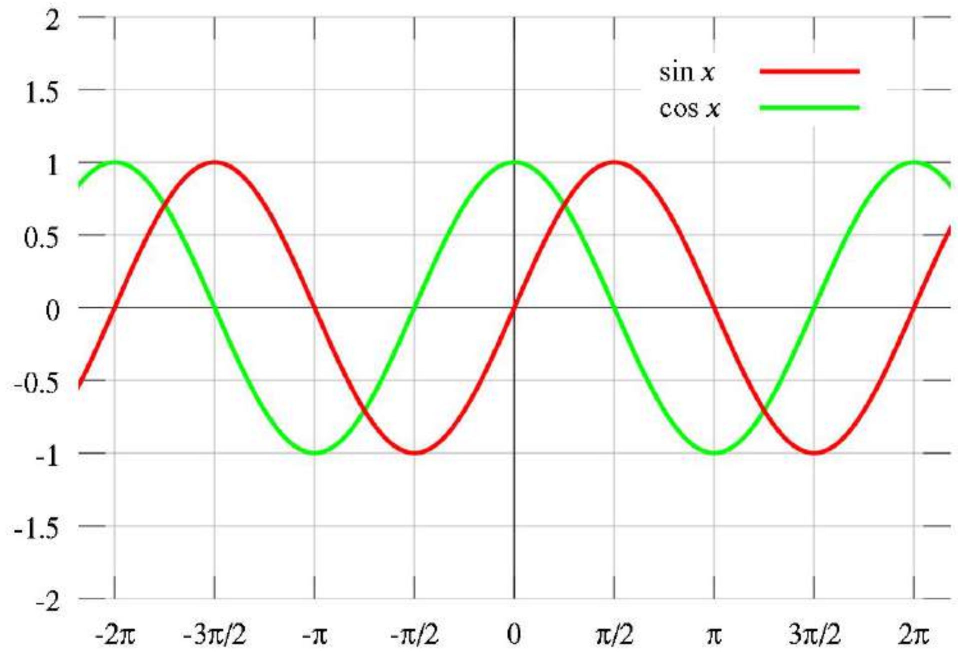
INDETERMINATE FORM

$$\frac{0}{0}, \frac{\infty}{\infty}, \infty - \infty, 0 \times \infty, 0^0, \infty^0, 1^\infty$$

L'HOSPITAL RULE

$$\lim_{x \to a} \frac{f(x)}{g(x)} = \lim_{x \to a} \frac{f'(x)}{g'(x)}$$

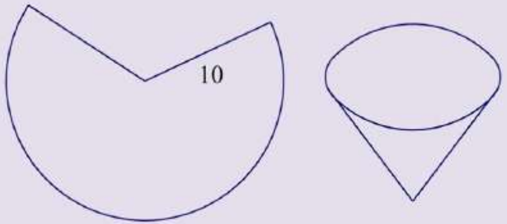
PERIODIC FUNCTION



OPTIMIZATION PROBLEM

EX: minimal cost, maximal profit, minimal error, optimal design, optimal management, variational principles

A cone-shaped drinking cup is made from a circular piece of paper of radius 10 inches by cutting out a sector and joining the two straight edges. Find the height of the cup that will maximize the capacity of such a cup.



CHECK-UP

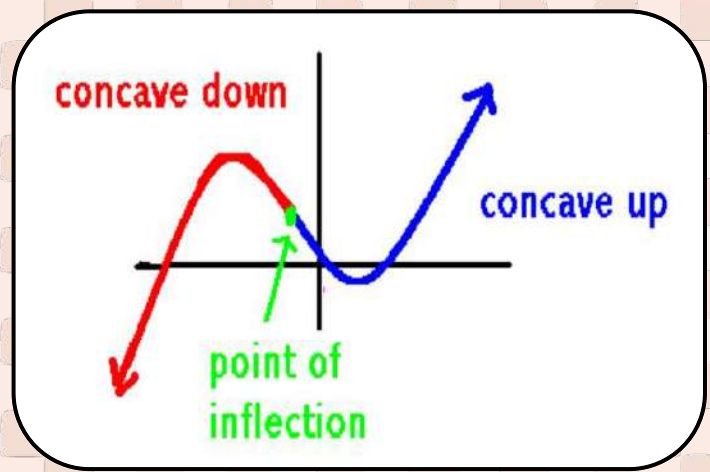
FIRST DERIVATIVE TEST

a process used to **find the critical point/ extrema**

SECOND DERIVATIVE TEST

a process used to **determine if a critical point** of a function is a **local minimum, maximum or saddle point**

CONCAVITY



relates to the **rate of change of a function's derivative**

INDETERMINATE FORM

an expression whose **limit cannot be determined** from the individual functions

L'HOSPITAL RULE

a **technique to evaluate limits of indeterminate forms**

PERIODIC FUNCTION

A function that **repeats its values** at regular intervals

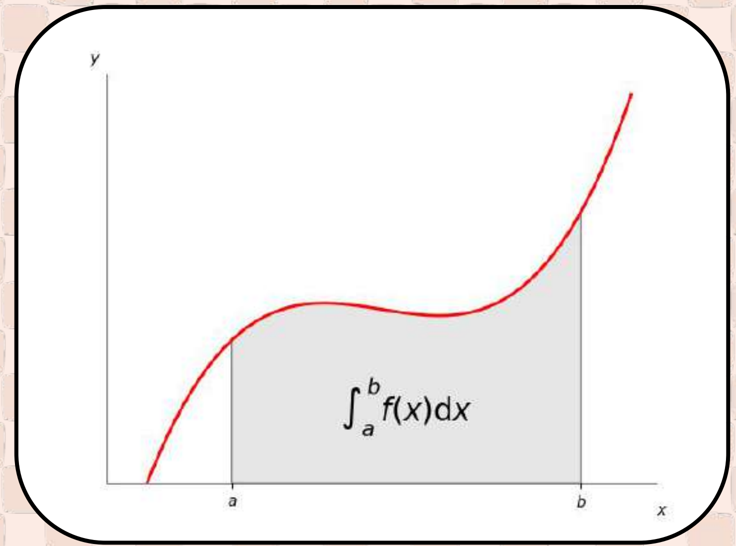
OPTIMIZATION PROBLEM

A problem that ask you to **minimize or maximize** function

$$f(x) = 3x^2 + 5$$

$$\frac{d}{dx} f(x) = 6x$$

$$\int 6x \, dx = 3x^2 + c$$



ANTIDERIVATIVE

functions whom derivative will be our original function

INTEGRAL

can be interpreted as **an area or a generalization of area**

DEFINITE INTEGRAL

$$\int_a^b f(x)dx$$

a **NUMBER** and represents the **area under the curve** *f(x)* from ***x=a* to *x=b***

INDEFINITE INTEGRAL

$$\int f(x)dx$$

a **FUNCTION** that can be interpreted as the **antiderivative of the function**

INTEGRABLE

when the integral is well defined

CHECK-UP

ANTIDERIVATIVE/ ANTIDIFFERENTIATION

a function that **reverses what the derivative does**

INTEGRAL/ INTEGRATION

can be interpreted as **an area** or a generalization of area

INTEGRABLE

when the **integral is well defined**

DEFINITE INTEGRATION

area under the curve *f(x)* from ***x*=*a* to *x*=*b***

$$\int_a^b f(x)dx$$

INDEFINITE INTEGRATION

antiderivatives of the function

$$\int f(x)dx$$

AWESOME JOB FOR TODAY!

Competition

Quizlet

1. You will received a number and a QRcode to enter the game.
2. Please use your group number + your student ID to be your name, we'll assign your teamates after you get in.
3. Rule: You will have 3~4 flashcards if the word on your screen can make as a pair, then choose the correct one. You may not get the right card in the same round, then just choose nothing and your teammates will choose the right one. If you choose the wrong one, your team will have to go to the begining.
4. Your teammates will be assigned randomly.

Live

ON A COMPUTER?

Go to www.quizlet.live

504-842

ON A DEVICE?

Open the Quizlet app

TAIWAN TECH

Welcome to

EMI 2021 SUMMER PROJECT

5nd session

01

CLASSROOM ENGLISH

P.33~37

引述xxx

Write down the author 作者 / title / page number of the quote.

Refer to 參考

If you don’t understand this part, there’s a section in the book you can refer to.

舉例

Take xxx as an example.

Let’s give an example of that.

I’ll show you an example.

超出時間

We are out of time.

01

Acronyms

Makes u type faster

BTW

BY THE WAY

BTW, don't forget to bring your wallet.

FYI

FOR YOUR INFORMATION

FYI, xxxxxxxxxxx.

TTYL

TALK TO YOU LATER

Sorry I got business, TTYL.

LOL

LAUGHING OUT LOUD

LOL, HAHAHAHAHA.

LMK

LET ME KNOW

If you have any information update, LMK.

NVM

NEVER MIND

SOMEBODY: I’m so sorry.

YOU: NVM.

IDK

I DON'T KNOW

SHE: How do I look today?

YOU: IDK.

ASAP

AS FAST AS POSSIBLE

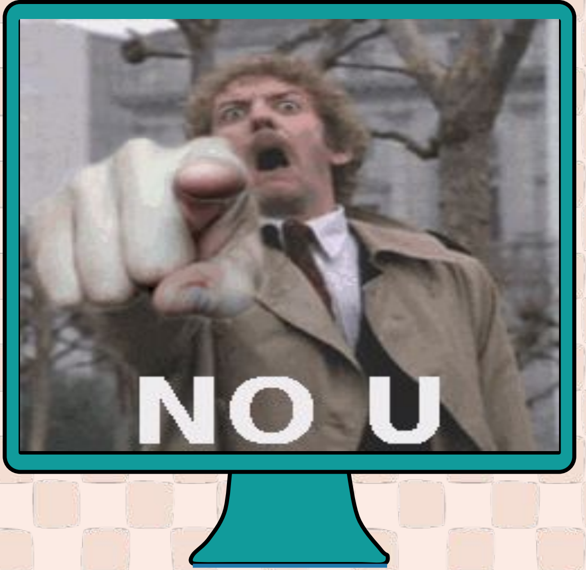
I'll meet you at the train station ASAP.

DM

DIRECT MESSAGE

If you have any question, just DM me.

Others



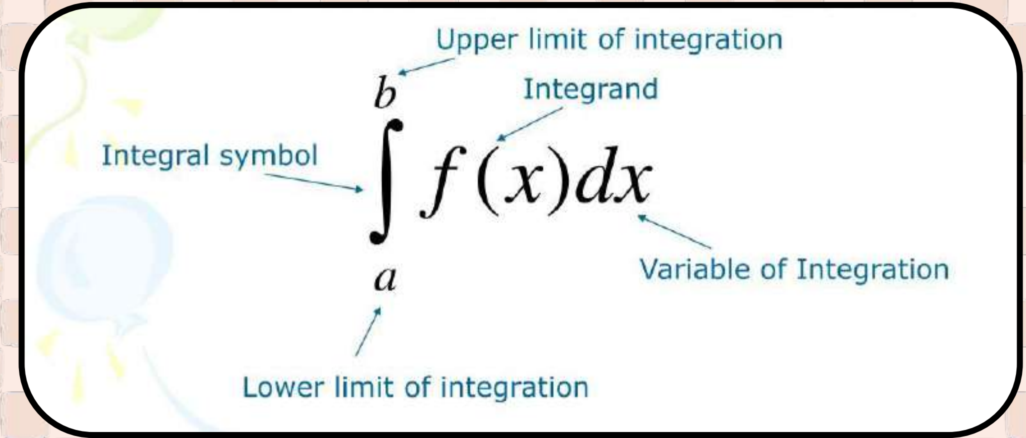
U = YOU
2 = TOO, TWO
B = BE
R = ARE
V = VERY
PPL = PEOPLE
GR8 = GREAT
F2F = FACE TO FACE

02

CALCULUS VOCABS

World-list 81 - 100

LOWER & UPPER LIMIT OF INTEGRATION

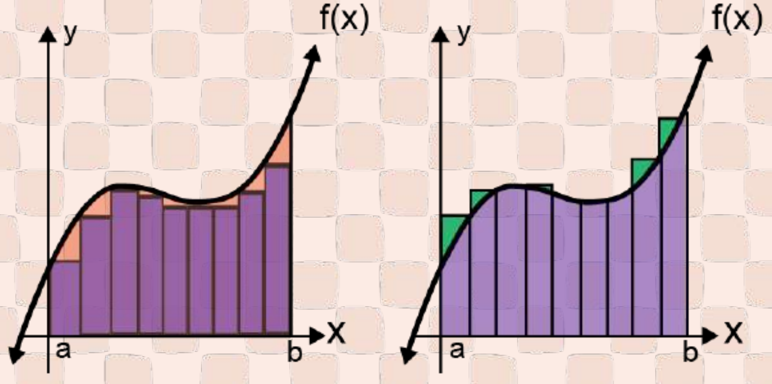


INTEGRAD

IMPROPER INTEGRAL

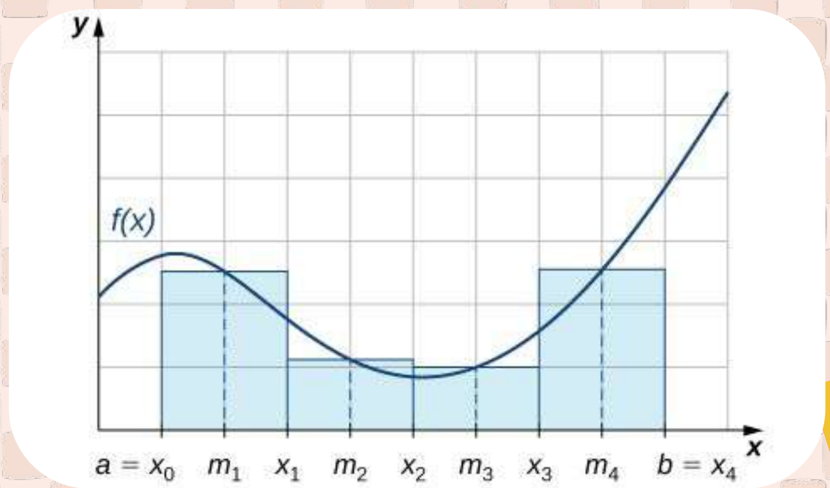
$$I_1 = \int_a^\infty f(x)dx$$
$$I_2 = \int_{-\infty}^a f(x)dx$$
$$I_3 = \int_{-\infty}^\infty f(x)dx$$

RIEMANN SUM



Lower Sum　　Upper Sum

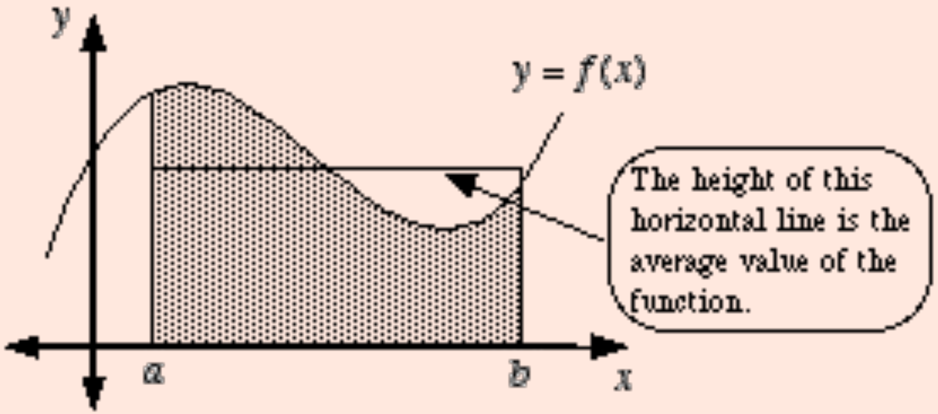
MIDPOINT RULE



AVERAGE VALUE

Average value $= \dfrac{1}{b-a}\int_a^b f(x)\,dx$

The rectangle has the same area as the shaded region under the curve.



FUNDAMENTAL THEOREM OF CALCULUS

F is a function defined on the closed interval [a,b]

$$F(x) = \int_a^x f(t)\,dt$$

If f is a function continuous on [a,b] then F is continuous on [a,b] and differentiable on the open interval (a,b) and

$$F'(x) = f(x)$$

for all x in (a,b)

Let f be a continuous function on $[a, b]$. Then

$$\int_a^b f(x)dx = F(b) - F(a)$$

Where F is any antiderivative of f.

NET CHANGE THEOREM

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

Where F' is a rate of change, the difference is the net change.

CIRCULAR CYLINDER

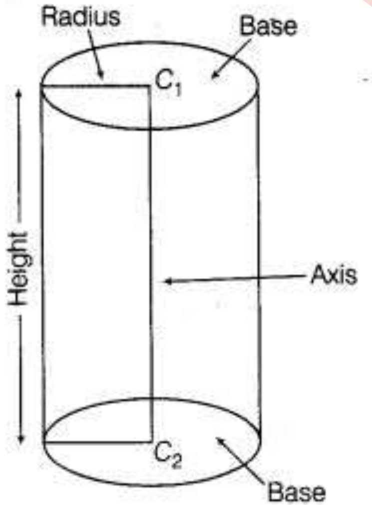
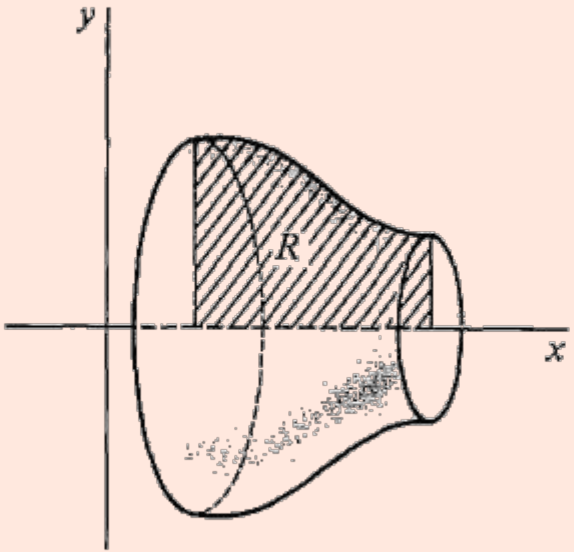
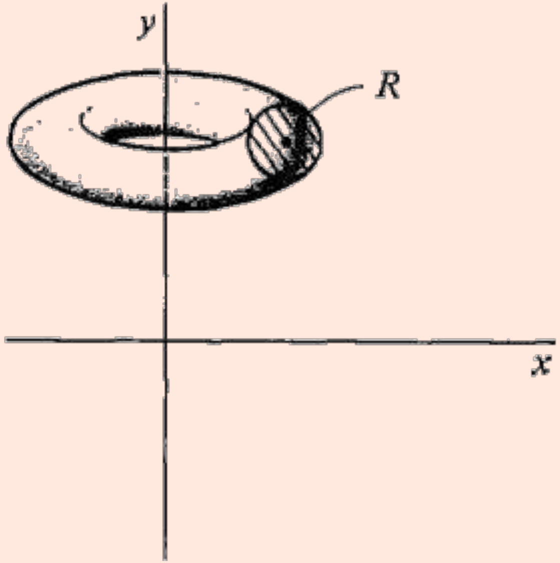


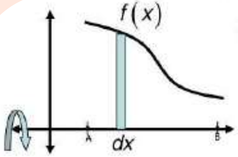
Fig. 29.2 Right Circular Cylinder

SOLID OF REVOLUTION



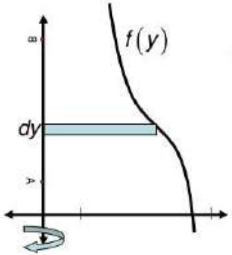


DISC/ RING METHOD



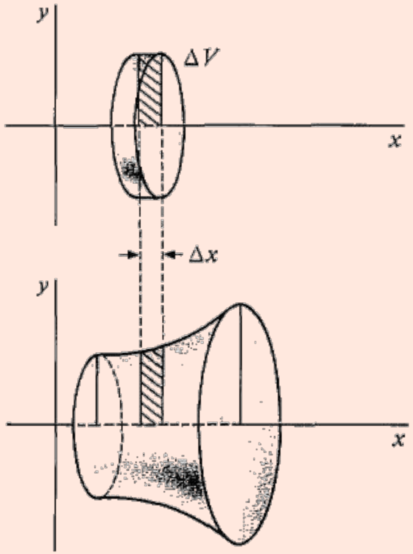
Horizontal Axis of revolution:

$$V = \pi \int_a^b \underbrace{[f(x)]^2}_{\text{radius}} \underbrace{dx}_{\text{width}}$$

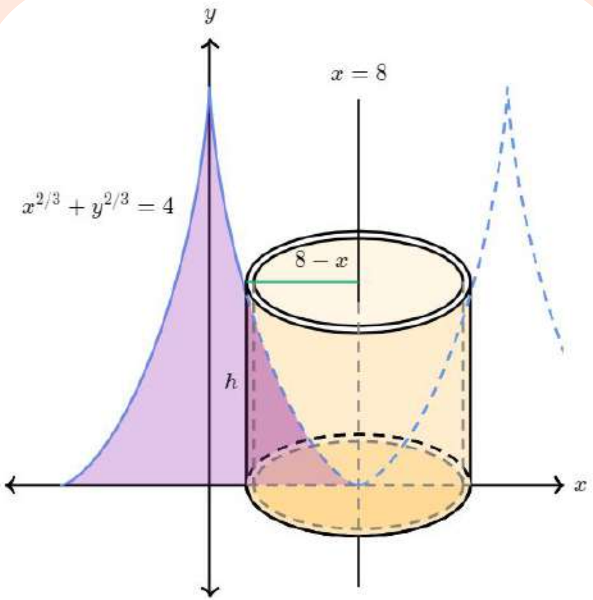


Vertical Axis of revolution:

$$V = \pi \int_a^b [f(y)]^2 \, dy$$



CYLINDRICAL SHELL METHOD



SUBSTITUTION RULE

If g' is continuous on $[a,b]$, and f is continuous on the range of $u=g(x)$ then

$$\int_a^b f(g(x))g'(x)dx = \int_{g(a)}^{g(b)} f(u)du$$

where $u = g(x)$

INTEGRATION BY PARTS

$$\int u\,dv = uv - \int v\,du$$

TRIGONOMETRIC INTEGRAL

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax$$

$$\int \tan ax \, dx = -\frac{1}{a}\log \cos ax$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax$$

$$\int \cot ax \, dx = \frac{1}{a}\log \sin ax$$

$$\int \sec ax \, dx = \frac{1}{a}\log(\sec ax + \tan ax) = \frac{1}{a}\log \tan\left[\frac{\pi}{4} + \frac{ax}{2}\right]$$

$$\int \csc ax \, dx = \frac{1}{a}\log(\csc ax - \cot ax) = \frac{1}{a}\log \tan \frac{ax}{2}$$

TRIGONOMETRIC SUBSTITUTION

For	substitute
$\sqrt{a^2 - x^2}$	$x = a\sin\theta$
$\sqrt{a^2 + x^2}$	$x = a\tan\theta$
$\sqrt{x^2 - a^2}$	$x = a\sec\theta$

RATIONALIZING SUBSTITUTION

Find $\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx$.

Let $u = \sqrt[6]{x} = x^{\frac{1}{6}}$, so $u^6 = x$. That means $6u^5\,du = dx$.

$$\int \frac{\sqrt{x}}{1+\sqrt[3]{x}}\,dx = \int \frac{u^3}{1+u^2}\cdot 6u^5\,du$$

$$= \int \frac{6u^8}{1+u^2}\,du \qquad \text{(use polynomial long division)}$$

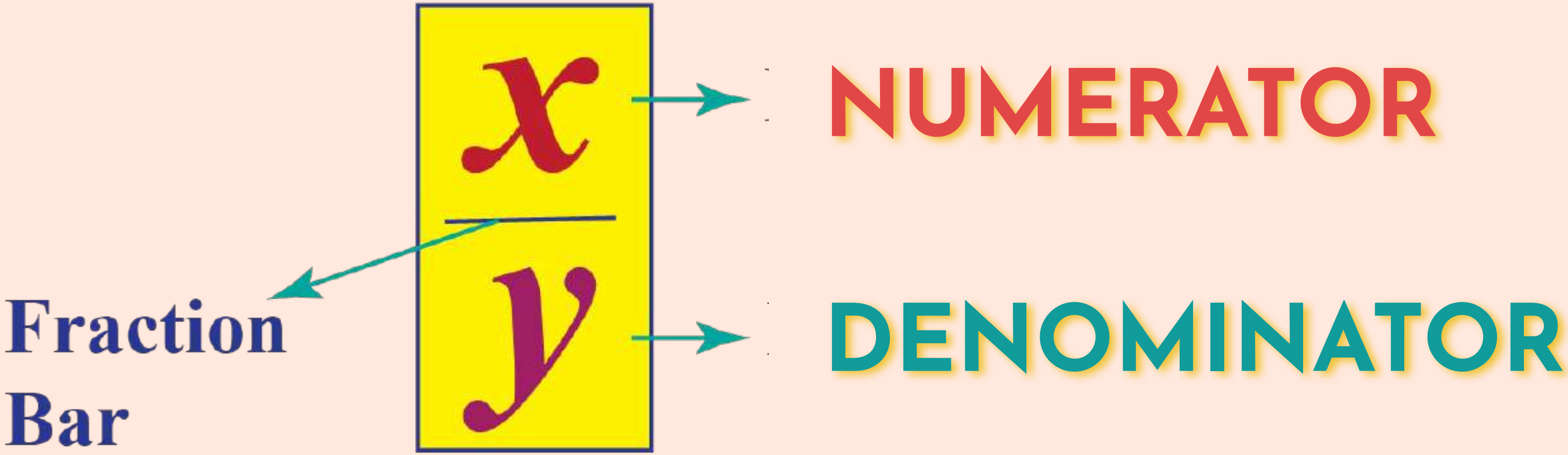
$$= \int \left(6u^6 - 6u^4 + 6u^2 - 6 + \frac{6}{1+u^2}\right) du$$

$$= \frac{6}{7}u^7 - \frac{6}{5}u^5 + 2u^3 - 6u + 6\tan^{-1}u + C$$

$$= \frac{6}{7}x^{\frac{7}{6}} - \frac{6}{5}x^{\frac{5}{6}} + 2x^{\frac{1}{2}} - 6x^{\frac{1}{6}} + 6\tan^{-1}x^{\frac{1}{6}} + C$$

PARTIAL FRACTION

Case	Fraction $\frac{N(x)}{D(x)}$	Form of denominator, D(x)	Partial Fraction Form (where A, B and C are unknown constants)
1	$\frac{N(x)}{(ax+b)(cx+d)}$	Linear Factors	$\frac{A}{ax+b}+\frac{B}{cx+d}$
2	$\frac{N(x)}{(ax+b)^2}$	Repeated Linear Factors	$\frac{A}{ax+b}+\frac{B}{(ax+b)^2}$
	$\frac{N(x)}{(ax+b)(cx+d)^2}$	Linear and Repeated Linear Factors	$\frac{A}{ax+b}+\frac{B}{cx+d}+\frac{C}{(cx+d)^2}$
3	$\frac{N(x)}{(ax+b)(x^2+c^2)}$	Linear and Quadratic (which cannot be factorised) Factors	$\frac{A}{ax+b}+\frac{Bx+C}{x^2+c^2}$



x
y
NUMERATOR
DENOMINATOR
Fraction Bar

LONG DIVISION

$$
\begin{array}{r}
x^3 + 2x^2 + 3x + 4 \qquad\qquad\quad \\
x-5\,\overline{\big)\,x^4 - 3x^3 - 7x^2 - 11x - 20} \\
\underline{x^4 - 5x^3} \qquad\qquad\qquad\qquad\quad \\
2x^3 - 7x^2 \qquad\qquad\qquad \\
\underline{2x^3 - 10x^2} \qquad\qquad\qquad \\
3x^2 - 11x \qquad\quad \\
\underline{3x^2 - 15x} \qquad\quad \\
4x \qquad\quad \\
\underline{4x - 20} \\
0 \qquad\qquad
\end{array}
$$

CHECK-UP

$$\int_a^b f(x)\,\mathrm{d}x$$

INTEGRAND

a **function you want to integrate**

LOWER/ UPPER LIMIT OF INTEGRATION

number **a and b** in the integral notation

IMPROPER INTEGRAL

either **a or b or both of them are infinity**

AVERAGE VALUE

mean of a function

MIDPOINT RULE

rule used to **approximate the area under a simple curve**

RIEMANN SUM

certain kind of **approximation of an integral by a finite sum**

FUNDAMENTAL THEOREM OF CALCULUS

theorem that **links the concept of differentiating** a function (gradient) with the concept of **integrating** a function (area under the curve)

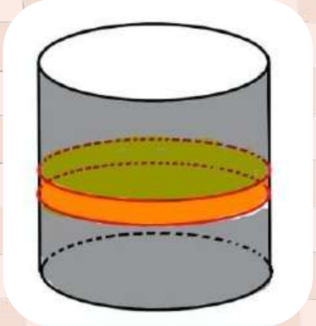
NET CHANGE THEOREM

theorem that **considers the integral of a rate of change** saying that when a quantity changes, the **new value equals the initial value plus the integral of the rate of change** of that quantity.

$$\int_a^b F'(x)\,dx = F(b) - F(a)$$

SOLID OF REVOLUTION

solid figure obtained by **rotating a plane curve** around the axis



DISK/ RING METHOD

a method that **integrates** along an axis that is **parallel to the axis of revolution**

CYLINDRICAL SHELL METHOD

a method that **integrates** along an axis **perpendicular to the axis of revolution**

SUBSTITUTION RULE

or **u-substitution**, is a method for **evaluating integrals and antiderivatives**

$$u = x^2 \qquad \frac{1}{2}du = xdx \qquad \frac{1}{2}\int_{x=0}^{x=2} e^u\,du$$
$$du = 2x$$

INTEGRATION BY PARTS

a special method of integration that is often useful when **two functions are multiplied together**

$$\int udv = uv - \int vdu$$

TRIGONOMETRIC SUBSTITUTION

a technique for evaluating integral by **substituting**

Trigonometric Integrals

Use this substitution	in this expression
x = a sin(t)	$\sqrt{a^2 - x^2}$

RATIONALIZING SUBSTITUTION

integration method which is used when the **integrand is a fraction** which includes **more than a type of root**

$$\int \frac{dx}{\sqrt[3]{x} + \sqrt{x}}$$

PARTIAL FRACTION

algebraic **fraction can be broken down** into simpler parts

$$\frac{x-3}{x(x+4)} = \frac{A}{x} + \frac{B}{x+4}$$

NUMERATOR how many **parts** of that whole are **being considered**

DENOMINATOR **total number** of parts created from the whole

LONG DIVISION

a method used for **dividing large numbers into groups or parts**

$$\begin{array}{r|l}
 & 2x^3 + 3x^2 - 1x - 5 \\
\hline
x - 2 & 2x^4 - 1x^3 - 7x^2 - 3x + 10 \\
 & 2x^4 - 4x^3 \\
\hline
 & 0 \quad 3x^3 - 7x^2 - 3x + 10 \\
 & \quad\; 3x^3 - 6x^2 \\
\hline
 & \quad\; 0 \quad -x^2 - 3x + 10 \\
 & \quad\quad\; -x^2 + 2x \\
\hline
 & \quad\quad\; 0 \quad -5x + 10 \\
 & \quad\quad\quad\; -5x + 10 \\
\hline
 & \quad\quad\quad\quad\; 0
\end{array}$$

AWESOME JOB FOR TODAY!

TAIWAN TECH

Welcome to

EMI 2021 SUMMER PROJECT

6nd session

01

CLASSROOM ENGLISH

P.38~45

總結

Sum up

Summarize

End up

文具

Compass 圓規

Correction tape 修正帶

Protractor 量角器

交卷

Hand in

Turn in

直行橫列

Row 列

Column 行

矇混

Don’t try to muddle your way through.

考試

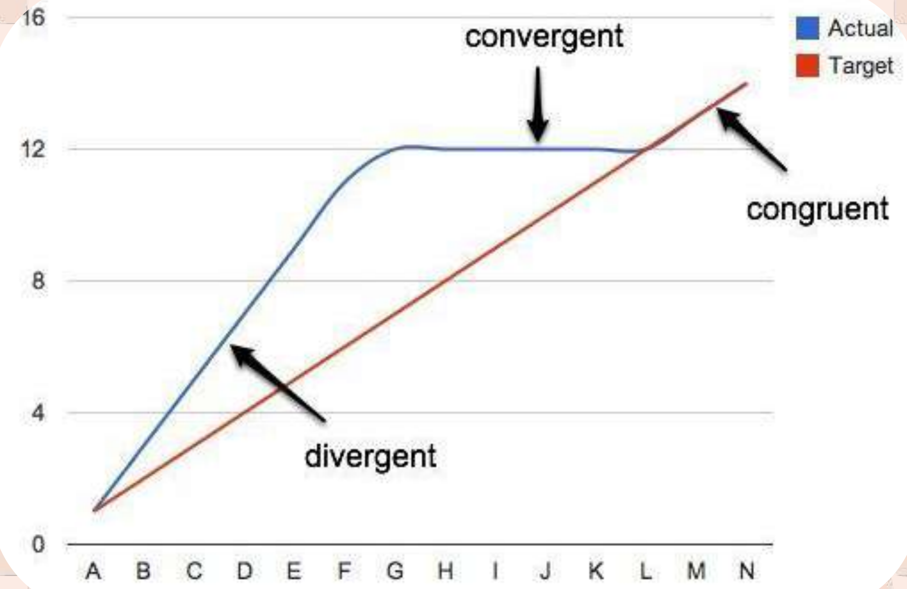
Mock exam 模擬考
Multiple choice 選擇題
Multiple selection 複選題

02

CALCULUS VOCABS

World-list 101 - 120

CONVERGENT CONVERGENCE VS DIVERGENT DIVERGENCE



DIRECT COMPARISON TEST

$$If\ a_n < b_n\ and\ \sum_{n=1}^{\infty} b_n\ converges,\ then\ \sum_{n=1}^{\infty} a_n\ converges$$

$$If\ a_n < b_n\ and\ \sum_{n=1}^{\infty} a_n\ diverges,\ then\ \sum_{n=1}^{\infty} b_n\ diverges$$

LIMIT COMPARISON TEST

If $a_n > 0$ and $b_n > 0$ for all $n \geq N$ (N a positive integer)

If $\lim_{n\to\infty} \frac{a_n}{b_n} = c \quad 0 < c < \infty$, then both $\sum a_n$ and $\sum b_n$ converge or both diverge.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ converges if $\sum b_n$ converges.

If $\lim_{n\to\infty} \frac{a_n}{b_n} = 0$, then $\sum a_n$ converges if $\sum b_n$ converges.

CHECK-UP

CONVERGENT/ CONVERGENCE

coming together

DIVERGENT/ DIVERGENCE

moving apart

1. If $\sum_{n=1}^{\infty} b_n$ converges, then $\sum_{n=1}^{\infty} a_n$ converges.
2. If $\sum_{n=1}^{\infty} a_n$ diverges, then $\sum_{n=1}^{\infty} b_n$ diverges.

DIRECT COMPARISON TEST (DCT)

a test providing a way of deducing the **convergence or divergence** of an **infinite series** or an **improper integral**

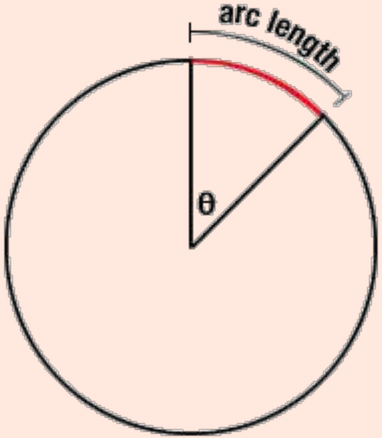
$$\lim_{n\to\infty}\left(\frac{a_n}{b_n}\right) = L$$

where L is *finite and positive*. Then the two series $\Sigma\, a_n$ and $\Sigma\, b_n$ either both converge or both diverge.

LIMIT COMPARISON TEST (LCT)

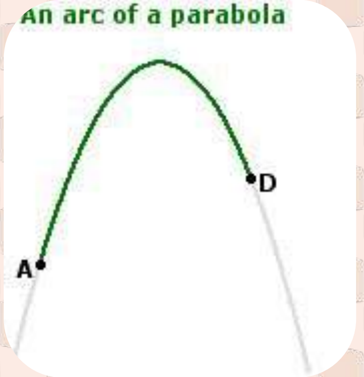
test used to find out if an **infinite series** of numbers **converges** (settles on a certain number) **or diverges**

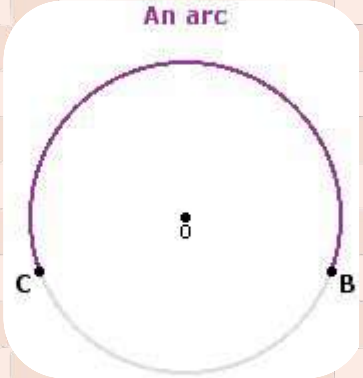
ARC LENGTH



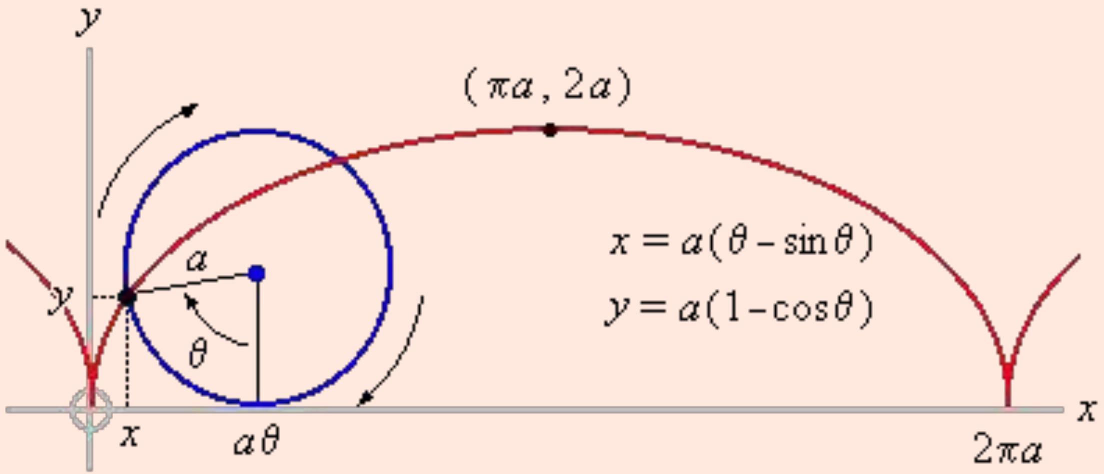
$$s = \int_a^b \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

ARCH

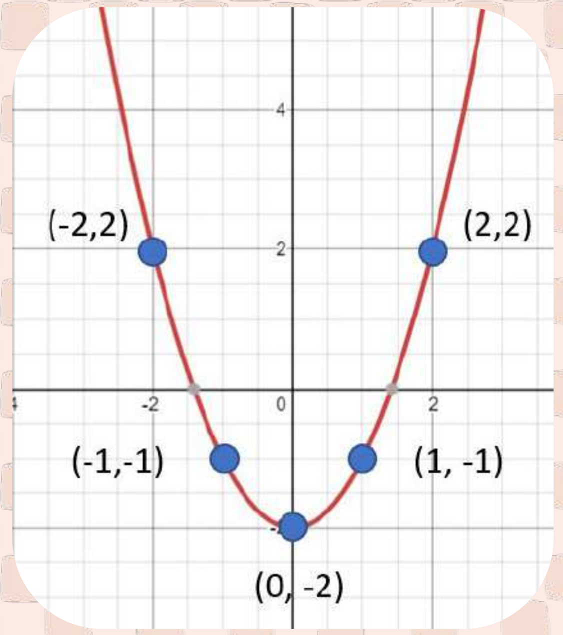


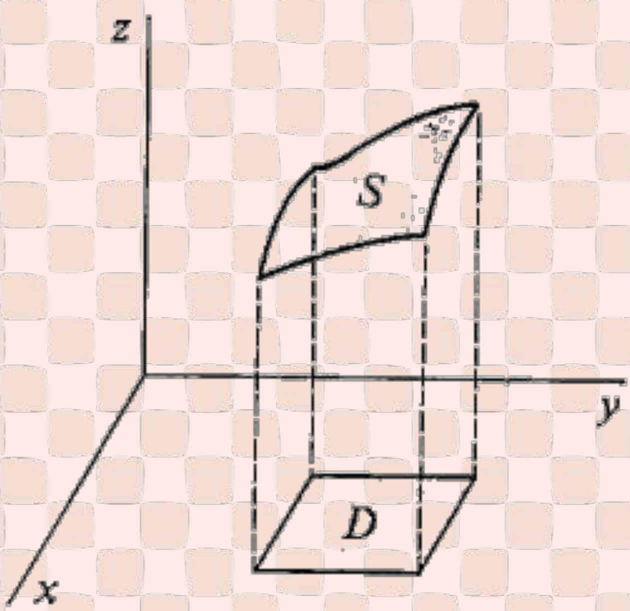


CYCLOID

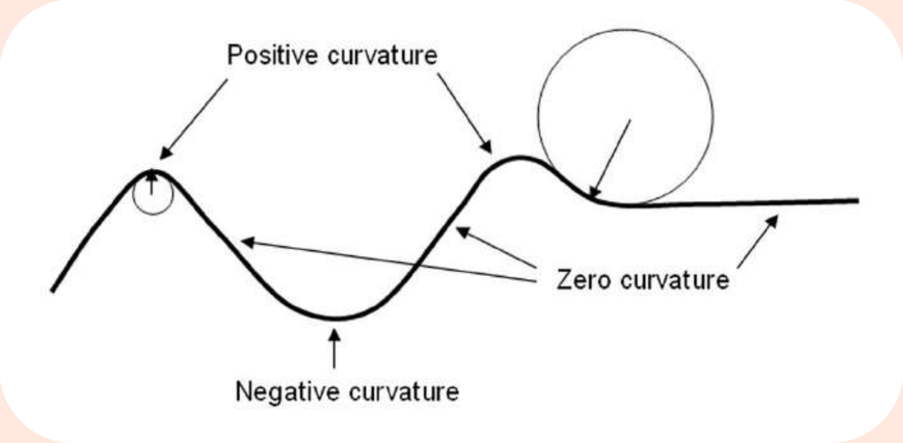


SMOOTH CURVE & SMOOTH SURFACE



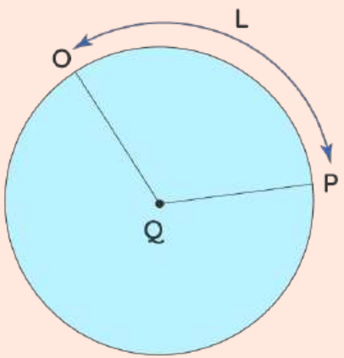


CURVATURE



$$\kappa = \frac{\left|f''(x)\right|}{\left(1+\left[f'(x)\right]^2\right)^{\frac{3}{2}}}$$

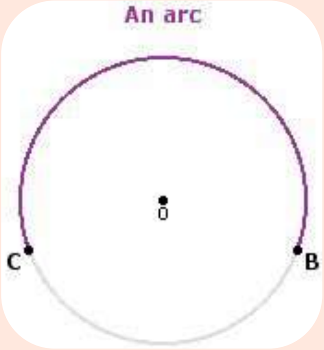
CHECK-UP



$$s(x) = \int_a^x \sqrt{1 + [f'(t)]^2}\, dt$$

ARC LENGTH

distance between two points along a section on the **curve**

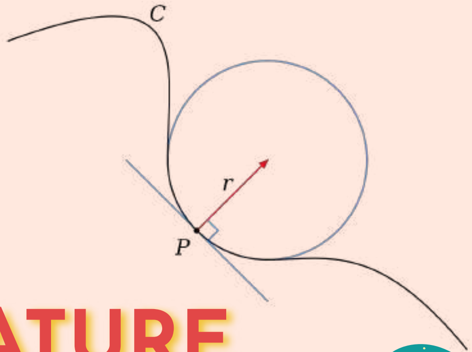


ARCH

a **curved member** that is used to span an opening and to **support loads** from above

CYCLOID

curve generated by a point on the circumference of **a circle that rolls along a straight line**



SMOOTH CURVE/SURFACE

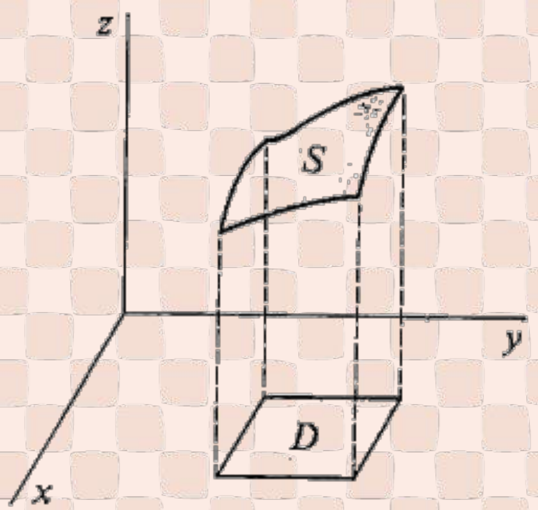
a curve which is a **smooth function** or a **continuous map *f*** from a one-dimensional space to an n-dimensional space

CURVATURE

measures **how fast a curve is changing direction** at a given point

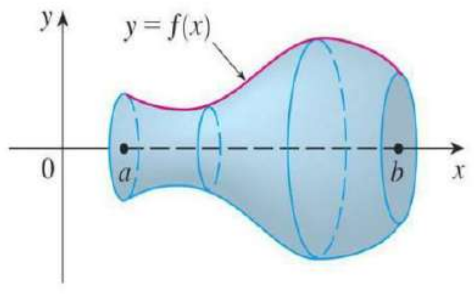
$$\kappa = \frac{|x'y'' - y'x''|}{(x'^2 + y'^2)^{\frac{3}{2}}},$$

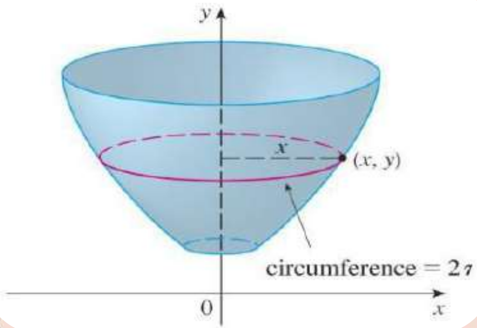
SURFACE AREA



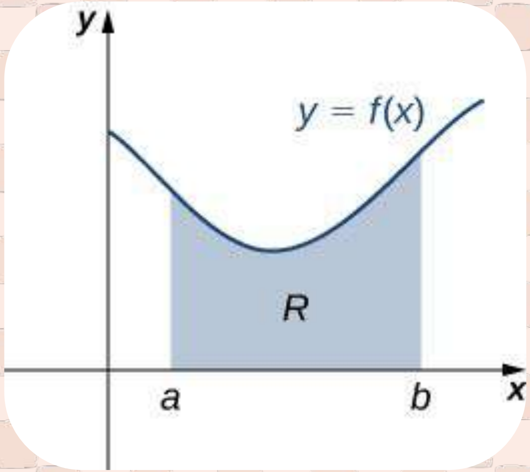
$$A = \int_a^b 2\pi f(x)\sqrt{1 + (\frac{df}{dx})^2}dx.$$

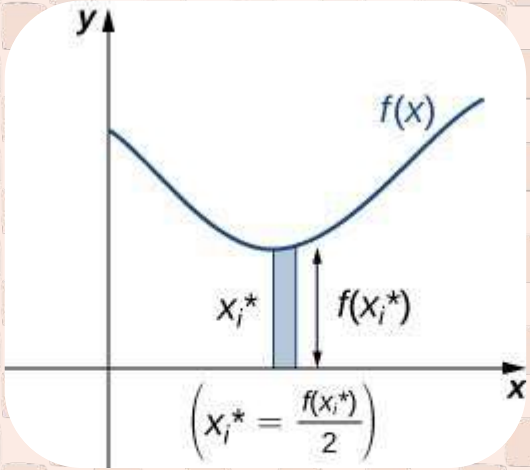
SURFACE OF REVOLUTION





CENTER OF MASS





$$x_{cm} = \lim_{\Delta m \to 0} \frac{\sum_{i=1}^{N} \Delta m_i x_i}{M} = \frac{\int_0^M x\,dm}{M}$$

MOMENT

$$M_y = \rho \int_a^b x f(x)\,dx$$

$$M_x = \frac{1}{2} \rho \int_a^b [f(x)]^2\,dx$$

CHECK-UP

SURFACE AREA

area of a given surface

SURFACE OF REVOLUTION

a **surface** generated by **rotating** a two-dimensional **curve about an axis**

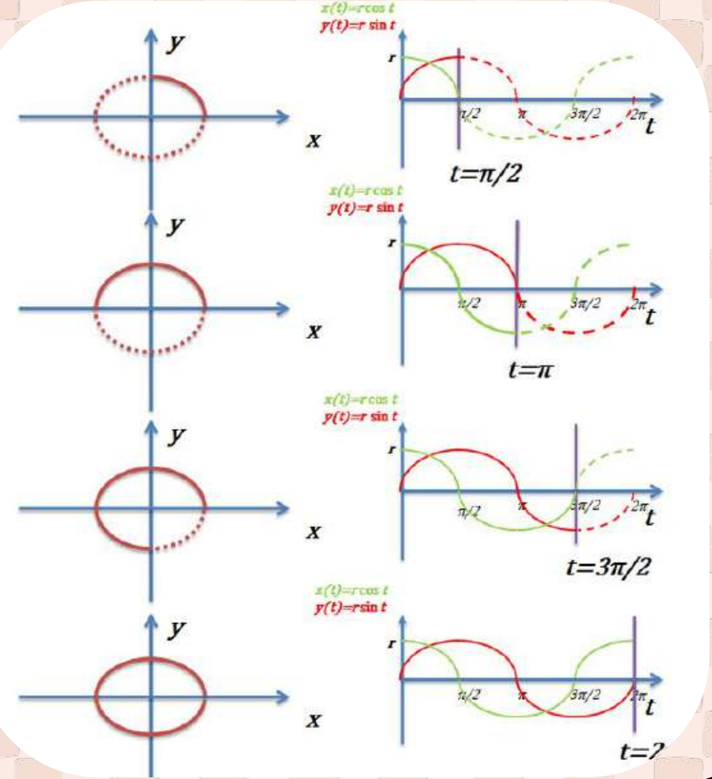
CENTER OF MASS

an **average position** of all the parts of the system

MOMENT

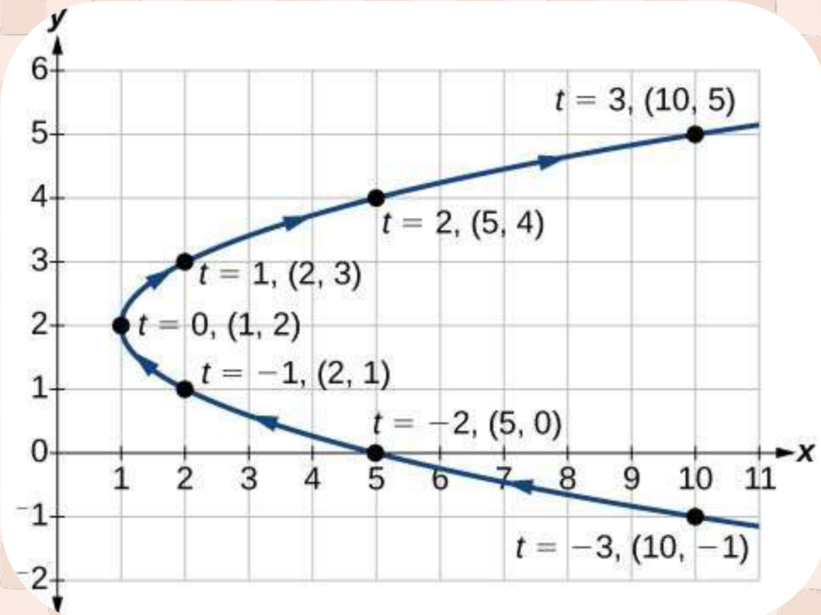
quantitative measures related to the **shape of the function's graph**

PARAMETRIC EQUATION & PARAMETRIC CURVE

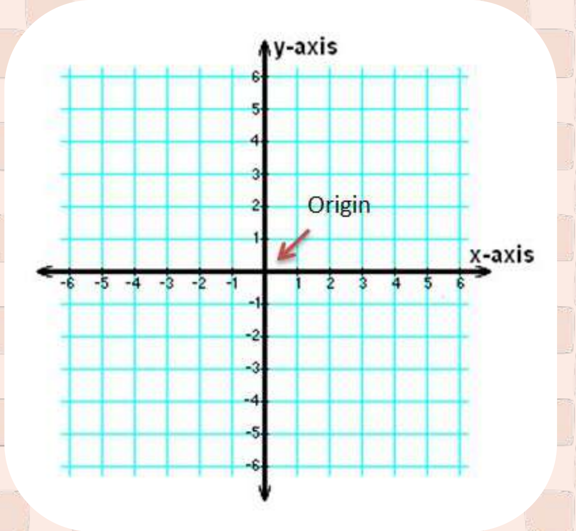


PARAMETER

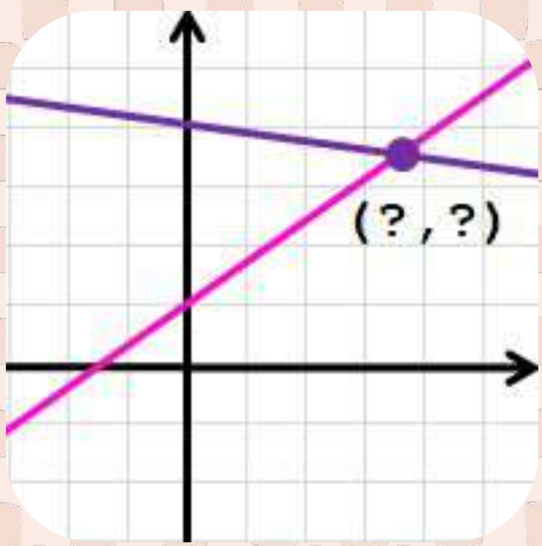
here **t** is a parameter



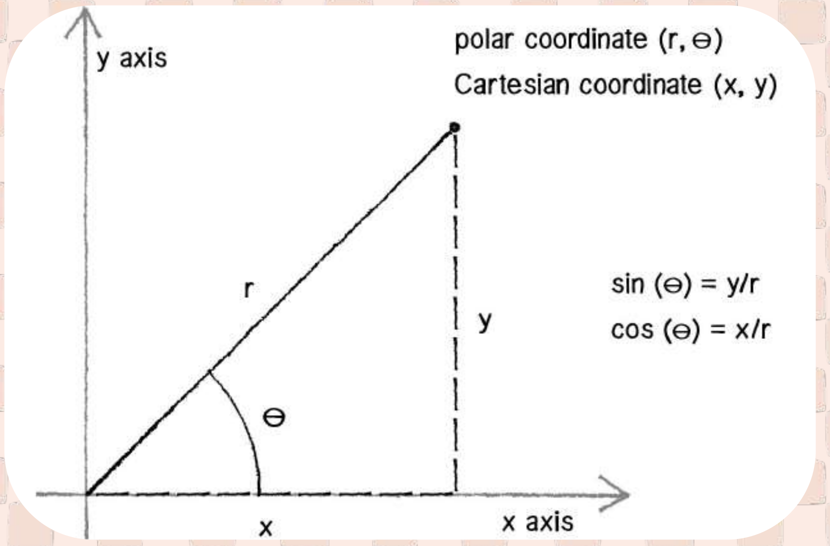
ORIGIN



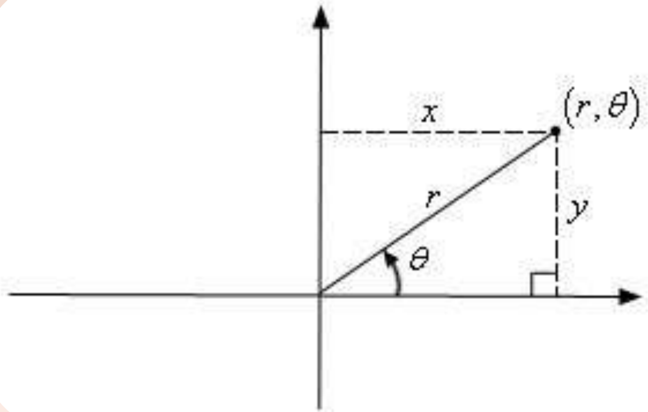
INTERSECTION



POLAR COORDINATE



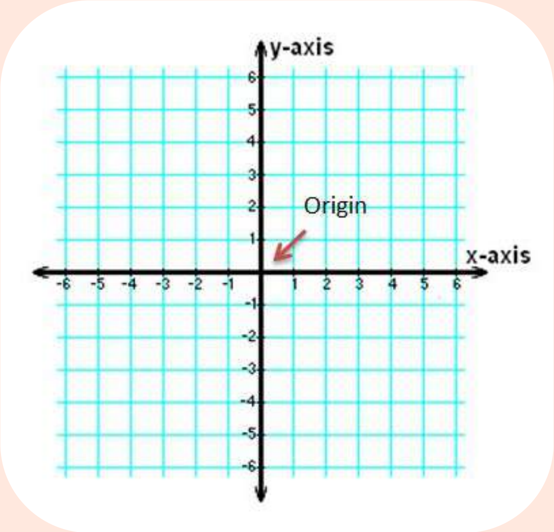
POLE



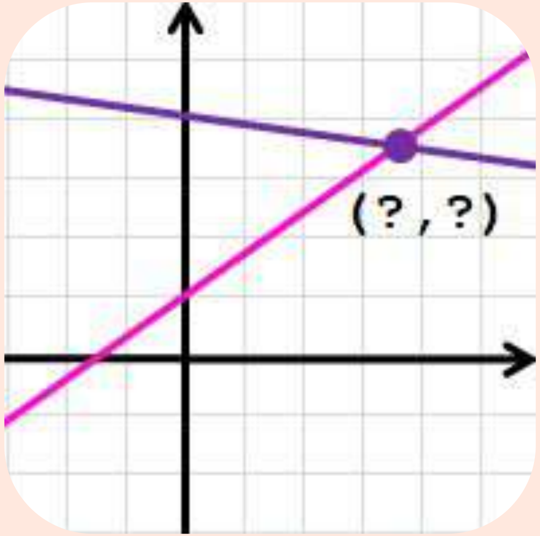
POLAR
AXIS

CHECK-UP

ORIGIN

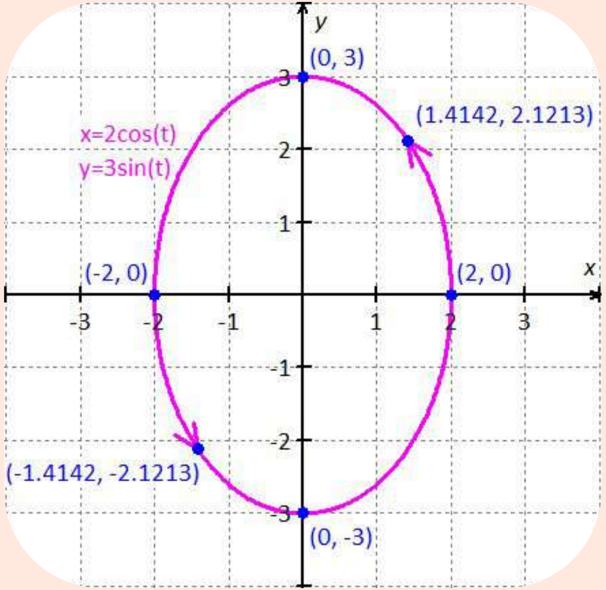


INTERSECTION



PARAMETRIC EQUATION/ CURVE

one of two or more equations **expressing the location** of a point on a curve or surface by **determining each coordinate separately**



PARAMETER

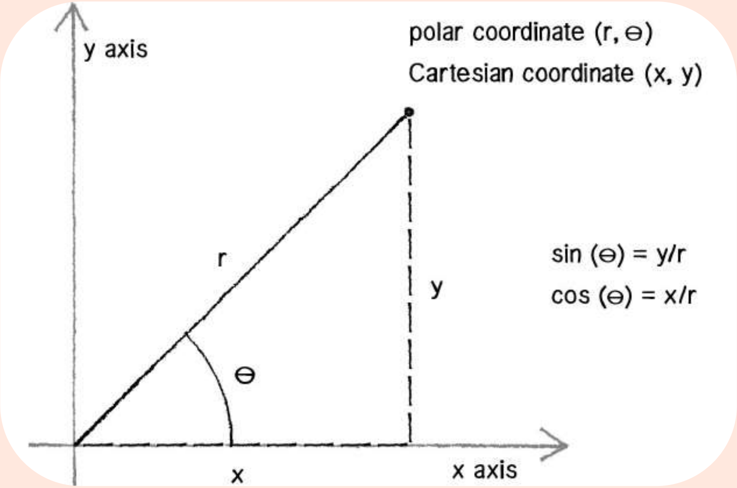
a **variable** for which the range of possible values **identifies a collection of distinct cases** in a problem

POLE

origin in cartesian system

POLAR COORDINATE

a **system locating the position** of a point in a plane, consists of the **length of the straight line (r)** connecting the point to the origin, and the **angle (θ)**



POLAR AXIS

the **line segment** ray **from the pole** or the centre in the reference direction

THANK U

BYE BYE