Question Paper Code : 5501

M.A./M.Sc. (Semester-I) Examination, 2019

STATISTICS

[First Paper]

(Real Analysis)

Time : Three Hours] [Maximum Marks : 70

Note : Answer **five** questions in all. Question **No. 1** is **compulsory**. Besides this, **one** question is to be attempted from each unit.

1. Answer all parts : [3x10=30]

   (a) Define closed set and prove that a set is closed if and only if its complement is open.

   (b) Define open set and prove that the intersection of any finite family of open sets is open.

   (c) Evaluate the upper and lower Riemann integrals of $f(x)$ over $[0,2]$, where

$$f(x) = x + x^2, \text{ when } x \text{ is rational.}$$

$$= x^2 + x^3, \text{ when } x \text{ is irrational.}$$

(d) Prove that every bounded monotonic function is Riemann integrable.

(e) Give an example of the bounded function which is not Riemann integrable over $[0,1]$.

(f) Find the radius of convergence of the following series :

$$x + \frac{x^2}{2^2} + \frac{\lfloor 2}{3^3}x^3 + \frac{\lfloor 3}{4^4}x^4 + \ldots\ldots$$

(g) If $f \in L^p[a,b]$, $g \in L^p[a,b]$ then show that $f + g \in L^p[a,b]$.

(h) Show that Bonnet's mean value theorem does not hold for $f(x) = g(x) = x^2$ on interval $[-1,1]$.

(i) Test the convergence of the integral

$$\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$$

(j) Define convex function and give an example.

UNIT-I [10]

2. (a) State and prove Bolzano-Weierstrass theorem.

(b) State and prove Representation theorem for open sets in R.

3. (a) Prove that zeros of $\sin\left(\frac{1}{x}\right)$ form a set of Ist order, zeros of $\sin\left(\frac{1}{\sin\frac{1}{x}}\right)$ form a set of IInd order and zeros of $\sin\left(\frac{1}{\sin\frac{1}{\sin\frac{1}{x}}}\right)$ form a set of IIIrd order.

(b) Prove that every closed and bounded set of real numbers is compact.

UNIT-II

[10]

4. (a) Determine the maximum and minimum values of the function

$$f(x,y) = 2x^2 + 3y^2 - 2x$$

subject to the constraints $x^2 + y^2 \leq 1$.

(b) Find the shortest distance from the point $\left(\frac{3}{2}, 0\right)$ to the parabola $y^2 = 4x$.

5. (a) Find the largest and smallest distance from $(0,0,0)$ to the ellipsoid :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1,$$

where $0 < a < b < c$.

(b) Prove that every bounded sequence has a limit point.

UNIT-III [10]

6. (a) State and prove the necessary and sufficient condition for Riemann integrability.

(b) State and prove IInd mean value theorem, and verify it for the functions :

$$f(x) = x \text{ and } g(x) = e^x \text{ in } [-1, 1].$$

7. (a) State and prove Abel's theorem for power series.

(b) Let $\sum a_n x^n$ be a power series with unit radius of convergence and let

$$f(x) = \sum_{n=0}^{\infty} a_n x^n \quad (-1 < x < 1)$$

if the series $\sum a_n$ converges, then

$$\lim_{x \to 1-0} f(x) = \sum_{n=0}^{\infty} a_n$$

UNIT-IV [10]

8. (a) State and prove Holder's inequality.

(b) State and prove Minkowski's inequality.

9. (a) Discuss the convergence of the Beta function.

(b) Show that the integral

$$\int_0^\infty e^{-a^2x^2} \cos bx \, dx$$

is absolutely convergent.

(c) Prove that $\int_0^1 \frac{dx}{\sqrt{x(1-x)}}$ converges.

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Question Paper Code : 5502

M.A./M.Sc. (Semester-I) Examination, 2019

STATISTICS

[Second Paper]

(Linear Algebra)

Time : Three Hours] **[Maximum Marks : 70**

Note : Answer **five** questions in all. Question **No. 1 of** short answer type is **compulsory**. Besides this, **one** question is to be attempted from each unit.

1. Answer all the parts : [3x10=30]

(a) Define Field.

(b) Define basis of Vector space.

(c) What do you understand by Linear transformation ?

(d) Define Kronecker Product.

(e) State the properties of inverse of a matrix.

(f) Define Symmetric, Idempotent and Singular matrices.

(g) What do you understand by Signature of quadratic form ?

(h) Define Jordan's Decomposition.

(i) Describe Matrix differentiation.

(j) Define Quadratic form.

UNIT-I

2\. (a) Define Vector space. [3]

(b) Determine the linearly independent vectors among following vectors : [7]

$$x_1 = \begin{bmatrix} 4 \\ 2 \\ 1 \\ -2 \\ 3 \end{bmatrix}, x_2 = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 2 \\ -1 \end{bmatrix}, x_3 = \begin{bmatrix} 2 \\ 1 \\ -2 \\ -4 \\ 4 \end{bmatrix}, x_4 = \begin{bmatrix} 2 \\ 1 \\ -7 \\ -10 \\ 9 \end{bmatrix}$$

3. (a) Describe Gram-Schmidt process of orthogonalization. [5]

(b) If : [5]

$$x = \begin{bmatrix} 3 \\ 1 \\ 4-i \\ 2+i \end{bmatrix}, y = \begin{bmatrix} 1+i \\ i \\ 2 \\ 3 \end{bmatrix}, \alpha = 2+i,$$

$p = 3+i$, then find -

$$< x, y >, < y, x >, \|y\|, \|x-y\|$$

and $< \alpha x, \beta y >$, where $< a, b >$ means inner product of a and b.

UNIT-II

4. (a) Prove that the dimension of the vector space formed by the set of all linear transformations $L(V_m, W_n)$ is nm. [5]

(b) Obtain the determinant of following matrix using Laplace expansion : [5]

$$A = \begin{bmatrix} 2 & 5 & 1 & 3 \\ 4 & 1 & 7 & 9 \\ 6 & 8 & 3 & 2 \\ 7 & 8 & 1 & 4 \end{bmatrix}$$

5. (a) Let V and W be the vertor spaces over field F and L be a linear transformation from V to W. If L is invertible then show that the inverse function of L will also be a linear transformation. [5]

(b) Obtain the inverse of following matrix using sweep out method : [5]

$$\begin{bmatrix} 2 & 5 & 1 \\ 4 & 1 & 7 \\ 6 & 8 & 3 \end{bmatrix}$$

UNIT-III

6. (a) Reduce the following quadratic form into diagonal form using Lagrange's method : [6]

$$q(x) = 2x_1^2 + 4x_2^2 + 8x_3^2 + 2x_1x_2 - 2x_1x_3 - 2x_2x_3$$

(b) Show that following matrix satisfy its own characteristic equation : [4]

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 0 & 0 & 4 \end{bmatrix}$$

7. (a) State about the definiteness of following quadratic form : [6]

$$q(x) = x_1^2 + 3x_2^2 + 6x_3^2 - 2x_1x_2 + 4x_1x_3$$

(b) Obtain the rank of the following matrix : [4]

$$A = \begin{bmatrix} 1 & 2 & 8 & 6 & 4 \\ 2 & 4 & 9 & 7 & 16 \\ 5 & 10 & 26 & 20 & 36 \\ 3 & 8 & 9 & 4 & 5 \\ 17 & 40 & 63 & 40 & 79 \end{bmatrix},$$

UNIT-IV

8. Obtain Singular value decomposition for following matrix : [10]

$$A = \begin{bmatrix} 3 & 2 & 2 \\ 2 & 3 & -2 \end{bmatrix}$$

9. Obtain the Eigenvalue decomposition for following matrix : [10]

$$A = \begin{bmatrix} 3 & 1 & 1 \\ 1 & 2 & 2 \\ 1 & 2 & 2 \end{bmatrix}$$

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Question Paper Code : 5503

M.A./M.Sc. (Semester-I) Examination, 2019

STATISTICS

[Third Paper]

(Measure Theory and Probability)

Time : Three Hours] [Maximum Marks : 70

Note : Answer **five** questions in all. Question **No. 1 of** short answer type is **compulsory**. Besides this, **one** question is to be attempted from each unit.

1. Answer all the parts : [3x10=30]

   (a) Define a ring and a field. Give an example which is not a field.

   (b) Define lim inf. A_n and lim sup. A_n for a sequence of sets $\{A_n\}_{n=1,\,2,\,......}$

   (c) Let B be a $\sigma-$field. If $\{A_1, A_2, ..., A_n, ...\}$ is a sequence of sets in B, then show that :

$$\bigcap_{n=1}^{\infty} A_n \in B$$

(d) If μ is a measure and $\{A_n\}_{n=1,2,.....}$ be a non-decreasing sequence of sets, then show that :

$$\underset{n \to \infty}{Lim} [\mu(A_n)] = \mu\left[\lim_{n \to \infty} A_n\right]$$

(e) If $\{X_n\}_{n=1,2,.....}$ is a converging sequence of measurable functions, show that $\lim_{n \to \infty} X_n$ is also measurable function.

(f) If $\{X_n\}_{n=1,2,......}$ be a sequence of non-negative measurable functions, prove that :

$$\int \Sigma X_n \, d\mu = \Sigma \int X_n d\mu$$

(g) Define convergence in probability and convergence in distribution.

(h) Define the distribution function of a random variable and state its properties.

(i) What is product space ? State a necessary and sufficient condition for a measurable sub-set to have a measure zero of product space.

(j) Define sub-martingales and super-martingales and state their properties.

UNIT-I

2. Define monotone class of sets and prove that : [10]

$$\overline{\lim}\left(A_n \cup B_n\right) = \overline{\lim} A_n \cup \overline{\lim} B_n$$

3. If X and Y are two measurable functions, show that $X+Y=Z$ and $XY=W$ are also measurable functions. [10]

UNIT-II

4. Define convergence in probability and show that : [10]

$$X_n \xrightarrow{P} C \Leftrightarrow X_n \xrightarrow{L} C$$

where C is a constant and symbols have their usual meanings.

5. Let $X_n \xrightarrow{P} X$ and $Y_n \xrightarrow{P} Y$, then prove that : [10]

(i) $aX_n \xrightarrow{P} aX \quad ('a' \text{ real})$

(ii) $X_n + Y_n \xrightarrow{P} X + Y$

UNIT-III

6. State and prove Helly-Bray theorem. [10]

7. Define expectation and conditional expectation using Radon-Nikodymn theorem, giving the statement of this theorem. [10]

UNIT-IV

8. State and prove Decomposition theorem. [10]

9. State and prove Schwartz's inequality. [10]

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Question Paper Code : 5504

M.A./M.Sc. (Semester-I) Examination, 2019

STATISTICS

[Fourth Paper]

(Sample Surveys)

Time : Three Hours] **[Maximum Marks : 70**

Note : Answer **five** questions in all. Question **No. 1 of** short answer type is **compulsory**. Besides this, **one** question is to be attempted from each unit.

1. Answer all the parts : [3x10=30]

(a) Define probability proportional to size (pps) sampling scheme. When does PPS sampling reduces to simple random sampling ?

(b) Discuss various procedures for selecting a pps sample.

(c) In πps sampling scheme, how should the probability of including i^{th} unit in the sample (π_i) be taken so that the estimator is more efficient ?

(d) Show that $\sum_{j}^{N}(\# i)\pi_{ij} = (n-1)\,\pi_{i}$, where π_{ij} is the probability of including i^{th} and j^{th} units in the sample of size n from a population of size N.

(e) Define Horvitz Thompson Estimator of $\widehat{\overline{V(Y_{HT})}}$. Does this estimator suffer from any drawback ? Discuss.

(f) Define Ordered sample and Ordered sampling design.

(g) While using Midzuno Sen Sampling Scheme, illustrate how using revised probabilities of selection P_i^*, the inclusion probabilities π_i are proportional to initial probabilities to selection (P_i) and how this limits the use of the sampling scheme.

(h) Discuss Sampford's sampling scheme.

(i) Discuss randomised response technique along with its utility.

(j) Discuss confidence interval for bias-adjusted Jackknife estimator.

UNIT-I

2. In varying probability sampling with replacement, obtain an unbiased estimator of population total and its variance. Also find an unbiased estimator of variance also. [10]

3. Define Desraj's ordered estimator for case of two draws for Probability Proportional to Size (PPSWOR) technique. Derive its variance. [10]

UNIT-II

4. Define Horvitz-Thompson Estimator of Population Mean. Show that its unbiased. Derive Yates-Grundy form of $\overline{V(\hat{Y}_{HT})}$. [10]

5. Explain Rao-Hartley-Cochran sampling scheme. What are its advantages ? Write down the estimator for population mean. Obtain its variance. [10]

UNIT-III

6. Describe the Jackknife estimate. Define Quenouille's bias estimator. Also obtain the estimate of variance. [10]

7. Describe bootstrap technique. How are bias and variance estimated ? Also give its algorithm. [10]

UNIT-IV

8. Discuss in detail Unrelated question randomised response technique. How is it different from the related randomised response technique ? [10]

9. Write short note on Sampling and Non-sampling errors. What are the sources of Non-sampling errors ? [10]

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Question Paper Code : 5505

M.A./M.Sc. (Semester-I) Examination, 2019

STATISTICS

[Fifth Paper (Elective)]

[Data Analysis Using SPSS]

Time : Three Hours] **[Maximum Marks : 70**

Note : Answer **five** questions in all. Question **No. 1 of** short answer type is **compulsory**. Besides this, **one** question is to be attempted from each unit.

1. Answer all the parts : [3x10=30]

(a) Discuss various types of variable *recode* options available in SPSS.

(b) What are the most popular graphical methods used in SPSS to check for outliers in the data ?

(c) Discuss different types of measurement scales in SPSS with the help of examples.

(d) What kind of robust measures of central tendencies are available in SPSS ? How to compute them ?

(e) Discuss Mc. Nemar's test and state how it is performed in SPSS ?

(f) In order to perform paired t-test in SPSS, what are the assumptions required and how are they checked ?

(g) Write brief note on Ordinal regression and state how it is performed in SPSS ?

(h) State clearly the steps for performing non-parametric equivalent of two samples (independent and paired) tests in SPSS.

(i) Discuss various procedures for clustering variables/cases in SPSS.

(j) When in Box's M-test of equality of covariance matrices applied ?

UNIT-I

2. Discuss in detail about any six procudres in SPSS for data file handling and transformation. [10]

3. How is the data set imported (and exported) from (to) an external source in SPSS ? State clearly all the steps. Also describe various files in the SPSS environment along with extension for each. [10]

UNIT-II

4. Discuss about various statistical computing that can be performed using Analyse → Descriptive Statistics → Crosstabs. [10]

5. Discuss in detail (along with sub-types) any five graphical options available in SPSS. Give examples. [10]

UNIT-III

6. Three sets of five mice were randomly selected to be placed in a standard maze but with different colour doors. The response is the time required to complete the maze. Appropriate analysis was performed to test if there is an effect due to door colour. The outputs of the analysis are given below : [10]

Test of Homogeneity of Variances

Maze Time

Levenue Statistic	df1	df2	Sig.
8522	2	12	5384

ANOVA

Maze Time

	df	Sum of Squares	Mean Squares	F	Sig
Between Groups	2	565.7333	282.9667	20.0142	.0002
Within Groups	12	169.6000	14.1333		
Total	14	735.3333			

Post Hoc Tests

Multiple Comparisons

Dependent variable Maze Time

Tukey HSD

(I) Door Colour	(J) Door Color	Mean Difference (1-J)	Std. Error	Sig.	99% confidence interval Lower Bound	Upper Bound
Red	Green	-11.4000*	2.3777	.0012	-19.8836	-2.9164
	Black	2.8000	2.3777	.4879	-5.6936	11.2836
Green	Red	11.4000*	2.3777	0.0012	2.9164	19.8836
	Black	14.2000*	2.3777	0.0002	5.7164	22.6836
Black	Red	-2.8000	2.3777	0.4879	-11.2836	5.6836
	Green	-14.2000	2.3777	0.0002	-22.6836	-5.7164

*The mean difference is significant at the 01 level

Answer the following questions :

(i) Identify the test procedure. State the assumptions. What were the steps performed to be able to produce this output ?

(ii) What will be the interpretation for each of the three tables given above ?

(iii) Why are post-hoc multiple comparisons performed ? Discuss about some post-hoc tests that are commonly used.

7. Discuss briefly any three Regression procedures in SPSS. State clearly all the steps/assumptions for performing Linear Regression in SPSS. [10]

UNIT-IV

8. Discuss in detail procedure in SPSS for Dimension Reduction. State clearly all the steps/assumptions for performing the same. [10]

9. Discuss briefly about discriminant analysis. Give its step by step analysis in SPSS along with details of various tests applied. Discuss its utility along with other multivariate techniques in SPSS. [10]

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