Rochester Institute of Technology

EE629 Antenna Theory

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Due Date: 10/13/2020

Re: PROJECT #2: Linear Arrays

Abstract

This project contains the analysis for the Uniform linear array pattern observed for different spacing when placed on z- axis and y - axis. The graphs are plotted using MATLAB code. Analysis of the graphs are done in the discussion section.

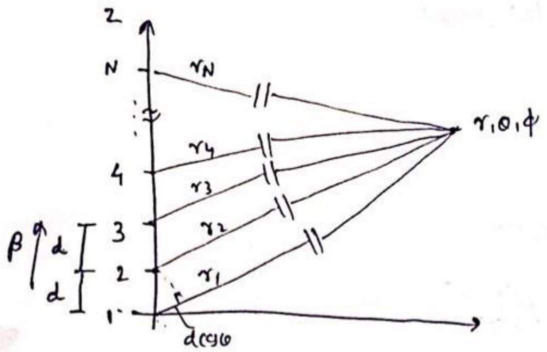
Theory

N element linear array is the generalized form of array factor. Figure 1 and 2 shows the derivation part for the N element linear array in which all elements have identical amplitudes and each succeeding element has progressive phase difference of β.

An array of identical elements all of the identical magnitude with progressive phase difference is known as uniform array.

Final equations in case when the elements are placed on z–axis and y–axis are shown in figure 1 and 2. By using that equations, main beam, nulls, side lobs and other quantities can be achieved.

* N-element uniform array of isotropic sources: (z-axis)



$\Rightarrow$ Spacing = d , progressive phase difference = β

For each element $E_p = \dfrac{1 \cdot e^{-jkr}}{r}$

$$AF = 1 + e^{j(kd\cos\theta + \beta)} + e^{j2(kd\cos\theta + \beta)} + \dots + e^{j(N-1)(kd\cos\theta + \beta)}$$

Let $kd\cos\theta + \beta = \psi$

$$AF = 1 + e^{j\psi} + e^{j2\psi} + e^{j3\psi} + \dots + e^{j(N-1)\psi} \quad \text{—— (1)}$$

multiplying (1) by $e^{j\psi}$

$$\text{so} \Rightarrow e^{j\psi}(AF) = e^{j\psi} + e^{j2\psi} + \dots + e^{jN\psi} \quad \text{—— (2)}$$

$$(2) - (1) \Rightarrow \quad AF\left[e^{j\psi} - 1\right] = e^{jN\psi} - 1$$

$$AF = \frac{1 - e^{jN\psi}}{1 - e^{j\psi}}$$

$$= \frac{e^{j\frac{N}{2}\psi}\left(e^{-j\frac{N}{2}\psi} - e^{j\frac{N}{2}\psi}\right)}{e^{j\frac{\psi}{2}}\left(e^{-j\frac{\psi}{2}} - e^{j\frac{\psi}{2}}\right)}$$

Figure 1. Derivation part 1

If elements are centered about the origin.

$$AF = \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\frac{\psi}{2}}$$

⇒ To Normalize the array factor,

$$AF\Big|_{\psi \to 0} = \lim_{\psi \to 0} \left[\frac{\cos\left(\frac{N\psi}{2}\right)\left(\frac{N}{2}\right)}{\cos\left(\frac{\psi}{2}\right)\left(\frac{1}{2}\right)} \right]$$

L'Hopital Rule

$$= N$$

so. $$\boxed{AF_N = \frac{1}{N} \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin(\psi/2)}}$$

, where $\psi = kd\cos\theta + \beta$

$k = \frac{2\pi}{\lambda}$

d : spacing

β = progressive phase difference.

⇒ When the N-elements are placed in y-axis,

$$\cos\gamma = \hat{a}_y \cdot \hat{a}_r = \sin\theta\sin\phi$$

$$\gamma = \cos^{-1}(\sin\theta \cdot \sin\phi)$$

$$\text{so } \psi = kd\cos(\sin\theta \cdot \sin\phi) + \beta$$

Figure 2. Derivation part 2

Results

Calculations:

Part – I when N-element uniform array placed on z-axis.

$$AF_N = \frac{1}{N} \cdot \frac{\sin\left(\frac{N\psi}{2}\right)}{\sin\left(\frac{\psi}{2}\right)}, \text{ when } \psi = kd\cos\theta + \beta.$$

(i) $\beta = 0 \Rightarrow$ (Broadside Array)

$d = \frac{\lambda}{4}$, $kd = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2}$. $\Rightarrow \psi = \frac{\pi}{2}\cos\theta$

Main beam $\Rightarrow$

$$\frac{\psi}{2} = \pm m\pi, \quad m = 0, 1, \ldots$$

$$\frac{\pi}{4}\cos\theta = \pm m\pi$$

$$\therefore \cos\theta = \pm 4m, \quad m = 0, 1, \ldots$$

$m = 0$, $\theta = 90^\circ \left(\frac{\pi}{2}\right) \Rightarrow$ Broadside array.

$m = 1$, $\theta \neq 0$ or π (Not possible)

For $d = \lambda$, $kd = \frac{2\pi}{\lambda} \cdot \lambda = 2\pi \Rightarrow \psi = 2\pi\cos\theta$

main beam $\Rightarrow$

$$\frac{\psi}{2} = \pm m\pi, \quad m = 0, 1, \ldots$$

$$\pi\cos\theta = \pm m\pi$$

$$\cos\theta = \pm m \quad m = 0, 1, \ldots$$

$m = 0$, $\theta = \frac{\pi}{2}$

$m = 1$, $\theta = 0$ or π.

Figure 3. Calculation 1

(ii) $\beta = -\frac{\pi}{4}$ (phase Array)

$$d = \frac{\lambda}{4}, \quad kd = \frac{\pi}{2}, \quad \psi = \frac{\pi}{2}\cos\theta - \frac{\pi}{4}$$

main beam ⇒

$$\frac{\psi}{2} = \pm m, \quad m = 0, 1 \ldots$$

$$\frac{\pi}{2}\cos\theta - \frac{\pi}{4} = \pm 2m$$

$$\frac{\pi}{2}\cos\theta = \pm 2m + \frac{\pi}{4}$$

$$\cos\theta = 2\left(\pm\frac{2m}{\pi} + \frac{1}{4}\right) = \left(\pm\frac{4m}{\pi} + \frac{1}{2}\right), \quad m = 0, 1 \ldots$$

$m = 0$, $\cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$m = 1$, $\cos\theta = \pm 1.27 + 0.5$

$\cos\theta = 0.77 \Rightarrow \theta = 39.64$

$m = 2$, not possible.

Figure 4. Calculation 2

(iii) $\beta = -\frac{\pi}{2}$, $(\theta = 0^\circ)$ so end fire array.

★ $x = \underline{d}$ $\quad d = \frac{\lambda}{4} \Rightarrow kd = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{4} = \frac{\pi}{2} \Rightarrow \psi = \frac{\pi}{2}\cos\theta - \frac{\pi}{2}$

$$= \frac{\pi}{2}(\cos\theta - 1)$$

Main beam

$$\frac{\psi}{2} = \pm m\pi, \quad m = 0, 1, \ldots$$

$$\frac{\pi}{2}(\cos\theta - 1) = \pm 2m\pi$$

$$\therefore \cos\theta = \pm 4m + 1$$

$m = 0$, $\cos\theta = 1 \Rightarrow \theta = 0^\circ$

$m = 1$ not possible.

$d = \frac{\lambda}{2} \Rightarrow kd = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{2} = \pi \Rightarrow \psi = \pi\cos\theta - \frac{\pi}{2}$

$$= \pi\left(\cos\theta - \frac{1}{2}\right)$$

main beam

$$\frac{\psi}{2} = \pm m\pi, \quad m = 0, 1, \ldots$$

$$\pi\left(\cos\theta - \frac{1}{2}\right) = \pm 2m\pi$$

$$\cos\theta = \pm 2m + \frac{1}{2}$$

$m = 0$, $\Rightarrow \cos\theta = \frac{1}{2} \Rightarrow \theta = \frac{\pi}{3}$

$m = 1$ Not possible.

$d = \lambda \Rightarrow kd = \frac{2\pi}{\lambda} \cdot \frac{\lambda}{\lambda} = 2\pi$, $\psi = 2\pi\cos\theta - \frac{\pi}{2} = \pi\left(2\cos\theta - \frac{1}{2}\right)$

main beam

$$\frac{\psi}{2} = \pm m\pi, \quad m = 0, 1, \ldots$$

$$\pi\left(2\cos\theta - \frac{1}{2}\right) = \pm 2m\pi$$

$$\therefore \cos\theta = \pm m + \frac{1}{4}$$

$m = 0 \Rightarrow \cos\theta = \frac{1}{4} \Rightarrow \theta = 75.52^\circ$

$m = 1 \Rightarrow \cos\theta = 0.75 \Rightarrow \theta = 41^\circ$ //

Figure 5. Calculation 3

- **MATLAB code for the rectangular plot on Z axis**

```
 % v is taken as Wavelength and assigned one for simplisity of calculation. It will cancel out eventually.
 v = 1;
 N = 10;
% Linear array , d is the spacing
for b = 0:-pi/4:-pi/2
for d = 0.25*v:0.25*v:v
% K is defined as constant
K = 2*pi/v;
x = 0:0.01:2*pi;
si = K*d*cos(x)+ b;
% equation for N element array with spacing d and phase difference b
F = abs((sin(N*si/2))./(N*sin(si/2)));
figure()
plot(x,F)
end
end
```

Figure 6. MATLAB code for rectangular plot for z axis

- **MATLAB code for the polar plot on Z axis**

```
 % v is taken as Wavelength and assigned one for simplisity of calculation. It will cancel out eventually.
 v = 1;
 N = 10;
% Linear array , d is the spacing
for b = 0:-pi/4:-pi/2
for d = 0.25*v:0.25*v:v
% K is defined as constant
K = 2*pi/v;
x = 0:0.01:2*pi;
si = K*d*cos(x)+ b;
% equation for N element array with spacing d and phase difference b
F = abs((sin(N*si/2))./(N*sin(si/2)));
figure()
plot(x,F)
end
end
```

Figure 7. MATLAB code for polar plot for z axis

- Array factor graphs for different spacing in rectangular coordinate system when placed on Z axis.

1) $\beta = 0$, $d = \lambda/4$

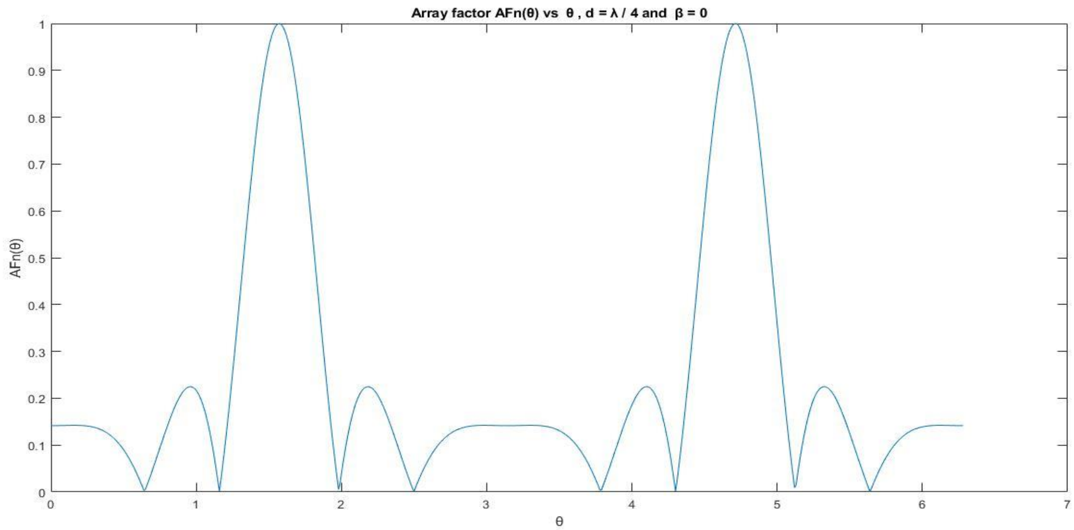


Figure 8. Waveform for $\beta = 0$, $d = \lambda/4$ (Rectangular)

2) $\beta = 0$, $d = \lambda$

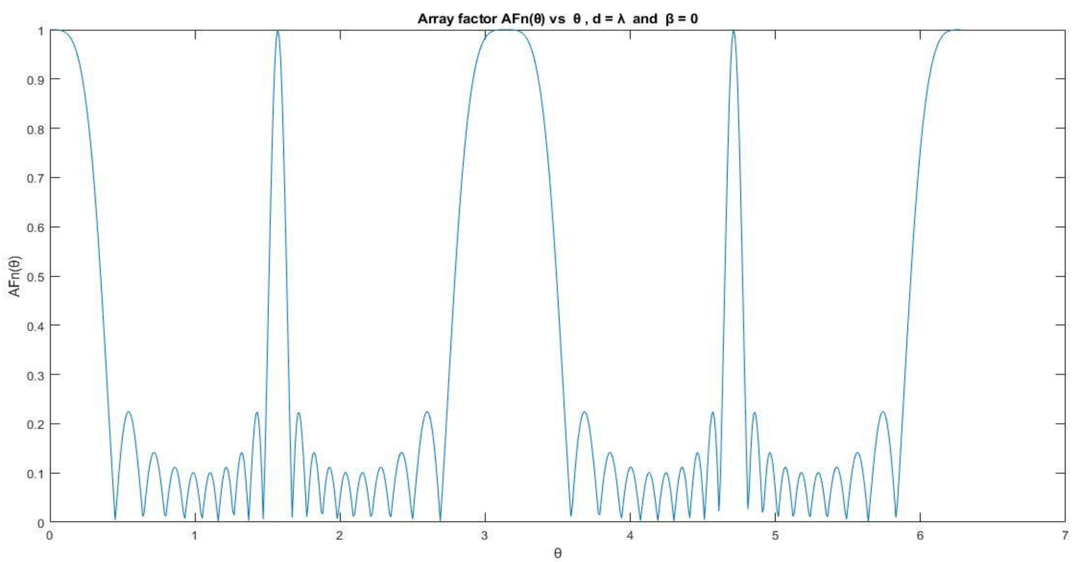


Figure 9. Waveform for $\beta = 0$, $d = \lambda$ (Rectangular)

3) $\beta = -\pi/4$, $d = \lambda/4$

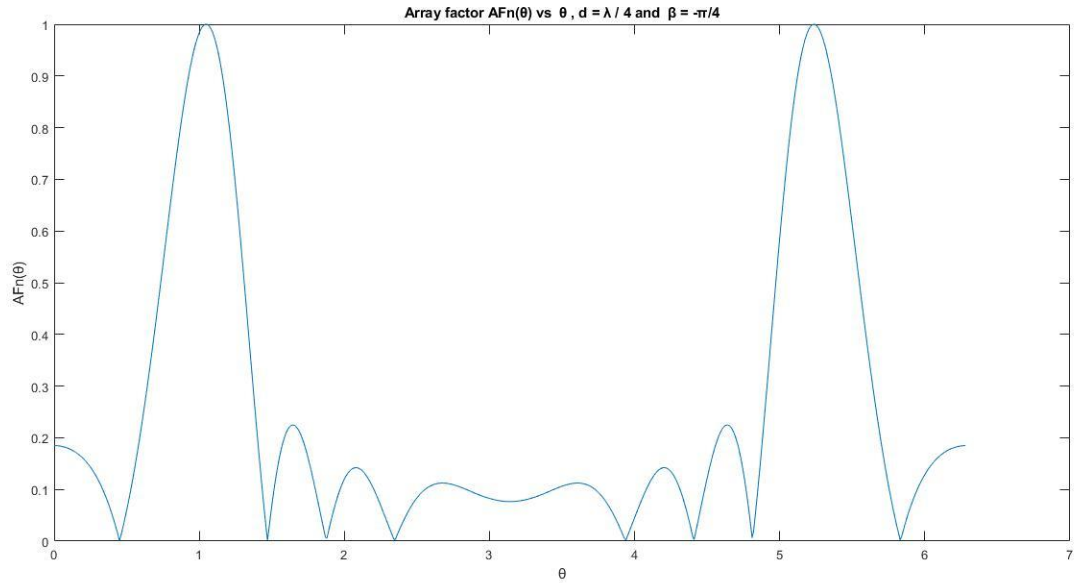


Figure 10.Waveform for $\beta = -\pi/4$, $d = \lambda/4$ (Rectangular)

4) $\beta = -\pi/2$, $d = \lambda/4$

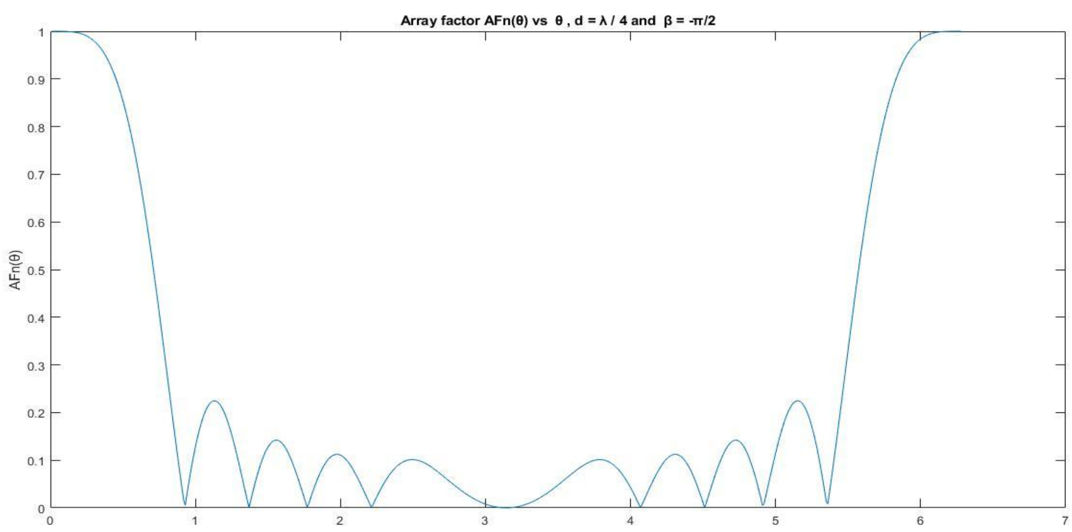


Figure 11. Waveform for $\beta = -\pi/2$, $d = \lambda/4$ (Rectangular)

5) $\beta = -\pi/2$, $d = \lambda/2$

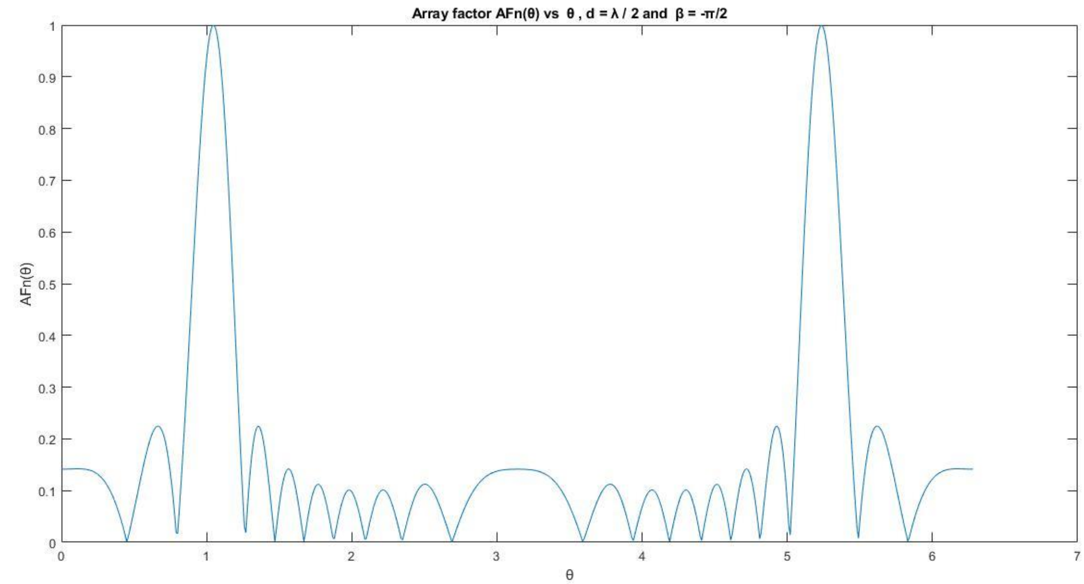


Figure 12. Waveform for $\beta = -\pi/2$, $d = \lambda/2$ (Rectangular)

6) $\beta = -\pi/2$, $d = \lambda$

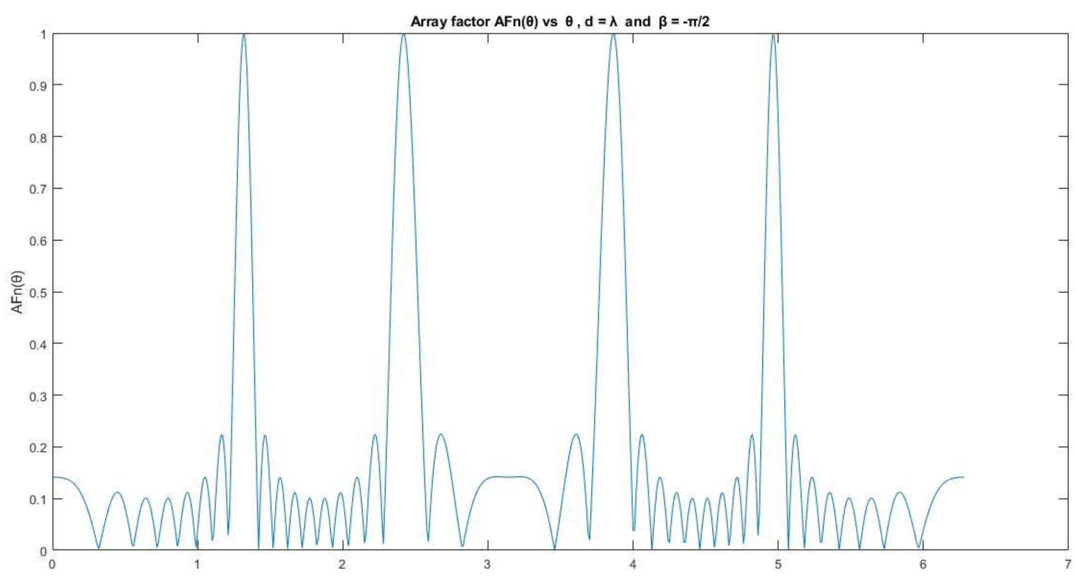


Figure 13. Waveform for $\beta = -\pi/2$, $d = \lambda$ (Rectangular)

Array factor graphs for different spacing in polar coordinate system when on placed on z axis

1) $\beta = 0$, $d = \lambda/4$

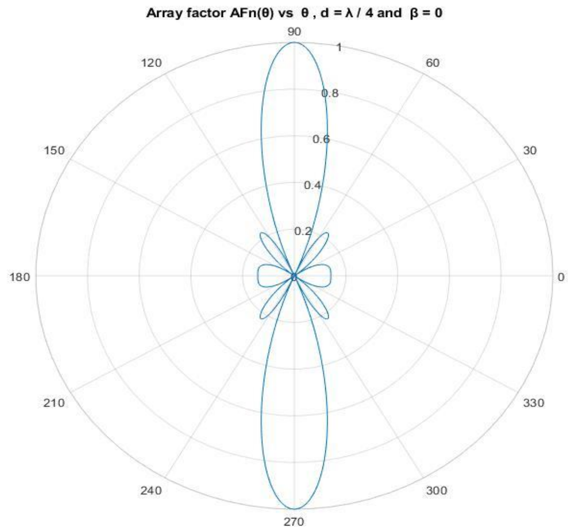


Figure 14.Waveform for $\beta = 0$, $d = \lambda/4$ (Polar)

2) $\beta = 0$, $d = \lambda$

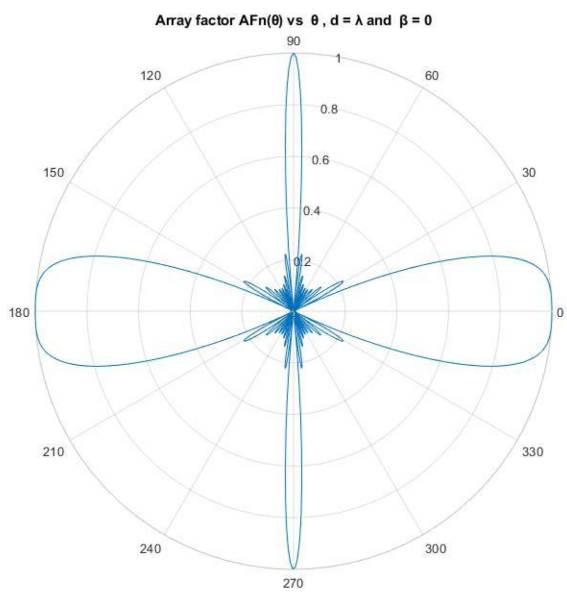


Figure 15. Waveform for $\beta = 0$, $d = \lambda$ (Polar)

3) $\beta = -\pi/4$, $d = \lambda/4$

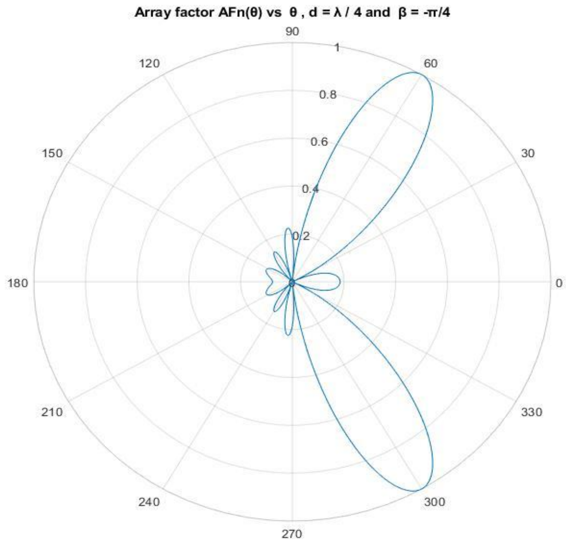


Figure 16. Waveform for $\beta = -\pi/4$, $d = \lambda/4$ (Polar)

4) $\beta = -\pi/2$, $d = \lambda/4$

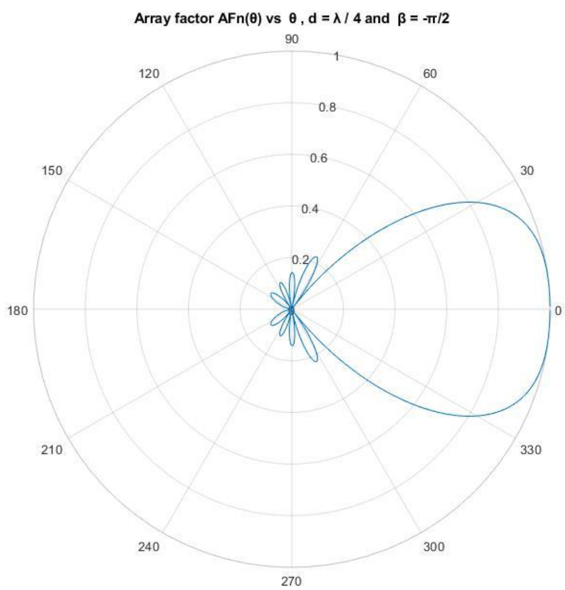


Figure 17. Waveform for $\beta = -\pi/2$, $d = \lambda/4$ (Polar)

5) $\beta = -\pi/2$, $d = \lambda/2$

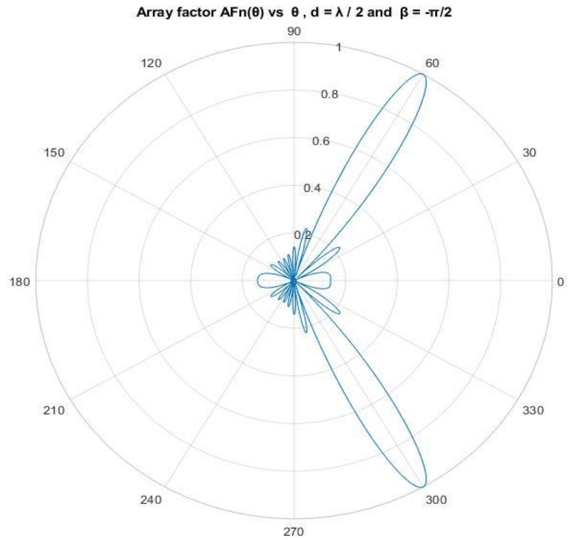


Figure 18.Waveform for $\beta = -\pi/2$, $d = \lambda/2$ (Polar)

6) $\beta = -\pi/2$, $d = \lambda$

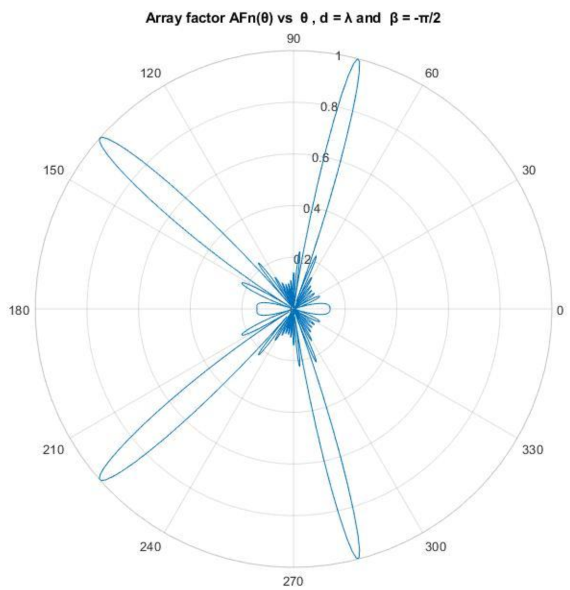


Figure 19. Waveform for $\beta = -\pi/2$, $d = \lambda$ (Polar)

- **MATLAB code for the rectangular plot on y axis**

```
% v is taken as Wavelength and assigned one for simplisity of calculation. It will cancel out eventually.
v = 1;
N = 10;
% Linear array , d is the spacing
for b = 0:-pi/4:-pi/2
for d = 0.25*v:0.25*v:v
% K is defined as constant
K = 2*pi/v;
x = 0:0.01:2*pi;
y = 0:0.01:2*pi;
q = sin(x).*sin(y);
si = K*d*cos(q)+ b;
% equation for N element array with spacing d and phase difference b
F = abs((sin(N*si/2))./(N*sin(si/2)));
figure()
plot(x,F)
end
end
```

Figure 20. MATLAB code foe element on y axis (Rectangular)

- **MATLAB code for the polar plot on y axis**

```
% v is taken as Wavelength and assigned one for simplisity of calculation. It will cancel out eventually.
v = 1;
N = 10;
% Linear array , d is the spacing
for b = 0:-pi/4:-pi/2
for d = 0.25*v:0.25*v:v
% K is defined as constant
K = 2*pi/v;
x = 0:0.01:2*pi;
y = 0:0.01:2*pi;
q = sin(x).*sin(y);
si = K*d*cos(q)+ b;
% equation for N element array with spacing d and phase difference b
F = abs((sin(N*si/2))./(N*sin(si/2)));
figure()
polarplot(x,F)
end
end
```

Figure 21. MATLAB code for element on y axis (Polar)

- Array factor graphs for different spacing in rectangular coordinate system when placed on y axis.

1) β = 0 , d = λ/4

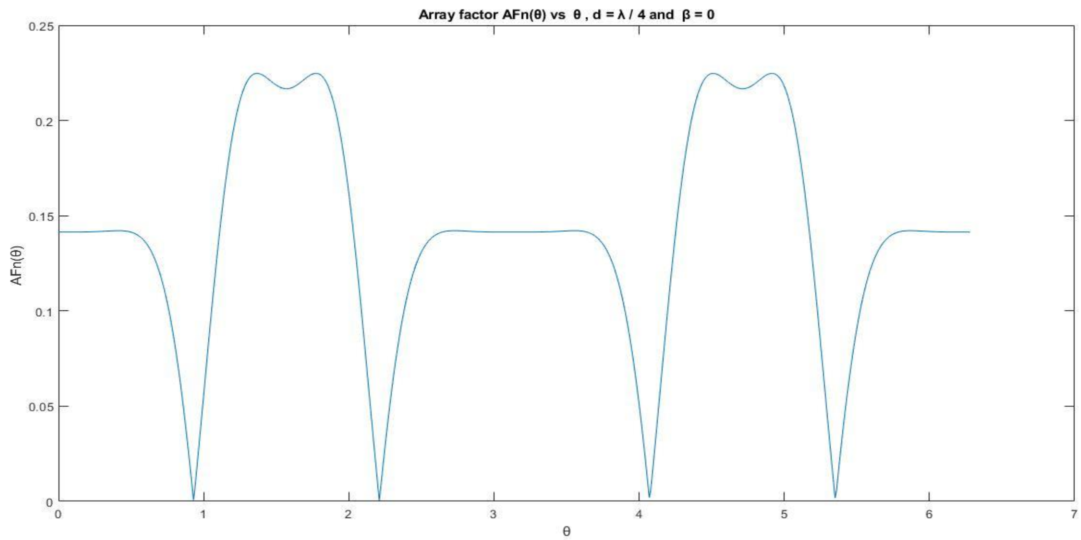


Figure 22. Waveform for β = 0 , d = λ/4 (Rectangular)

2) β = 0 , d = λ

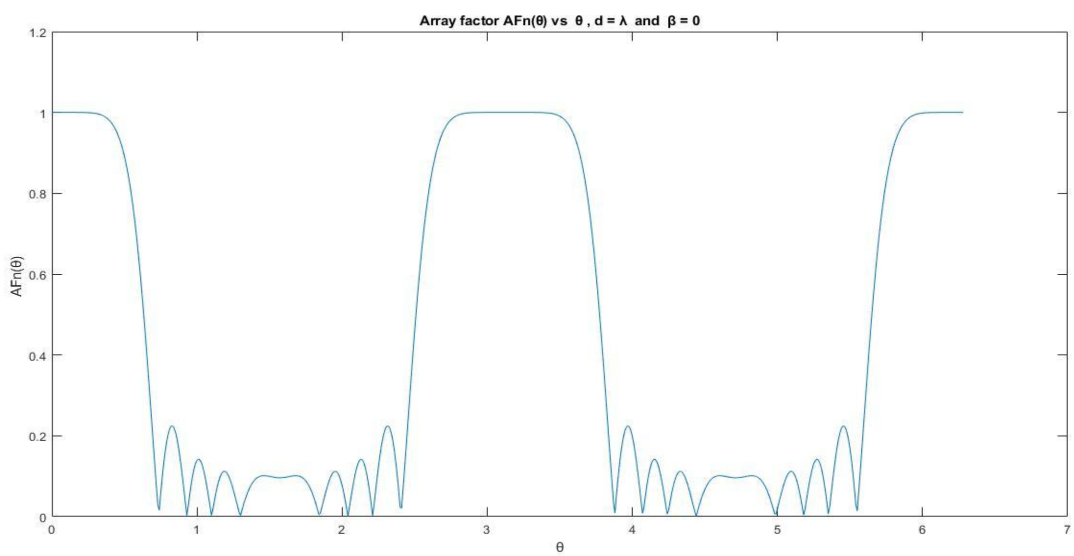


Figure 23. Waveform for β = 0 , d = λ (Rectangular)

3) $\beta = -\pi/4$, $d = \lambda/4$

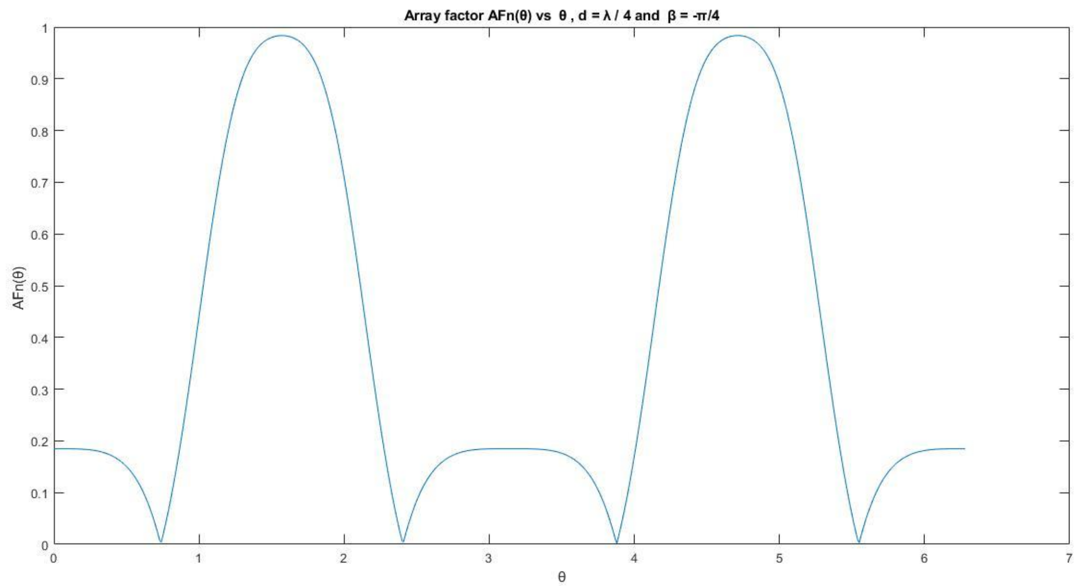


Figure 24. Waveform for $\beta = -\pi/4$, $d = \lambda/4$ (Rectangular)

4) $\beta = -\pi/2$, $d = \lambda/4$

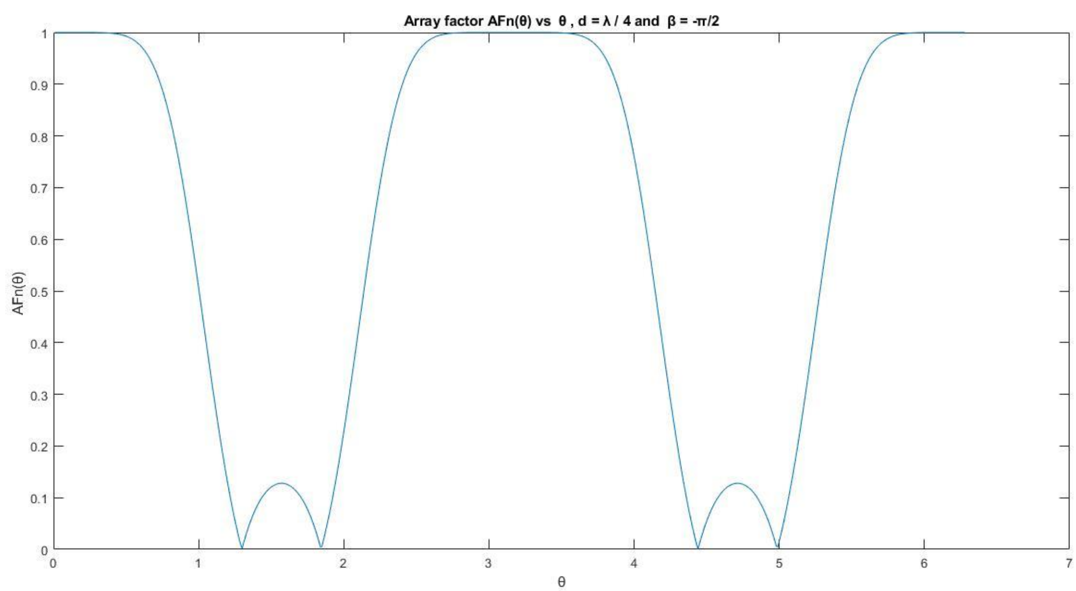


Figure 25. Waveform for $\beta = -\pi/2$, $d = \lambda/4$ (Rectangular)

5) $\beta = -\pi/2$, $d = \lambda/2$

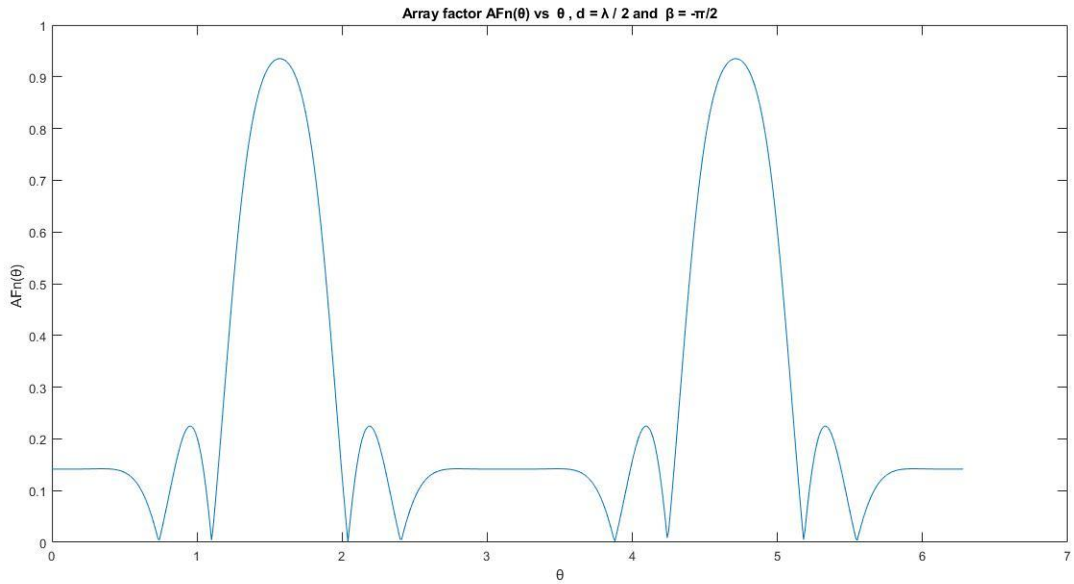


Figure 26. Waveform for $\beta = -\pi/2$, $d = \lambda/2$ (Rectangular)

6) $\beta = -\pi/2$, $d = \lambda$

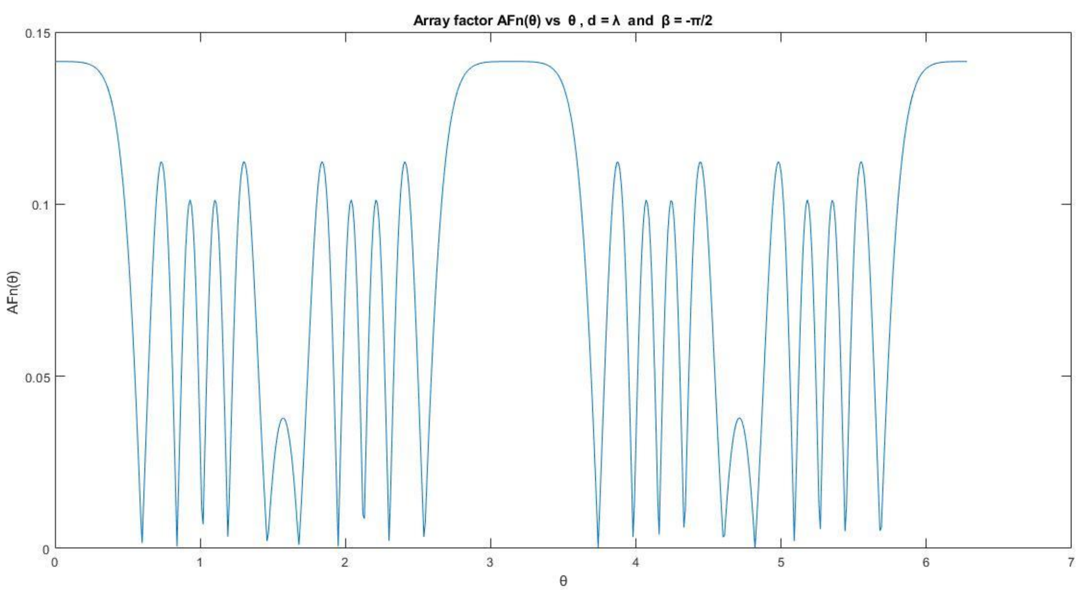


Figure 27. Waveform for $\beta = -\pi/2$, $d = \lambda$ (Rectangular)

- Array factor graphs for different spacing in polar coordinate system when placed on y axis.

1) $\beta = 0$, $d = \lambda/4$

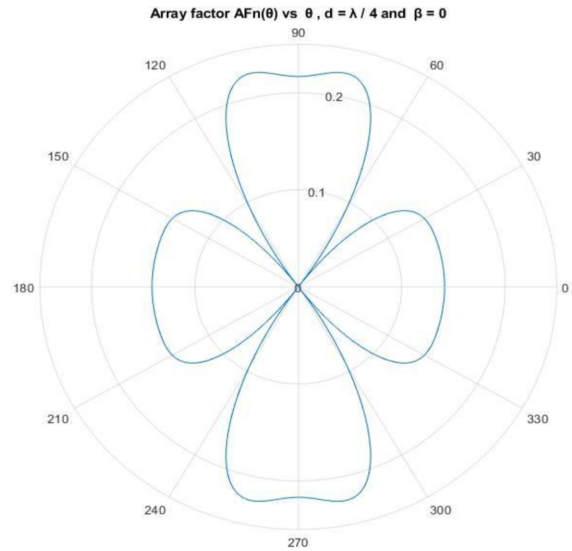


Figure 28. Waveform for $\beta = 0$, $d = \lambda/4$ (Polar)

2) $\beta = 0$, $d = \lambda$

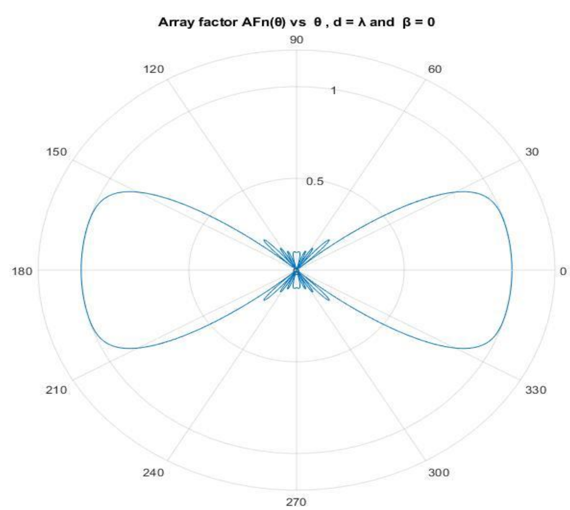


Figure 29. Waveform for $\beta = 0$, $d = \lambda$ (Polar)

3) $\beta = -\pi/4$, $d = \lambda/4$

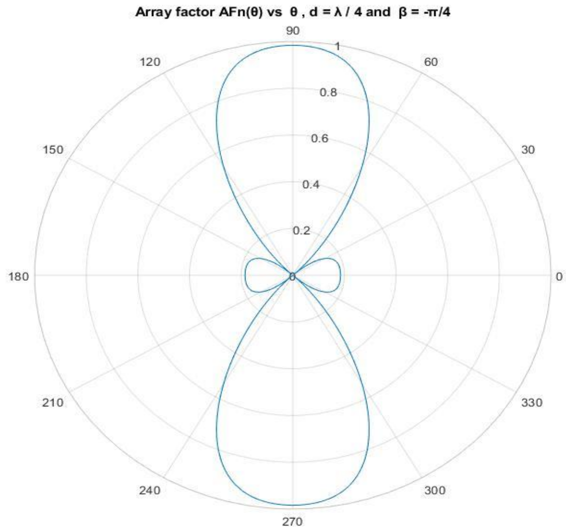


Figure 30. Waveform for $\beta = -\pi/4$, $d = \lambda/4$ (Polar)

4) $\beta = -\pi/2$, $d = \lambda/4$

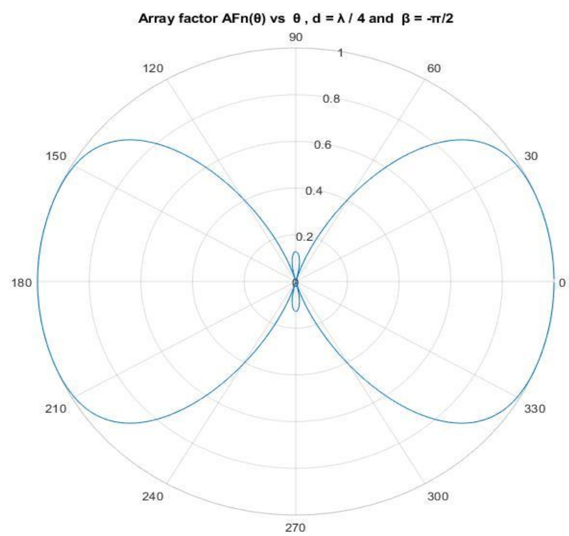


Figure 31. Waveform for $\beta = -\pi/2$, $d = \lambda/4$ (Polar)

5) $\beta = -\pi/2$, $d = \lambda/2$

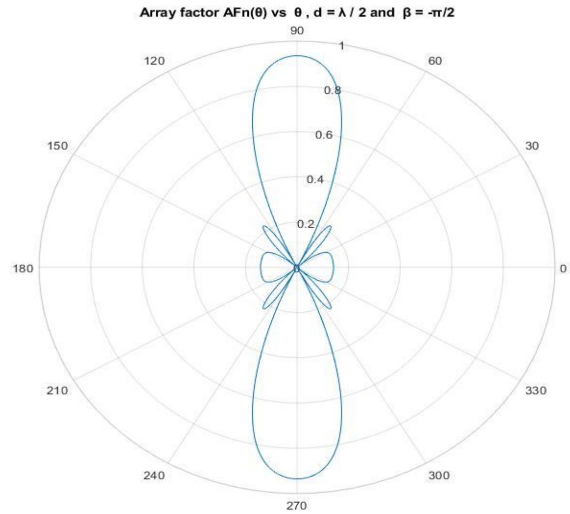


Figure 32. Waveform for $\beta = -\pi/2$, $d = \lambda/2$ (Polar)

6) $\beta = -\pi/2$, $d = \lambda$

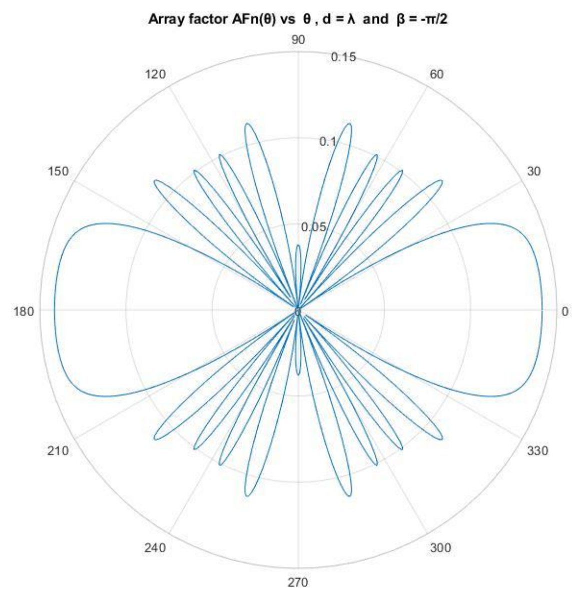


Figure 33. Waveform for $\beta = -\pi/2$, $d = \lambda$ (Polar)

Discussion

- For a particular phase difference β, if the spacing is increased, HPBW get reduced.
- For a given spacing d, if the magnitude of phase difference increased, the position of maximum beam gets changed.
- For elements on Y axis or X axis, the nature of beams does not change.