Homework #2
Due by Friday 1/21 at 11:59 pm

NETID: xw1921 Name: Sophia Wang Email: xw1921@nyu.edu

Question 5:
a) 1. Exercise 1.12.2, sections b, e

1.12.2(b)
$p \rightarrow (q \land r)$
$\underline{\neg q}$

$\therefore \neg p$

1	$p \rightarrow (q \land r)$	hypothesis
2	$p \rightarrow p$	Simplification 1 (rule of inference)
3	$\neg q$	hypothesis
4	$\neg p$	Modus Tollens 2 & 3

1.12.2 (e)
$p \lor q$
$\neg p \lor r$

$\underline{\neg p}$

$\therefore r$

1	$p \lor q$	hypothesis
2	$\neg p \lor r$	hypothesis
3	$q \lor r$	Resolution 1&2
4	$\neg p$	hypothesis
5	r	Disjunctive syllogism

2. Exercise 1.12.3, section c

1.12.3 (c)

One of the rules of inference is Disjunctive syllogism :

$p \lor q$

$\neg p$

$\therefore q$

1	$p \lor q$	hypothesis
2	$\neg\neg p \lor q$	Double negation 1
3	$\neg p \to q$	Conditional identities 2
4	$\neg p$	hypotheses
5	q	Modus ponens 3&4

3. Exercise 1.12.5, sections c, d
1.12.5(c)
I will buy a new car and a new house only if i get a job
I am not going to get a job
∴I will not buy a new car

a: i will get a job
b:i will buy a new car
C: i will get a new house

Argument:
$(b \land c) \to a$
$\neg a$

$\therefore \neg b$

b	c	a	Conclusion $\neg b$
True	False	False	False

The argument is not valid.

1.12.5(d)
I will buy a new car and a new house only if i get a job
I am not going to get a job
I will buy a new house.
∴I will not buy a new car.

A: I will get a job
B: i will buy a new car
C: I will buy a new house

Argument:
$(B \land C) \to A$
$\neg A$
C

$\therefore \neg B$

1	$(B \land C) \to A$	hypothesis
2	$\neg(B \land C) \lor A$	Conditional identity 1
3	$A \lor \neg(B \land C)$	Commutative Law 2
4	$\neg A$	hypothesis
5	$\neg(B \land C)$	Disjunctive syllogism, 3&4
6	$\neg B \lor \neg C$	De Morgan Law 5
7	$\neg C \lor \neg B$	Commutative Law 6
8	C	Hypothesis
9	$\neg\neg C$	Double negation law 8
10	$\neg B$	Disjunctive syllogism, 7&9

b) Solve the following questions from the Discrete Math zyBook:

1. Exercise 1.13.3, section b
   (b)

$\exists x(P(x) \vee Q(x))$

$\exists x \neg Q(x)$

$\therefore \exists x P(x)$

	p	q
a	F	F
b	F	T

$\exists x(P(x) \vee Q(x))$ is true when Q(b) is true.

$\exists x \neg Q(x)$ is true when Q(a) is false

However, P(a)=P(b)=F, $\exists x(P(x) \vee Q(x))$ is false

Therefore, both hypothesis are true. The argument is false.

2. Exercise 1.13.5, sections d, e

1.13.5(d)
Every student who missed class got a detention
Penelope is a student in the class
Penelope did not miss class

Penelope did not get a detention.

M(X): x missed class

D(X): x got detention

S(X): x is a student in the class

P: Penelope

$\forall x(S(x) \land M(x) \rightarrow D(x))$

$S(P)$

$\neg M(P)$

$\therefore \neg D(P)$

The argument is invaild. Once M(P)=F and D(P)=F, the the first hypothesis is true , the third hypothesis is true. The conclusion will be true .

1.13.5(e)Every student who missed class got a detention did not get an A
Penelope is a student in the class
Penelope got an A

Penelope did not get a detention.

M(x): x missed the class D(x): x got a detention S(x): x is a student in the class
A(x) : got an A

$\forall x(S(x) \land M(x) \lor D(x)) \rightarrow \neg A(x))$

S(penelope)

A(penelope)

$\therefore \neg D(penelope)$

1	$\forall x(S(x) \land M(x) \lor D(x)) \rightarrow \neg A(x))$	hypothesis
2	$(S(penelope) \land (M(penelope) \lor D(penelope))) \rightarrow \neg A(penelope)$)	Universal instantial 1
3	A(penelope)	hypothesis
4	$\neg(S(penelope) \land (M(penelope) \lor D(penelope)))$	Module Tollens 2&3
5	$\neg(S(penelope) \lor (\neg M(penelope) \land \neg D(penelope)))$	De Morgan Law 4

6	S(penelope)	hypothesis
7	$\neg M(penelope) \land \neg D(penelope)$	Disjunctive syllogism 5&6
8	$\neg D(penelope)$	simplification 7

<u>Question 6:</u>
Solve Exercise 2.4.1, section d;
2.4.1(d)
The product of two odd integers is an odd integer.

Proof: Let x and y be two odd integers.
$xy=(2m+1)(2n+1)$ m and n are integers
$=4mn+2m+2n+1$
$=2(2mn+m+n)+1$
$=2p+1$ which is odd
Hence the product of two odd integers(x,y) is an odd integer

Exercise 2.4.3, section b, from the Discrete Math zyBook:

2.4.3 (b) If x is a real number and x≤3, then 12−7x+x^2≥0.

Proof: Assume that for real number x, x≤3. We will show that

$$x^2 - 7x + 12 \geq 0$$

(x-3)(x-4)≥0

X≥3 or x≥4

It means that whatever x≤ 3, then x^2-7x+12≥0.

Question 7:
Solve Exercise 2.5.1, section d; Exercise

For every integer n, if $n^2 - 2n + 7$ is even, then n is odd.

Proof: Assume: n is even, So n=2m where m is an integer
Plugging n=2m into $n^2 - 2n + 7$ gives
$n^2 - 2n + 7 = 2m^2 - 2(2m) + 7$
$= 4m^2 - 4m + 7$
$= 2(2m^2 - 2m + 3) + 1$

$= 2k + 1$ $[k = 2m^2 - 2m + 3]$

2k+1 is odd number

If n is even, then $n^2 - 2n + 7$ is even
Thus, by contrapositive method,
For every integer n, if $n^2 - 2n + 7$ is even, then n is odd

2.5.4, sections a, b;
2.5.4(a)
For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2 y + y^3$, then x≤y.

Proof: Suppose it is not true that x≤y, so x > y
Then x-y > 0. Multiply both sides of x-y > 0 by the value $(x^2 + y^2)$

So x > y
→x $(x^2 + y^2)$ > y $(x^2 + y^2)$
→ $x^3 + xy^2 > x^2y + y^3$

Thus, by contrapositive method,
For every pair of real numbers x and y, if $x^3 + xy^2 \leq x^2 y + y^3$, then x≤y.

2.5.4 (b)
For every pair of real numbers x and y, if x+y>20, then x>10 or y>10.

Proof: Assume x, y are integers
If x≤10 and y≤10 to prove x+y≤20
Adding both inequalities x≤10 and y≤10 together
x≤10
x+y ≤10+10
x+y ≤20

Thus, by contrapositive method,
For every pair of real numbers x and y, if x+y>20, then x>10 or y>10.

Exercise 2.5.5, section c, from the Discrete Math zyBook:
2.5.5(c)
For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.

Proof: if x is rational, then $\frac{1}{x}$ is also rational.

Assume that $\frac{1}{x}$ is rational and x≠0, then $\frac{1}{x} = \frac{p}{q}$ where q≠0

$\frac{1}{x}$≠0 because $\frac{1}{x}$ *x≠0*x →1≠x*0 → p≠0

P≠0, then x=$\frac{1}{1/x}$=$\frac{1}{p/q}$=$\frac{q}{p}$

X can be written as a quotation of two integers with p≠0 , x is rational
Thus, by contrapositive method: For every non-zero real number x, if x is irrational, then $\frac{1}{x}$ is also irrational.

Question 8:
Solve Exercise 2.6.6, sections c, d, from the Discrete Math zyBook:

2.6.6 (c) The average of three real numbers is greater than or equal to at least one of the numbers.

Proof: Suppose the average of three real numbers is less than at least one of the numbers.

$\frac{a+b+c}{3} < a$, $\frac{a+b+c}{3} < b$, $\frac{a+b+c}{3} < c$

$\frac{a+b+c+a+b+c+a+b+c}{3} < a+b+c$

$\frac{3a+3b+3c}{3} < a+b+c$

$\frac{3(a+b+c)}{3} < a+b+c$

$a+b+c < a+b+c$

The fact of the equation that it contradicts, the assumption is wrong
Thus, by the proof of contradiction method, an average of three real numbers is less than at least one of the numbers. Our claim is true

2.6.6 (d)There is no smallest integer.

Proof: Suppose there is no smallest integer. We assume it is x

If x is an integer, then x-1 is an integer.

However, x-1＜x

This contradicts the fact that there is no smallest integer.
Thus, by the proof of contradiction method, There is no smallest integer.
Our claim is true.

Question 9:

Solve Exercise 2.7.2, section b, from the Discrete Math zyBook:

2.7.2(b) If integers x and y have the same parity, then x+y is even. The parity of a number tells whether the number is odd or even. If x and y have the same parity, they are either both even or both odd.

Let integer x and y have the same parity, then there are two case to consider:

Case 1: If x =even and y= even

Since x is even, x=2r for some integer r and y=2s for some integer s

Therefore x+y=2r+2s=2(r+s). so x+y =even

Case 2: If x =odd and y=odd

Since x is odd, x=2r+1 for some integer r and y=2s+1 for some integer s

Therefore x+y=2r+1+2s+1=2r+2s+2=2(r+s)+2. so x+y =even

If x =even and y=even, x+y=even

If x=odd and y=odd, x+y=even

Conclusion: If x and y have the same parity, they are all even