Homework 3: Problem 6

William Svoboda (wsvoboda)

Collaborators: Epi Torres-Smith, Leslie Kim

Problem 6:
We wish to show that a simple graph is 2-edge-connected if and only if it is connected and every edge of the graph is traversed by a cycle.

First, let us assume that we have a 2-edge-connected simple graph that is *disconnected*. This is a contradiction, because by definition a k-edge-connected graph must be connected.

Let us then assume that we have a 2-edge-connected simple graph where at least one edge is *not* traversed by a cycle. By definition, a cycle is a path where the first and last vertices are repeated (creating a loop). If there is an edge not traversed by a cycle, then it is possible to remove that single edge and create a disconnection in that part of the graph (since the vertices that covered that edge would no longer be in the path). This is also a contradiction, since by definition it should be possible to remove $2 - 1 = 1$ edge from a 2-edge-connected graph without disconnecting it.

Therefore, a simple graph is 2-edge-connected if and only if it is connected and every edge of the graph is traversed by a cycle.