Homework 1: Problem 5

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Problem 5:
We wish to find the minimum number of elements that must be picked from the set $A = \{1, 3, 5, \ldots, 99\}$ of all the odd positive numbers less than 100 such that there is always a pair of picked numbers that sum up to 100.

(Proof) Intuitively, we know that the possible pairs that sum to 100 are the 25 pairs where each number in the pair is the same distance from the end of the range $[1, 99]$. For example, A can be split into another set that contains all these pairs:

$$\{(1, 99), (3, 97), (5, 95), \ldots, (49, 51)\}$$

The Pigeonhole Principle states that if n items are put into m containers and $n > m$ then at least one container contains more than one item. If we take m equal to the 25 pairs that sum to 100 we can write the inequality

$$n > 25$$

where n is the number of integers picked. Thus picking a minimum of $n = 26$ integers from A will guarantee a pair that sum up to 100.