Homework 1: Problem 4

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Collaborators: None

Problem 4:
We wish to prove

$$P(n) : \text{There are always two sums that are equal}$$

out of the row, column, and diagonal sums for a $n \times n$ board.

(Base Case) We first show $P(1)$ is true, i.e. that the formula is valid when $n = 1$. If $n = 1$, then the table is made up of only a single entry. Because there is only one entry, the rows, columns, and diagonals of the table are the same (they all cover only the single square). No matter what value the entry has the sums will be equal and $P(1)$ holds.

(Proof) There will always be $2n + 2$ sums from an $n \times n$ board. This can be concluded from the observation that the two available diagonals is constant for all square boards, and that there are n rows and n columns.

We wish to choose two sums out of the $2n + 2$ available sums from the board that are equal. This can be represented by

$$\binom{(2n+2)+2-1}{2} = \binom{2n+3}{2}$$

Because repetitions are allowed and the order is not important.