

Homework 5: Problem 2

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Problem 2:

We wish to show the reduction $\text{PERFECT-MATCHING} \leq_P \text{DISJOINT-PATHS}$.

We start by considering a simple, bipartite graph G as an input. To transform this input into one for the disjoint paths decision problem, we can add two vertices s and t to make the graph G' . Each vertex in part A will be connected to s , and each vertex in part B will be connected to t . G' will then have k disjoint paths connecting s and t if and only if G is a perfect matching where $|A| = |B| = k$. If, for example, G did not have a perfect matching, the maximum possible number of disjoint paths in G' would be lower; a vertex would exist in G without a matching, so G' would have paths that shared the same edge.

Likewise, if G' has k disjoint paths the added vertices s and t and their edges can be ignored. This returns the graph to G , which we know must have a perfect matching.