Homework 3: Problem 4

William Svoboda (wsvoboda)

Collaborators: Epi Torres-Smith, Leslie Kim

Problem 4:
We wish to determine if there is an ℓ-regular graph $G = (V, E)$ where ℓ is an odd positive integer and $|V|$ is also an odd positive integer for $\ell \geq 1$.

The handshaking theorem states that

$$\sum_{v \in V} deg(V) = 2 \cdot |E|$$

If we had both an odd positive number of vertices and all vertices had degree ℓ that was also odd positive, then the summation $\sum_{v \in V} deg(V)$ would be odd (an odd number added odd times is still odd). However, we know that this summation must be equal to $2 \cdot |E|$ which is always an even positive number.

This is a contradiction, so by the handshaking theorem there is no ℓ-regular graph $G = (V, E)$ with $|V|$ an odd positive integer for $\ell \geq 1$.