Homework 3: Problem 3

William Svoboda (`wsvoboda`)

Collaborators: Epi Torres-Smith, Leslie Kim

Problem 3:

We wish to show that any simple graph $G = (V, E)$ with $\delta(G) > \frac{1}{2} \cdot (|V| - 2)$ is connected.

We can partition G into two subgraphs with $\frac{|V|}{2}$ vertices each. If we take any vertex v from either of the subgraphs, the maximum degree it could have is $\frac{|V|}{2} - 1$ since it could be connected to every vertex in the subgraph besides itself (creating a connected component). There could not be more edges from the vertex because in a simple graph self loops and multiple edges are not allowed. However, it is given that $\delta(G) > \frac{1}{2} \cdot (|V| - 2) = \frac{|V|}{2} - 1$, so there must actually be one additional edge from v to the other connected component. Therefore, the graph G must actually be connected.