Computer Science 340
Reasoning about Computation
Princeton University, Fall 2020

Assignment 1 Handout
Due 9/16/2020, 9:00PM

No collaboration allowed on all problems in this problem set

Problem 1 [20 points]
(No collaboration)
Consider the sequence defined below

$$\begin{aligned} T_0 &= T_1 = T_2 = 1 \\ T_n &= T_{n-1} + T_{n-2} + T_{n-3}, \quad \text{for any integer } n \geq 3. \end{aligned}$$

Prove by strong induction that

$$T_n \leq 2^n, \quad \text{for any nonnegative integer } n$$

Problem 2 [20 points]
(No collaboration)
Prove by induction on y the following inequality

$$(1 + a)^y \geq 1 + ay,$$

where $y \in \mathbb{N}$ and $a \geq -1$ is a real number.
(For this problem you should assume that $0 \in \mathbb{N}$)

Problem 3 [20 points]
(No collaboration)
Show that at a party of $n \geq 2$ people, there are two people who have the same number of friends in the party. (Assume that friendship is symmetric,

so if a person p is friends with a person q, for p, q different people, then q is friends with p. Furthermore, a person cannot be friends with themselves.)

Problem 4 [20 points]
(No collaboration)
Consider a $d \times d$ table A, with $d \geq 1$, where each of its entries contains exactly one element of the set $\{0, 1, 2\}$. Index the rows of the table starting from 0 for the top row, and the columns of the table starting from 0 from the left column. Hence, each element of the table is referenced by $A_{i,j}$, where i is the index of the row the element belongs in, and j is the index of the column the element belongs in.
Furthermore, consider the sums of the entries of the table per row (namely $R_i = \sum_{k=0}^{d-1} A_{i,k}$ for $i = 0, 1, \ldots, d-1$), the sums of the entries of the table per column (namely $C_j = \sum_{k=0}^{d-1} A_{k,j}$ for $j = 0, 1, \ldots, d-1$) and the sums across the diagonals of the table (namely $D_1 = \sum_{k=0}^{d-1} A_{k,k}$ and $D_2 = \sum_{k=0}^{d-1} A_{k,d-1-k}$). Show that there are always two sums (out of the row, column and diagonal sums described above) that are equal.

Problem 5 [20 points]
(No collaboration)
Consider the set $A = \{1, 3, 5, \ldots, 99\}$ of all the odd positive numbers less than 100. What is the minimum number of distinct elements that must be picked from A, so that there is always a pair of the numbers that are picked that sum up to 100?

Problem 6 [20 points]
(No collaboration)
Prove by induction that, for all integers $n > 0$,

$$\frac{1}{1 \times 3} + \frac{1}{3 \times 5} + \cdots + \frac{1}{(2n-1)(2n+1)} = \frac{n}{2n+1}.$$