Homework 5: Problem 3

William Svoboda (wsvoboda)

Collaborators: Epi Torres-Smith, Leslie Kim

Problem 3:

(A) Compute $3^n \mod 11$ for $n = 1, 2, \ldots, 10$.

- $3^1 \mod 11 = 3$
- $3^2 \mod 11 = 9$
- $3^3 \mod 11 = 27 \mod 11 = 5$
- $3^4 \mod 11 = (3^3 \mod 11 \cdot 3^1 \mod 11) \mod 11 = (5 \cdot 3) \mod 11 = 4$
- $3^5 \mod 11 = (3^4 \mod 11 \cdot 3^1 \mod 11) \mod 11 = (4 \cdot 3) \mod 11 = 1$
- $3^6 \mod 11 = (3^5 \mod 11 \cdot 3^1 \mod 11) \mod 11 = (1 \cdot 3) \mod 11 = 3$
- $3^7 \mod 11 = (3^6 \mod 11 \cdot 3^1 \mod 11) \mod 11 = (3 \cdot 3) \mod 11 = 9$
- $3^8 \mod 11 = (3^4 \mod 11 \cdot 3^4 \mod 11) \mod 11 = (4 \cdot 4) \mod 11 = 5$
- $3^9 \mod 11 = (3^8 \mod 11 \cdot 3^1 \mod 11) \mod 11 = (5 \cdot 3) \mod 11 = 4$
- Because 11 is a prime, and $10 = 11 - 1$, by Fermat's Little Theorem $3^{10} \mod 11 = 1$

(B) Compute $3^{2324238942348721381245} \mod 11$.

$$= (3^{2324238942348721381240} \mod 11 \cdot 3^5 \mod 11) \mod 11 \quad (1)$$
$$= ((3^{10} \mod 11)^{232423894234872138124} \cdot 3^5 \mod 11) \mod 11 \quad (2)$$
$$= (1^{23242389423487213812 4} \mod 11 \cdot 1) \mod 11 \quad (3)$$
$$= 1 \quad (4)$$

(C) Compute $15^{3248723482156246732128312537123} \pmod{11}$.

$$= (3 \cdot 5)^{3 \ldots 3} \bmod 11 \tag{5}$$
$$= (3^{3 \ldots 3} \bmod 11 \cdot 5^{3 \ldots 3} \bmod 11) \bmod 11 \tag{6}$$
$$= ((3^{3 \ldots 0} \bmod 11 \cdot 3^3 \bmod 11) \cdot (5^{3 \ldots 0} \bmod 11 \cdot 5^3 \bmod 11)) \bmod 11 \tag{7}$$
$$= (((3^{10} \bmod 11)^{3 \ldots} \bmod 11 \cdot 3^3 \bmod 11) \cdot ((5^{10} \bmod 11)^{3 \ldots} \bmod 11 \cdot 5^3 \bmod 11)) \bmod 11 \tag{8}$$
$$= ((1 \bmod 11 \cdot 5 \bmod 11) \cdot (1 \bmod 11 \cdot 4 \bmod 11)) \tag{9}$$
$$= (5 \cdot 4) \bmod 11 \tag{10}$$
$$= 9 \tag{11}$$
$$\tag{12}$$