Homework 2: Problem 3

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Problem 3:

A:

We wish to find the number of different m-word sentences that can be constructed using an alphabet of n letters where each of the letters appears exactly once. We note that when order is important and there should be no repetitions that there will be $\frac{n!}{(n-n)!} = n!$ ways to arrange the letters. If there are m words in the sentence there are $m-1$ spaces in that sentence since there is no space before the first character. The number of ways to put the spaces in the sentence is then $\binom{n-1}{m-1}$ since $n-1$ is the available length with which to insert spaces between each word. Following this, the total number of combinations will be

$$n! \cdot \binom{n-1}{m-1}$$

B:

From the previous question, we know there are $n! \cdot \binom{n-1}{m-1}$ ways to use an alphabet of n letters to construct different m-word sentences where each of the letters appears exactly once. Now that we wish to create paragraphs, we know that the dividers are now either spaces or periods. Since this is a binary choice for each divider picked, and there are $m-1$ dividers in a paragraph, there are 2^{m-1} combinations of dividers for a paragraph. We can then multiply this by the number of ways to create different sentences to find that the number of different paragraphs is

$$n! \cdot \binom{n-1}{m-1} \cdot 2^{m-1}$$