COS 445 - PSet 5, Problem 3

Odysseus

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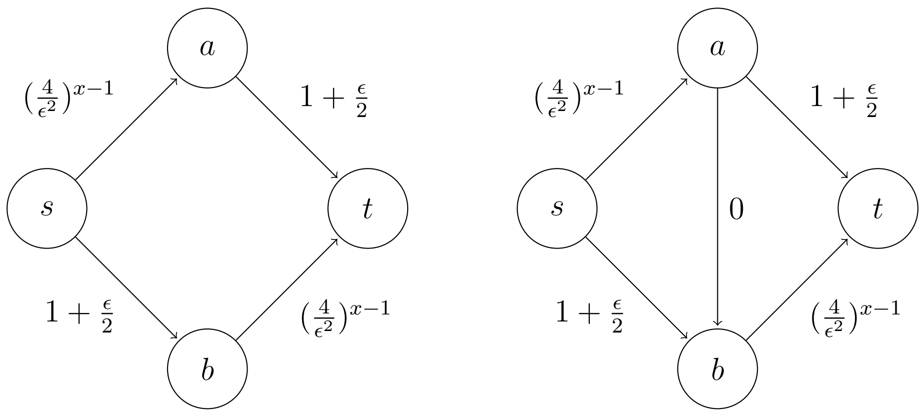
Problem 3: Bigger Badder Braess' Paradox

We wish to show two networks G_ϵ and G'_ϵ, cost functions $c_e(\cdot)$ for each edge, and $\vec{r}$ where r_{ab} units of traffic want to travel from a to b for all nodes a, b such that all given conditions are met. We present the following networks:



(a) G_ϵ (b) G'_ϵ

We note that the nodes in G_ϵ and G'_ϵ are the same, that is they each contain the nodes s, a, b, and t. Additionally, all edges in G_ϵ are also present in G'_ϵ, with G'_ϵ only having one additional edge between nodes a and b. Each cost function on these edges is continuous, monotone non-decreasing, and non-negative.

We also observe that the unique equilibrium flow in G_ϵ has total cost at most $1+\epsilon$. For example, if $\epsilon = \frac{1}{2}$ the resulting cost (no matter which direction is taken from s) is equal to $1 + \frac{\frac{1}{2}}{2} + (\frac{4}{(\frac{1}{2})^2})^{-\frac{1}{2}}$ or $1 + \frac{1}{2} = 1 + \epsilon$.