COS 445 - PSet 1, Problem 2

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Problem 2: Both Sides Propose

We wish to either prove that Both-Proposing Deferred Acceptance always terminates in a stable matching or provide an example of preferences and order of proposals such that BPDA does not output a stable matching.

Let there be two students, S_1 and S_2, and two universities U_1 and U_2. Let their preferences be the following:

Element	Preferences
S_1	$U_1 > U_2$
S_2	$U_1 > U_2$
U_1	$S_1 > S_2$
U_2	$S_1 > S_2$

Since students propose first, let S_2 begin and propose to U_1, forming a pair. The remaining unmatched university, U_2, then proposes to the remaining student S_1 because they are their first choice. The algorithm terminates, since there are no more unmatched students. However, the resulting matching (S_2, U_1) and (S_1, U_2) is not stable. Both S_1 and U_2 prefer each other (in fact they are each other's first choice) over the matching they got, meaning (S_1, U_1) is a blocking pair and would be strictly happier together. Therefore, BPDA does not always terminate in a stable matching.