

Agenda

- * Binomial Distribution
- * Bernoulli Distribution => Special case
- * Population vs Sample
- (*) Sample statistics
- (*) Sampling techniques
- * Uniform Distribution

**

Assignments

Empirical vs Theoretical





Binomial Distribution

=> Discrete probability distribution of the no of success in n independent experiments.

x → no of red balls

=> Binomial RV

$\Rightarrow n \Rightarrow$ no of independent trials $n = 4$

$p \Rightarrow$ probability of success in one trial $p = 3/5$

$$P(X = k) = {}^n C_k * p^k * (1-p)^{n-k}$$

$$P(X = 0) = {}^4 C_0 * \left(\frac{3}{5}\right)^0 * \left(\frac{2}{5}\right)^4$$

X

0

$4c_0$

$(\frac{2}{5})^4 \Rightarrow 4c_0 * (\frac{2}{5})^4 (\frac{3}{5})^0$

1

$4c_1$

$(\frac{3}{5}) * (\frac{2}{5})^3 \Rightarrow 4c_1 * (\frac{3}{5})^1 * (\frac{2}{5})^3$

2

$4c_2$

$\vdots \Rightarrow 4c_2 * (\frac{3}{5})^2 * (\frac{2}{5})^2$

3

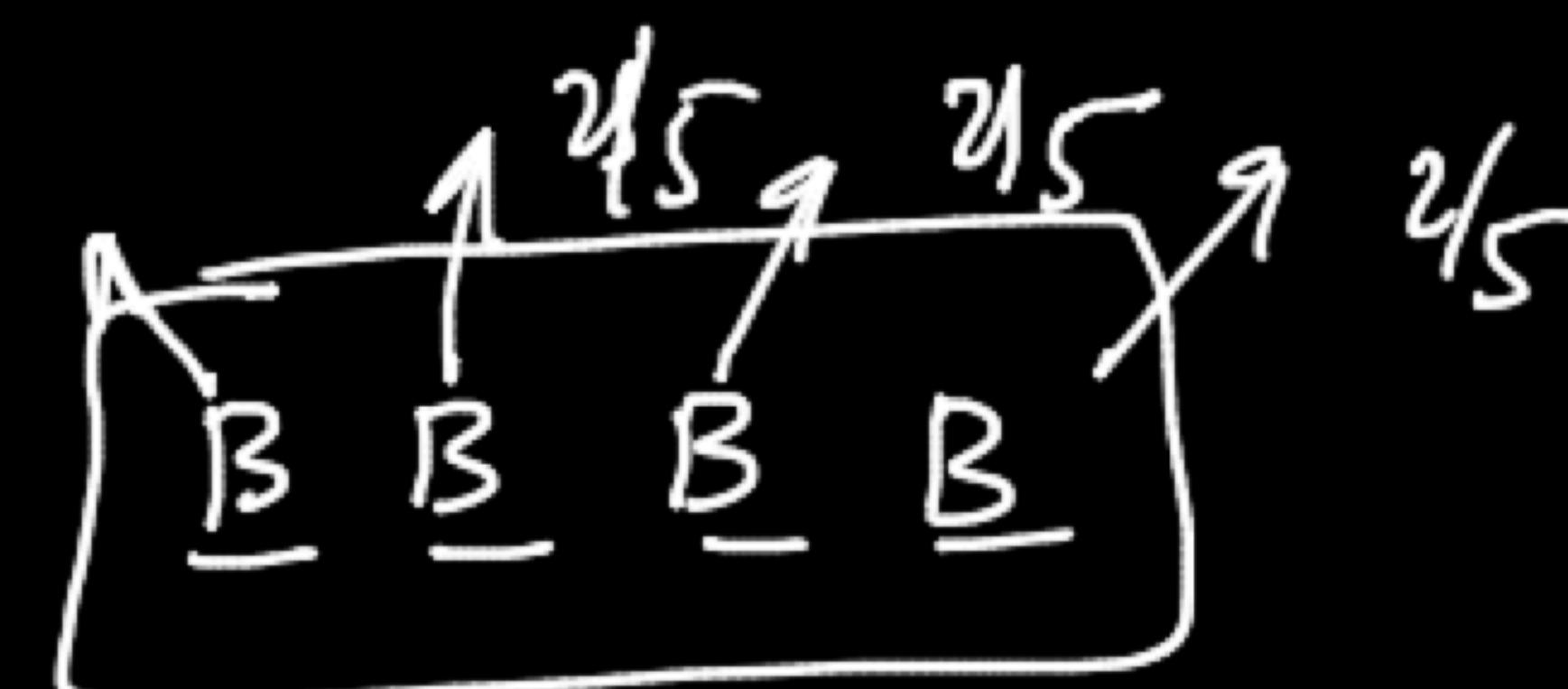
$4c_3$

\vdots

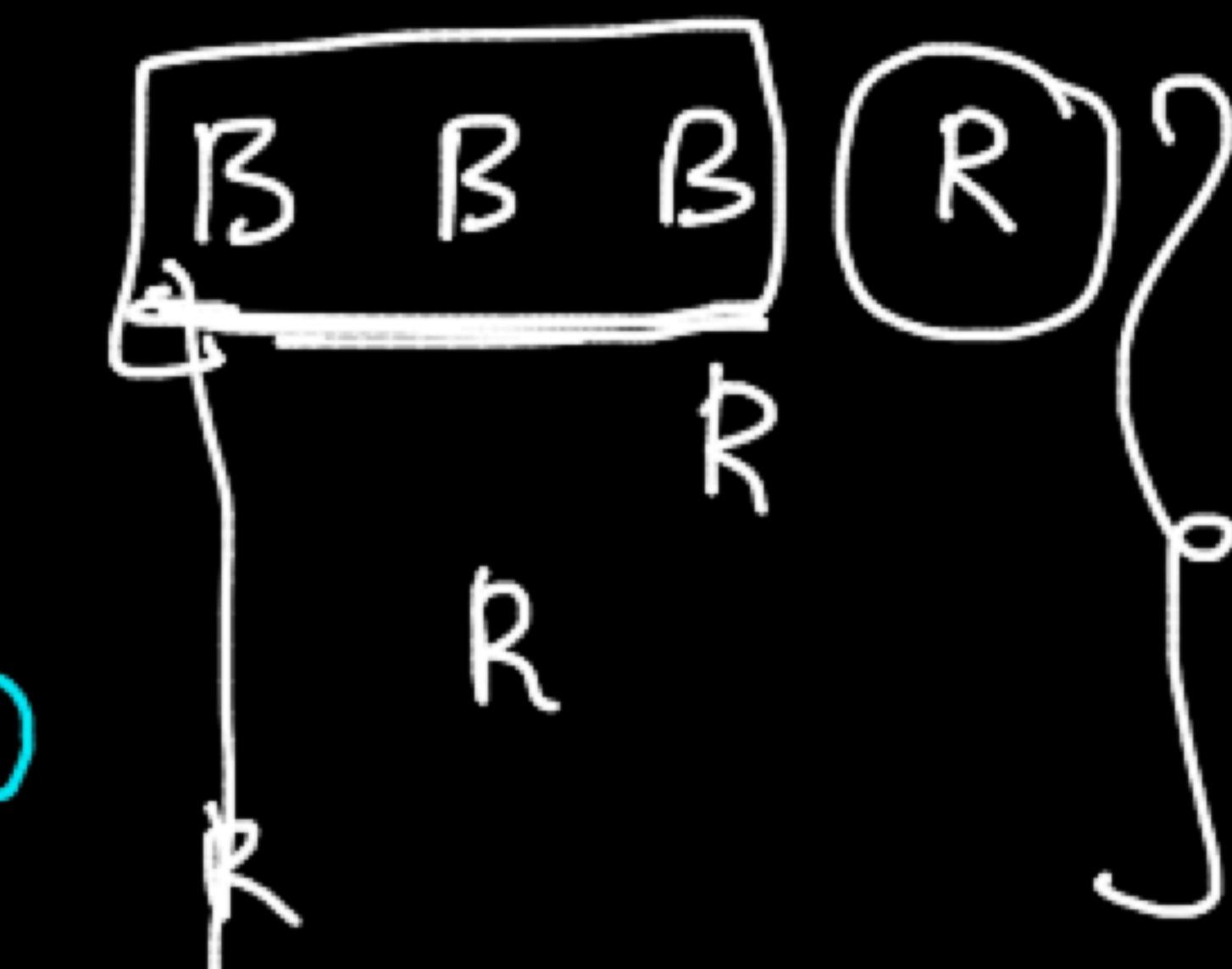
4

$4c_4$

\vdots

 $P(x)$ $\frac{2}{5}$ 

$$\vdots$$



$P(R) = \frac{3}{5}$

$P(B) = \frac{2}{5}$

x

0

$4c_0$

$(\frac{2}{5})^4$

 $P(x)$

$4c_0 * (\frac{2}{5})^4 (\frac{3}{5})^0$

 $\frac{2}{5}$

1

$4c_1$

$(\frac{3}{5}) * (\frac{2}{5})^3$

$4c_1 * (\frac{3}{5}) * (\frac{2}{5})^3$

 $\frac{3}{5}$

2

$4c_2$

:

$4c_2 * (\frac{3}{5}) * (\frac{2}{5})^2$

:

3

$4c_3$

:

4

$4c_4$

:

$(\frac{2}{5})^4$

$4c_0 * (\frac{2}{5})^4 (\frac{3}{5})^0$

 $\frac{2}{5}$

$(\frac{3}{5}) * (\frac{2}{5})^3$

$4c_1 * (\frac{3}{5}) * (\frac{2}{5})^3$

 $\frac{3}{5}$

$4c_2 * (\frac{3}{5}) * (\frac{2}{5})^2$

:

$4c_3 * (\frac{3}{5}) * (\frac{2}{5})^1$

:

$4c_4 * (\frac{3}{5}) * (\frac{2}{5})^0$

:

$P(R) = \frac{3}{5}$

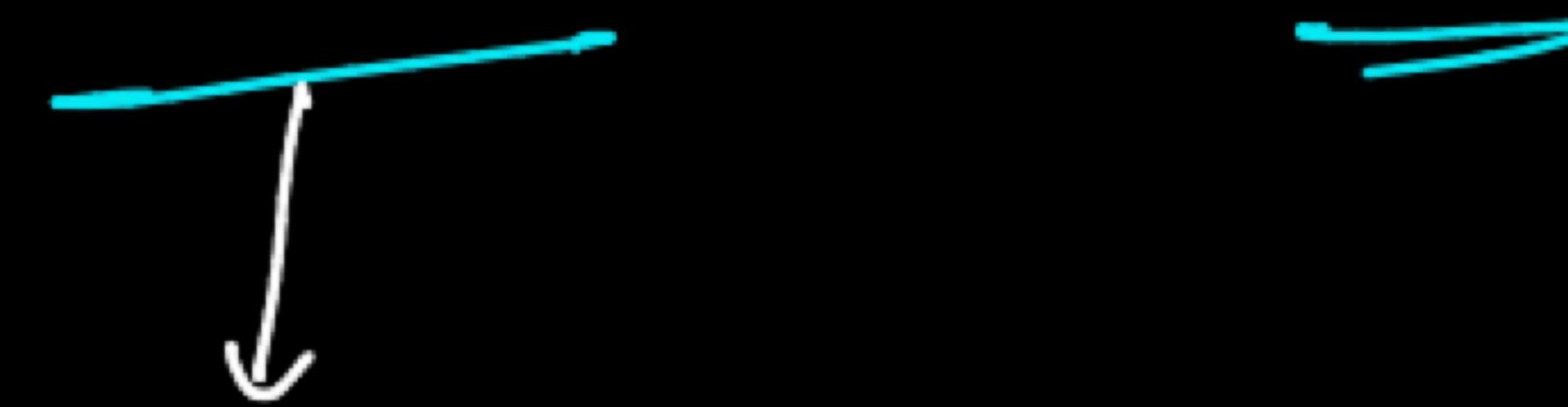
$P(B) = \frac{2}{5}$

Conditions

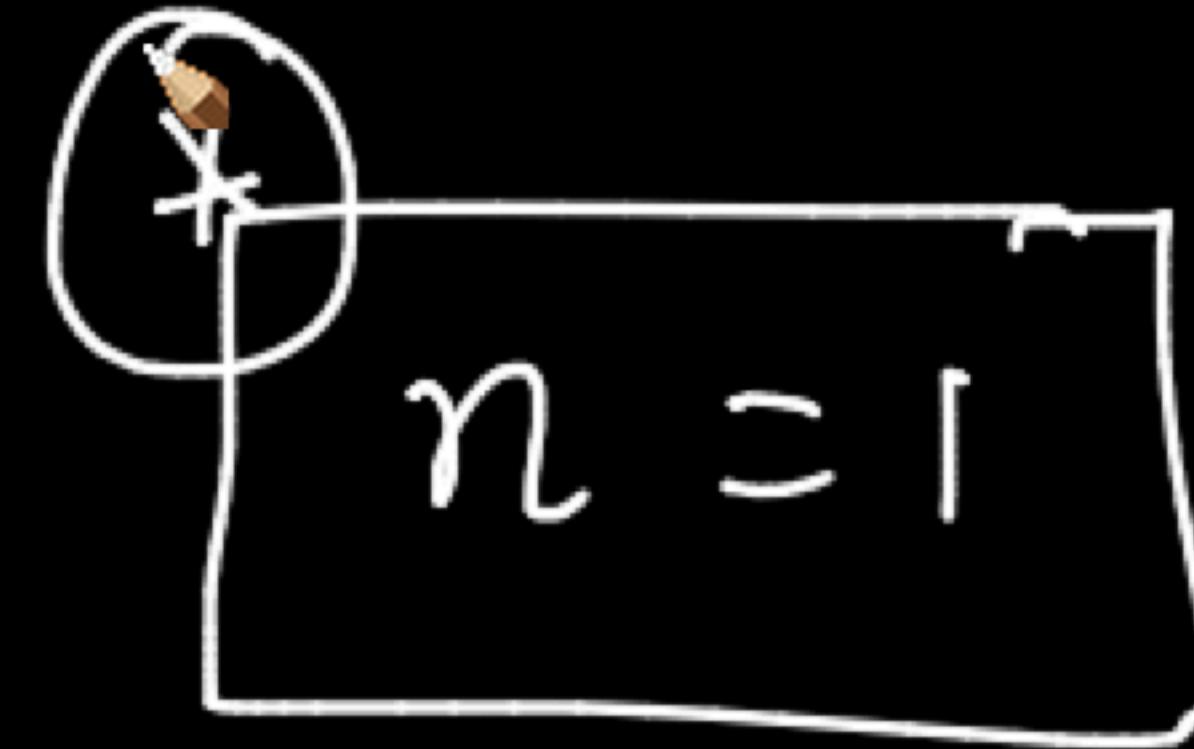
$$* \textcircled{n} = u$$

1. Experiment must contains fixed no of trials ; with
only 2 possible outcomes
2. Individual trials are **identical** and independent.
3. R.V denotes the no of success in n trials

Bernoulli Distribution



Special Binomial



Success / Fail

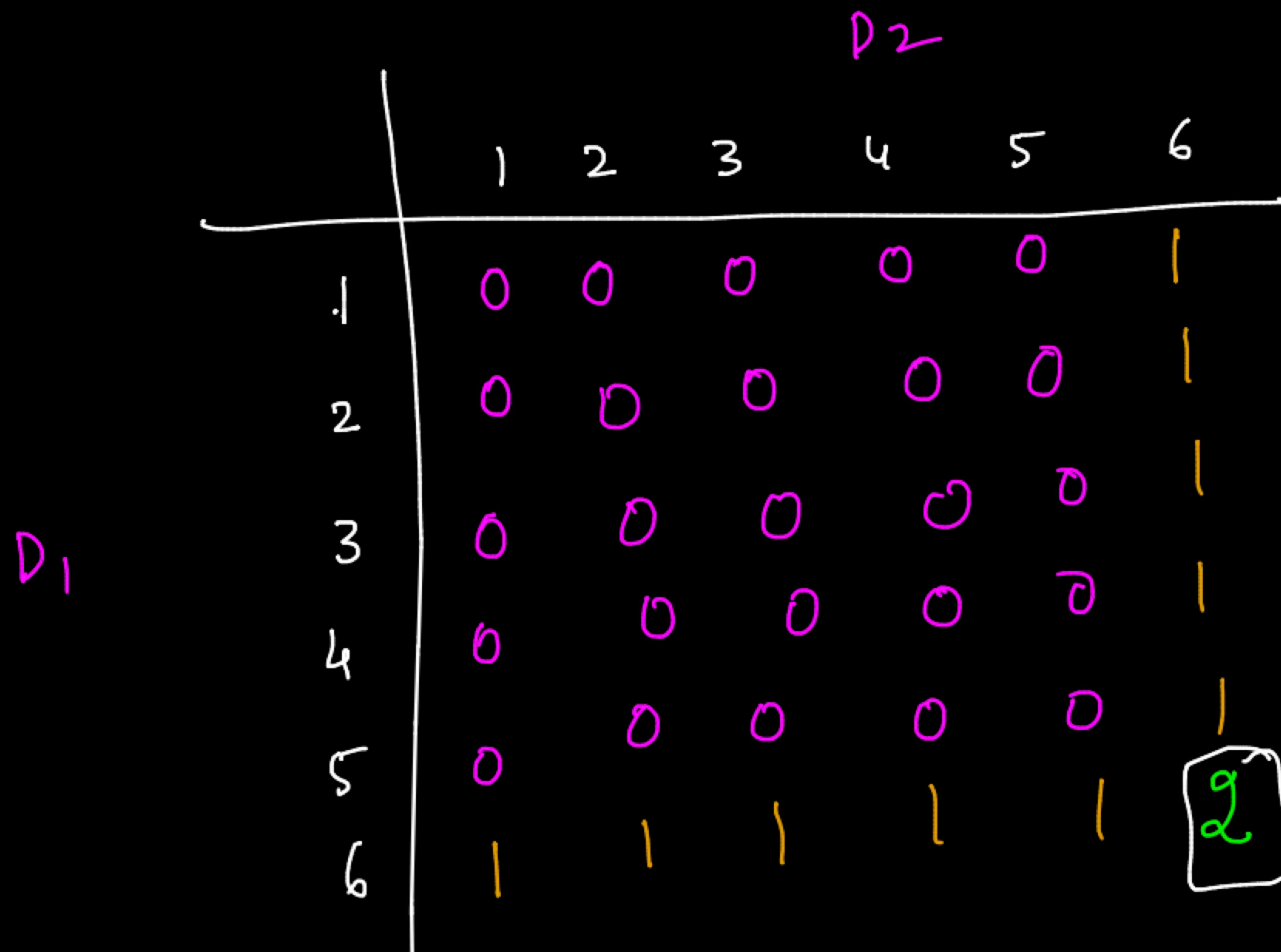
Dice Example

P(R)

⇒ You throw 2 dice. If both dice are 6, you get Rs 2.
else if one dice is 6 you get Rs 1 otherwise 0.

x → amount of money won

$$x = \underline{\{0, 1, 2\}}$$



$$P(X \geq 0) =$$

$$\frac{25}{36}$$

$$P(X = 1) = \frac{10}{36}$$

$$P(X = 2) = \frac{1}{36}$$

$$\text{P}(X=k) = {}^n C_k * (p)^k * (1-p)^{n-k}$$

$$n = 2$$

$$p \Rightarrow$$

probability of success in one trial

$$\Rightarrow p = 1/6$$

$$1-p = 5/6$$

$$P(X=0) = {}^2 C_0 * \left(\frac{1}{6}\right)^0 * \left(\frac{5}{6}\right)^2 = 25/36$$

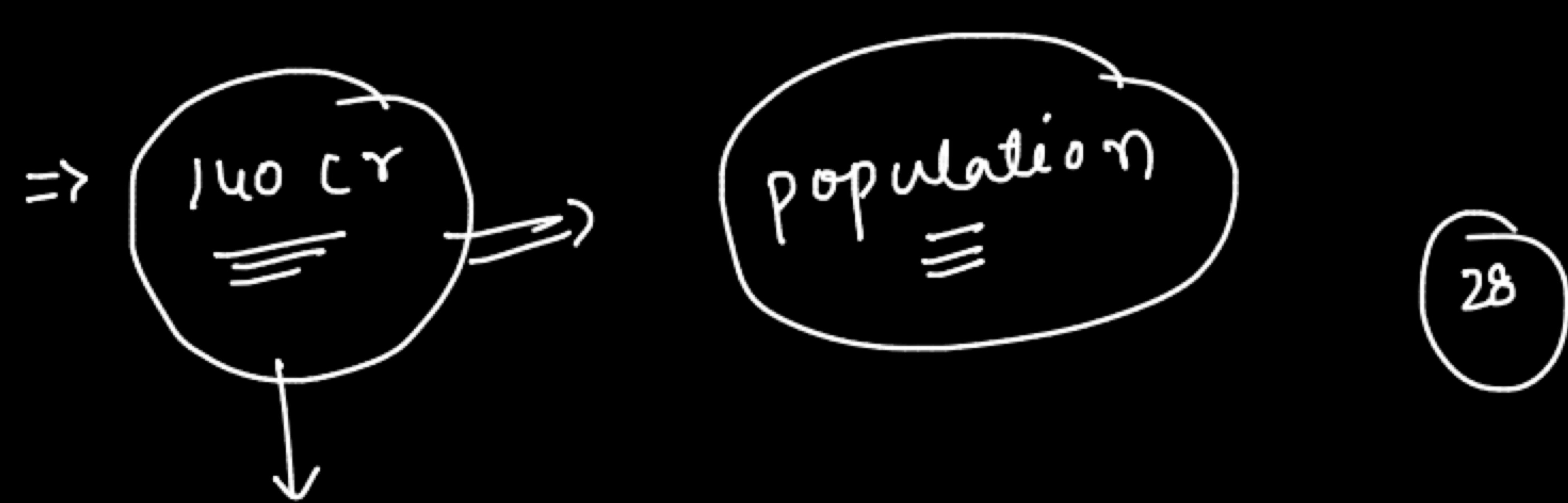
$$P(X=1) = {}^2 C_1 * \left(\frac{1}{6}\right)^1 * \left(\frac{5}{6}\right)^1 \Rightarrow 20/36$$

$$P(X=2) = {}^2 C_2 * \left(\frac{1}{6}\right)^2 * \left(\frac{5}{6}\right)^0 = 1/36 \checkmark$$

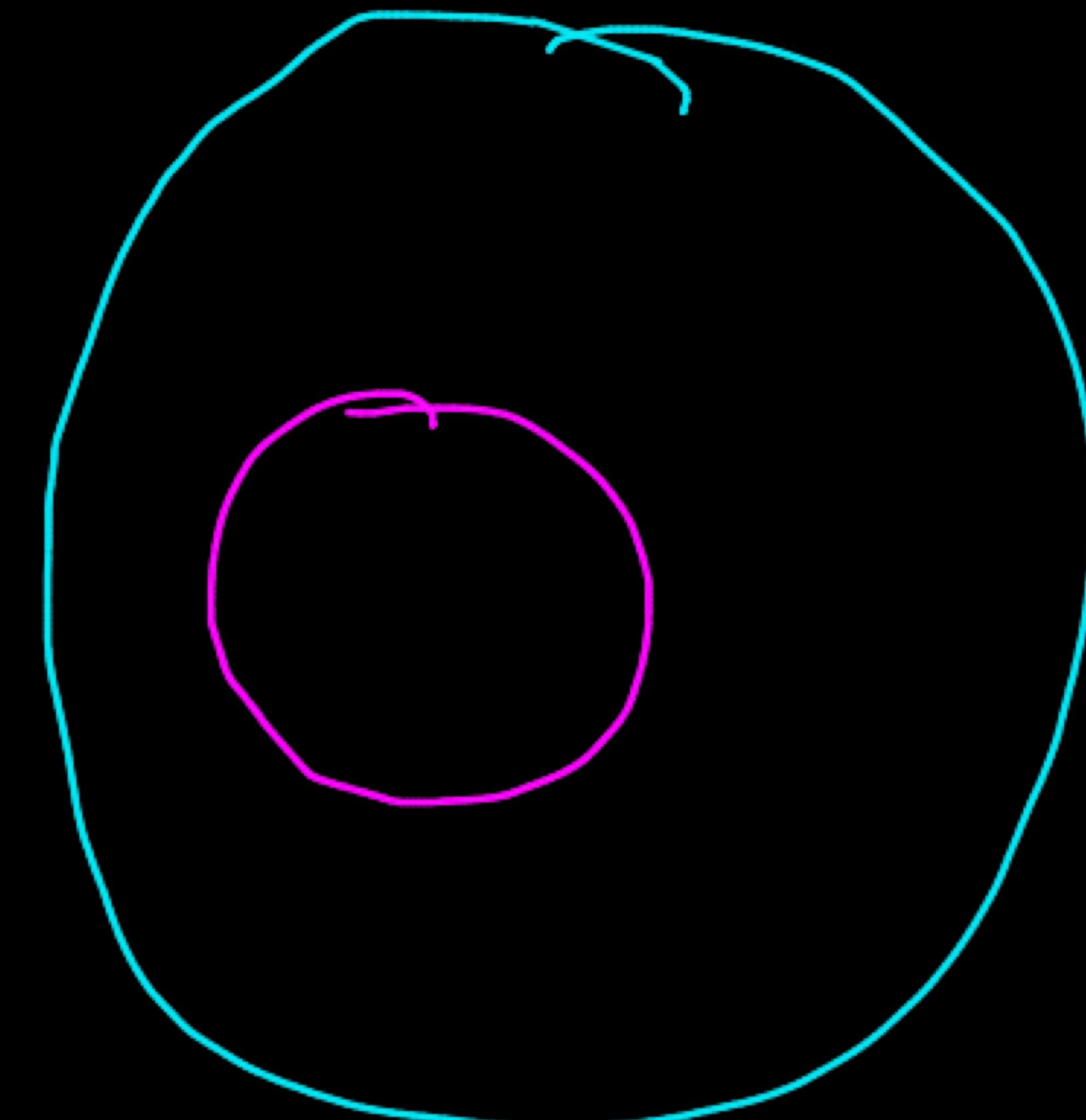
Population vs

Sample

inferential



Avg Unbiased Subset of the population

Sample : 

=> why should we use sample instead of population
↳

(i) Practicality :

10000



Sample Statistics

population

$$\text{Population mean } (\mu) = \frac{\sum x_i}{N}$$

$$\text{Population variance } (\sigma^2) = \frac{n}{\sum_{i=1}^n} \frac{(x_i - \mu)^2}{N}$$

$$\text{Population St. D } (\sigma) = \sqrt{\sigma^2}$$

Sample (n)

$$\text{mean} \Rightarrow \bar{x} = \frac{\sum x_i}{n}$$

$$\text{Variance} \Rightarrow s^2 = \frac{\sum_{i=1}^n (x_i - \bar{x})^2}{n-1}$$

 $s \cdot D$

$$= \sqrt{s^2}$$

~~Bessel's correction~~

correct

$n-1, n-2, \dots$ bias

Quiz #1

45 students

$N = \underline{45}$

5 students were randomly

$n = 5$

[131, 150, 140, 142, 152]

Sample mean

variance.

$$\bar{x} = \frac{131 + 150 + 140 + 142 + 152}{5}$$

$$715 / 5 = 143$$

↑ 1

$$s^2 = \frac{(131 - 143)^2 + (150 - 143)^2 + (140 - 143)^2 + (142 - 143)^2}{4}$$

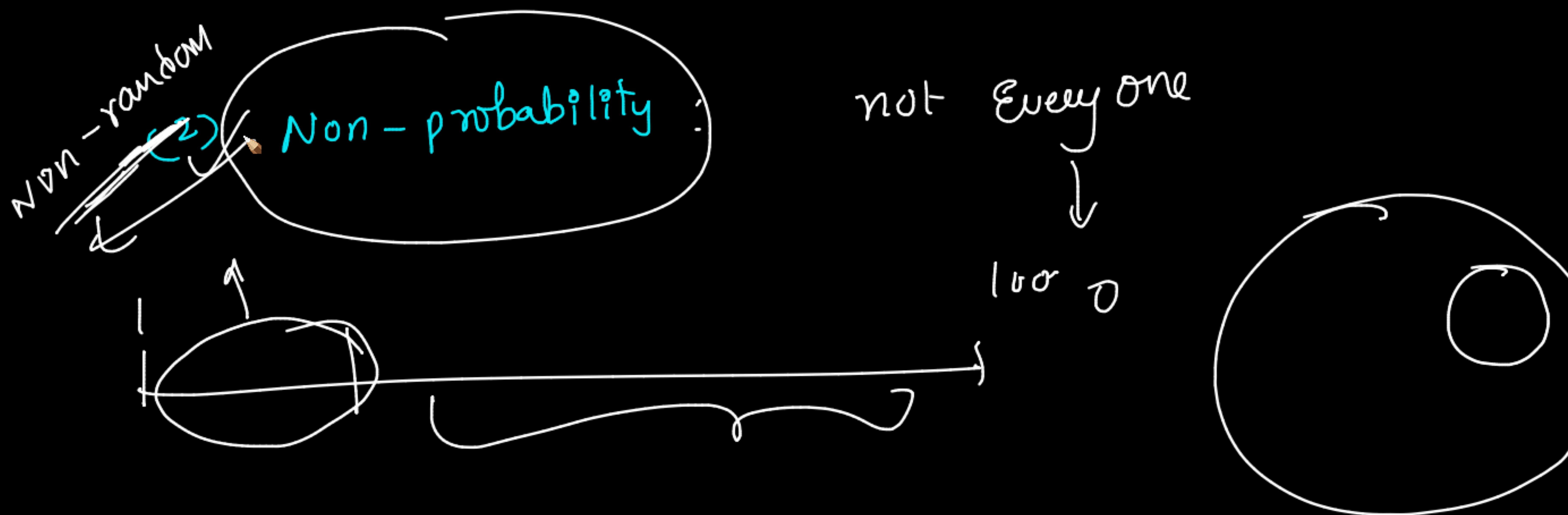
Sampling

Techniques

Probability Sampling: chance being selected

(1) random

(2) non-random



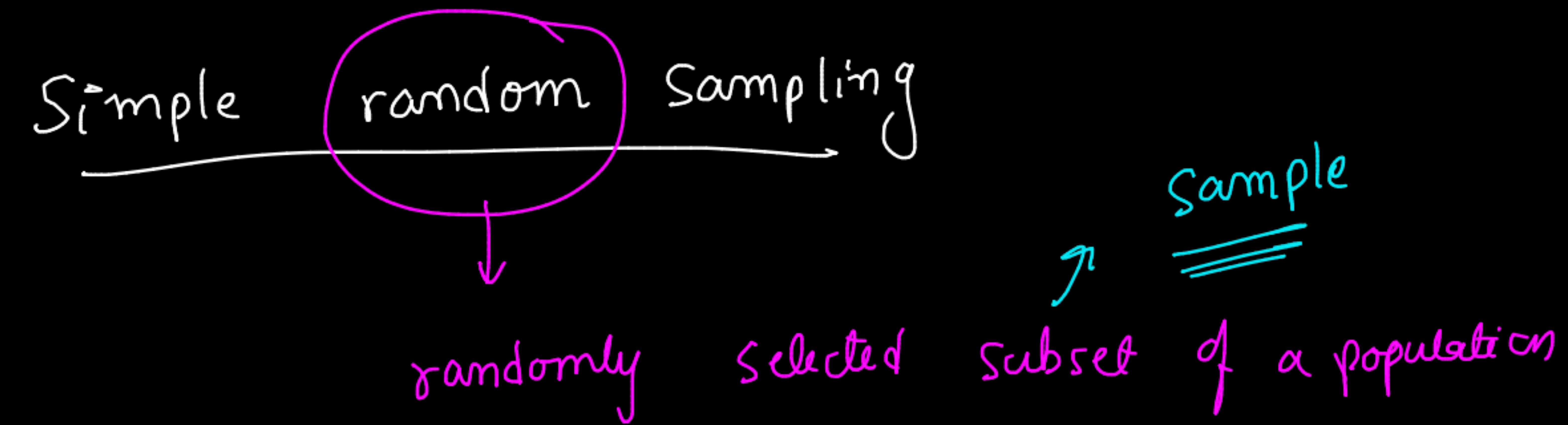


Probability Sampling techniques

- 1. Simple random ✓
- 2. Systematic
- 3. Stratified ✓
- 4. cluster ✓



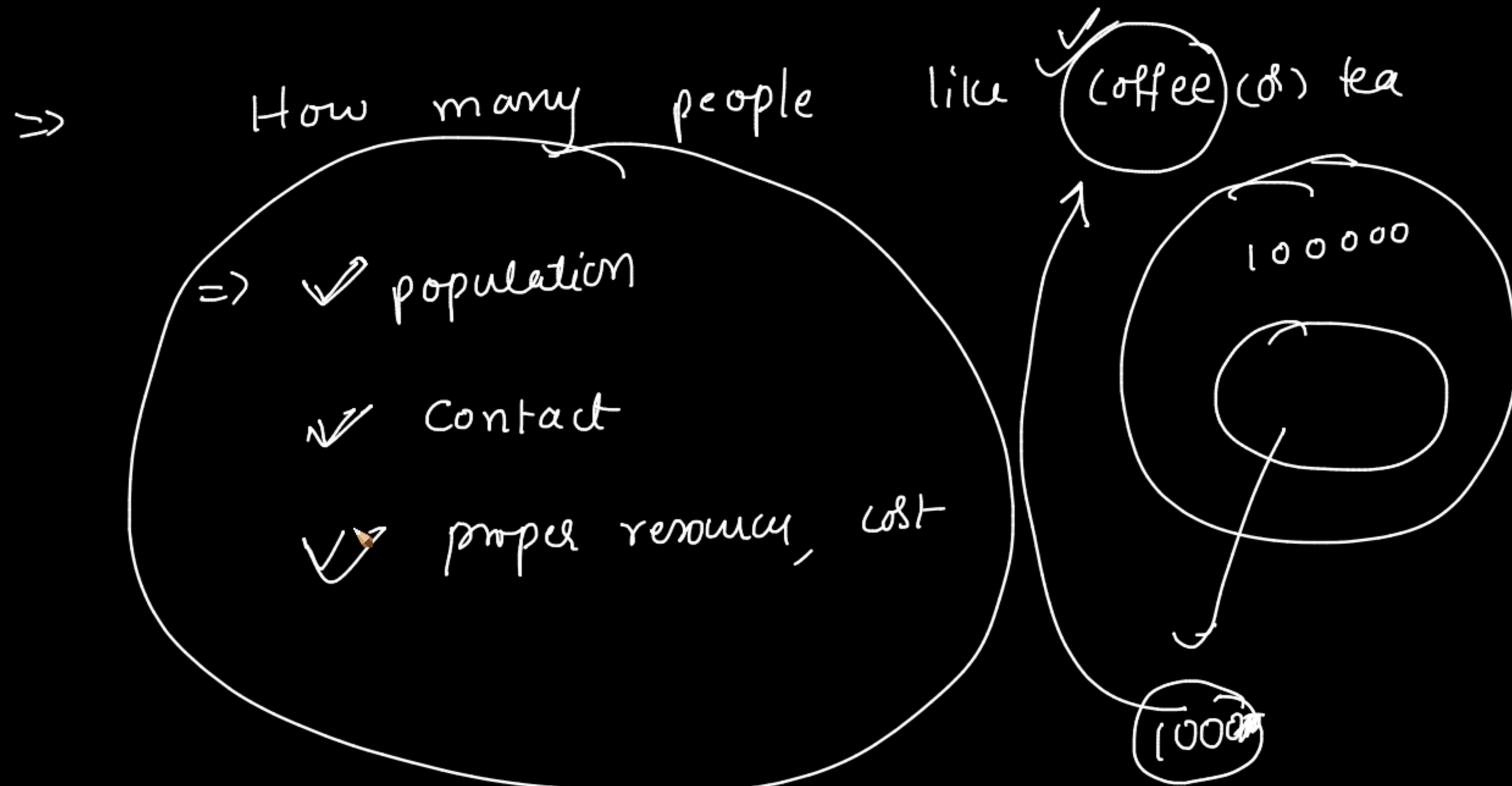
ML



⇒ | 0 0 0 0 0

⇒

1 0 0 0



✓ Simple random Sampling

1. Define population => 100000
2. Determine sample size => 1000
3. Randomly select your sample
4. Collect data from sample

1.
2.
3.
4.
:
1000

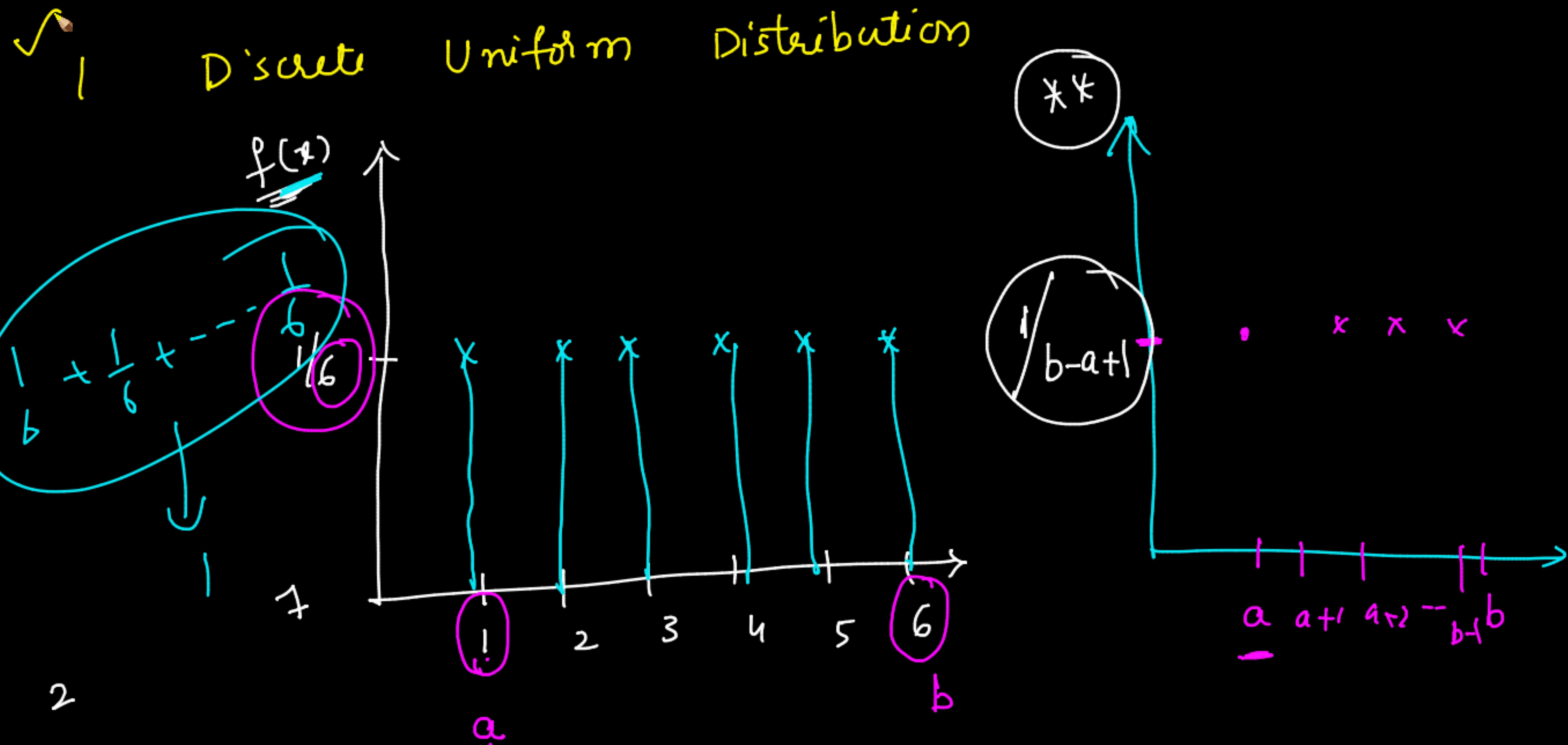
Uniform Distribution

$$\text{---} \quad S = \{ 1, 2, 3, 4, 5, 6 \}$$

$$P(\{1\}) = \frac{1}{6}$$
$$P(\{2\}) = \frac{1}{6}$$
$$\vdots$$
$$P(\{6\}) = \frac{1}{6}$$

1. Discrete uniform
2. Continuous " Distr

Pen selected



$$\frac{1}{\max(a,b)}$$

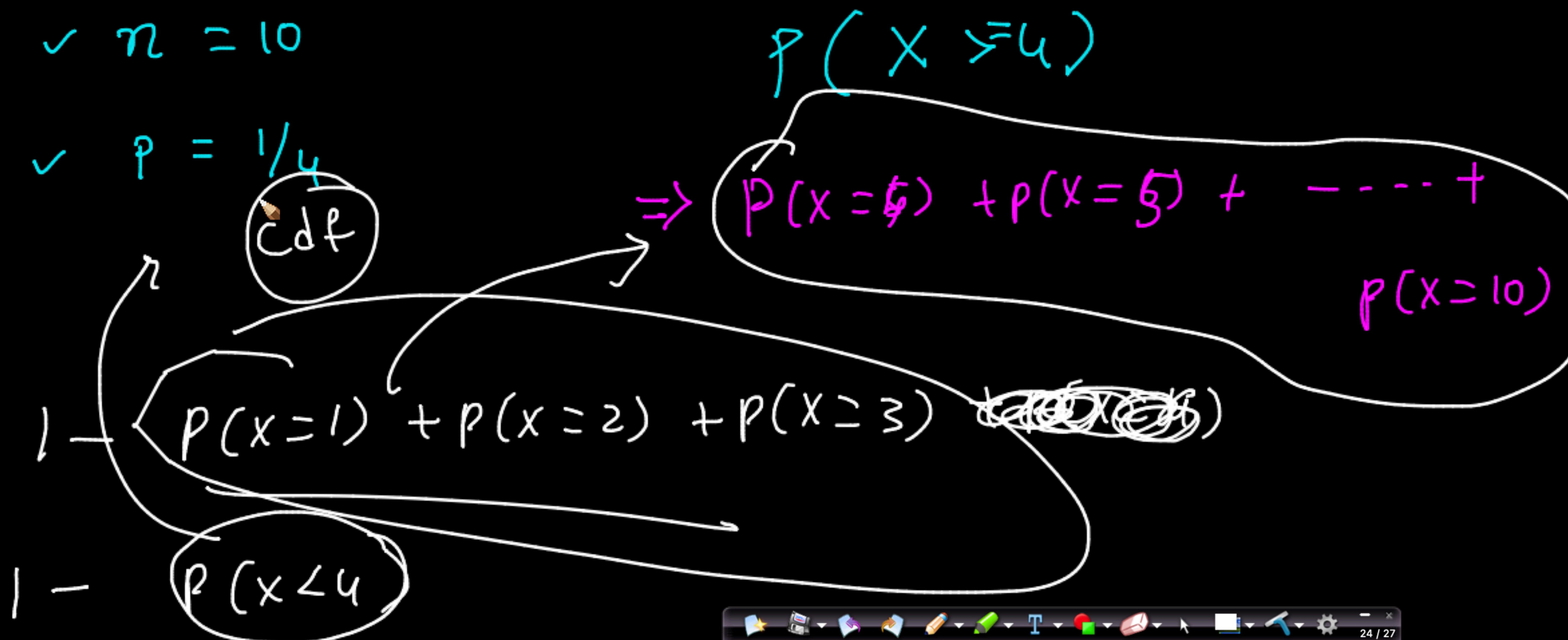
$$b-a+1$$

$$6-1$$

⇒ we have 10 quizzes, with 4 options each
only 1 option is correct, and we are guessing the answer.
what is the probability that we will get at least 4 answers

$$\checkmark n = 10$$

$$\checkmark p = 1/4$$



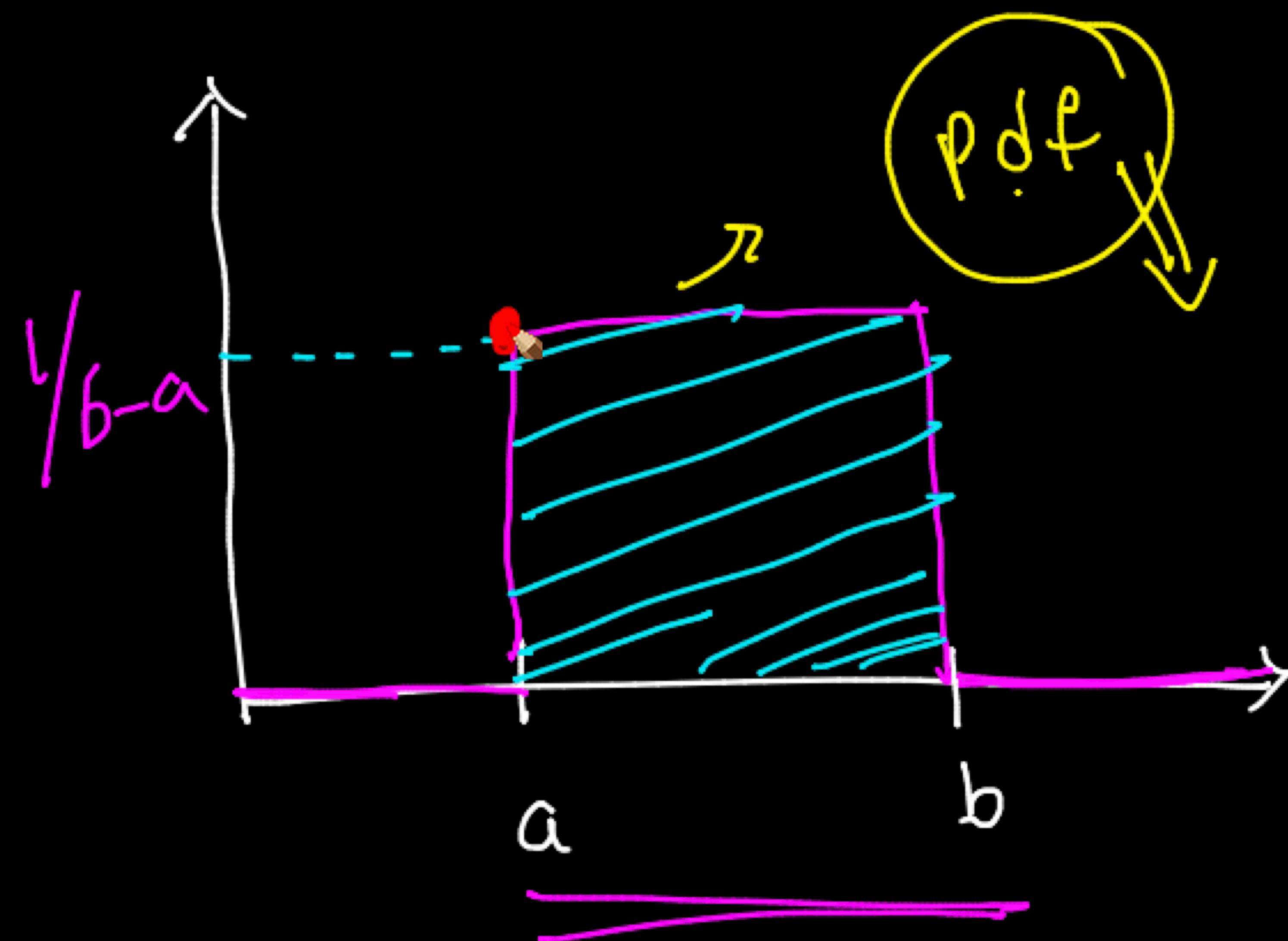
the probability that we will get exactly
2 answers.

$$n = 10$$

$$p = 1/4$$

$$P(X=2) = {}^{10}C_2 * \left(\frac{1}{4}\right)^2 * \left(\frac{3}{4}\right)^8$$

Pdf.



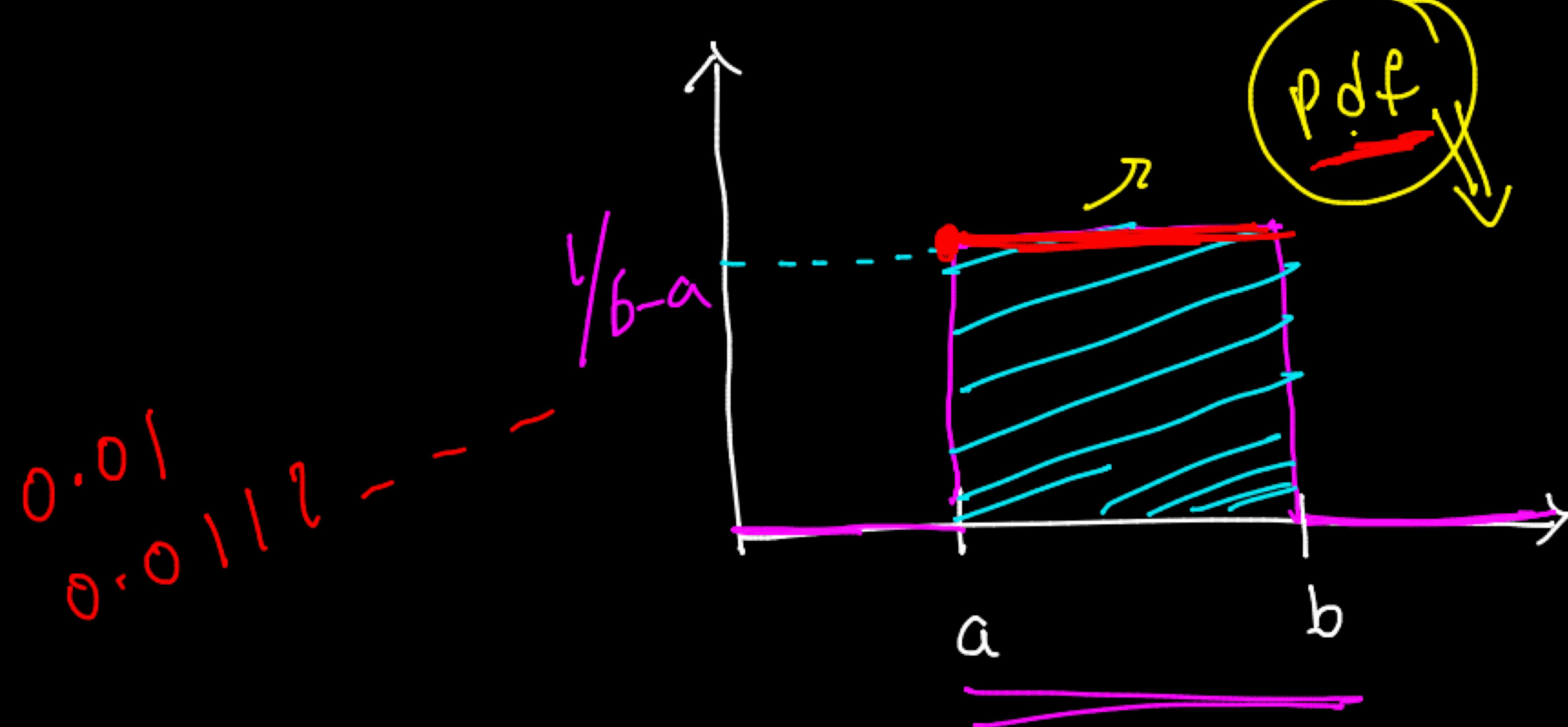
Continuous R.V

$$P(a \leq x \leq b)$$

$$P = (b-a) * f(x)$$

$$f(x) = \frac{1}{b-a}$$

$$p.d.f(x) = \frac{1}{b-a}$$



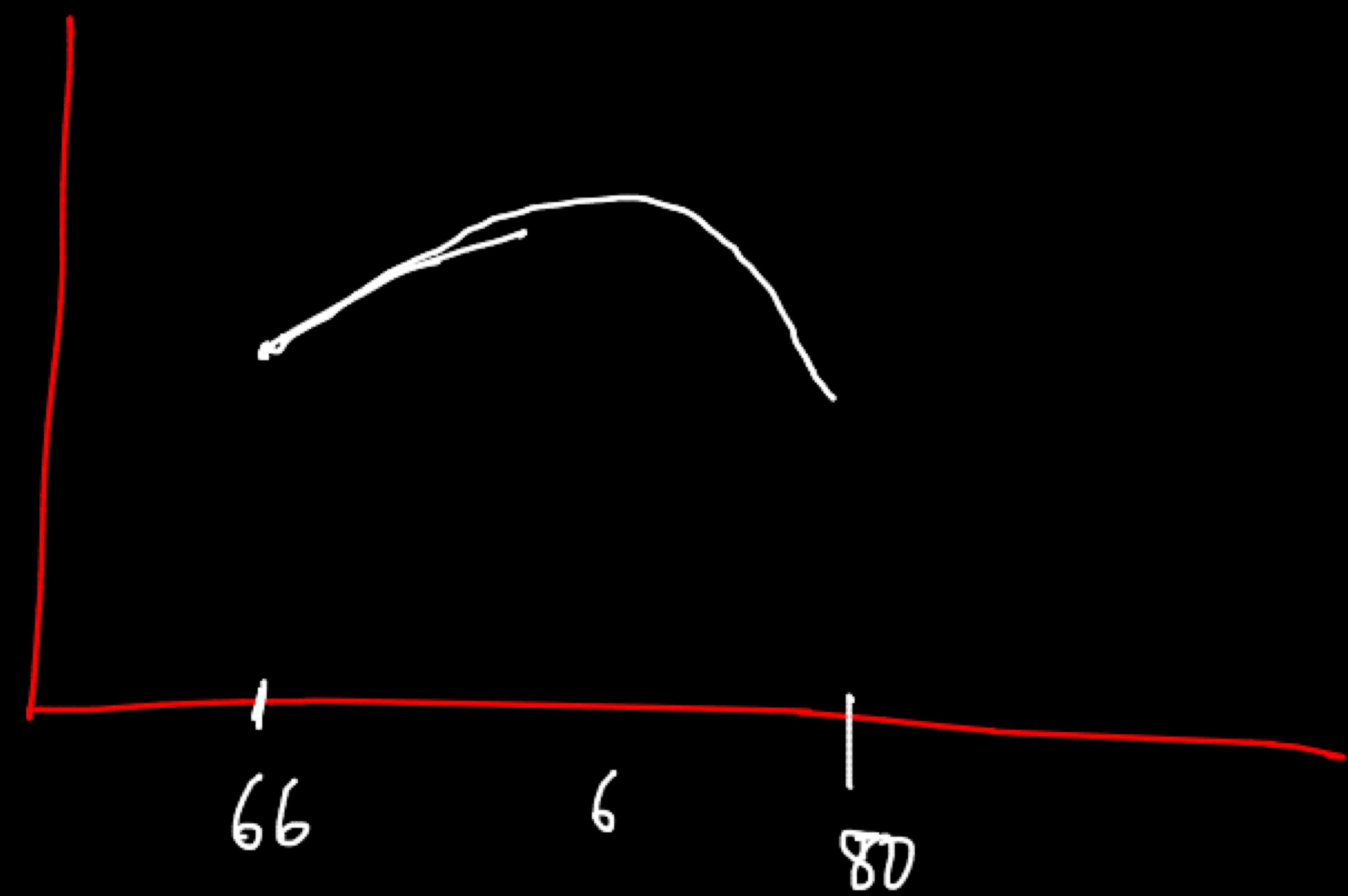
Continuous R.V

$P(a \leq x \leq b)$

$$P(a \leq x \leq b) = (b-a) * f(x)$$
$$f(x) = \frac{1}{b-a}$$

$$P \text{ df}(x) = \frac{1}{b-a}$$

66.125
66.25



Brilliant.org

Cursor selected