Linear Algebra - 3

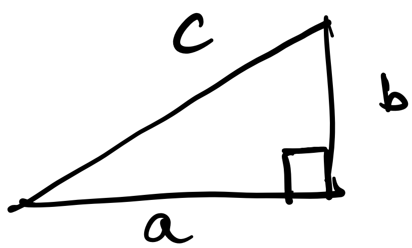
~ Aditya Jain

(AJ)

(1) $\cos\theta = \dfrac{\vec{x}^T * \vec{y}}{||x|| * ||y||}$

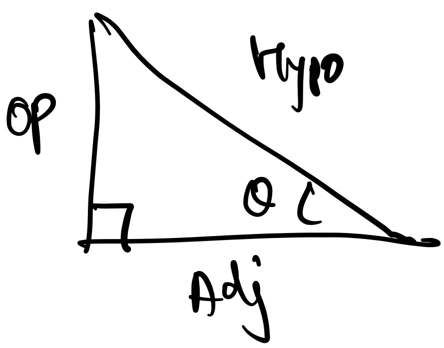
(2) $\hat{y} = \dfrac{\vec{y}}{||\vec{y}||}$

* Trignometry basics



Pythagoras Thm

$$\boxed{c^2 = a^2 + b^2}$$



$$\cos\theta = \frac{Adj}{Hypo}$$

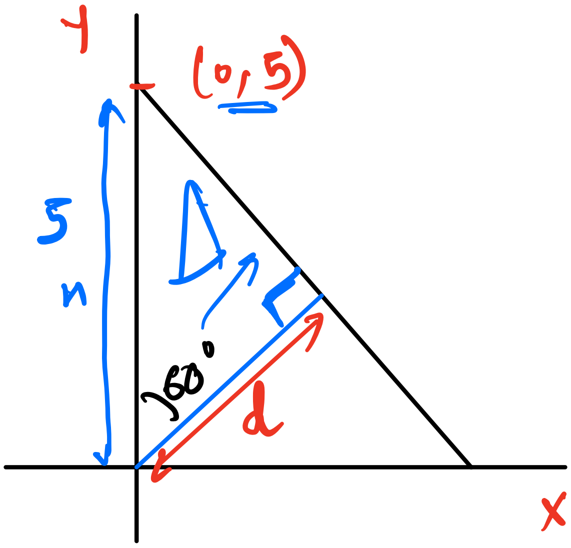
$$\sin\theta = \frac{OP}{Hypo}$$

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{OP}{Adj}$$

$\underline{\underline{\cos\theta}}$

$0°$	$30°$	$45°$	$60°$	$90°$
1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0

(9)



$d = ?$

$H = 5$

$$\cos 60° = \frac{d}{H}$$

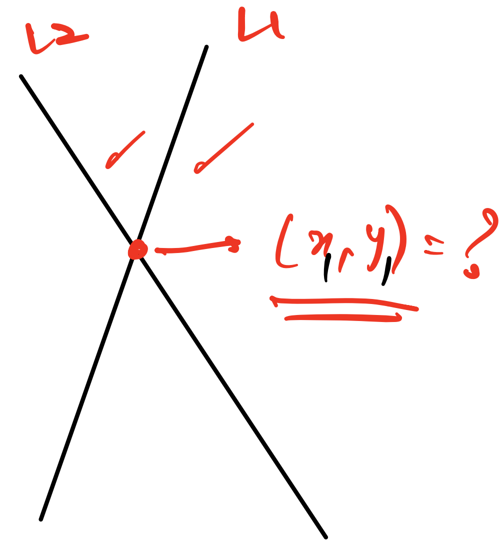
$$d = H \times \cos 60$$

$$= 5 \times \frac{1}{2}$$

$$\boxed{d = 2.5}$$

(Q.2) $L1: \; 3x - y + 7 = 0$

$L2: \; 2x + 2y = 0$



Point of intersection = ?

Assume $P \longrightarrow (x_1, y_1)$

$$3x - y + 7 = 0$$

$$3x_1 - y_1 + 7 = 0 \quad \text{--- (1)}$$

$$2x + 2y = 0$$

$$2x_1 + 2y_1 = 0$$

$$x_1 = -y_1 \quad \text{--- (2)}$$

$$3(-y_1) - y_1 + 7 = 0$$

$$-4y_1 = -7$$

$$\boxed{y_1 = 7/4}$$

$$x_1 = -y_1 = -\left(7/4\right)$$

$$x_1 = -7/4$$

$$P : (x_1, y_1) \rightarrow \left(-7/4, 7/4\right)$$

$L: \ ax + by + \cancel{c} = 0$

y- intercept = ?

$y = mx + c$

$ax + by + c = 0$

$by = -ax - c$

$y = \left(-\frac{a}{b}\right)x + \left(-\frac{c}{b}\right)$

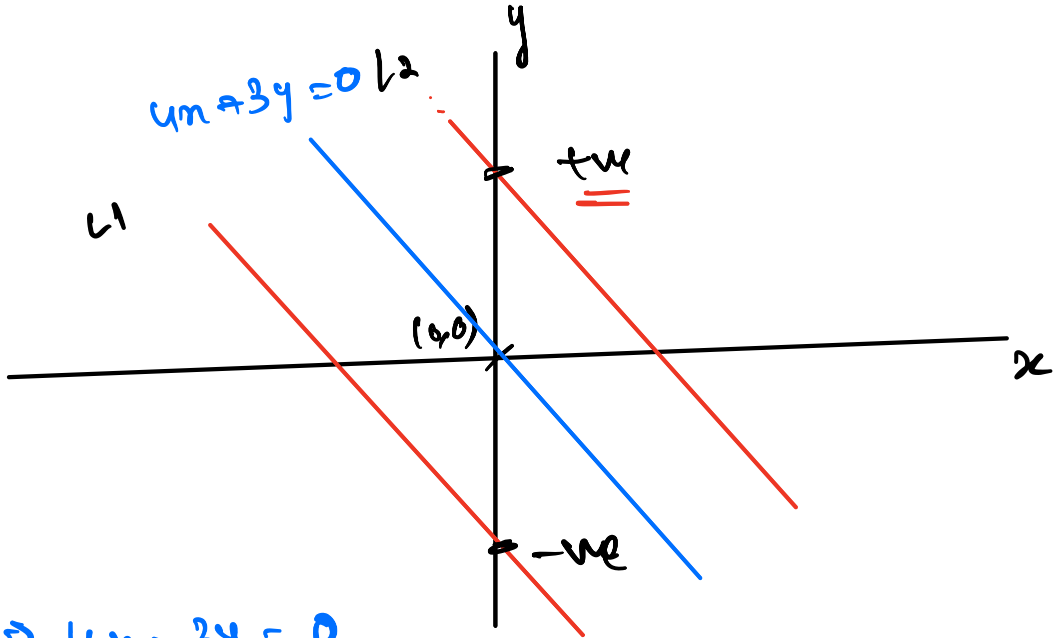
$\boxed{\text{y-intercept} \Rightarrow -\frac{c}{b}}$

$x \qquad y$

$w_1x_1 + w_2x_2 + w_0 = 0$

$$x_2 = \left(-\frac{w_1}{w_2}\right)x_1 + \left(-\frac{w_0}{w_2}\right)$$

(Q1)



$L \Rightarrow 4x+3y=0$

L1 —

L2 —

① $4x+3y+2=0$ —

② $4x+3y-5=0$ —

① $4x+3y+2=0$

$$y=\left(-\frac{4}{3}\right)x+\left(-\frac{2}{3}\right)$$

L1

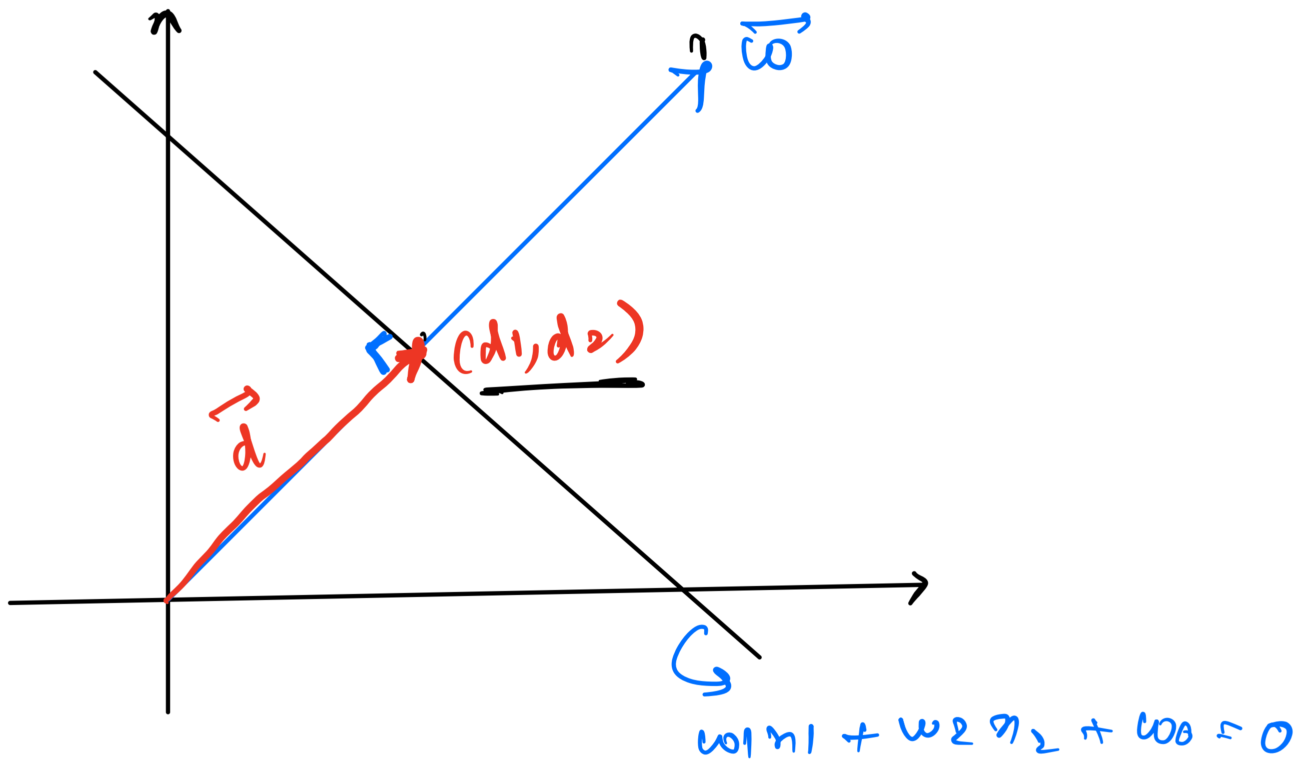
(2) $4x + 3y - 5 = 0$

$$y = \left(-\frac{4}{3}\right)x + \left(\frac{5}{3}\right)$$

+ve

L2

(7) Proof of weight vector is always perpendicular to the Hyperplane.



$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

$$\|\vec{w}\| = \sqrt{w_1^2 + w_2^2} \longrightarrow (1)$$

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix}$$

$$\|\vec{d}\| = \sqrt{d_1^2 + d_2^2}$$

$$L: w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\downarrow$$

$$\underline{w_1 d_1 + w_2 d_2 + w_0 = 0} \text{ --- } (2)$$

$$\hat{w} = \frac{\vec{w}}{\|\vec{w}\|}$$

$$\vec{d} = \begin{bmatrix} d_1 \\ d_2 \end{bmatrix} \longrightarrow \begin{bmatrix} K * \hat{w}_1 \\ K * \hat{w}_2 \end{bmatrix}$$

$$\hat{w}_1 = \frac{w_1}{\|\vec{w}\|}, \quad \hat{w}_2 = \frac{w_2}{\|\vec{w}\|}$$

$$d_1 = \frac{K * w_1}{\|\vec{w}\|} \quad , \quad d_2 = \frac{K * w_2}{\|\vec{w}\|} \quad (3)$$

using (3) in (2)

$$w_1 d_1 + w_2 d_2 + w_0 = 0$$

$$w_1 \left(\frac{K * w_1}{\|\vec{w}\|} \right) + w_2 * \left(\frac{K * w_2}{\|\vec{w}\|} \right) + w_0 = 0$$

$$K \left(\frac{w_1^2}{\|\vec{w}\|} \right) + K \left(\frac{w_2^2}{\|\vec{w}\|} \right) + w_0 = 0$$

$$K \left(\frac{w_1^2}{\|\vec{w}\|} \right) + K \left(\frac{w_2^2}{\|\vec{w}\|} \right) = -w_0$$

$$K \left(\frac{w_1^2 + w_2^2}{\|\vec{w}\|} \right) = -w_0$$

$$K = \frac{-w_0}{\left(\frac{w_1^2 + w_2^2}{\|\vec{w}\|}\right)}$$

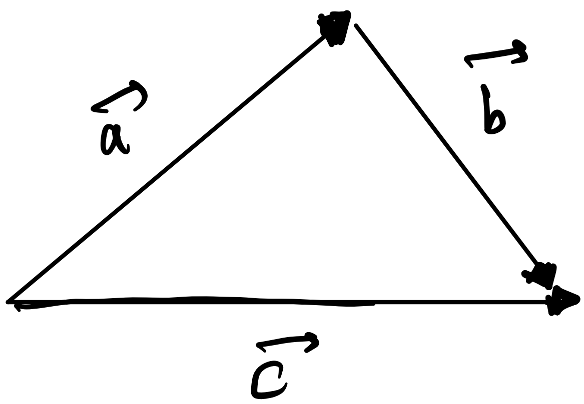
$$\frac{a}{\left(\frac{b}{c}\right)} \implies \frac{a * c}{b}$$

$$\boxed{K = \frac{-w_0 * \|\vec{w}\|}{w_1^2 + w_2^2}}$$

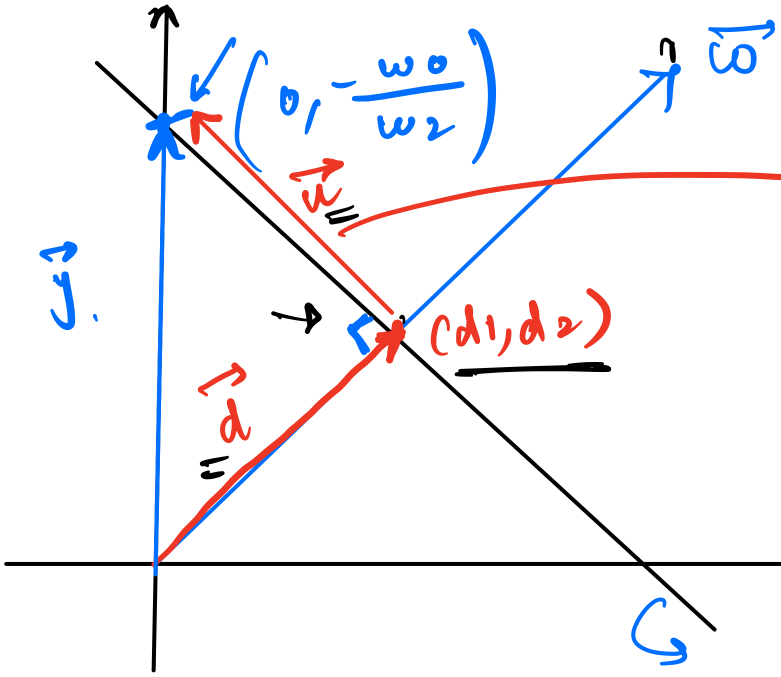
Put K in $\vec{d}$

$$\vec{d} = \begin{bmatrix} \frac{-w_0 * \cancel{\|\vec{w}\|}}{w_1^2 + w_2^2} * \frac{w_1}{\cancel{\|\vec{w}\|}} \\ \frac{-w_0 * \cancel{\|\vec{w}\|}}{w_1^2 + w_2^2} * \frac{w_2}{\cancel{\|\vec{w}\|}} \end{bmatrix}$$

$$\vec{d} = \begin{bmatrix} \dfrac{-w_0 * w_1}{w_1^2 + w_2^2} \\[2ex] \dfrac{-w_0 * w_2}{w_1^2 + w_2^2} \end{bmatrix}$$



$$\boxed{\vec{c} = \vec{a} + \vec{b}}$$



$$\vec{y} = \vec{d} + \vec{u}$$

$$\star \ \boxed{\vec{u} = \vec{y} - \vec{d}}$$

$$w_1 x_1 + w_2 x_2 + w_0 = 0$$

$$\left(-\frac{w_0}{w_2}\right)$$

$$\vec{u} = \vec{y} - \vec{d}$$

$$= \begin{bmatrix} 0 - \left(\dfrac{-w_0 * w_1}{w_1^2 + w_2^2} \right) \\[2em] -\dfrac{w_0}{w_2} - \left(\dfrac{-w_0 * w_2}{w_1^2 + w_2^2} \right) \end{bmatrix}$$

$$
= \begin{bmatrix} \dfrac{w_0 * w_1}{w_1^2 + w_2^2} \\[2em] \dfrac{-w_0 (w_1^2 + w_2^2) + w_2^2 * w_0}{w_2 * (w_1^2 + w_2^2)} \end{bmatrix}
$$

$\vec{d} * \vec{u}$

$\downarrow$

$\vec{d} * (\vec{y} - \vec{d})$

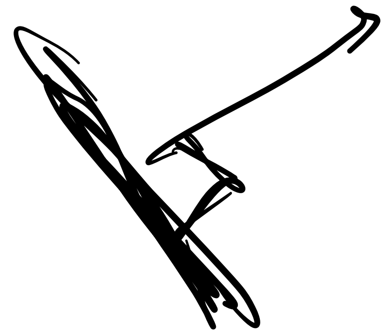
$$\begin{bmatrix} \frac{-w_1 w_0}{w_1^2 + w_2^2} & \frac{-w_2 w_0}{w_1^2 + w_2} \end{bmatrix} *$$

$$\boxed{\vec{d} * \vec{u} = 0}$$

$$\cos\theta = \frac{\vec{d}^T * \vec{u}}{\|\vec{d}\| * \|\vec{u}\|} \rightarrow 0$$

$\cos\theta = 0$

$\boxed{\theta = 90^\circ}$



Shifting 2D lines

$\rightarrow \quad w_1 x_1 + w_2 x_2 + w_0 = 0$

① 'a' units to the left

$\downarrow$

$$w_1 (x_1 + a) + w_2 x_2 + w_0 = 0$$

② 'a' $\rightarrow$ right

$$w_1 (x_1 - a) + w_2 x_2 + w_0 = 0$$

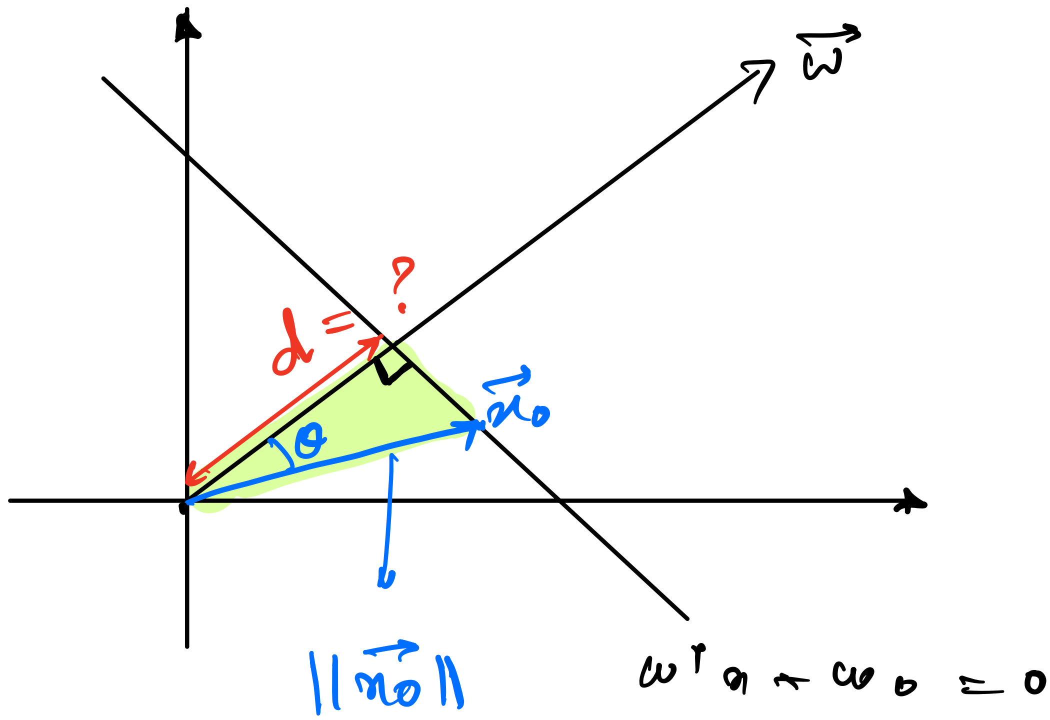
③ 'a' $\rightarrow$ up

$$w_1 x_1 + w_2 (x_2 - a) + w_0 = 0$$

④ 'a' $\rightarrow$ down

$$w_1 x_1 + w_2 (x_2 + a) + w_0 = 0$$

⊛ Distance between origin & line :-

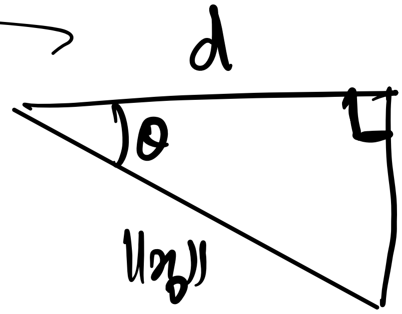


$\vec{w}^T \vec{x} + w_0 = 0$

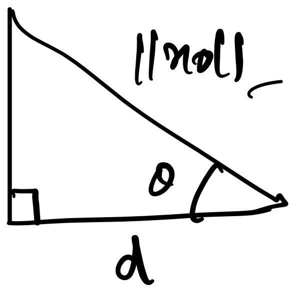
$\vec{w}^T \vec{x}_0 + w_0 = 0$

$\boxed{\vec{w}^T \vec{x}_0 = -\underline{w_0}}$ —— ①

$$\cos\theta = \frac{\vec{w}^{T} * \vec{x}_0}{\|\vec{w}\| * \|x\|} \quad \text{—— (2)}$$



$$\cos\theta = \frac{d}{\|x_0\|} \quad \text{—— (3)}$$



equate (2) & (3)

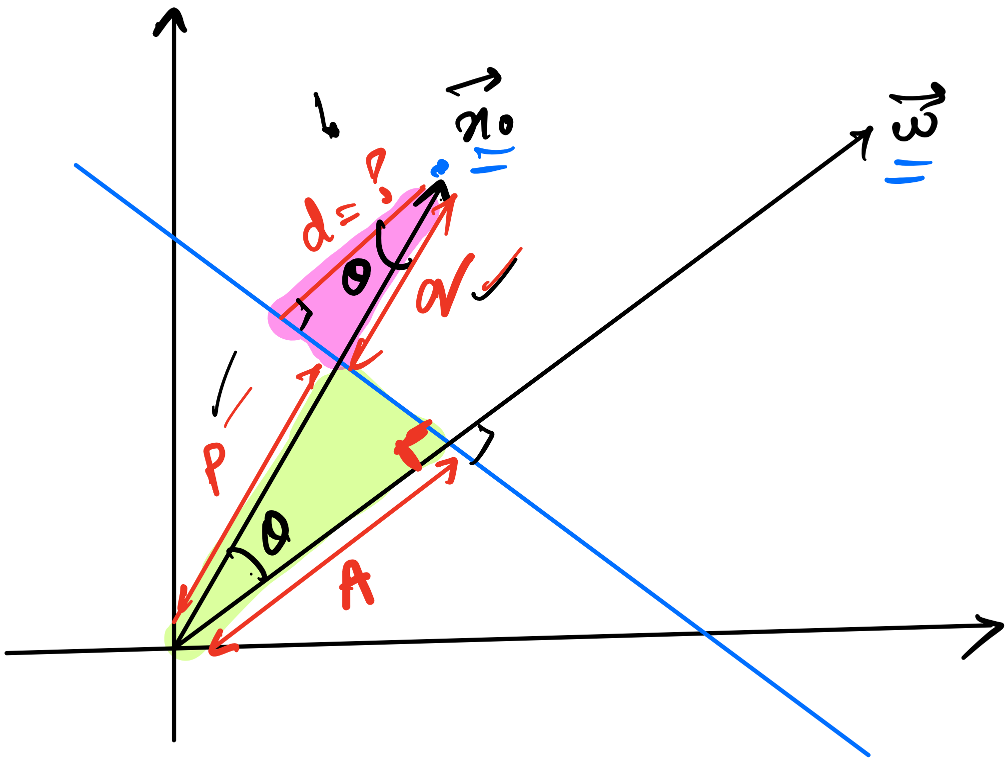
$$\frac{\vec{w}^{T} * \vec{x}_0}{\|w\| * \cancel{\|x_0\|}} = \frac{d}{\cancel{\|x_0\|}}$$

$$d = \frac{\vec{w}^{T} * \vec{x}_0}{\|\vec{w}\|} \quad \text{—— (4)}$$

using 1 in 4

** $$\boxed{d = \frac{-w_0}{\|\vec{w}\|}}$$

⊛ Distance between Point and Line



$w^T x + w_0 = 0$

$$A = \frac{-w_0}{\|\vec{w}\|} \quad \text{(1)}$$

$$\cos\theta = \frac{A}{P}$$

$$\boxed{P = \frac{A}{\cos\theta}} \quad \text{(2)}$$

$$\underline{p} + \underline{q} = \|\vec{x}_0\|$$

$$q = \|\vec{x}_0\| - p$$

$$\boxed{q = \|\vec{x}_0\| - \frac{A}{\cos\theta}} \longrightarrow 3$$

$$\cos\theta = \frac{d}{q}$$

$$d = q * \cos\theta$$

$$d = \left(\|\vec{x}_0\| - \frac{A}{\cos\theta} \right) * \cos\theta$$

$$\boxed{\cos\theta = \frac{w^T x_0}{\|w\|\|x_0\|}} \longrightarrow 4$$

$$d = \|\vec{x}_0\| \cos\theta - A$$

from ④

$$d = \|\cancel{\vec{x}_0}\| * \frac{w^T x_0}{\|w\| * \|\cancel{x_0}\|} - A$$

$$= \frac{w^T x_0}{\|w\|} - A$$

from eqn ①

$$= \frac{w^T x_0}{\|w\|} - \left(-\frac{w_0}{\|w\|} \right)$$

$$\boxed{d = \frac{\vec{w}^T * \vec{x}_0 + w_0}{\|\vec{w}\|}}$$

(Q)

$H: \; x + y + z = 0$

point $\longrightarrow$ $(30, 45, 0)$

what is the dist of point from the given H = ?

$$\vec{w} = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

$$w_0 = 0$$

$$d = \frac{w^T x + w_0}{\|w\|}$$

$$= \frac{1 \times 30 + 1 \times 45 + 1 \times 0 + 0}{\sqrt{1^2 + 1^2 + 1^2}}$$

$$= \boxed{\frac{75}{\sqrt{3}}}$$