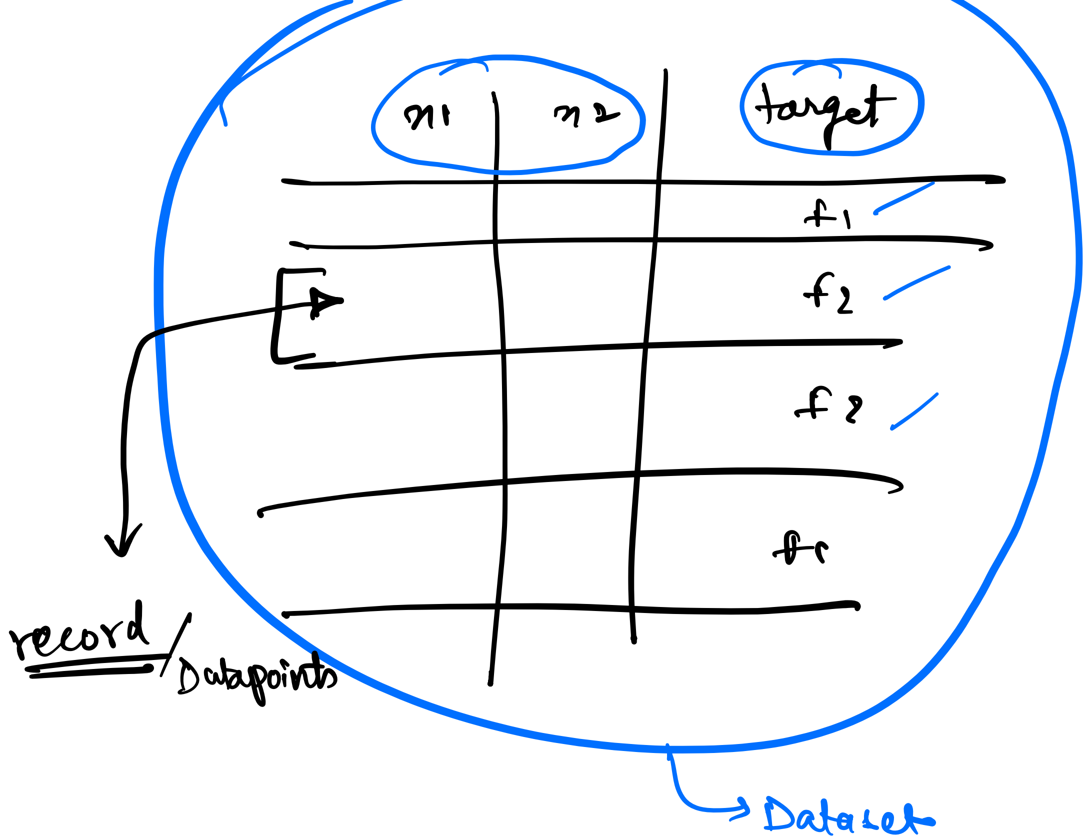
Linear Algebra - 2

1 features / indep

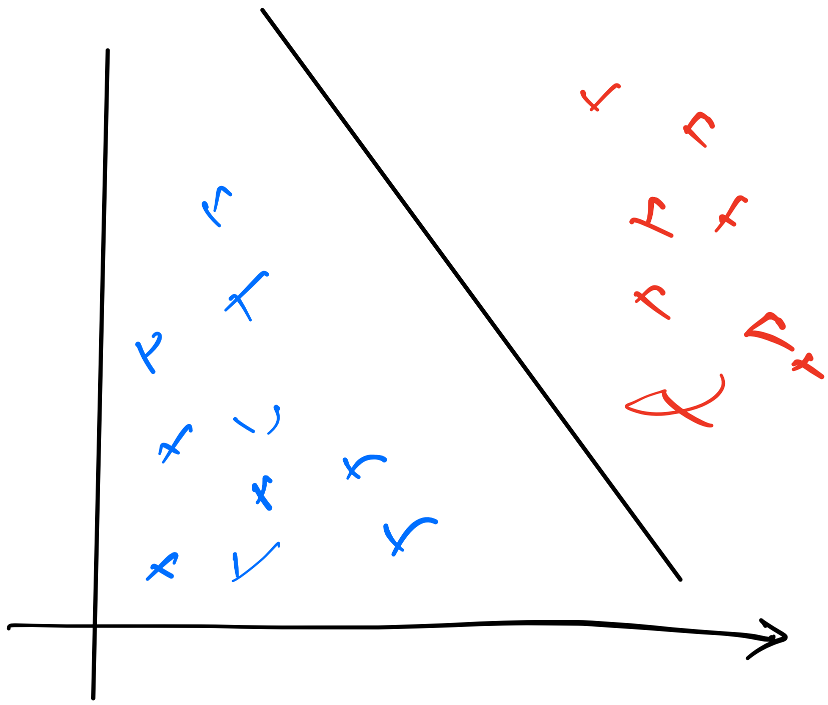
2 Target / label

dependent

3 Dataset



5 Classifier



$$y = mx + c$$

Slope

y-intercept

$$w_1x_1 + w_2x_2 + w_0 = 0$$

weight

features

2D Hyperplane

$$w_1x_1 + w_2x_2 + w_3x_3 + w_0 = 0$$

↳ 3D Hyperplane.

$$\vdots$$

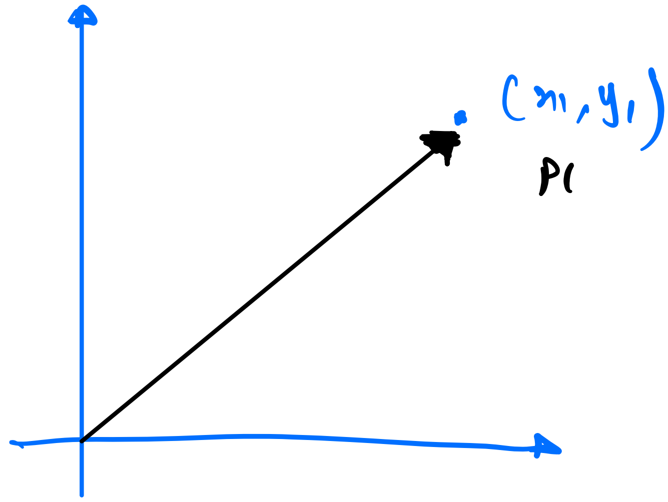
$$w_1x_1 + w_2x_2 + \dots \dots w_dx_d + w_0 = 0$$

↳ nD

* Vectors

what is a vector?

→ Both magnitude and direction



How do I represent it?

$\vec{x} = [1, 2, 5, 4, 3]$

By-default → Column vector

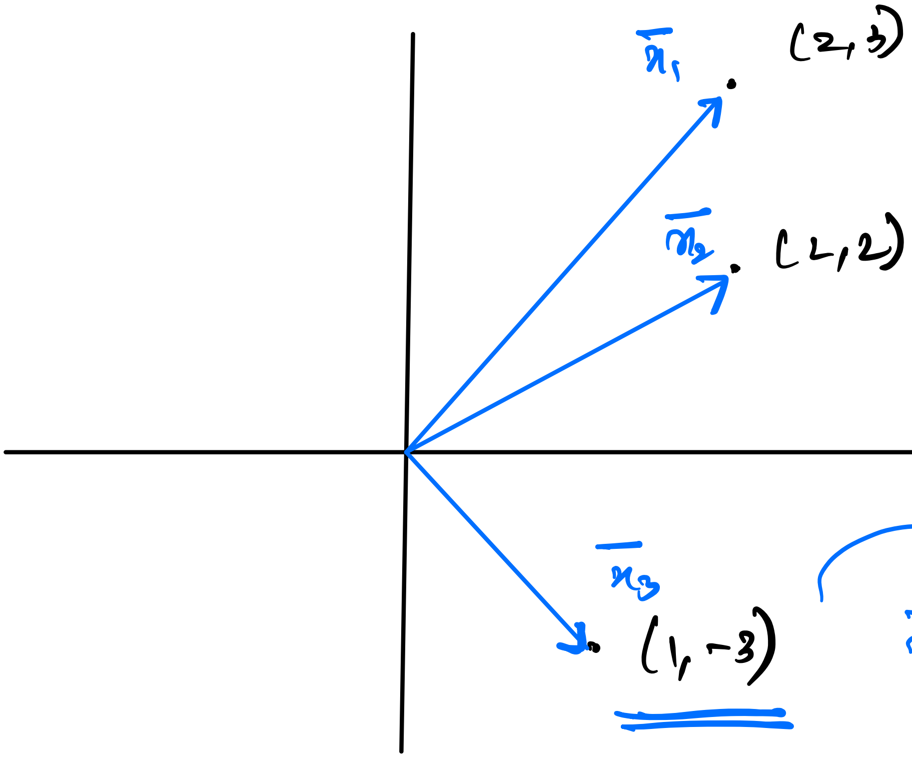
$$\vec{x} = \begin{bmatrix} 1 \\ 2 \\ 5 \\ 4 \\ 3 \end{bmatrix}$$

5x1

→ Column vector

$$\bar{x}^T = \begin{bmatrix} 1 & 2 & 5 & 4 & 3 \end{bmatrix}$$

↳ <u>row vector</u>



$$\bar{x}_1 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$$

$$\bar{x}_2 = \begin{bmatrix} 2 \\ 2 \end{bmatrix}$$

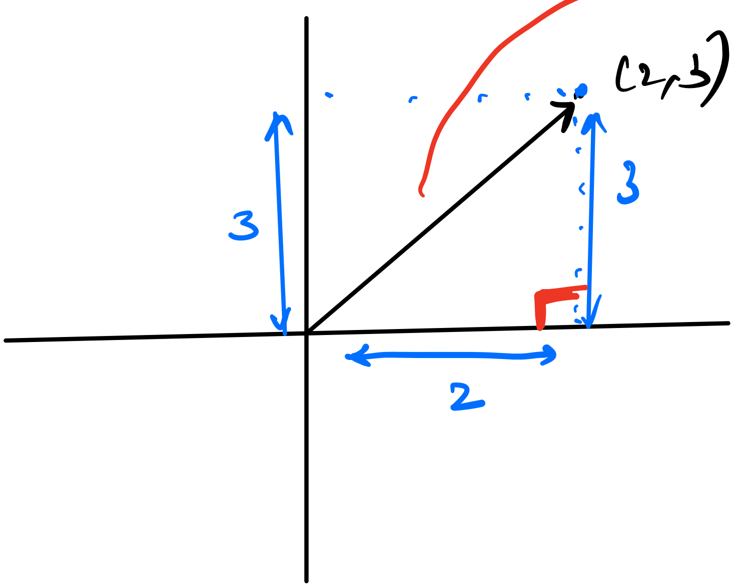
$$\bar{x}_3 = \begin{bmatrix} 1 \\ -3 \end{bmatrix}$$

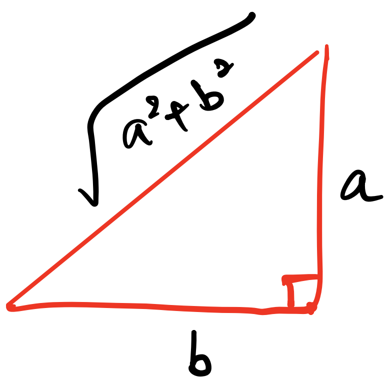
(Q) What is the magnitude of a vector?

$$\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$$

for this 2D vector: $\sqrt{x_1^2 + x_2^2}$

$\sqrt{2^2 + 3^2}$





(4) what will be magnitude for a d-dimensional vector?

2D : $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}$

magnitude $(\vec{x}) = \|\vec{x}\| = \sqrt{x_1^2 + x_2^2}$

3D: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_2 \end{bmatrix} \Rightarrow |\vec{x}| = \sqrt{x_1^2 + x_2^2 + x_3^2}$

d D: $\vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix} \Rightarrow |\vec{x}| = \underline{\underline{\sqrt{x_1^2 + x_2^2 + \cdots + x_d^2}}}$

① for parallel lines $\longrightarrow$

$L1: y = m_1 x + c_1$

$L2: y = m_2 x + c_2$

$\Downarrow$

$m_1 = m_2$

② for perpendicular lines $\longrightarrow$

$\Downarrow$

$\boxed{m_1 * m_2 = -1}$

Quiz:

$\boxed{m_1 = m_2}$

L1: $3x - 2y + 6 = 0$

L2: $9x - 6y - 18 = 0$

$3/2$

Slope

L1: $3x - 2y + 6 = 0$

$\boxed{y = mx + c}$

$3x - 2y + 6 = 0$

$2y = 3x + 6$

$y = \left(\frac{3}{2}\right)x + \frac{6}{2}$

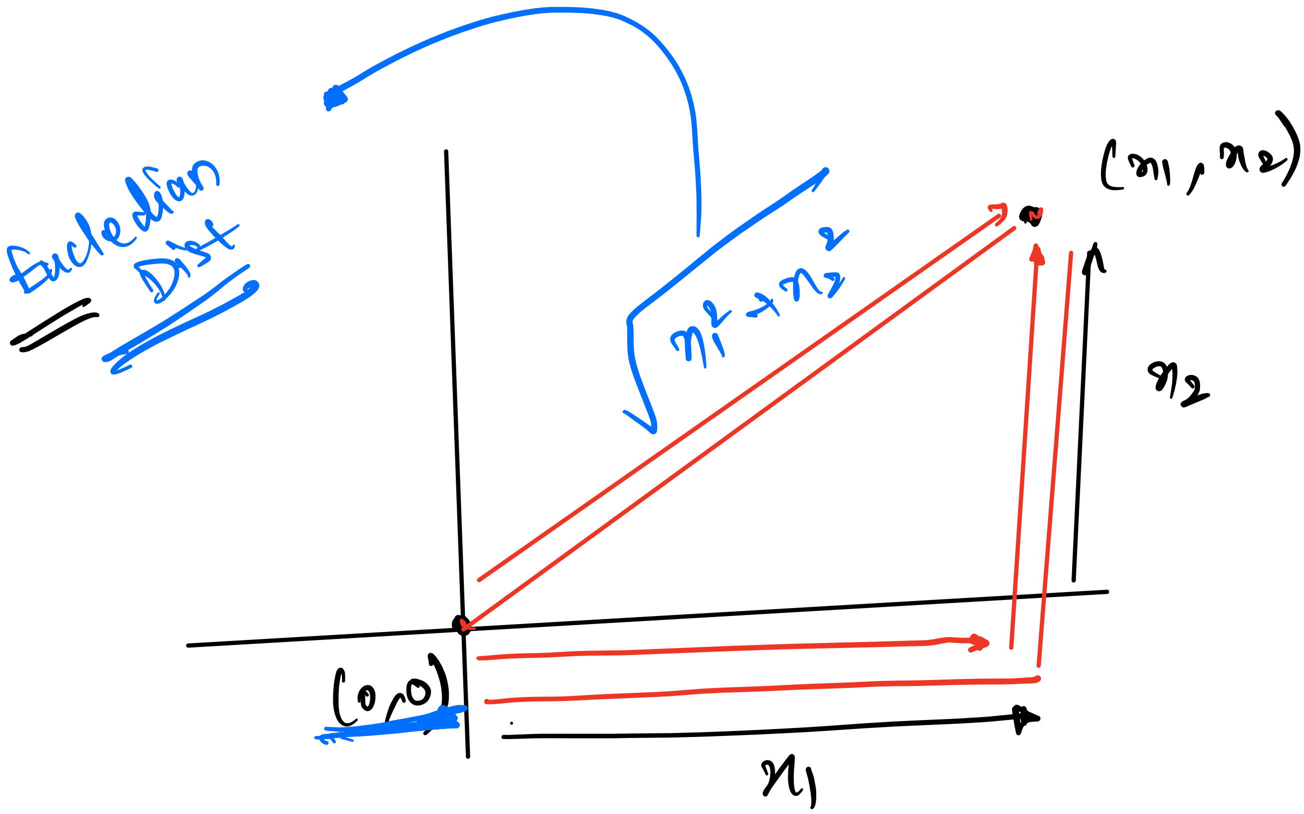
$\boxed{y = \left(3/2\right)x + 3}$

$y = mx + c$

$\boxed{m = 3/2}$

* Norm of a vector → its length

→ magnitude



$\boxed{|x_1| + |x_2|}$ → manhattan dist

$|-3| \rightarrow 3$

$|+3| \rightarrow 3$

magnitude $(\vec{x}) = \sqrt{x_1^2 + x_2^2}$

1. L1 Norm $\Rightarrow \|\vec{x}\|_1 = |x_1| + |x_2|$

↓

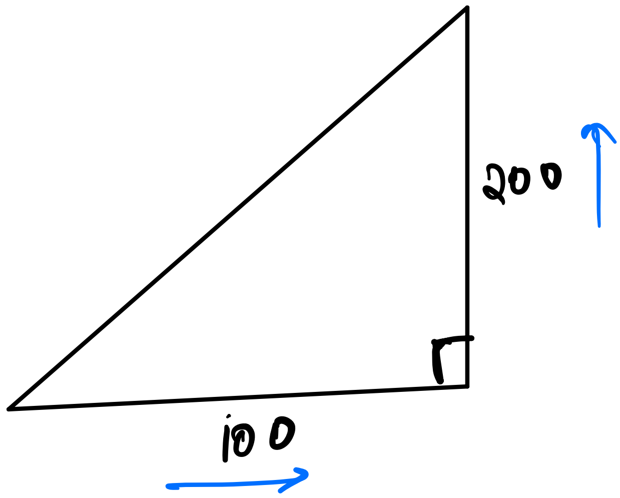
Manhattan Dist

2. L2 Norm $\Rightarrow \|\vec{x}\|_2 \longrightarrow \sqrt{x_1^2 + x_2^2}$

↓

Euclidian Dist

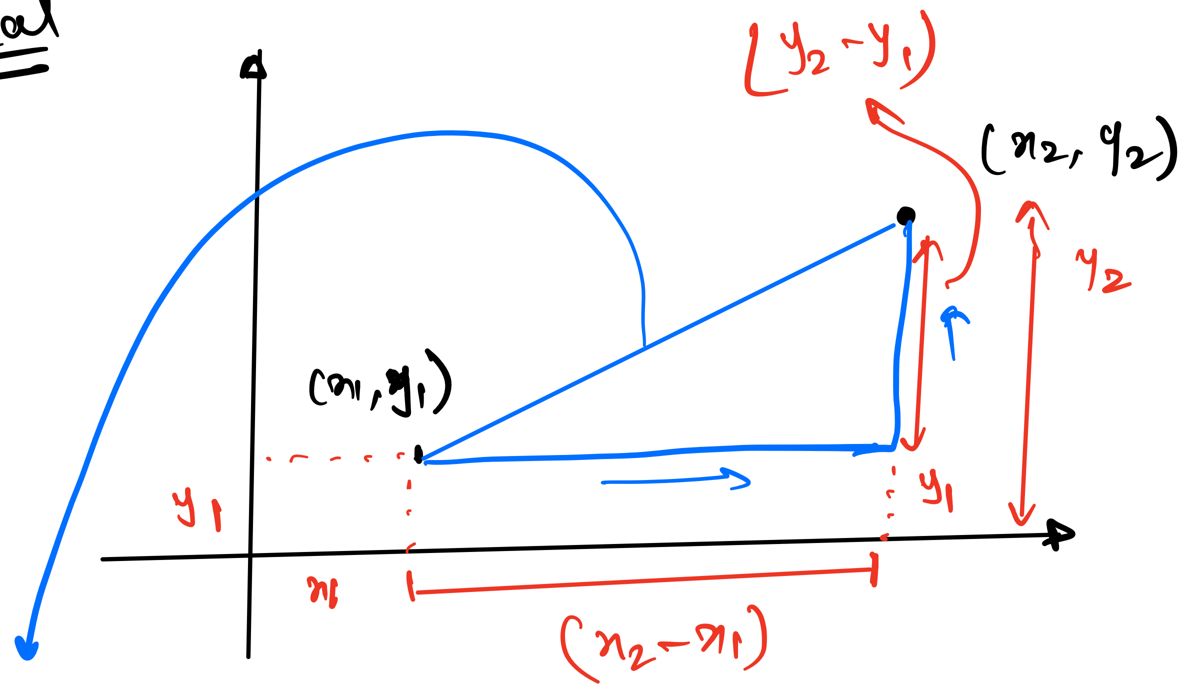
eg: 1



1 Euclidian $\longrightarrow \sqrt{(100)^2 + (200)^2} = \underline{\underline{223.6}}$

2 manhattan $\longrightarrow |100| + |200| = \underline{\underline{300}}$

* **General**



$$\sqrt{(x_2-x_1)^2+(y_2-y_1)^2} \longrightarrow \text{eucledian}$$

$$\text{manhattan} - |x_2-x_1| + |y_2-y_1|$$

* Matrix Multiplication

$$A_{m * n} \quad * \quad B_{p * q}$$

① Condition : $\boxed{n = p}$ ✓

② Resultant Matrix → $C_{m \times q}$

eg: $A = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}_{2 \times 2}$ $\quad B = \begin{bmatrix} 5 & 6 \\ 7 & 8 \end{bmatrix}_{2 \times 2}$

✓ $A * B$

$$\begin{bmatrix} 1*5 + 2*7 & 1*6 + 2*8 \\ 3*5 + 4*7 & 3*6 + 4*8 \end{bmatrix}_{2 \times 2}$$

* Angle between 2 vectors

$\vec{x} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}_{2 \times 1}$ $\qquad$ $\vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2 \times 1}$

$\bar{x}^T * \bar{y}$

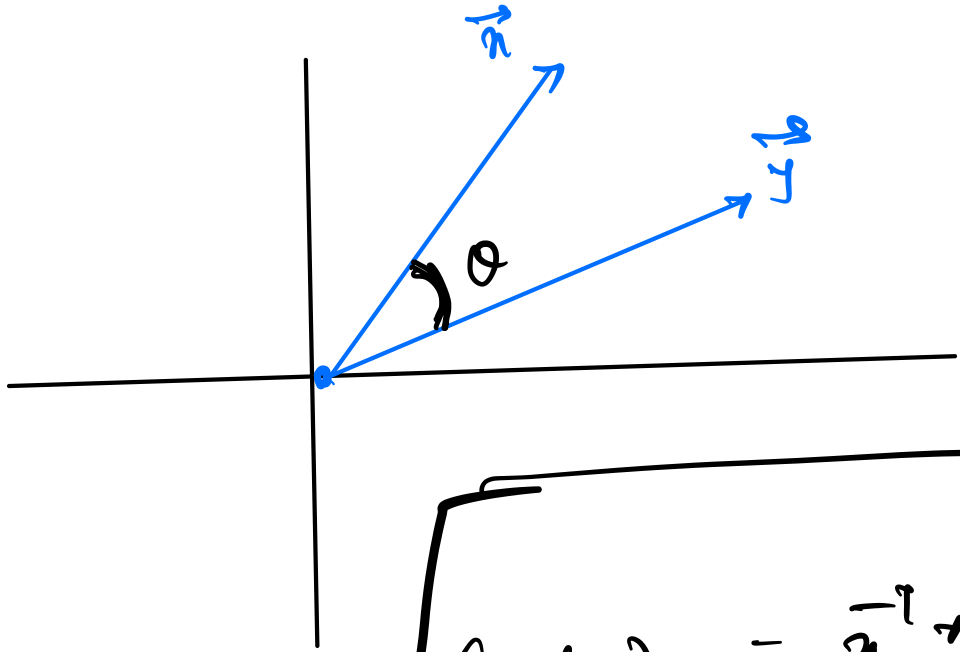
$\bar{x}^T = \begin{bmatrix} 1, & 2 \end{bmatrix}_{1 \times 2}$ $\qquad$ $\vec{y} = \begin{bmatrix} 3 \\ 4 \end{bmatrix}_{2 \times 1}$

$\bar{x}^T * \bar{y} = [1 \times 3 + 2 \times 4]$

$= [11]$

Dot product

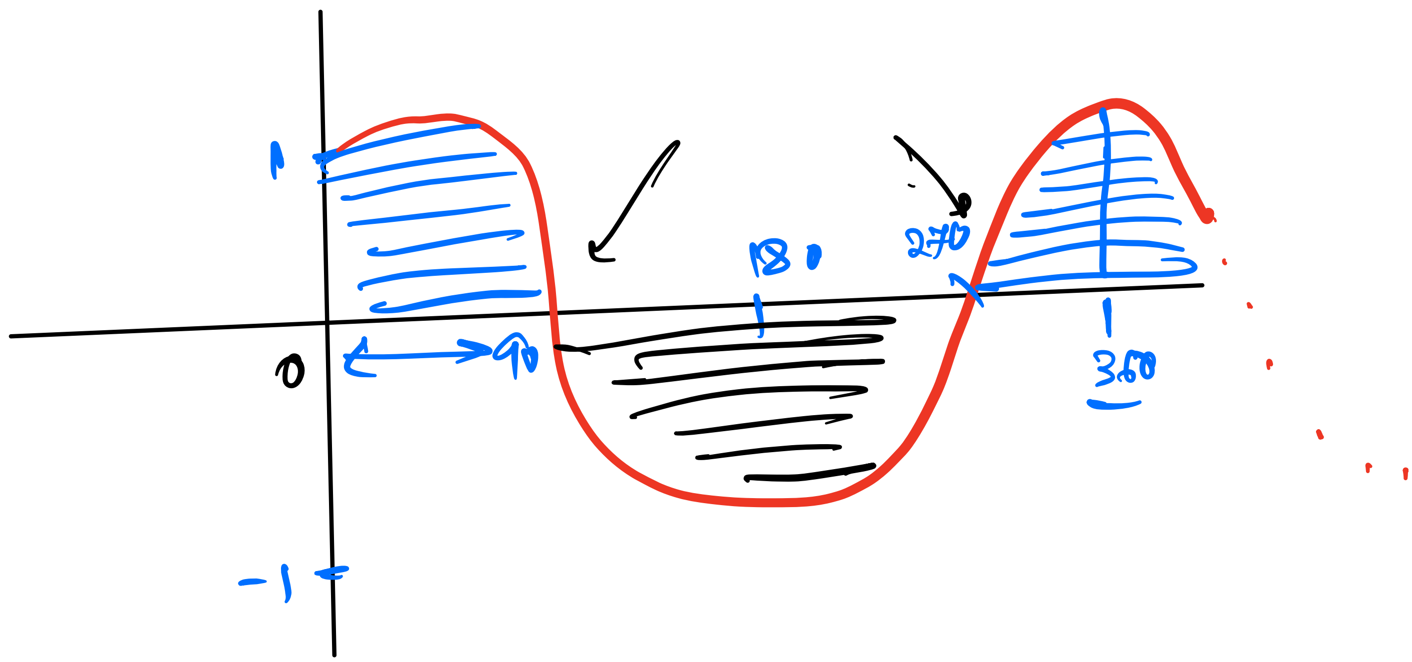
$x * y$



$$\boxed{\underline{\underline{\cos(\theta)}} = \frac{\vec{x}^{T} * \vec{y}}{\|\vec{x}\|_2 * \|\vec{y}\|_2}}$$

$$\underline{\underline{\theta = \cos^{-1}\Big(\qquad \Big)}}$$

$$\cos(0) = 1$$



$\cos\theta > 0 \rightarrow 0 < \theta < 90$

or

$270 < \theta < 360$

$\cos\theta < 0 \Rightarrow \boxed{90 < \theta < 270}$

$$w_1 x_1 + w_2 x_2 + w_3 x_3 \; . \; . \; . \; w_d x_d + w_0 = 0$$

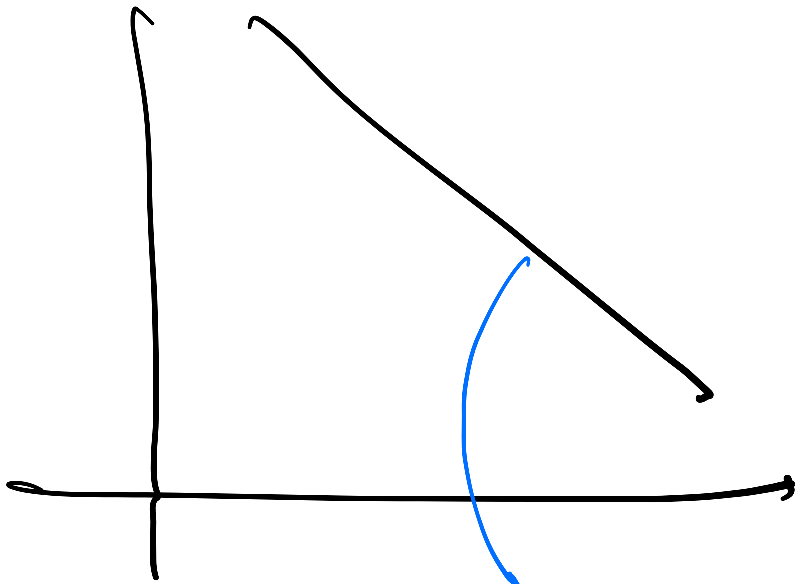
$$\Downarrow$$

$$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \\ w_3 \\ \vdots \\ w_d \end{bmatrix} \qquad \vec{x} = \begin{bmatrix} x_1 \\ x_2 \\ \vdots \\ x_d \end{bmatrix}$$

weight vector

feature vector

$$\boxed{\bar{w}^T \bar{x} + w_0 = 0}$$



$1 x_1 + 1 x_2 - 98 = 0$

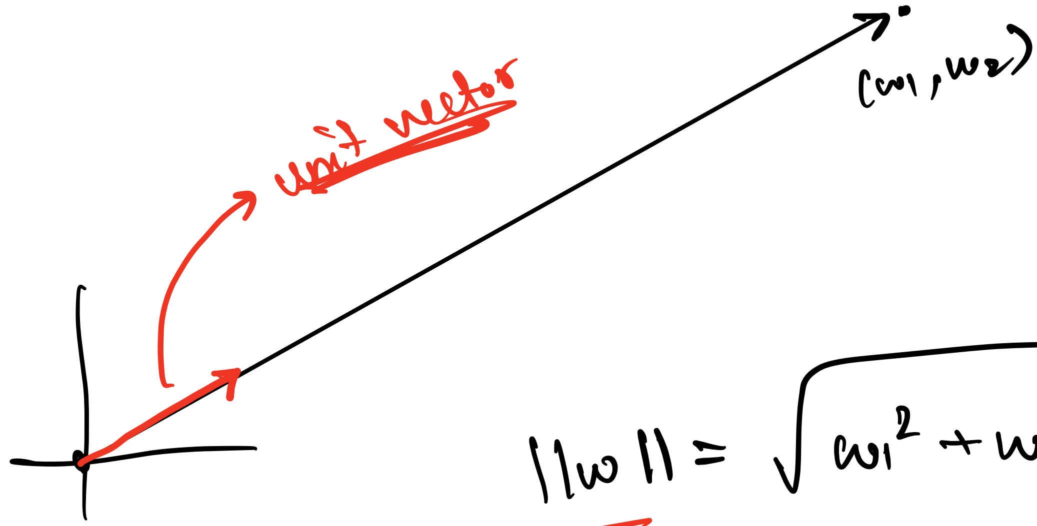
$w_1 x_1 + w_2 x_2 + w_0 = 0$

$\vec{w} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ ✓

$w_0 = -98$

★ Unit vector

$\vec{w} = \begin{bmatrix} w_1 \\ w_2 \end{bmatrix}$



$$||w|| = \sqrt{w_1^2 + w_2^2}$$

$$\boxed{\hat{w} = \frac{\vec{w}}{||\vec{w}||}}$$

extra :

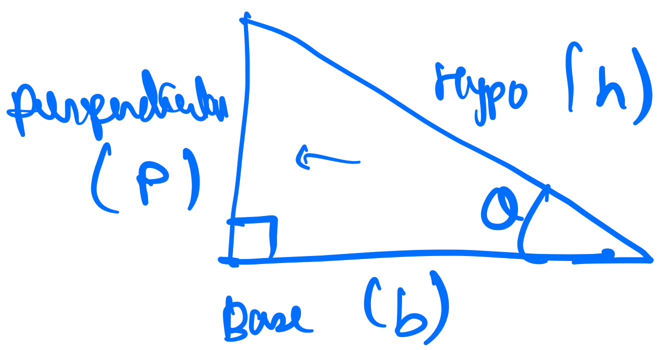
$$\hat{w} = \begin{bmatrix} \dfrac{w_1}{\sqrt{w_1^2 + w_2^2}} \\ \dfrac{w_2}{\sqrt{w_1^2 + w_2^2}} \end{bmatrix}$$

$$\|\hat{w}\| = \sqrt{\left(\frac{w_1}{\sqrt{w_1^2+w_2^2}}\right)^2 + \left(\frac{w_2}{\sqrt{w_1^2+w_2^2}}\right)^2}$$

$$= \sqrt{\frac{\cancel{w_1^2+w_2^2}}{\cancel{w_1^2+w_2^2}}}$$

$$= \sqrt{1} = 1$$

★ Basic Trignometry

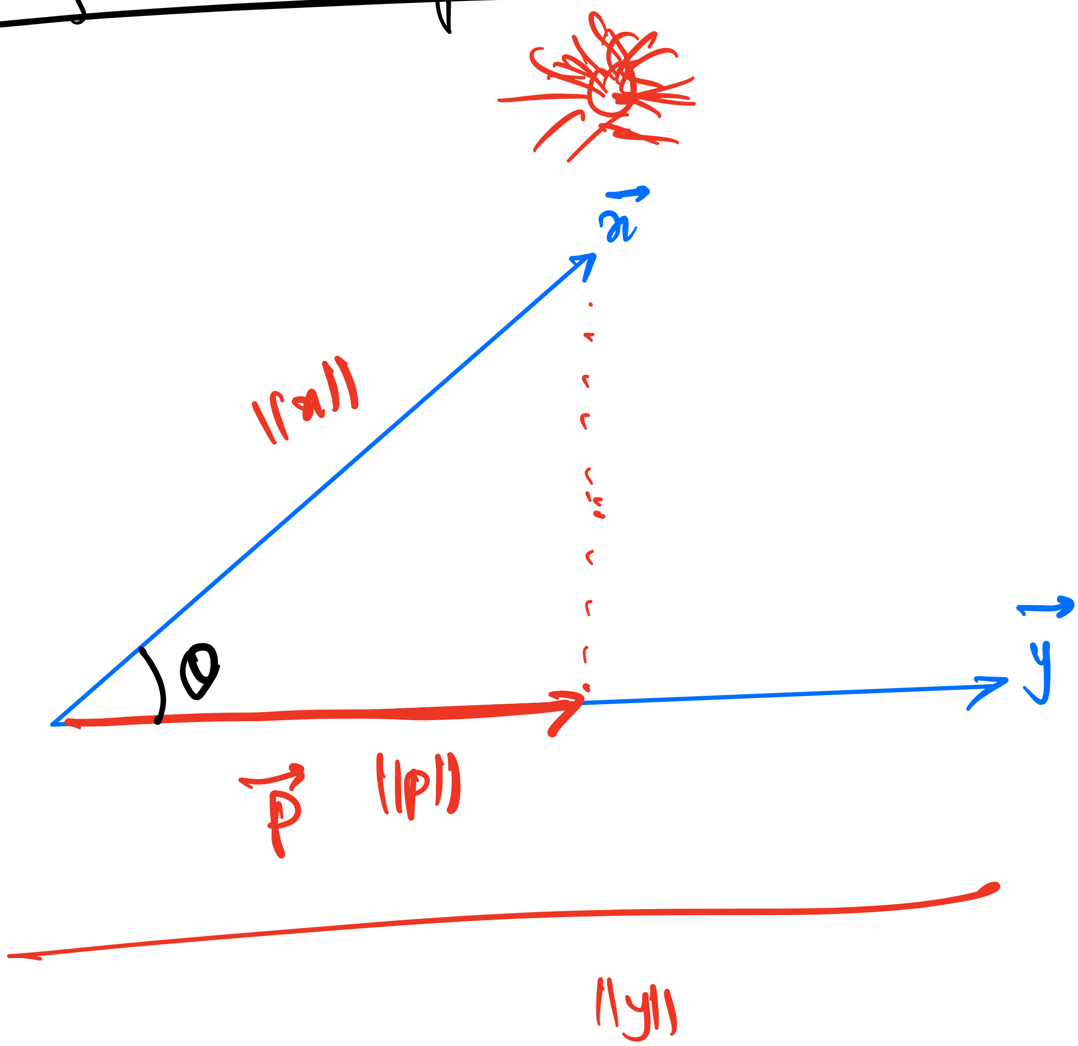


$$\boxed{\sin\theta = \frac{p}{h}}$$

$$\boxed{\cos\theta = \frac{b}{h}}$$

$$\boxed{\tan\theta = \frac{p}{b}}$$

* Projection of a vector



$\vec{p} \rightarrow$ projection of $\vec{x}$ on $\vec{y}$

$$\cos\theta = \frac{\bar{x}^T * \vec{y}}{\|x\| * \|y\|} \longrightarrow 1$$

from Trigno:

$$\cos\theta = \frac{\|p\|}{\|x\|} \longrightarrow 2$$

from 1 & 2

$$\frac{\|p\|}{\cancel{\|x\|}} = \frac{\bar{x}^T * \bar{y}}{\cancel{\|x\|} * \|y\|}$$

$$\|p\| = \frac{\vec{x}^{T} * \vec{y}}{\|y\|}$$

$$\boxed{\|p\| = \vec{x} * \hat{y}}$$