

**Note: Please solve the question and be ready with your answer. I will launch the quiz at 09:10 PM**

Consider the data, in the following table, recorded over a month with 30 days:  
On each day I recorded, whether it was sunny, (S), or not, (NS), and whether my girlfriend's mood was good, G, or not (NG).

		Whether	
		Sunny	Not Sunny
Mood	Good	9	6
	Not Good	1	14

$$P(\text{mood} \mid \text{sunny})$$

- If I pick a random day, what's the probability of my girlfriend being in good mood given that the day is sunny?
- Is her mood dependent on the whether condition?

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Mood		Whether	
		Sunny	Not Sunny
	Good	9	6
	Not Good	1	14

Handwritten notes on the table:  
 - A red arrow points down to the 'Sunny' column header.  
 - A red arrow points from the 'Good' row header to the cell containing '9'.  
 - The cell containing '9' is circled in red.  
 - A horizontal line connects the '9' and '6' cells, with a circled '15' at the end.  
 - A vertical line connects the '9' and '1' cells, with a '10' written below it.  
 - The value '9' is also written in red at the top right of the page.

$$P(\text{Good and Sunny}) = \frac{9}{30}$$

$$P(\text{Good} | \text{Sunny}) = \frac{P(\text{Good and Sunny})}{P(\text{Sunny})} = \frac{9/30}{10/30} = \frac{9}{10} = 0.9$$

This additional info is useless

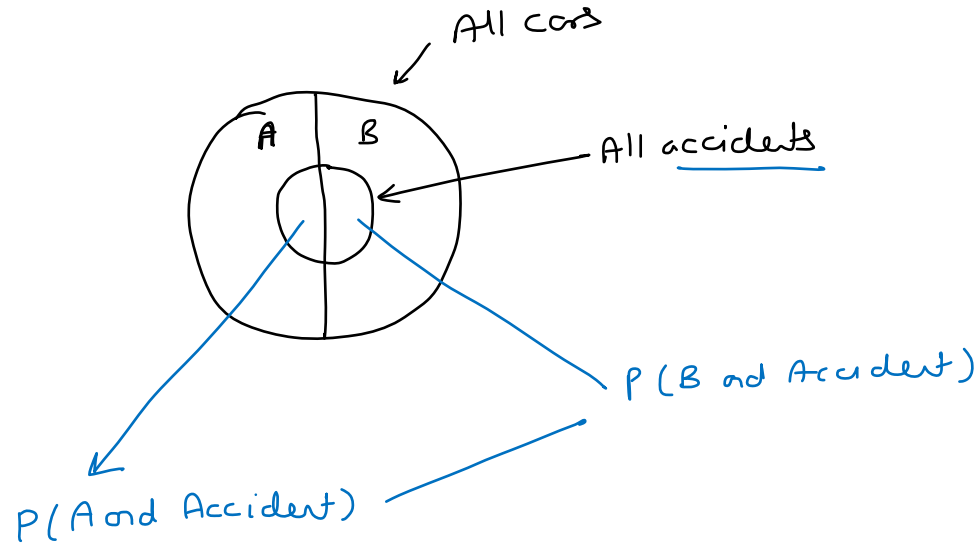
$$P(A|B) = P(A)$$

$$P(\text{Good}) = \frac{15}{30} = 0.5$$

whether condition and her mood is dependent

$$\underbrace{P(\text{Good} | \text{Sunny})}_{0.9} \neq \underbrace{P(\text{Good})}_{0.5}$$

## Recap



$P(A \text{ and } B)$

$$P(A|B) = \frac{P(A \cap B)}{P(B)}$$

$$P(AB) = P(A|B)P(B)$$

$$P(A \text{ and Accident}) + P(B \text{ and Accident})$$

$$P(A) \times P(\text{Accident}|A) + P(B) \times P(\text{Accident}|B)$$

$$0.06 \times 0.03 + 0.4 \times 0.06$$

$$0.018 + 0.024 = 0.042$$

Quiz1:

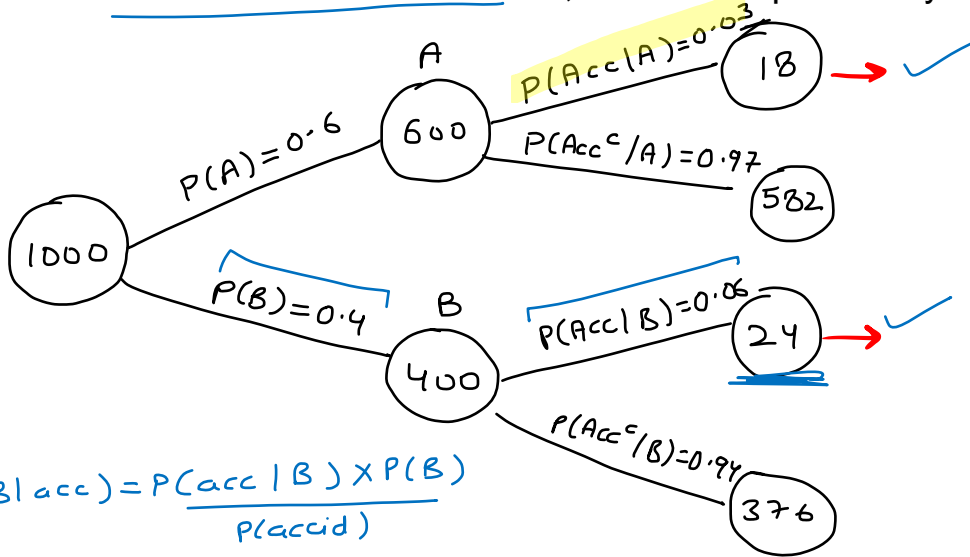
Accident  $\rightarrow$  Acc

A city has 2 taxi companies, A and B.

A has 60% of the taxis in the city and B has 40%.

A's taxis are involved in accidents 3% of the time, B's are involved in 6%.

If a taxi is involved in an accident, what is the probability that it belongs to Company B?



$$P(B|acc) = \frac{P(acc|B) \times P(B)}{P(accid)}$$
$$= \frac{0.06 \times 0.4}{0.042} = 0.57$$

$$P(Accident) = \frac{18 + 24}{1000} = \frac{42}{1000} = 0.042$$

$$P(B|accident) = \frac{Bondaccident}{accident}$$
$$= \frac{24}{18 + 24} = 0.57$$

## Quiz2:

It is known that 30% of emails are spam, and 70% are not spam.

The word "purchase" occurs in 80% of spam emails.

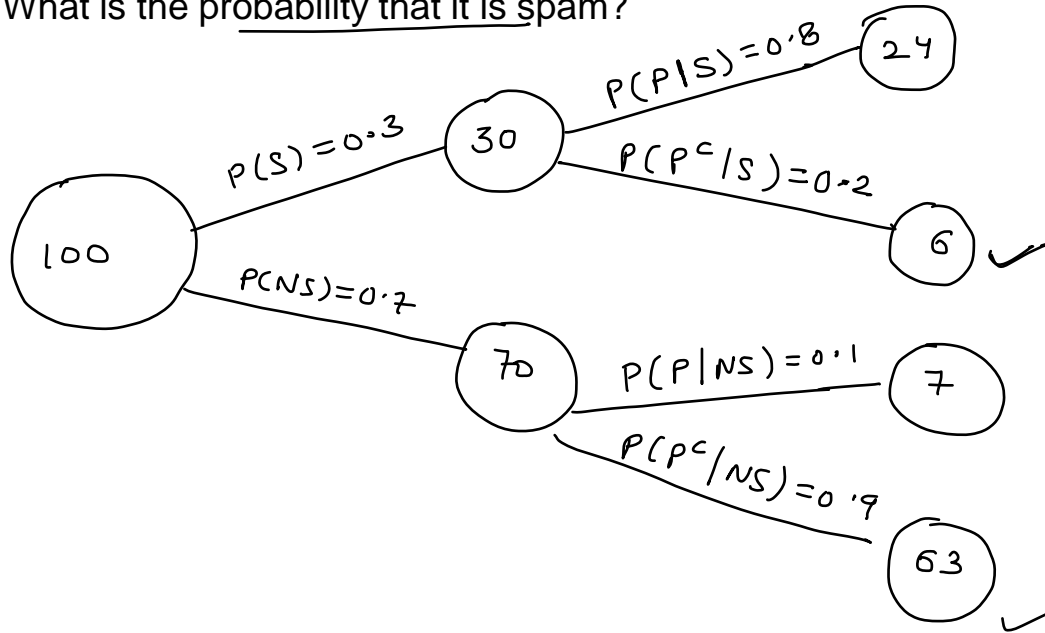
It also occurs in 10% of non-spam emails | A new mail does not have the word "purchase".

What is the probability that it is spam?

$S \rightarrow \text{spam}$

$NS \rightarrow \text{not spam}$

$P \rightarrow \text{word purchase}$



$$P(S | P^c) = \frac{6}{6 + 63} = 0.08$$

$P = \text{premium}$   
 $NP = \text{non-premium}$

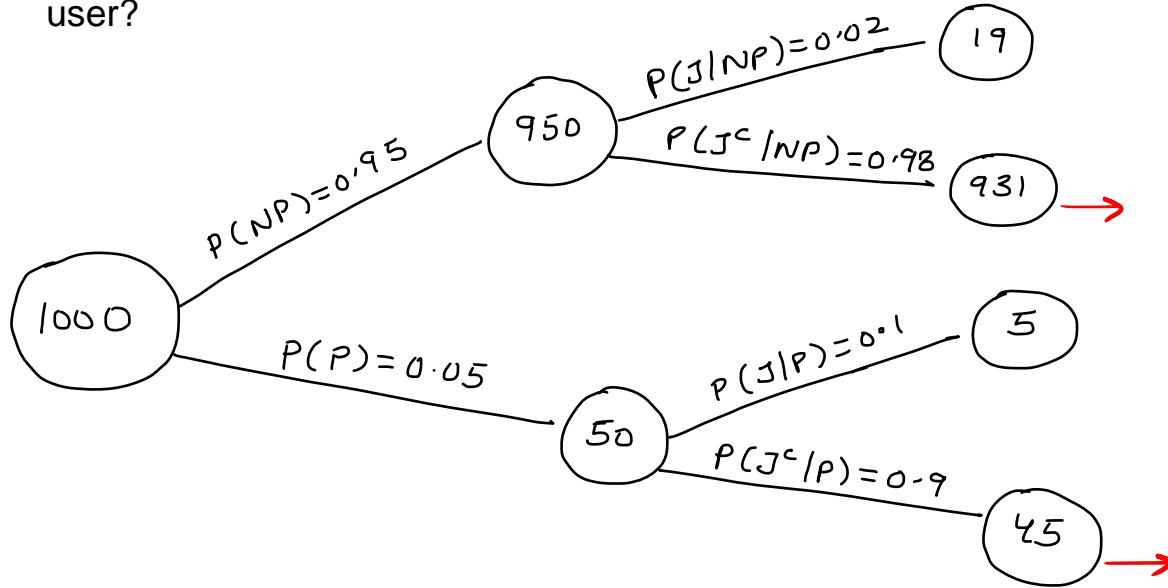
$J = \text{Looking for Job}$

Quiz3:

5% of all LinkedIn users are premium users. 10% of premium users are seeking new jobs.

2% of non-premium users are seeking new jobs.

A randomly chosen person is NOT seeking new jobs. What is the probability that he is a premium user?



$$P(P | J^c) = \frac{45}{931 + 45}$$

$$= 0.046$$

Quiz4:

An website shows two types of ads:

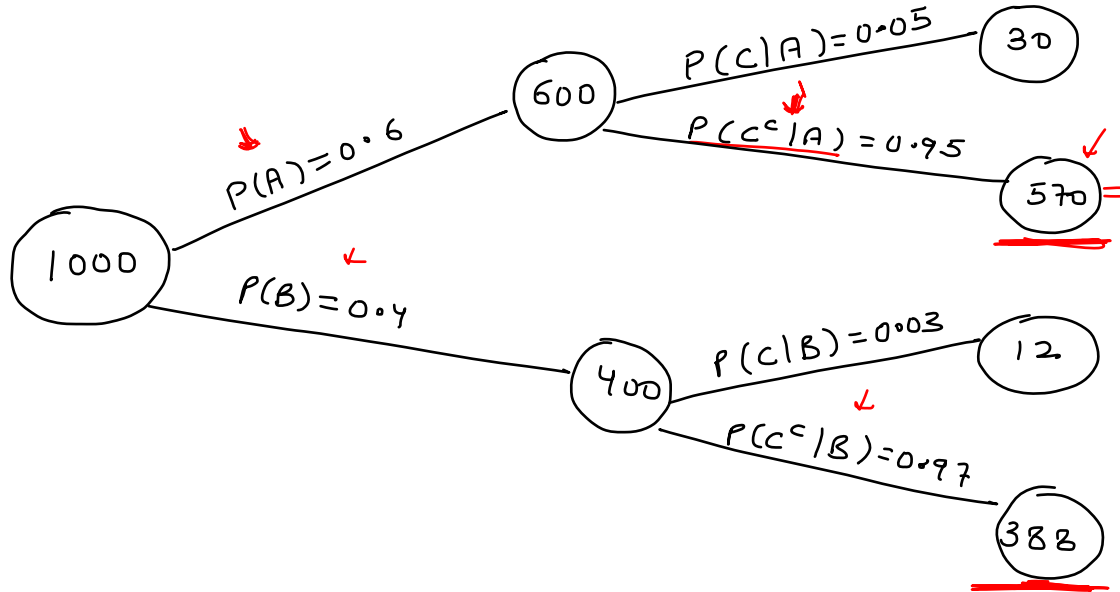
60% of the visitors see Type A ads, and 40% visitors see Type B ads.

The click-through rate for A is 5%, and for B is 3%.

A visitor to the website does not click the ad. What is the probability that he saw Type A ad?

$A \rightarrow \text{Type-A}$   
 $B \rightarrow \text{Type-B}$

$C \rightarrow \text{click through rate}$



$$P(A | C^c) = \frac{570}{570 + 388} = 0.59$$

$$P(A | C^c) = \frac{P(C^c | A) \times P(A)}{P(C^c | A) \times P(A) + P(C^c | B) \times P(B)}$$

Quiz5:

$$\frac{5}{100} = 0.05$$

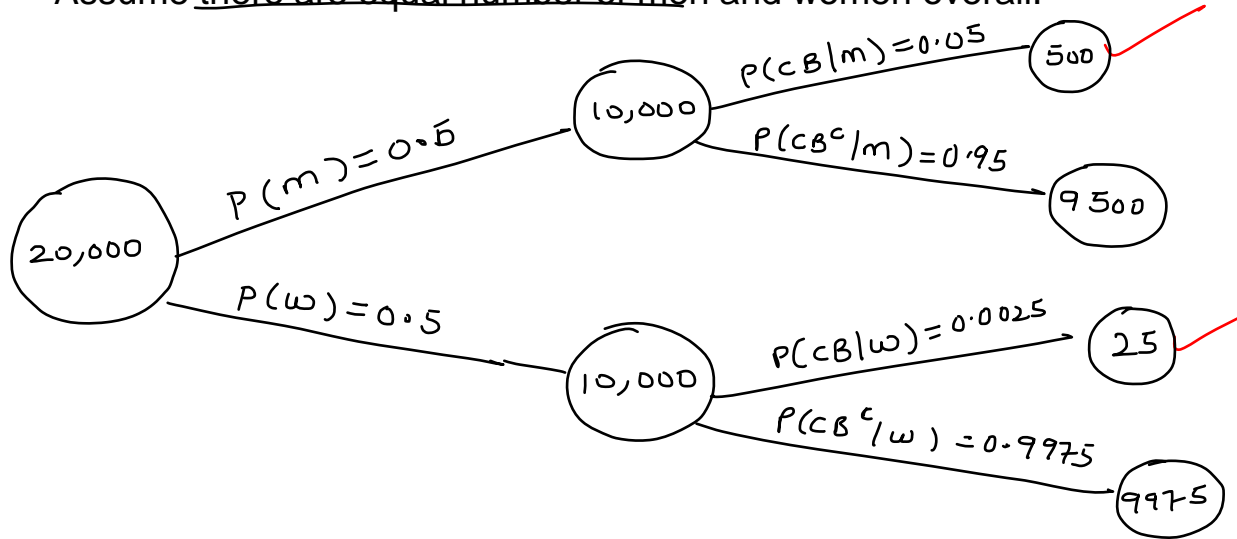
men  $\rightarrow$  m  
women  $\rightarrow$  w  
color-blind  $\rightarrow$  CB

Suppose 5 percent of men and 0.25 percent of the women are color-blind.

A random color-blind person is chosen.

What is the probability of this person being male?

Assume there are equal number of men and women overall.



$$P(M | CB) = \frac{500}{500 + 25}$$
$$= 0.95$$
$$=$$



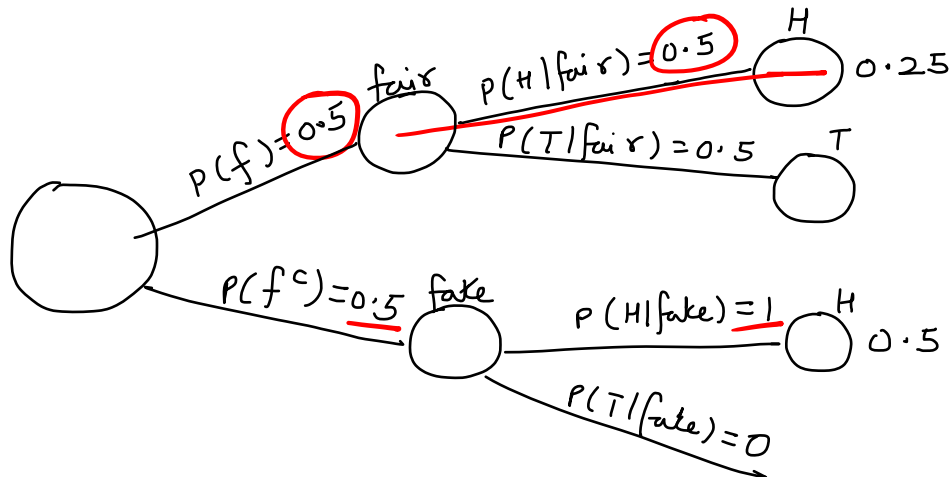
# Quiz6

$\textcircled{H}^T$

$\textcircled{H}^H$

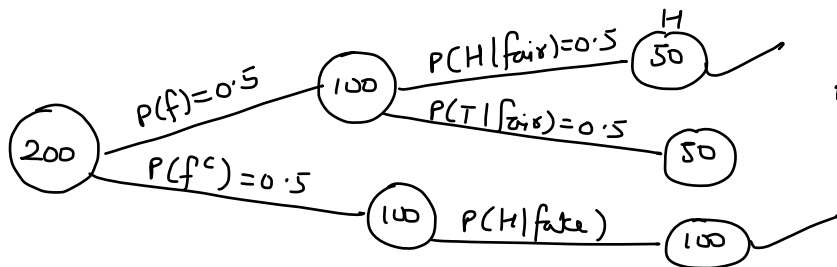
fair = f  
fake = w

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and flips it. It lands heads. Compute probability that it is fair coin.



$$P(f | H) = \frac{P(\text{fair and } H)}{P(\text{fair and } H) + P(\text{fake and } H)}$$

$$= \frac{0.25}{0.25 + 0.5}$$



$$P(f | H) = \frac{50}{50 + 100} = \frac{50}{150} = \left(\frac{1}{3}\right)$$

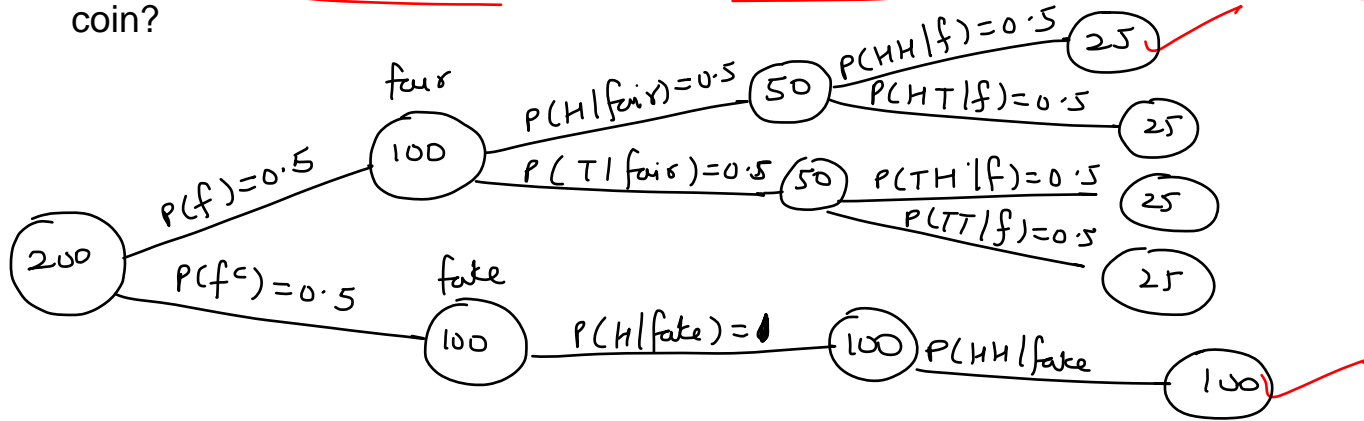
$$= \frac{0.25}{0.75}$$

$$= \frac{1}{3}$$

$$= \underline{\underline{0.33}}$$

## Quiz7:

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it twice. It shows heads both the times. What is the probability that it is the fair coin?



$$P(f|HH) = \frac{25}{100 + 25} = \frac{25}{125} = \frac{1}{5}$$

H H

A gambler has in his pocket a fair coin and a two-headed coin. He selects one of the coins at random, and he flips it twice. It shows HT both the times. What is the probability that it is the fake coin?

$$p(\text{fake} \mid HT) = 0$$

# Quiz8:

A family has 2 children, at least one of them is a girl. What is the probability that both are girls?

$$\Omega = \{ \underset{c_1 c_2}{GB}, \underset{c_1 c_2}{BG}, \underset{c_1 c_2}{BB}, \underset{c_1 c_2}{GG} \}$$

$$A \cap B = \{ \overset{\downarrow}{GG} \}$$
$$P(A \cap B) = 1/4$$

$$A = \text{At least one of them is a girl} = \{ GB, BG, \overset{\downarrow}{GG} \}$$
$$P(A) = 3/4$$
$$B = \text{Both are girls} = \{ GG \}$$

$$P(B|A) = \frac{P(A \cap B)}{P(A)} = \frac{1/4}{3/4} = \left( \frac{1}{3} \right)$$

$\equiv$

$$P(\overset{\downarrow}{GG} | \text{at least one child is girl}) = \frac{1}{3}$$