PH307: Introduction to Numerical Analysis
Tutorial Sheet 7

Note: This tutorial sheet deals with the numerical solutions of a second-order initial value ODE by converting it into a pair of coupled first-order ODEs.

1. Consider the second-order differential equation

$$y'' + y' + y^2 = x, \ \ y(0) = 1, \ \ y'(0) = 0$$

First, convert it into two simultaneous first-order differential equations by making the substitutions $y_1 = y$ and $y_2 = \frac{dy}{dx}$. Now the initial conditions will be $y_1(0) = 1, \ \ y_2(0) = 0$. Develop a computer program to solve the resulting equations by:

(a) Fourth-order Taylor Expansion approach

(b) Fourth-order Runge-Kutta (RK4) method

Your program should prompt the user for: (a) which method to use, (b) value of h, and (c) the value of x_{max} ($0 \leq x \leq x_{max}$).